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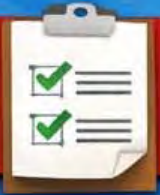


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CHAPTER 1

Units of Measurements

SUMMARY

1. INTRODUCTION

Measurement of a physical quantity is the comparison of the quantity with another quantity of same kind which is taken as a reference quantity.

When we say that length of a room is 5 metres it means that it is five times conventionally chosen unit of length metre (m). Therefore, measurement of the length of a room is just in comparison with another length. Hence measurement of a physical quantity is just a comparison.

In physics, we try to express many natural phenomena in terms of relationships among the quantities involved. These relations may be stated in sentences, but these are better expressed in terms of mathematical equations. To verify these equations or laws of physics it becomes necessary to measure the physical quantities involved. Hence we need measurement of physical quantities in order to express physical laws mathematically and to verify the laws.

2. UNIT

Unit of a physical quantity is defined as the reference standard of measurement of that quantity.

Measurement involves comparison of the quantity to be measured with a reference standard quantity. This reference standard of measurement is what we call a unit.

To determine the magnitude of a physical quantity completely, we should know

- the unit (u) in which it is measured and
- number of times that unit is contained in the quantity. i.e., numerical value (n).

If the length of a room is 5 metre, then unit is metre and numerical value is 5. Here metre is taken as a standard of measurement. This statement implies that one metre is contained five times in the

length of the room. In general, if n is the numerical value when unit is u for a physical quantity Q , then

$$Q = nu$$

Since Q is always same for a given quantity, therefore, the bigger the unit, the smaller is the numerical value.

$$n_1 u_1 = n_2 u_2$$

2.1 Fundamental and Derived Units

Fundamental units :

Fundamental units are those units, which can neither be derived from one another, nor they may be further resolved into any other units. e.g. length, mass and time.

Derived units :

The units of all such physical quantities, which can be obtained from fundamental units, are called derived units. e.g. area is a derived unit as it can be derived from unit of length, similarly velocity can be derived from unit of length and time.

3. SYSTEM OF UNITS

A system of units is the set of units with particular basic units. The common system

of units are :

- CGS system of units
- FPS system of units
- MKS system of units
- SI units.

Unit of length :

The unit of length in SI unit is metre. One metre is the distance between the lines marked on platinum-iridium rod kept at constant temperature of 273.16 K at one bar pressure in the International Bureau of Weights and Measures at Sevres, near Paris in France.

All other 'metres' had to be calibrated against this metre. This makes it very inconvenient. So in 1960, the Eleventh General Conference of Weights and Measures decided to redefine metre by adopting atomic standards.

On atomic standards, one metre is defined as to be equal to 1650,763.3 wavelengths in vacuum of the radiation emitted due to transition between the levels $2p^{10}$ and $5d^5$ of the isotope of Krypton having mass number 86.

This definition is considered to be more accurate because wavelength of light emitted by a source is easily reproducible and does not change with time. Units frequently used for expressing distances are : For short lengths, the following units are used :

$$1 \text{ angstrom} \quad (1\text{\AA}) = 10^{-10} \text{ m}$$

$$1 \text{ fermi} \quad (1 \text{ fm}) = 10^{-15} \text{ m}$$

$$1 \text{ micron} \quad (1 \mu\text{m}) = 10^{-6} \text{ m}$$

For large distances, the following units are used :

(i) Light year

It is the distance that light travels in one year in vacuum.

$$S = c.t.$$

(where c = velocity of light = $3 \times 10^8 \text{ ms}^{-1}$)

1 light year

$$= 9.46 \times 10^{15} \text{ m}$$

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

Astronomical Unit. It is the mean distance between sun and the earth.

1 Astronomical Unit (1 A.U.)

$$= 1.496 \times 10^{11} \text{ m}$$

(ii) Parsec (Parallactic second)

It is the distance corresponding to an annual parallax of one second of arc due to rotation of earth.

$$1 \text{ parsec} = 3.26 \text{ light year}$$

$$= 3.26 \times 9.46 \times 10^{15} \text{ m}$$

$$= 3.08 \times 10^{16} \text{ m}$$

Units of Mass :

In S.I. the unit of mass is kilo-gram (kg).

Kilogram

One kilogram is the mass of a platinum-iridium cylinder (90% Pt and 10% Ir) kept in International Bureau of Weights and Measures at Sèvres near Paris in France.

Weight

Weight is a force with which the body is pulled towards the centre of earth. It is the product of the mass of the body and the acceleration due to gravity of earth on the body.

Weight = mass $\times$ acceleration due to gravity

$$W = m \times g$$

Weight is a vector quantity and is measured by a spring balance and its unit is newton. It acts towards the centre of the earth. Since the value of g is different at different places, hence weight is also different at different places and at different planets. On the surface of earth, the value of g is minimum at the equator and maximum at the poles, so weight is also minimum at the equator and maximum at the poles. At the centre of earth, weight of a body is zero.

In space, away from all gravitational attraction, the body will have negligible weight even though it has the same mass.

1 amu. One atomic mass unit (1 amu) is defined as the $1/12$ of the mass of the carbon isotope ^{12}C (including the mass of the electrons).

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

It is used to express the masses of atoms or molecules.

4. DIMENSIONAL ANALYSIS

Dimensions of a physical quantity are the powers to which the fundamental units are raised to express the unit of that physical quantity.

e.g. Force = $m \times a = [M^1 L^1 T^{-2}]$

So the dimensions of F are (1) in mass, (1) in length and (-2) in time.

Dimensional formula is the mathematical expression showing how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity.

e.g., dimensional formula of force is $[M^1 L^1 T^{-2}]$

4.1 Dimensional Equation

When the dimensional formula is equated with its physical quantity, the equation is called dimensional equation.

e.g., the dimensional equation of force is

$$[F] [M^1 L^1 T^{-2}]$$

4.2 Principle of Homogeneity

It states that dimensions on both the sides of an equation for all terms are separately same.

e.g. For equation

$$s = ut + \frac{1}{2}at^2 \quad \dots(1)$$

Dimensional formula of

$$s = [L]$$

Dimensional formula of

$$ut = [LT^{-1}][T] = [L]$$

Dimensional formula

$$\frac{1}{2}at^2 = [LT^{-2}][T^2] = [L]$$

Thus, we find that the dimensions of each term on right hand side of equation (i) are same and that on the left hand side of the equation.

4.3 Uses of Dimensional Analysis

- (i) Conversion of one system of units into another systems of units. The formula used is

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

- (ii) Checking the correctness of the given physical relation. Here we use principle of homogeneity.
(iii) To derive relationship between various physical quantities. Here we must know the factors on which a physical quantity depends

5. SIGNIFICANT FIGURES

The significant figures are the numbers of digits used to express the physical quantity with precision. The number of significant digits in a physical quantity depends on the accuracy of the measuring instrument. If significant digits are more, the percentage error is less and vice versa.

5.1 Rules for Significant Figures

1. All non-zero digits are significant. e.g. 54695 has 5 significant figures. 2.62456 has 6 significant figures.
2. All zeroes occurring between non-zero digits are significant e.g. 540059 has 6 significant figures. 2.0059 has 5 significant figures.
3. All zeroes to the right of decimal point and to the left of a non-zero digits are not significant. e.g. 0.0545 has 3 significant figures. 000545 has 3 significant figures.
4. All zeroes to the right of a non-zero digit in a number written with a decimal point are

significant. e.g. 5.4500 has 5 significant figures. 0.04500 has 4 significant figures. 5.00 has 3 significant figures.

5. (a) All zeroes to the right of the last non-zero digits are not significant. e.g. 2020 has 3 significant figures. 2.64 $\times 10^{24}$ kg has 3 significant figures.
(b) All zeroes to the right of last non-zero digits are significant, if they come from a measurement. e.g. 2020 m has 4 significant figures or 202000 cm has 4 significant figures. or 2.020 km has 4 significant figures and 0.082400 Nm^{-2} has 5 significant figures.

NCERT TEXT-BOOK QUESTIONS

1. Fill in the blanks :
(a) The volume of a cube of side 1 cm is equal to _____ m^3 .
(b) The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to _____ $(\text{mm})^2$
(c) A vehicle moving with a speed of 18 km/h covers _____ m in 1 s.
(d) The relative density of lead is 11.3. Its density is _____ g cm^{-3} or _____ kg m^{-3}

Ans :

- (a) The volume of a cube of side 1 cm is equal to

$$= 1 \times 1 \times 1 \text{ cm}^3$$

$$= 1 \times \left(\frac{1}{100} \right)^3 \text{ m}^3 = 10^{-6} \text{ m}^3$$
- (b) Area, $A = 2\pi r^2 + 2\pi rh$

$$= 2\pi r(r + h)$$

$$r = 20 \text{ mm}, h = 100 \text{ mm}$$

$$A = 2 \times \frac{22}{7} \times 20(20 + 100)$$

$$= 15099 \text{ mm}^2$$

$$= 1.5 \times 10^4 \text{ mm}^2$$
- (c) $18 \text{ km h}^{-1} = \frac{18 \times 1000}{60 \times 60} = 5 \text{ m s}^{-1}$
A vehicle moving with a speed of 18 km h^{-1} covers 5 m in 1 s.
- (d) The relative density of lead is 11.3. Its density is 11.3 g cm^{-3} or 11300 kg m^{-3} . This is as follows :

$$1 \text{ g} = 10^{-3} \text{ kg}, 1 \text{ cm} = 10^{-2} \text{ m}$$

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$$\begin{aligned}
 11.3 \text{ g cm}^{-3} &= 11.3(10^{-3} \text{ kg})(10^{-2} \text{ m})^{-3} \\
 &= 11.3 \times 10^{-3} \times 10^6 \text{ kg m}^{-3} \\
 &= 11.3 \times 10^3 \text{ kg m}^{-3} \\
 &= 1.13 \times 10^4 \text{ kg m}^{-3}
 \end{aligned}$$

(a) 10^{-6} , (b) 1.5×10^4 , (c) 5, (d) 11.3, 11.3×10^4 .

2. Fill in the blanks by suitable conversion of units :

- (a) $1 \text{ kg m}^2 \text{ s}^{-2} = \dots\dots\dots \text{g cm}^2 \text{ s}^{-2}$
 (b) $1 \text{ m} = \dots\dots\dots \text{light year}$
 (c) $3 \text{ ms}^{-2} = \dots\dots\dots \text{km h}^{-2}$
 (d) $G = 6.67 \times 10^{-11} \text{ Nm}^2 (\text{kg})^{-2} = \dots\dots\dots (\text{cm})^3 \text{ s}^{-2} \text{ g}^{-1}$

Ans :

- (a) $1 \text{ kg m}^2 \text{ s}^{-2} = 1(10^3 \text{ g})(10^2 \text{ cm})^2 \text{ s}^{-2}$
 $= 10^7 \text{ g cm}^2 \text{ s}^{-2}$
 (b) $1 \text{ m} = 1.057 \times 10^{-16} \text{ light year}$
 $\simeq 10^{-16} \text{ light year}$
 (1 light year $= 9.46 \times 10^{15} \text{ m}$)
 (c) $3 \text{ ms}^{-2} = 3\left(\frac{1}{1000} \text{ km}\right)\left(\frac{1}{(60 \times 60)} \text{ hr}\right)^{-2}$
 $= 10.8 \text{ km h}^{-2}$
 $= 3 \times 10^{-3} \times (3600)^2$
 $= 3.888 \times 10^4 \text{ km h}^{-2}$
 $= 3.9 \times 10^4 \text{ km h}^{-2}$
 (d) $G = 6.67 \times 10^{-11} (10^5 \text{ dyn})$
 $(100 \text{ cm})^2 (1000 \text{ g})^{-2}$
 $= 6.67 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}$
 $= 6.67 \times 10^{-8} \text{ g cm s}^{-2} \cdot \text{cm}^2 \text{ g}^{-2}$
 $= 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}$
 (a) 10^7 ; (b) 10^{-16} (c) 3.9×10^4 (d) 6.67×10^{-8}

3. Calorie is a unit of heat or energy and it equals about 4.2 J where $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$. Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length equal $\beta \text{ m}$, the unit of time is $\gamma \text{ s}$. Show that calorie has a magnitude $4.2 \alpha^{-1} \beta^{-2} \gamma^{+2}$ in terms of the new units.

Ans :

For a given physical quantity, q , if n_1 , n_2 are the numerical values when the fundamental units are (M_1, L_1, T_1) and (M_2, L_2, T_2) respectively, then

$$\begin{aligned}
 q &= n_1 u_1 = n_2 u_2 \\
 n_1 [M_1^a L_1^b T_1^c] &= n_2 [M_2^a L_2^b T_2^c]
 \end{aligned}$$

where a , b , c are the dimensions of q in mass, length and time respectively.

Now

$$1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$$

$$a = 1, b = 2, c = -2$$

$$M_1 = 1 \text{ kg}, L_1 = 1 \text{ m}, T_1 = 1 \text{ s} \quad (\text{S.I. units})$$

$$M_2 = \alpha \text{ kg}, L_2 = \beta \text{ m}, T_2 = \gamma \text{ s} \quad (\text{Units in new system})$$

$$n_2 = ? \quad n_1 = 4.2$$

$$\begin{aligned}
 n_2 &= n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c \\
 &= 4.2 \left(\frac{1 \text{ kg}}{\alpha \text{ kg}}\right)^1 \left(\frac{1 \text{ m}}{\beta \text{ m}}\right)^2 \left(\frac{1 \text{ s}}{\gamma \text{ s}}\right)^{-2} \\
 n_2 &= 4.2 \alpha^{-1} \beta^{-2} \gamma^{+2}
 \end{aligned}$$

4. Explain this statement clearly : “To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison”. In view of this, reframe the following statements wherever necessary:

- (a) atoms are very small objects
 (b) a jet plane moves with great speed
 (c) the mass of Jupiter is very large
 (d) the air inside this room contains a large number of molecules
 (e) a proton is more massive than an electron
 (f) the speed of sound is much smaller than the speed of light.

Ans :

- (a) Atoms are very small objects of the order of 10^{-10} m .
 (b) A jet plane moves with great speed of the order of 10^4 m^{-1} .
 (c) The mass of Jupiter is very large of the order of 10^{25} kg .
 (d) The air inside this room contains about 10^{23} molecules.
 (e) The proton is 1840 times massive than that of an electron.
 (f) The speed of sound is 10^{-6} times smaller than the speed of light.

5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?

Ans :

We know that

$$n_1 u_1 = n_2 u_2$$

For velocity,

$$n_1 [L_1 T_1^{-1}] = n_2 [L_2 T_2^{-1}]$$

$$n_1 = 3 \times 10^8, L_1 = 1 \text{ m}, T_1 = 1 \text{ s}$$

$$n_2 = 1, L_2 = ?, T_2 = 1 \text{ s}$$

$$1 \text{ ms}^{-1} = 1 \times L_2 \times 1 \text{ s}^{-1}$$

New unit of length,

$$L_2 = 3 \times 10^8 \text{ m}$$

$$\text{Distance} = \text{speed} \times \text{time}$$

$$= 3 \times 10^8 \text{ ms}^{-1} \times 500 \text{ s}$$

$$\text{or Distance} = 500(3 \times 10^8 \text{ m})$$

$$= 500 L_2$$

$$\text{Distance} = 500 \text{ new units.}$$

6. Which of the following is the most precise device for measuring length ?

- A vernier callipers with 20 divisions on the sliding scale.
- A screw gauge of pitch 1 mm and 100 divisions on the circular scale.
- An optical instrument that can measure length to within a wavelength of light ?

Ans :

(a) Least count of vernier callipers

$$= 1 \text{ M.S.D.} - 1 \text{ V.S.D}$$

$$= \left(1 - \frac{19}{20}\right) \text{SD}$$

$$= 0.005 \text{ cm} \approx 10^{-5} \text{ m}$$

(b) Least count of screw gauge

$$= \frac{1}{100} \text{ mm}$$

$$= 0.001 \text{ cm} = 10^{-5} \text{ m}$$

(c) Wavelength of light

$$\lambda \approx 10^{-6} \text{ m}$$

Hence, an optical instrument that can measure length to within a wavelength of light (which is approx. equal to 10^{-6} m) is the most precise device for measuring length.

7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate of the thickness of hair ?

Ans :

Estimate of the thickness of hair

$$= \frac{3.5 \text{ mm}}{100} = 0.035 \text{ mm}$$

8. Answer the following :

- You are given a thread and a metre scale. How will you estimate the diameter of this thread ?
- A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gauge arbitrarily by increasing the number of divisions on the circular scale ?
- The mean diameter of a thin brass rod is to be measured by a vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Ans :

- Wind the thread in close turns on a round pencil. Count the number of turns. Find the average length of the coiled thread using the metre scale. Divide this average length by the number of turns

$$\text{i.e. diameter} = \frac{\text{Average length}}{\text{Number of turns}}$$

It gives the diameter of the thread.

- Yes, the accuracy of the gauge increases by increasing the number of divisions on the circular scale because the least count of the instrument becomes less, according to the formula,

Least count

$$= \frac{\text{Pitch}}{\text{Number of division on the circular scale}}$$

- By taking a large number of observations (100) and then taking their mean for the result, greatly reduces the random errors as compared by taking only five observations and then taking their mean. Therefore, a set of 100 measurements of the diameter is expected to yield a more reliable estimate than a set of 5 measurements only.

9. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2 . What is the linear magnification of the projector-screen arrangement ?

Ans :

Magnification of area

$$= \frac{1.55 \times 100 \times 100}{1.75}$$

$$= 8857$$

$$\text{Linear magnification} = \sqrt{8857} = 94.1$$

10. State the number of significant figures in the following:

- (a) 0.007 m^2
 (b) $2.64 \times 10^{24} \text{ kg}$
 (c) 0.2370 g cm^{-2}
 (d) 6.320 J
 (e) 6.032 Nm^{-2}
 (f) 0.0006032

Ans :

- (a) 1, (b) 3, (c) 4, (d) 4, (e) 4, (f) 4.

11. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m , 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Ans :

Length = 4.234 m , Breadth = 1.005 m , Thickness = 0.0201 m

$$\begin{aligned} \text{Area} &= 2(lb + bh + hl) \\ &= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \\ &= 2[4.255 + 0.0202 + 0.0851] \end{aligned}$$

Since the least accuracy in the given data is upto three places of decimals.

$$\text{Area} = 8.720 \text{ m}^2$$

$$\begin{aligned} \text{Volume} &= \text{Length} \times \text{breadth} \times \text{thickness} \\ &= 4.234 \times 1.005 \times 0.0201 \\ &= 0.0855 \text{ m}^3 \end{aligned}$$

Since the least accuracy in the given data is upto three places of decimals.

$$\text{Volume} = 0.0855 \text{ m}^3$$

12. The mass of a box measured by a grocer's balance is 2.3 kg . Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?

Ans :

- (a) Total mass of box
 $= 2.3 + 0.02015 + 0.02017$
 $= 2.34032 \text{ kg} = 2.3 \text{ kg}$

- (b) Difference in the masses of pieces
 $= 20.17 - 20.15 = 0.02 \text{ g}$

13. A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c . (This relation first arose as a consequence of special relativity theory due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c . He writes :

$$m = \frac{m_0}{(1 - v^2)^{1/2}} \text{ Guess where to put the missing } c.$$

Ans :

To make the dimensions on both sides equal to $[M^1]$ the denominator must be dimensionless, 1 is dimensionless factor at ' v^2 ' must be dimensionless i.e., there must be v^2/c^2 .

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

14. The unit of length convenient on the atomic scale is known as an angstroms and is denoted by $\text{\AA}$. $1 \text{\AA} = 10^{-10} \text{ m}$. The size in $\text{\AA}$ of a hydrogen atom is about $0.5 \text{\AA}$, what is the total atomic volume in m^3 of a mole of hydrogen atom ?

Ans :

Volume of 1 hydrogen atom

$$\begin{aligned} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3 \end{aligned}$$

Since one mole of hydrogen contains 6.02×10^{23} atoms

Atomic volume

$$= 6.02 \times 10^{23} \times \frac{4}{3} \times 3.14 \times 1.25 \times 10^{-30}$$

Atomic volume

$$\begin{aligned} &= \frac{602 \times 4 \times 314 \times 125 \times 10^{-14}}{3} \\ &= 3.151 \times 10^{-7} \text{ m}^3 \cong 3 \times 10^{-7} \text{ m}^3 \end{aligned}$$

15. One mole of an ideal gas at standard temperature and pressure occupies 22.4 litres (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about $1 \text{\AA}$). Why is the ratio so large ?

Ans :

$$\begin{aligned} 22.4 \text{ litres} &= 22.4 \times 10^{-3} \text{ m}^3 \\ (1 \text{ litre} &= 10^{-3} \text{ m}^3) \end{aligned}$$

Volume of 1 hydrogen atom

$$\begin{aligned} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times 3.14 \times (1 \times 10^{-10})^3 \end{aligned}$$

$$= \frac{4}{3} \times 3.14 \times 10^{-30} \text{ m}^3$$

Volume of 1 mole of hydrogen atoms

$$= 6.02 \times 10^{23} \times \frac{4}{3} \times 3.14 \times 10^{-30} \text{ m}^3$$

$$= 6.02 \times \frac{4}{3} \times 3.14 \times 10^{-7} \text{ m}^3$$

$$= \frac{6.02 \times 4 \times 314}{3} \times 10^{-11} \text{ m}^3$$

$$= 2.52 \times 10^{-6} \text{ m}^3$$

$$\frac{\text{Molar volume}}{\text{Atomic volume of a mole of hydrogen}} = \frac{22.4 \times 10^{-3} \text{ m}^3}{2.52 \times 10^{-6} \text{ m}^3} \simeq 10^4$$

This ratio is so large because inter-atomic separation in a gas is much larger than the size of an atom.

16. Explain this common observation clearly. If you look out of the window of a fast-moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the moon, the stars etc.) seem to be stationary.

Ans :

Since $\theta = l/r$, the distant objects appear to have lesser angular displacement relative to the observer in the train the distant objects appear to be stationary. The relative angular displacement for nearer objects is not negligible.

17. The sun is a hot plasma (ionized matter) with its inner core at temperature exceeding 10^7 K , and its outer surface at a temperature of about 6000 K . At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be ? In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the Sun $= 2.0 \times 10^{30} \text{ kg}$, radius of the Sun $= 7.0 \times 10^8 \text{ m}$.

Ans :

Mass density of Sun should be $1.4 \times 10^3 \text{ kg m}^{-3}$. The mass density of the Sun is in the range of densities of solids and not gases. This high density of the hot plasma arises due to inward gravitational layers due to layers of the Sun.

$$\text{Radius of Sun} = 7.0 \times 10^8 \text{ m}$$

$$\begin{aligned} \text{Volume} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times 3.14 \times (7.0 \times 10^8)^3 \\ &= \frac{4}{3} \times 3.14 \times 49 \times 7 \times 10^{24} \end{aligned}$$

$$\text{Mass of Sun} = 2.0 \times 10^{30} \text{ kg}$$

$$\text{Density} = \frac{M}{V}$$

$$= \frac{2.0 \times 10^{30} \times 3}{4 \times 3.14 \times 49 \times 7 \times 10^{24}}$$

$$= \frac{3000 \times 10^3}{2 \times 3.14 \times 49 \times 7}$$

$$= 1.393 \times 10^3 \text{ kg m}^{-3}$$

$$= 1.4 \times 10^3 \text{ kg m}^{-3}$$

Therefore, nucleus density is 10^5 times atomic density of matter.

OBJECTIVE QUESTIONS

1. What is a unit in the context of measuring a physical quantity?
- (A) A numerical value representing the magnitude of the quantity.
- (B) A basic, arbitrarily chosen, internationally accepted reference standard.
- (C) A derived quantity obtained from fundamental quantities.
- (D) The instrument used for measuring the physical quantity.

Ans :

- (B) A basic, arbitrarily chosen, internationally accepted reference standard.

A unit is a fundamental, globally agreed-upon standard used for comparing and expressing the measurements of physical quantities. This standard allows for consistent and universally understood measurements across different contexts and locations.

How is the result of a measurement of a physical quantity typically expressed?

- (A) By a unit only.
- (B) By a number (or numerical measure) only.
- (C) By a number (or numerical measure) accompanied by a unit.
- (D) By the name of the physical quantity only.

Ans :

- (C) By a number (or numerical measure) accompanied by a unit.

The measurement of a physical quantity is conveyed by a numerical value indicating its magnitude, followed by a unit that provides context and standardizes the measurement against an accepted reference.

3. What are fundamental or base units?

- (A) Units derived from combinations of other units.
- (B) Units for expressing all physical quantities.
- (C) Units for the fundamental or base quantities.
- (D) Units that are the smallest possible measure.

Ans :

(C) Units for the fundamental or base quantities.
Fundamental or base units are the foundational units defined for basic physical quantities, from which other units (derived units) are formed. They serve as the building blocks for the entire system of measurement.

4. What are derived units?

- (A) Units that are arbitrarily chosen and internationally accepted.
- (B) Units that represent fundamental or base quantities.
- (C) Units obtained as combinations of the base units for derived quantities.
- (D) Units that are only used in specific scientific fields.

Ans :

(C) Units obtained as combinations of the base units for derived quantities.
Derived units are units for physical quantities that are not fundamental. They are expressed by combining the fundamental or base units according to the mathematical relationships between the physical quantities involved in the system.

5. What constitutes a system of units?

- (A) Only the set of fundamental or base units.
- (B) Only the set of derived units.
- (C) A complete set of both base units and derived units.
- (D) The historical units used in different countries.

Ans :

(C) A complete set of both base units and derived units.

A system of units is a comprehensive collection that includes both the fundamental (base) units for basic quantities and the derived units for all other physical quantities, forming a coherent measurement framework.

6. Which of the following were common systems of units used extensively until recently?

- (A) SI, MKS, and CGS systems.
- (B) CGS, FPS (British), and MKS systems.
- (C) Metric, Imperial, and SI systems.

(D) Atomic, Astronomical, and British systems.

Ans :

(B) CGS, FPS (British), and MKS systems.
Before the widespread adoption of the SI system, the CGS (centimetre-gram-second), FPS (foot-pound-second or British), and MKS (metre-kilogram-second) systems were predominantly used by scientists in various countries for measurements.

7. What were the base units for length, mass, and time in the CGS system?

- (A) Metre, kilogram, and second.
- (B) Foot, pound, and second.
- (C) Centimetre, gram, and second.
- (D) Yard, slug, and minute.

Ans :

(C) Centimetre, gram, and second.
The CGS system of units is based on the centimetre as the unit of length, the gram as the unit of mass, and the second as the unit of time.

8. What is the full name of the SI system of units?

- (A) System of International Units.
- (B) Système Internationale d'Unités.
- (C) Standard International Units.
- (D) Scientific International Units.

Ans :

(B) Système Internationale d'Unités.
The SI system, which is the internationally accepted system for measurement, is an abbreviation for *Système Internationale d'Unités*, a French term meaning International System of Units. It was developed by the BIPM.

9. How many base units are there in the SI system?

- (A) Five
- (B) Six
- (C) Seven
- (D) Nine

Ans :

(C) Seven
The International System of Units (SI) is built upon seven fundamental or base units for quantities such as length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.

10. Besides the base units, SI defines units for which other two types of angles?

- (A) Acute angle and obtuse angle.
- (B) Plane angle and solid angle.

- (C) Right angle and straight angle.
 (D) Reflex angle and full angle.

Ans :

(B) Plane angle and solid angle.

In addition to the seven base units, the SI system defines units for plane angles (radian) and solid angles (steradian), which are dimensionless quantities derived from geometric ratios.

11. What is the SI unit for plane angle and its symbol?

- (A) Degree, deg
 (B) Radian, rad
 (C) Gradian, grad
 (D) Steradian, sr

Ans :

(B) Radian, rad

The SI unit for measuring a plane angle is the radian, symbolized as 'rad'. It is defined as the ratio of the length of an arc to its radius.

12. What is the SI unit for solid angle and its symbol?

- (A) Radian, rad
 (B) Degree, deg
 (C) Steradian, sr
 (D) Candela, cd

Ans :

(C) Steradian, sr

The steradian, with the symbol 'sr', is the SI unit for measuring a solid angle. It is defined as the ratio of the intercepted spherical surface area to the square of the radius.

13. Why is reporting the result of a measurement with more digits than the significant figures considered misleading?

- (A) It makes the number too long and difficult to read.
 (B) It implies a greater precision than the measurement actually has.
 (C) It does not affect the understanding of the measurement's precision.
 (D) It is a common practice to show all calculated digits.

Ans :

(B) It implies a greater precision than the measurement actually has.

Including digits beyond the significant figures gives a false impression of the measurement's accuracy and precision, suggesting a level of detail that was not actually achieved by the measuring instrument or process.

14. Which of the following statements about significant figures is incorrect?

- (A) All non-zero digits are significant.
 (B) All zeros between two non-zero digits are significant.
 (C) Zeros on the right of a decimal point but to the left of the first non-zero digit are significant.
 (D) The trailing zero(s) in a number with a decimal point are significant.

Ans :

(C) Zeros on the right of a decimal point but to the left of the first non-zero digit are significant.

Zeros located to the right of the decimal point and to the left of the first non-zero digit are not significant; they serve only as placeholders to indicate the magnitude. For example, in 0.0023, the underlined zeros are not significant.

15. Which statement accurately describes how to handle trailing zeros in significant figures?

- (A) Trailing zeros in a number without a decimal point are always significant.
 (B) Trailing zeros in a number with a decimal point are not significant.
 (C) Trailing zeros in a number without a decimal point are not significant unless indicated otherwise.
 (D) Trailing zeros are never significant in any number.

Ans :

(C) Trailing zeros in a number without a decimal point are not significant unless indicated otherwise. In a number without a decimal point, trailing zeros (e.g., in 12300 cm) are generally not considered significant. However, if the number includes a decimal point (e.g., 3.500), the trailing zeros are significant.

16. How does changing units affect the number of significant figures in a measurement?

- (A) It increases the number of significant figures.
 (B) It decreases the number of significant figures.
 (C) It does not change the number of significant figures.
 (D) It depends on whether the new unit is larger or smaller.

Ans :

(C) It does not change the number of significant figures.

The number of significant figures indicates the precision of a measurement and is independent of

the unit chosen. Converting a measurement from one unit to another (e.g., cm to m) does not alter its precision or its significant figures.

17. Which of the following is NOT a rule for arithmetic operations with significant figures?

- (A) In multiplication or division, the final result should retain as many significant figures as in the original number with the least significant figures.
- (B) In addition or subtraction, the final result should retain as many decimal places as in the number with the least decimal places.
- (C) In addition or subtraction, the final result should retain as many significant figures as in the original number with the least significant figures.
- (D) Exact numbers in formulae have an infinite number of significant figures.

Ans :

(C) In addition or subtraction, the final result should retain as many significant figures as in the original number with the least significant figures. For addition or subtraction, the rule is based on the number of decimal places, not the number of significant figures. The result should have the same number of decimal places as the measurement with the fewest decimal places.

18. How does the rule for rounding off differ when the insignificant digit to be dropped is exactly 5, preceded by an even digit versus an odd digit?

- (A) The preceding digit is always raised by 1 in both cases.
- (B) The preceding digit is always left unchanged in both cases.
- (C) If the preceding digit is even, it's dropped; if odd, it's raised by 1.
- (D) If the preceding digit is even, it's raised by 1; if odd, it's dropped.

Ans :

(C) If the preceding digit is even, it's dropped; if odd, it's raised by 1. By convention, if the digit to be dropped is 5, and the preceding digit is even, the insignificant digit 5 is simply dropped. If the preceding digit is odd, it is raised by 1. For example, 2.745 becomes 2.74, and 2.735 becomes 2.74.

19. What are the "dimensions" of a physical quantity?

- (A) The specific units used to measure the physical quantity.
- (B) The numerical magnitude of the physical quantity.

- (C) The powers to which the base quantities are raised to represent that quantity.
- (D) The physical size or extent of the quantity.

Ans :

(C) The powers to which the base quantities are raised to represent that quantity.

The dimensions of a physical quantity describe its nature by indicating how it relates to the fundamental (base) quantities (like length [L], mass [M], time [T], etc.) through the powers or exponents of these base quantities.

20. How are the dimensions of base quantities like length, mass, and time denoted?

- (A) L, M, T
- (B) [L], [M], [T]
- (C) (L), (M), (T)
- (D) {L}, {M}, {T}

Ans :

(B) [L], [M], [T]

The dimensions of base quantities are conventionally denoted using square brackets around the symbol for the quantity. For instance, length is denoted as [L], mass as [M], and time as [T].

21. What are the dimensions of force?

- (A) $[MLT^{-1}]$
- (B) $[ML^2T^{-2}]$
- (C) $[MLT^{-2}]$
- (D) $[ML^{-1}T^{-2}]$

Ans :

(C) $[MLT^{-2}]$

Force is defined as mass times acceleration. The dimensions of mass are [M], length are [L], and time are [T]. Acceleration has dimensions $[LT^{-2}]$. Therefore, the dimensions of force are $[M][LT^{-2}] = [MLT^{-2}]$.

22. What is a dimensional formula of a physical quantity?

- (A) An equation relating the physical quantity to its numerical value.
- (B) An expression showing how and which base quantities represent the dimensions of the quantity.
- (C) A formula used to convert the physical quantity from one unit to another.
- (D) The SI unit of the physical quantity.

Ans :

(B) An expression showing how and which base quantities represent the dimensions of the quantity.

A dimensional formula is an expression that indicates the powers to which the fundamental (base) quantities must be raised to obtain the dimensions of that particular physical quantity. For example, the dimensional formula for velocity is $[M^0 L T^{-1}]$.

23. What is a dimensional equation?

- (A) An equation that includes only dimensionless constants.
- (B) An equation obtained by equating a physical quantity with its dimensional formula.
- (C) An equation that relates different systems of units.
- (D) An equation involving only base quantities.

Ans :

(B) An equation obtained by equating a physical quantity with its dimensional formula.

A dimensional equation is formed by setting a physical quantity equal to its dimensional formula. For instance, the dimensional equation for force $[F]$ is $[F] = [MLT^{-2}]$.

24. According to the principle of homogeneity of dimensions, what is true about the terms in a correct physical equation?

- (A) All terms must have different dimensions.
- (B) Only terms on one side of the equation must have the same dimensions.
- (C) All terms on both sides of the equation must have the same dimensions.
- (D) The dimensions of the terms do not matter for the correctness of an equation.

Ans :

(C) All terms on both sides of the equation must have the same dimensions.

The principle of homogeneity of dimensions states that in any valid physical equation, the dimensions of each term on both sides of the equation must be identical. This principle is used to check the dimensional consistency of equations.

25. What can be said about an equation if it fails the dimensional consistency test?

- (A) It might still be correct.
- (B) It is proved wrong.
- (C) It is proved right.
- (D) Its correctness is uncertain.

Ans :

(B) It is proved wrong.

If an equation is dimensionally inconsistent (i.e., the dimensions on both sides do not match), it must be

incorrect. However, dimensional consistency alone does not guarantee that an equation is correct.

26. Which of the following statements regarding dimensional analysis is incorrect?

- (A) Dimensionally consistent equations are always physically correct.
- (B) Physical quantities can only be added or subtracted if they have the same dimensions.
- (C) The arguments of trigonometric functions must be dimensionless.
- (D) Dimensional analysis can be used to deduce relations among physical quantities.

Ans :

(A) Dimensionally consistent equations are always physically correct.

While a dimensionally consistent equation is a necessary condition for correctness, it is not sufficient. A dimensionally correct equation might still be physically incorrect due to missing dimensionless constants or incorrect functional relationships.

27. Which of the following statements about dimensional analysis is correct?

- (A) Dimensional analysis can determine the value of dimensionless constants in a formula.
- (B) If an equation passes the dimensional consistency test, it is definitively proven right.
- (C) A dimensionally wrong or inconsistent equation must be wrong.
- (D) Velocity can be added to force if their numerical values are the same.

Ans :

(C) A dimensionally wrong or inconsistent equation must be wrong.

Dimensional analysis is a powerful tool for checking equations. If an equation is found to be dimensionally inconsistent (meaning the dimensions on both sides don't match), it is definitively incorrect.

28. When using the method of dimensions to deduce a relation among physical quantities, what is a limitation of this method?

- (A) It cannot handle quantities with fractional dimensions.
- (B) It cannot determine dimensionless constants involved in the relation.
- (C) It only works for equations involving exactly three physical quantities.
- (D) It cannot be applied if the quantities are interdependent.

Ans :

(B) It cannot determine dimensionless constants involved in the relation.

While dimensional analysis can help deduce the form of a relationship between physical quantities, it cannot determine the numerical value of any dimensionless constants (like 2π or $1/2$) that might be part of the exact formula.

29. Which of the following is NOT one of the seven fundamental or base quantities in the SI system?

(A) Length
(B) Mass
(C) Force
(D) Time

Ans :

(C) Force

The seven base quantities in the SI system are length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity. Force is a derived quantity, not a base quantity.

30. Compare the precision indicated by the measurements 1.02 ± 0.01 g and 9.89 ± 0.01 .

(A) 1.02 g is measured with higher relative error.
(B) 9.89 g is measured with higher relative error.
(C) Both measurements have the same relative error.
(D) The absolute error determines the higher precision.

Ans :

(A) 1.02 g is measured with higher relative error. The relative error in 1.02 g is $(\pm 0.01/1.02) \times 100\% \approx \pm 1\%$. The relative error in 9.89 g is $(\pm 0.01/9.89) \times 100\% \approx \pm 0.1\%$. Thus, the measurement 1.02 g has a higher relative error.

31. Assertion : The number 0.002308 m has four significant figures.

Reason : Zeros to the right of the decimal point but to the left of the first non-zero digit are not significant.

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(C) Assertion is true but Reason is false.
(D) Assertion is false but Reason is true.

Ans :

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

In 0.002308 m, the digits 2, 3, 0, 8 are significant. The two zeros after the decimal point and before the digit 2 are not significant as they only locate the decimal point. Thus, there are four significant figures.

32. Assertion : The equation $x = x_0 + v_0 t + (1/2) at^2$ is dimensionally correct.

Reason : All terms in a physically correct equation must have the same dimensions.

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(C) Assertion is true but Reason is false.
(D) Assertion is false but Reason is true.

Ans :

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Each term in the equation (x , x_0 , $v_0 t$, and $(1/2) at^2$) has the dimension of length [L]. Since all terms have the same dimensions, the equation is dimensionally correct, which is a requirement for any physically valid equation.

33. Assertion : When a body is in uniform motion, no net external force acts on it.

Reason : According to dimensional analysis, force and acceleration must have the same dimensions.

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(C) Assertion is true but Reason is false.
(D) Assertion is false but Reason is true.

Ans :

(C) Assertion is true but Reason is false.

The assertion is a direct consequence of Newton's first law of motion. The reason is false; force has dimensions $[MLT^{-2}]$ and acceleration has dimensions $[LT^{-2}]$. They do not have the same dimensions.

34. Consider the following statements regarding units:
1. Fundamental units are derived from other units.
 2. The SI system has seven base units.
 3. Radian is the SI unit for solid angle.
 4. Derived units are combinations of base units.
- Which of the above statements are correct?

(A) 1 and 3
(B) 2 and 4
(C) 1, 2 and 3
(D) 2, 3 and 4

Ans :

(B) 2 and 4

Statement 1 is incorrect; fundamental units are the basic, independently defined units. Statement 3 is incorrect; radian is the unit for plane angle, while steradian is for solid angle. Statements 2 and 4 are correct descriptions of the SI system and derived units.

35. Regarding significant figures, which of the following statements are true?

1. All non-zero digits are significant.
2. Zeros between non-zero digits are not significant.
3. Trailing zeros in a number with a decimal point are significant.
4. Changing units does not change the number of significant figures.

(A) 1, 2, and 3

(B) 1, 3, and 4

(C) 2, 3, and 4

(D) 1, 2, 3, and 4

Ans :

(B) 1, 3, and 4

Statement 2 is false; zeros between non-zero digits (e.g., in 2.03) are significant. Statements 1, 3, and 4 correctly describe rules and properties related to significant figures in measurements.

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36. Identify the correct statements about dimensional analysis:

1. If an equation is dimensionally correct, it must be physically correct.
2. Physical quantities can be added only if they have the same dimensions.
3. The dimensions of force are $[MLT^{-1}]$.
4. Dimensional analysis can be used to check the consistency of equations.

(A) 1 and 3

(B) 2 and 4

(C) 1, 2, and 4

(D) 2, 3, and 4

Ans :

(B) 2 and 4

Statement 1 is false; dimensional correctness doesn't guarantee physical correctness. Statement 3 is false; the dimensions of force are $[MLT^{-2}]$. Statements 2 and 4 are true principles of dimensional analysis.

37. Match the physical quantities in Column I with their corresponding SI base units or derived units with special names in Column II.

	Column I		Column II
1.	Length	A.	kilogram (kg)
2.	Mass	B.	second (s)
3.	Time	C.	metre (m)
4.	Force	D.	newton (N)

(A) 1-C, 2-A, 3-B, 4-D

(B) 1-A, 2-C, 3-D, 4-B

(C) 1-C, 2-B, 3-A, 4-D

(D) 1-D, 2-A, 3-B, 4-C

Ans :

(A) 1-C, 2-A, 3-B, 4-D

Length is measured in metres (m), mass in kilograms (kg), and time in seconds (s); these are base SI units. Force is a derived quantity, and its SI unit is the newton (N).

38. Match the system of units in Column I with their base units for length, mass, and time in Column II.

	Column I		Column II
1.	CGS System	A.	Foot, Pound, Second
2.	FPS System	B.	Metre, Kilogram, Second
3.	MKS System	C.	Centimetre, Gram, Second
4.	SI System (subset)	D.	Metre, Kilogram, Second

(A) 1-C, 2-A, 3-B, 4-D

(B) 1-A, 2-C, 3-B, 4-D

(C) 1-C, 2-B, 3-A, 4-D

(D) 1-B, 2-A, 3-D, 4-C

Ans :

(A) 1-C, 2-A, 3-B, 4-D

The CGS system uses centimetre, gram, second. The FPS system uses foot, pound, second. The MKS system uses metre, kilogram, second. The SI system also uses metre, kilogram, second as base units for these quantities.

39. Match the dimensional formulae in Column I with the physical quantities in Column II.

	Column I		Column II
1.	$[M^0 L^1 T^{-1}]$	A.	Acceleration
2.	$[M^1 L^1 T^{-2}]$	B.	Volume
3.	$[M^0 L^1 T^{-2}]$	C.	Velocity
4.	$[M^0 L^3 T^0]$	D.	Force

- (A) 1-C, 2-D, 3-A, 4-B
 (B) 1-A, 2-B, 3-C, 4-D
 (C) 1-C, 2-A, 3-D, 4-B
 (D) 1-B, 2-D, 3-A, 4-C

Ans :

- (A) 1-C, 2-D, 3-A, 4-B

Velocity has dimensions $[LT^{-1}]$. Force has dimensions $[MLT^{-2}]$. Acceleration has dimensions $[LT^{-2}]$. Volume has dimensions $[L^3]$. The given options align these correctly.

- 40.** Which of the following pairs correctly matches a physical quantity with its SI base unit?
 (A) Luminous intensity - Lumen
 (B) Amount of substance - Gram
 (C) Thermodynamic temperature - Kelvin
 (D) Electric current - Volt

Ans :

- (C) Thermodynamic temperature - Kelvin

The SI base unit for thermodynamic temperature is the kelvin (K). Lumen is the unit for luminous flux, gram is a unit of mass (kilogram is the base unit), and volt is the unit for electric potential difference.

- 41.** Identify the correct pair of a quantity and its dimensional formula.
 (A) Density - $[ML^{-2}T^0]$
 (B) Speed - $[M^0LT^{-2}]$
 (C) Acceleration - $[M^0LT^{-2}]$
 (D) Work - $[MLT^{-2}]$

Ans :

- (C) Acceleration - $[M^0LT^{-2}]$

Acceleration has dimensions of length divided by time squared, $[LT^{-2}]$. Density is mass/volume, $[ML^{-3}]$. Speed is length/time, $[LT^{-1}]$. Work is force x distance, $[ML^2T^{-2}]$.

- 42.** Which of the following pairs incorrectly matches a physical quantity with its nature (scalar/vector)?
 (A) Mass - Scalar
 (B) Velocity - Vector
 (C) Speed - Vector
 (D) Displacement - Vector

Ans :

- (C) Speed - Vector

Speed is a scalar quantity as it only has magnitude and no direction. Velocity is a vector quantity because it has both magnitude (speed) and direction. Mass and displacement are correctly identified.

- 43.** Identify the incorrect pairing of a system of units and one of its base units.

- (A) CGS system - Gram
 (B) FPS system - Second
 (C) MKS system - Foot
 (D) SI system - Ampere

Ans :

- (C) MKS system - Foot

The MKS system uses Metre, Kilogram, and Second as its base units for length, mass, and time, respectively. Foot is the unit of length in the FPS system. The other pairs are correct.

- 44.** Arrange the following steps for checking the dimensional consistency of an equation in the correct order:

1. Write down the dimensions of each term in the equation.
2. Simplify the dimensions on both sides of the equation.
3. If the dimensions of all terms are the same, the equation is dimensionally consistent.
4. Identify the physical quantities involved in the equation.

- (A) 4, 1, 2, 3
 (B) 1, 4, 2, 3
 (C) 4, 2, 1, 3
 (D) 1, 2, 4, 3

Ans :

- (A) 4, 1, 2, 3

First, identify all physical quantities. Then, write down their dimensions. Next, simplify these dimensions for each term on both sides. Finally, compare them to check for consistency.

- 45.** What is the logical progression in the development of measurement systems mentioned in the text?

1. Adoption of the SI system.
2. Use of CGS, FPS, and MKS systems.
3. Realization of the need for a limited number of units.

- (A) 3, 2, 1
 (B) 2, 3, 1
 (C) 1, 2, 3
 (D) 3, 1, 2

Ans :

- (A) 3, 2, 1

The text first introduces the general concept of measurement and the idea that a limited number of units are needed. It then discusses earlier systems

like CGS, FPS, and MKS, and finally describes the current internationally accepted SI system.

46. A student measures the length of a rod as 1.20 meters and its breadth as 0.85 meters. What is the area of the rod reported to the correct significant figures?
- (A) $1.0200m^2$
 (B) $1.02m^2$
 (C) $1.0m^2$
 (D) $1.020m^2$

Ans :

- (B) $1.02m^2$

The length of the rod is given as 1.20m (3 significant figures) and its breadth is 0.85m (2 significant figures). When multiplying these values to find the area, the result should be reported with the same number of significant figures as the measurement having the least number of significant figures. Area = $1.20m \times 0.85m = 1.020m^2$. Rounding to two significant figures (due to 0.85m), the area is $1.0m^2$. However, typical representation might retain $1.02m^2$ if 1.20 implies precision to two decimal places and 0.85 to two decimal places, leading to a product of 1.02. The rule is the result should have no more significant figures than the factor with the fewest significant figures. So 0.85 (2 sig figs) dictates the result has 2 sig figs. 1.02 rounded to 2 sig figs is 1.0. If 1.02 is the intended answer, it means retaining 3 significant figures.

47. A car accelerates from rest to a velocity of $20m/s$ in 10 seconds. Its acceleration is $2m/s^2$. Which of the following equations is dimensionally consistent with this scenario? (x = displacement, v = final velocity, u = initial velocity, a = acceleration, t = time)
- (A) $v = u + at^2$
 (B) $x = ut + \frac{1}{2}at$
 (C) $v^2 = u^2 + 2ax$
 (D) $x = v_0t + \frac{1}{2}a$

Ans :

- (C) $v^2 = u^2 + 2ax$

Let's check dimensions:

- (A) $[LT^{-1}] = [LT^{-1}] + [LT^{-2}T^2] = [LT^{-1}] + [L]$.
 Inconsistent.
 (B) $[L] = [LT^{-1}T] + [LT^{-2}T] = [L] + [LT^{-1}]$.
 Inconsistent.
 (C) $[(LT^{-1})^2] = [(LT^{-1})^2] + [LT^{-2}L]$
 $[L^2T^{-2}] = [L^2T^{-2}] + [L^2T^{-2}]$. Consistent.
 (D) $[L] = [LT^{-1}T] + [LT^{-2}]$. Inconsistent.
 Only $v^2 = u^2 + 2ax$ is dimensionally consistent.

48. What is the term for a reference standard used for comparison in the measurement of a physical quantity?
- (A) Magnitude
 (B) Dimension
 (C) Unit
 (D) System

Ans :

- (C) Unit

A unit is defined as a certain basic, arbitrarily chosen, internationally accepted reference standard with which measurements of a physical quantity are compared. This ensures consistency and universal understanding in science.

49. Which organization was responsible for developing the SI system of units, including symbols and abbreviations, in 1971 and revising it in November 2018?
- (A) International Union of Pure and Applied Physics (IUPAP)
 (B) Bureau International des Poids et Mesures (BIPM)
 (C) National Institute of Standards and Technology (NIST)
 (D) International Organization for Standardization (ISO)

Ans :

- (B) Bureau International des Poids et Mesures (BIPM)

The Système Internationale d'Unités (SI) was developed and is maintained by the Bureau International des Poids et Mesures (BIPM), also known as The International Bureau of Weights and Measures. This body ensures global uniformity in measurements.

50. What is the SI unit of luminous intensity, and what is its symbol?
- (A) Lumen, lm
 (B) Lux, lx
 (C) Candela, cd
 (D) Watt, W

Ans :

- (C) Candela, cd

The SI base unit for luminous intensity is the candela, and its symbol is 'cd'. It measures the power emitted by a light source in a particular direction, weighted by the luminous efficiency function.

51. Which of the following numbers has exactly three significant figures?

(A) 0.0500
(B) 2.308
(C) 12300 (ambiguous without context or decimal)
(D) 4.700×10^2

Ans :

(A) 0.0500

In 0.0500, the leading zeros are not significant, but the trailing zeros after the non-zero digit and decimal point are significant, making it three significant figures (5, 0, 0). 2.308 has four. 12300 typically has three (1, 2, 3) unless specified otherwise. 4.700×10^2 has four.

52. If the mass of an object is 4.237 g (four significant figures) and its volume is 2.51 cm^3 (three significant figures), to how many significant figures should its density be reported?

(A) Two
(B) Three
(C) Four
(D) Eleven

Ans :

(B) Three

When multiplying or dividing measured quantities, the result should retain as many significant figures as are present in the original number with the least number of significant figures. Here, volume (2.51 cm^3) has three significant figures, which is less than mass (four significant figures).

53. The diameter of the Earth is approximately $1.28 \times 10^7 \text{ m}$. What is the order of magnitude of the Earth's diameter?

(A) 10^6 m
(B) 10^7 m
(C) 10^8 m
(D) 7

Ans :

(B) 10^7 m

The order of magnitude is the power of 10 when the number is expressed in scientific notation ($a \times 10^b$, where $1 \leq a < 10$). For $1.28 \times 10^7 \text{ m}$, since 1.28 is less than or equal to 5, the order of magnitude is 10^7 m . The order of magnitude itself is the exponent, 7.

54. What is the dimensional formula for volume?

(A) $[M^1 L^2 T^0]$
(B) $[M^0 L^3 T^0]$
(C) $[M^0 L^2 T^{-1}]$

(D) $[M^1 L^3 T^{-1}]$

Ans :

(B) $[M^0 L^3 T^0]$

Volume is the product of three lengths (length $\times$ breadth $\times$ height). Therefore, its dimensions are $[L] \times [L] \times [L] = [L^3]$, or expressed with all base dimensions, $[M^0 L^3 T^0]$.

55. Can the method of dimensions be used to determine the value of a dimensionless constant like 'k' in an equation such as $T = k\sqrt{l/g}$?

(A) Yes, it can determine all constants.
(B) No, it cannot determine dimensionless constants.
(C) Yes, but only if 'k' is less than 10.
(D) No, because 'k' depends on the units used.

Ans :

(B) No, it cannot determine dimensionless constants. Dimensional analysis can verify the consistency of an equation or deduce the relationship between physical quantities based on their dimensions, but it cannot find the numerical value of dimensionless constants (like k, π , etc.).

56. The side of a cube is measured to be 2.00 m. What is its volume expressed with the correct number of significant figures?

(A) 8.000 m^3
(B) 8.00 m^3
(C) 8.0 m^3
(D) 8 m^3

Ans :

(B) 8.00 m^3

The side length 2.00 m has three significant figures. The volume of a cube is side $\times$ side $\times$ side. So, $(2.00 \text{ m})^3 = 8.000000 \text{ m}^3$. Since the input measurement has three significant figures, the result should also be rounded to three significant figures, which is 8.00 m^3 .

57. Which of the following statements about the SI system is incorrect?

(A) It uses a decimal system, making conversions convenient.
(B) It has seven base units.
(C) The unit for plane angle, radian, is a base unit.
(D) It is the internationally accepted system for measurement.

Ans :

(C) The unit for plane angle, radian, is a base unit. The radian (for plane angle) and steradian (for solid angle) are supplementary units in the SI system,

not base units. The seven base units are metre, kilogram, second, ampere, kelvin, mole, and candela.

58. Which of the following correctly describes the rule for addition with significant figures?

- (A) The result should have the same number of significant figures as the input with the most significant figures.
- (B) The result should have the same number of decimal places as the input with the fewest decimal places.
- (C) The result should have the total number of significant figures from all inputs.
- (D) The result should always be rounded to two significant figures.

Ans :

(B) The result should have the same number of decimal places as the input with the fewest decimal places.

In addition (or subtraction), the final result should retain as many decimal places as are present in the number with the least number of decimal places, indicating the precision of the combined measurement.

59. The 'order of magnitude' of a physical quantity expressed as $a \times 10^b$ (where a is rounded to 1 or 10) refers to:

- (A) The value of 'a'.
- (B) The exponent 'b'.
- (C) The product $a \times b$.
- (D) The unit of the quantity.

Ans :

(B) The exponent 'b'.

When a physical quantity is expressed in scientific notation as $a \times 10^b$, and 'a' is rounded to 1 (if $a \leq 5$) or 10 (if $5 < a \leq 10$), the exponent 'b' of 10 is called the order of magnitude of that physical quantity.

60. What is the primary purpose of using scientific notation for reporting measurements?

- (A) To make numbers look more complex.
- (B) To remove ambiguities in determining the number of significant figures and simplify large/small numbers.
- (C) To convert all measurements to SI units.
- (D) To ensure all measurements have at least five significant figures.

Ans :

(B) To remove ambiguities in determining the number of significant figures and simplify large/small numbers.

Scientific notation ($a \times 10^b$) clearly indicates significant figures (all digits in 'a' are significant) and provides a concise way to represent very large or very small numbers, simplifying numerical computation and interpretation.

ONE MARK QUESTIONS

61. Name the fundamental units of measurement.

Ans :

They are metre, kilogram, second, kelvin, ampere, candela and mole.

62. What is 1 A.U. ?

Ans :

1 A.U. or astronomical unit is the unit of length and is equal to average distance between the sun and the earth. It is equal to 1.496×10^{11} m.

63. What is the unit shake ?

Ans :

It is a unit of time and is hundred millionth of a second.

$$1 \text{ shake} = 10^{-8} \text{ s}$$

- 64.** (a) What is micron ?
(b) Express $0.53 \text{ \AA}$ in m.

Ans :

(a) $1 \text{ micron} = 10^{-6} \text{ m}$

(b) $0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$

65. Express one parsec second in terms of light years.

Ans :

Since,

$$1 \text{ Parsec} = 3.08 \times 10^{16} \text{ m}$$

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

Number of light years in 1 parsec

$$= \frac{3.08 \times 10^{16}}{9.46 \times 10^{15}} \\ = 3.256$$

66. Can a body have zero mass and zero weight ?

Ans :

No, A body cannot have zero mass, but its weight can be zero (when $g = 0$).

- 67.** The mass of a body as measured by two students is given as 1.2 kg and 1.23 kg. Which of the two is more accurate and why ?

Ans :

The measurement 1.23 is more accurate because it has been made to the second decimal place.

- 68.** What is the difference between mass and weight ?

Ans :

Mass is the quantity of matter contained in the body and weight of a body is the force with which it is attracted by the earth towards its centre.

- 69.** Define light year.

Ans :

Light year is the distance travelled by light in one year in vacuum.

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

- 70.** What do you mean by least count?

Ans :

Least count is the minimum value which can be measured by an instrument.

- 71.** Define significant figures.

Ans :

Significant figures are the digits, whose values are accurately known in a particular measurement.

- 72.** What is a unit ?

Ans :

A unit is the chosen standard of the same kind taken as reference in order to measure a physical quantity.

- 73.** Name the device used for measuring the mass of atoms and molecules.

Ans :

The mass of atoms and molecules is measured by 'Mass spectrograph'.

- 74.** Express unified atomic mass unit in kg.

Ans :

$$1 \text{ amu} (u) = 1.67 \times 10^{-27} \text{ kg}$$

- 75.** Why length, mass and time are chosen as base quantities in mechanics ?

Ans :

Length, mass and time are chosen as base quantities

in mechanics because all other quantities can be expressed in terms of M , L and T by simple relations.

- 76.** If the universe was to shrink to the size of earth, how large would be the Earth on this scale ?

Ans :

10^{-11} m (size of an atom).

Do the inertial and gravitational masses of ordinary objects differ in magnitude ?

Ans :

No.

- 78.** Which is the most accurate clock ?

Ans :

Cesium atomic clock.

- 79.** Name two types of masses.

Ans :

(i) Inertial mass, (ii) Gravitational mass.

- 80.** What are dimensions of Angular momentum ?

Ans :

Angular momentum = $ML^2 T^{-1}$

- 81.** Of which quantity joule is SI unit ?

Ans :

Work.

- 82.** Of which quantity watt is SI unit ?

Ans :

Power.

- 83.** Name the quantity associated with the dimensions $[L^2 MT^{-2}]$.

Ans :

Work or Torque.

- 84.** Name two quantities which are dimensionless.

Ans :

Strain and angle.

- 85.** What do you mean by a physical quantity ?

Ans :

A quantity which can be measured directly or indirectly is called a physical quantity.

86. How will you define the mass ?

Ans :

Mass is a measure of the quantity that the object contains. It does not depend on the temperature, pressure or location of the object in space.

87. Name seven basic physical quantities.

Ans :

The seven basic physical quantities are : length, mass, time, electric current, temperature, luminous intensity and amount of substance.

88. What is the size of atom and size of nucleus ?

Ans :

Size of atom : 10^{-10} m

Size of nucleus : 10^{-14} m

89. Name three fundamental units.

Ans :

Mass, length and time are three fundamental units.

90. Name four physical quantities having the same dimensional formula.

Ans :

Length, displacement, circumference of a circle and radius of gyration. All have dimensional formula $[L]$.

91. Give two examples of dimensional constants.

Ans :

Gravitational constant and Planck's constant.

92. Give two examples of dimensionless constants.

Ans :

π and 21. (i.e. any number)

93. Write three physical quantities having dimensions $[M^1 L^{-1} T^{-2}]$.

Ans :

Pressure, Young's modulus of elasticity and stress.

94. Can a quantity have units but still be dimensionless?

Ans :

Yes, plane angle has no dimensions but has unit like radian.

95. Do all physical quantities have dimensions ? If no, name three physical quantities which are dimensionless.

Ans :

No. All physical quantities do not have dimensions. For examples, angle, strain and relative density are physical quantities and are dimensionless quantities.

96. What are significant figures?

Ans :

The significant figures are the number of digits used to express the physical quantity with precision.

97. How many significant figures are there in the number 0.04500 ?

Ans :

Four

98. What do you mean by least count error ?

Ans :

This is the error associated with the resolution of the instrument.

TWO MARKS QUESTIONS

99. What are derived units ?

Ans :

The units of all such quantities which can be obtained from fundamental units are called derived units e.g. area and velocity.

100. Why do we have different units for the same physical quantity ?

Ans :

Because the same physical quantity differs in order of magnitude. For example if we take the distance between two quantities, its order is different as for inter-atomic distances, it is of the order of angstroms, the interstellar distances are of the order of light year and the distances between two cities is of the order of kilometres.

101. Comment, on statement : "To define a physical quantity for which no method of measurement is given or known has no meaning."

Ans :

This is incorrect. Every physical quantity must have a physical meaning. If it can be measured by any

direct method, there must be some indirect method for its measurement.

e.g. Entropy is such physical quantity.

- 102.** If $x = a + bt + ct^2$, where x is in metre and t in second; then what is the unit of c ?

Ans :

$$\begin{aligned}\text{Since } [x] &= [ct^2] \\ [c] &= \left[\frac{x}{t^2} \right] \\ \text{Unit of } c &= \text{ms}^{-2}\end{aligned}$$

- 103.** Sometimes angular measurements are very helpful. Explain.

Ans :

For determining large distances like the height of a mountain, the distance of moon or any planet, we cannot measure directly and hence indirect methods are used and in indirect methods, the angular method is more useful.

By knowing the elevation, we can easily find the inaccessible distances.

- 104.** What is the difference between Angstroms and A.U.?

Ans :

$$\begin{aligned}1 \text{ Angstrom} &= 10^{-10} \text{ m} \\ \text{and } 1 \text{ A.U.} &= 1.496 \times 10^{11} \text{ m}\end{aligned}$$

- 105.** Write down the dimensional formula of gravitational constant (G).

Ans :

$$\begin{aligned}\text{Since } F &= \frac{Gmm}{r^2} \\ G &= \frac{Fr^2}{m^2} = \frac{[MLT^{-2}][L^2]}{[M^2]} \\ &= [M^{-1}L^3T^{-2}]\end{aligned}$$

- 106.** Write the dimensional formula of kinetic energy.

Ans :

$$\begin{aligned}KE &= \frac{1}{2}mv^2 \\ &= M[L T^{-1}]^2 = [ML^2 T^{-2}]\end{aligned}$$

So dimensional formula of KE is $[M^1 L^2 T^{-2}]$

- 107.** Name the quantities represented by the dimensional formula :

- (a) $[M^1 L^2 T^{-1}]$ and
(b) $[M^1 L^{-3} T^0]$

Ans :

$$\begin{aligned}\text{(a) } [M^1 L^2 T^{-1}] &= [ML^2 T^{-2}][T] = \text{joule sec} \\ &= \text{Planck's constant}\end{aligned}$$

$$\text{(b) } [M^1 L^{-3} T^0] = \frac{[M]}{[L^3]} = \frac{M}{V} = \text{density}$$

- 108.** Check the correctness of the equation $s = vt$

Ans :

Dimensional formula of

$$s = [L]$$

Dimensional formula of

$$vt = [L T^{-1}][T] = [L]$$

Since dimensional formula of

$$\text{L.H.S.} = \text{dimensional formula of R.H.S.}$$

So the equation is correct.

- 109.** What do you mean by absolute error?

Ans :

The magnitude of the difference between the value of the quantity and the individual measurement value is called the absolute error of the measurement.

- 110.** Solve with due regard to significant figures.
 $\sqrt{6.5 - 6.32}$

Ans :

$$\begin{aligned}\sqrt{6.5 - 6.32} &= \sqrt{0.18} \\ &= \sqrt{0.2} \quad (\text{upto one decimal place}) \\ &= 0.447 \\ &= 0.44 \quad (\text{least significant figure is 2})\end{aligned}$$

- 111.** Round off 3.250 and 3.750 to one decimal place of decimal.

Ans :

$$\begin{aligned}3.250 &\rightarrow 3.2 \\ 3.750 &\rightarrow 3.8\end{aligned}$$

- 112.** Add $6.75 \times 10^3 \text{ cm}$ to $4.52 \times 10^2 \text{ cm}$

Ans :

Since

$$\begin{aligned}4.52 \times 10^2 \text{ cm} &= 0.452 \times 10^3 \text{ cm} \\ &= (6.75 \times 10^3 + 0.452 \times 10^3) \text{ cm} \\ &= 7.202 \times 10^3 \text{ cm} \\ &= 7.20 \times 10^3 \text{ cm} \\ &\quad (\text{upto two decimal places})\end{aligned}$$

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- 113.** Though the number of physical quantities is very large yet we do not have as many different units, why ?

Ans :

We use some basic units in terms of which the units of other quantities can be expressed. These units are called derived units. The advantage is that we have not to name and remember so many units.

- 114.** What type of phenomena can be used as measure of time ? Give two examples of it.

Ans :

Any phenomena in nature which repeats itself after a definite interval of time can be used as a measure of time e.g. Oscillation of a pendulum, the rotation of earth about its axis.

- 115.** What is a coherent system of units ? Give an example of coherent system of units.

or

Define coherent system of units.

Ans :

A coherent system of units is that which is based on certain set of fundamental units from which all derived units are obtained by simple multiplication or division. SI system and M.K.S. system of units are coherent system of units.

- 116.** 'Sometimes angular measurements are very helpful'. Discuss.

Ans :

Large distances, such as height of a mountain or distance of a planet, cannot obviously be measured directly with a metre scale. We have to use some other methods. For them, 'angular displacements' are very helpful.

- 117.** Bring out the difference between 2.0 and 2.000.

Ans :

2.0 has 2 significant figures while 2.000 has got four significant figures. In another way, we can say that measurement 2.0 has been made accurate upto first decimal only, while the measurement 2.000 has been made accurate upto fourth decimal place.

- 118.** How is systematic error eliminated ?

Ans :

An error which is always of the same sign is called a systematic error. This error can be eliminated by detecting the source of error and the rule governing this error.

- 119.** What is the role of measurement in physics? Explain its significance.

Ans :

Measurement is fundamental to physics, a quantitative science. It involves comparing a physical quantity with a basic, internationally accepted reference standard called a unit. Accurate measurements are crucial for verifying theories and establishing reliable scientific laws, forming the basis of experimental physics.

- 120.** Differentiate between fundamental units and derived units. Provide one example for each.

Ans :

Fundamental or base units are the units for fundamental or base quantities like length, mass, and time. For example, the metre is a fundamental unit. Derived units are expressed as combinations of base units for derived quantities like speed or force. An example is metre per second (m/s).

- 121.** Describe the CGS, FPS, and MKS systems of units briefly.

Ans :

These are earlier systems of units. In the CGS system, the base units for length, mass, and time were centimetre, gram, and second. In the FPS system, they were foot, pound, and second. In the MKS system, they were metre, kilogram, and second respectively.

- 122.** Why is the SI system of units preferred in scientific work internationally?

Ans :

The SI system, with its standard scheme of symbols, units, and abbreviations, is internationally accepted for its convenience and consistency in scientific, technical, industrial, and commercial work. SI units use the decimal system, making conversions within the system quite simple and convenient.

- 123.** Define radian and steradian. What physical quantities do they measure?

Ans :

The radian (rad) is the SI unit for plane angle, defined as the ratio of the length of an arc to its radius. The steradian (sr) is the SI unit for solid angle, defined as the ratio of the intercepted spherical surface area to the square of its radius.

- 124.** List the seven base quantities in the SI system along with their respective SI units.

Ans :

The seven base quantities in SI are:

- Length (metre, m)
- Mass (kilogram, kg)
- Time (second, s)
- Electric current (ampere, A)
- Thermodynamic temperature (kelvin, K)
- Amount of substance (mole, mol)
- Luminous intensity (candela, cd)

- 125.** What is the importance of specifying elementary entities when using the 'mole' unit?

Ans :

When the mole is used, the elementary entities must be specified. These entities may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. This specification ensures clarity and avoids ambiguity in quantifying the amount of substance.

- 126.** Explain the importance of significant figures in scientific measurements.

Ans :

Significant figures in a measurement include all digits known reliably plus the first uncertain digit. They indicate the precision of the measurement, which depends on the least count of the measuring instrument. Reporting results with correct significant figures avoids giving a misleading idea about measurement accuracy.

- 127.** State the general rules for determining if zeros in a number are significant.

Ans :

Zeros between two non-zero digits are significant. Zeros on the right of the decimal point but to the left of the first non-zero digit are not significant if the number is less than 1. Trailing zeros in a number with a decimal point are significant.

- 128.** How does using scientific notation (e.g., $a \times 10^b$) improve the reporting of significant figures?

Ans :

Scientific notation clearly indicates significant figures by showing all significant digits in the base number 'a'. It removes ambiguities associated with trailing zeros in numbers without a decimal point, as the power of 10 is irrelevant to determining significant figures.

- 129.** Explain the rules for rounding off uncertain digits when the last digit to be dropped is 5.

Ans :

If the insignificant digit to be dropped is 5, the convention is : if the preceding digit is even, the insignificant digit is simply dropped. If the preceding digit is odd, it is raised by 1. This helps in consistent rounding of numbers.

- 130.** Describe the rule for determining the number of significant figures in the result of addition or subtraction.

Ans :

In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places. This ensures the result's precision is consistent with the least precise measurement involved in the calculation.

- 131.** Describe the rule for determining the number of significant figures in the result of multiplication or division.

Ans :

In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures. This ensures the calculated result does not imply a greater precision than the input data.

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- 132.** What is meant by the order of magnitude of a physical quantity? How is it determined?

Ans :

The order of magnitude is an exponent of 10 representing an approximate idea of a number. When a number is expressed in scientific notation $a \times 10^b$, if 'a' is rounded to 1 (for $a \leq 5$) or 10 (for $5 < a \leq 10$), the quantity is of the order of 10^b .

- 133.** How are exact numbers, like the '2' in $r = d/2$, treated when performing arithmetic operations concerning significant figures?

Ans :

Exact numbers, which are not measured values but defined quantities or counting numbers (like 2 in $r = d/2$ or in 2π), are considered to have an infinite number of significant digits. They do not limit the number of significant figures in a calculation.

- 134.** Why is it important to retain one extra digit in intermediate steps of multi-step calculations involving significant figures?

Ans :

Retaining one more digit than the significant digits in intermediate steps of complex calculations helps to avoid the accumulation of rounding errors. This ensures that the final rounded result is more accurate and closer to the true value.

- 135.** What are the dimensions of a physical quantity? How do they help in understanding its nature?

Ans :

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities (like length, mass, time) are raised to represent that quantity. They describe the fundamental nature of the quantity, irrespective of the specific units used for its measurement.

- 136.** Explain what dimensional formulae and dimensional equations are. Give the dimensional formula for acceleration.

Ans :

A dimensional formula shows how and which base quantities represent the dimensions of a physical quantity. For example, acceleration's dimensional formula is $[M^0 L T^{-2}]$. A dimensional equation is obtained by equating a physical quantity with its dimensional formula, like $[a] = [M^0 L T^{-2}]$.

- 137.** Can a quantity have units but be dimensionless? Explain with an example.

Ans :

Yes, a quantity can have units but be dimensionless. For example, a plane angle has the unit radian, and a solid angle has the unit steradian. However, both plane angle (ratio of arc length to radius) and solid angle (ratio of area to radius squared) are dimensionless quantities.

- 138.** Explain the principle of homogeneity of dimensions. How is it used to check an equation's correctness?

Ans :

The principle of homogeneity states that physical quantities can be added or subtracted only if they have the same dimensions. This principle is used to check an equation's correctness : if the dimensions of all terms on both sides are not identical, the equation is dimensionally wrong.

- 139.** Discuss two limitations of dimensional analysis when deriving or verifying physical equations.

Ans :

Dimensional analysis has limitations:

- It cannot determine dimensionless constants in a physical relation; it only provides the form of the equation.
- It cannot distinguish between physical quantities that have the same dimensions (e.g., work and torque).
- It does not guarantee the absolute correctness of an equation, only its dimensional consistency.

- 140.** Outline the general steps involved in deducing a relation among physical quantities using dimensional analysis.

Ans :

To deduce a relation, first identify the physical quantities involved and assume the dependence as a product of these quantities raised to some powers. Write the dimensional formula for each quantity. Then, equate the powers of $[M]$, $[L]$, and $[T]$ on both sides of the dimensional equation to find the exponents, thus establishing the relation.

- 141.** Why is it stated that a dimensionally correct equation need not be an actually correct physical equation?

Ans :

A dimensionally correct equation ensures that the dimensions on both sides are consistent. However, it does not provide information about dimensionless constants or functions that might be part of the actual physical relationship. Thus, while dimensional correctness is necessary, it's not sufficient for an equation to be physically exact.

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- 142.** What is the primary advantage of using dimensional analysis for checking consistency over merely checking units?

Ans :

Dimensional analysis offers a more fundamental consistency check as it is independent of the specific system of units chosen. We don't need to commit to particular units (like SI or CGS) or worry about conversions between multiples and sub-multiples of units, simplifying the verification process.

- 143.** How does the concept of “unit” relate to the process of “measurement”?

Ans :

Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called a unit. The unit provides a consistent basis for quantifying physical properties, making measurements meaningful and comparable.

THREE MARKS QUESTIONS

- 144.** Underline the rules for significant figures.

Ans :

- (i) All non zero digits are significant.
- (ii) All zeroes occurring between non-zero digits are significant.
- (iii) All zeroes to the right of decimal point and to the left of a non zero digits are not zero digits are not significant.
- (iv) All zeroes to the right of a non-zero digit in a number written with a decimal point are significant.

- 145.** What is measurement of a physical quantity ?

Ans :

Measurement of a physical quantity is the comparison of the quantity with another quantity of same kind which is taken as a reference quantity. When we say that length of a room is 5 metres, it means that it is five times conventionally chosen unit of length metre (m). Therefore, measurement of the length of a room is just in comparison with another length. Hence measurement of a physical quantity is just a comparison.

- 146.** What is the need of measurement?

Ans :

In physics, we try to express many natural phenomena in terms of relationships among the quantities involved. These relations may be stated in sentences, but these are better expressed in terms of mathematical equations. To verify these equations or laws of physics, it becomes necessary to measure the physical quantities involved. Hence we need measurement of physical quantities in order to express physical laws mathematically and to verify the laws.

- 147.** What is the SI unit of length ? Define ‘metre’ in different ways. Which is the most accurate ?

Ans :

Unit of length. In SI, the unit of length is metre.

One metre is the distance between the lines marked on platinum-iridium rod kept at constant temperature of 273.16 K at one bar pressure in the International Bureau of Weights and Measures at Sevres, near Paris in France.

All other ‘metres’ had to be calibrated against this metre. This makes it very inconvenient. So in 1960, the Eleventh General Conference of Weights and Measures decided to redefine metre by adopting atomic standards.

On atomic standards, one metre is defined as to be equal to 1,650,763.3 wavelengths in vacuum of the radiation emitted due to transition between the levels $2p^{10}$ and $5d^5$ of the isotope of Krypton having mass number 86.

This definition is considered to be more accurate because wavelength of light emitted by a source is easily reproducible and does not change with time.

- 148.** What are the characteristics of a physical standard?

Ans :

Characteristics of physical standard

- (i) Unit must be well defined.
- (ii) Unit must be easily reproducible every where.
- (iii) Unit should be invariant with time.
- (iv) Unit should be imperishable.
- (v) Unit must not change with physical conditions.
- (vi) Unit must be of proper size.

- 149.** Check the correctness of the relation $S = ut + \frac{1}{2}at^2$ using dimensional analysis.

Ans :

Given $S = ut + \frac{1}{2}at^2$

Dimensional formula of S

$$= [L] = [M^0 L^1 T^0]$$

Dimensional formula of

$$ut = [LT^{-1}][T] \\ = [L] = [M^0 L^1 T^0]$$

Dimensional formula of

$$\frac{1}{2}at^2 = [LT^{-2}][T^2] \\ = [L] = [M^0 L^1 T^0]$$

Since dimensions of mass, length and time on each

and every term on either side is the same, hence the given formula is correct.

- 150.** What do you mean by dimensions of a physical quantity ? Give examples to show how the dimensions of a physical quantity are determined.

Ans :

Dimensions. Dimensions of physical quantity are defined as the powers to which the base units are raised to obtain the unit of that quantity.

The dimensions are normally written in a square bracket.

In other words, the dimensions of a physical quantity show how its unit is related to the base units.

Example

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Dimensions of density

$$= \frac{M}{[L]^3} = [M][L]^{-3}$$

- 151.** In defining the standard of length we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity if another physical quantity, temperature has to be specified in choosing a standard ?

Ans :

We are not justified in calling length a fundamental quantity because temperature has to be specified in choosing a standard of length. The standard of length may be chosen at a particular temperature and length of an object may be measured at some other temperature.

- 152.** What is the advantage of expressing metre in terms of krypton-86 wavelength ?

Ans :

The standard metre defined in terms of wavelength of Krypton-86 is well defined. It is more satisfactory as Krypton atoms are easily available in any laboratory. Moreover, wavelength of light is not affected by temperature, time and environment. Thus such a standard is invariable.

1 m = 1650,763.73 wavelengths of orange-red light of Krypton-86

- 153.** What is the advantage in choosing the wavelength of light radiation as a standard of length ?

Ans :

Following are the advantages in choosing the wavelength of light radiation as a standard of length.

1. Wavelength can be accurately defined.
2. The wavelength of light is not affected by time and environment.
3. The wavelength of light is not affected by changes in place.
4. It is easily reproducible in any standard laboratory.

- 154.** Given below is an equation of a wave :
 $y = r \sin \omega \left(\frac{x}{v} - k\pi \right)$.
 What are the dimensions of x and k ? Here ω is angular velocity and v linear velocity.

Ans :

Since $\omega(x/v - k\pi)$ is an angle, it is a dimensionless quantity. In other words, each of ω , x/v and $\omega k\pi$ is dimensionless.

$$\begin{aligned} \text{Dimension of } x &= \frac{v}{\omega} = [L T^{-1}] / T^{-1} \\ &= [M^0 L T^0] \end{aligned}$$

$$\begin{aligned} \text{and dimension of } k &= 1/\pi\omega = \frac{1}{[1/T]} \\ &= [M^0 L^0 T] \end{aligned}$$

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- 155.** Find the dimensions of 'a' and 'b' in the following equation : $(P + a/V^2)(V - b) = RT$

Ans :

- (i) $\frac{a}{V^2}$ must have units of pressure

$$P = \frac{a}{V^2} \text{ or } a = PV^2$$

$$[a] = [M L^{-1} T^{-2}] [L^3]^2$$

$$= [M^1 L^5 T^{-2}]$$

- (ii) b must have units of volume, because only volume is added to volume, $[b] = [L^3]$

- 156.** If $x = a + bt + ct^2$, where x is in the metres and t in seconds, what are the units of a , b and c ?

Ans :

$$x = a + bt + ct^2$$

From the homogeneity of the equations, we have

$$(i) \quad a = [L^1] \quad \text{units of } a \text{ is metre}$$

$$(ii) \quad [b][T^1] = [L] \quad b = [L^1 T^{-1}]$$

unit of ' b ' is ms^{-1}

$$(iii) \quad [c][T^2] = [L^1], \quad [c] = [L^1 T^{-2}]$$

unit of c is ms^{-2}

- 157.** Mention some of the time intervals of importance in physics.

Ans :

The ranges of time intervals of importance are given below :

$$\text{Period of atomic vibrations} = 10^{-15} \text{ s}$$

$$\text{Time for light to cross a room} = 10^{-4} \text{ s}$$

$$\text{Wink of eye} = 10^{-1} \text{ s}$$

$$\text{Time of heart beat} = 1 \text{ s}$$

$$\text{Day} = 10^5 \text{ s}$$

$$\text{Man's life} = 10^9 \text{ s}$$

$$\text{Life of earth} = 10^{17} \text{ s}$$

- 158.** State principle of homogeneity of dimensions and its use in dimensional analysis.

Ans :

In a correct physical relation, the dimensions of all terms on left hand of an equation must be equal to the dimensions of all terms on right hand side.

This principle is used to :

- (i) check the accuracy of a given relation.
- (ii) derive a relation for a given physical quantity.

- 159.** Define dimensions, dimensional formula and dimensional equation of a physical quantity.

Ans :

Dimensions of a physical quantity are power to which the fundamental or basic units are raised to express to obtain its units.

For example,

$$\begin{aligned} \text{Velocity} &= \frac{\text{distance}}{\text{Time}} \\ &= \frac{L}{T} (M^0 L^1 T^{-1}) \end{aligned}$$

So dimensions of velocity are zero in mass, one in length and -1 in time.

A dimensional formula is an expression showing how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity.

For example, the dimensional formula for force is $[M^1 L^1 T^{-2}]$.

This shows that a unit of mass with dimension one, unit of length with dimension one and the unit of time with dimension negative two gives the dimensional formula for force.

Dimensional equation

When the dimensional formula is equated with its physical or derived quantity in the form of an equation, the equation is called a dimensional equation.

Thus dimensional equation for force will be :

$$[F] = [M^1 L^1 T^{-2}]$$

- 160.** Define the following -

- (i) Dimensional variables
- (ii) Dimensional constants
- (iii) Non-dimensional variables
- (iv) Non-dimensional constants.

Ans :

- (i) Dimensional variables are the quantities which are variable and have dimensions e.g. acceleration, velocity, force etc.
- (ii) Dimensional constants are the quantities which have constant values and yet have dimensions e.g. gravitational constant, coefficient of viscosity, gas constant, Planck's constant, velocity of light etc.
- (iii) Non-dimensional variables are the quantities which are variable and yet have no dimensions e.g. specific gravity, strain, angle etc.
- (iv) Non-dimensional constants are mere numbers like 2, 3, π etc.

- 161.** What is the difference between Mass and weight ?

Ans :

The difference between Mass and weight are as follows

	Mass	Weight
1.	Its is a scalar quantity	It is a vector quantity
2.	Mass of a body is always constant	It varies from place to place
3.	It is the measurement of inertia or gravitation	It is the measurement of gravity
4.	It can never be zero	It is zero at the centre of the earth

- 162.** What are the limitations of dimensional analysis ?

Ans :

Following are the limitations of dimensional analysis:

- (1) Since there are only three quantities namely M , L and T , whose powers can be compared on

the two sides of an equation, the dimensional analysis fails for formulae involving more than three independent factors.

- (2) Dimensionless constants occurring in the physical formula cannot be determined. The value of the constants has to be determined experimentally.
- (3) The method fails in case of the derivation of composite equations such as $v^2 - u^2 = 2as$.
- (4) Since trigonometric functions have no dimensions, the dimensional analysis fails to give information about these.

163. How does scientific notation help in unambiguously representing significant figures? Provide an example.

Ans :

Scientific notation ($a \times 10^b$) is the best way to report measurements unambiguously regarding significant figures.

- All digits in the base number 'a' are considered significant.
- It eliminates confusion with trailing zeros. For example, 4.700m can be written as 4.700×10^2 cm or 4.700×10^3 mm, each clearly showing four significant figures. The power of 10 is irrelevant to counting significant figures.

164. Explain how the precision of a measurement relates to its significant figures and the instrument's least count.

Ans :

The precision of a measurement indicates how closely repeated measurements agree with each other and is primarily determined by the least count of the measuring instrument. Significant figures reflect this precision.

- The reported result includes all reliably known digits plus the first uncertain digit (the significant figures).
- A smaller least count allows for more significant figures, implying higher precision in the measurement.

165. Discuss the rules for determining the uncertainty in results of arithmetic calculations, particularly for multiplication and subtraction.

Ans :

Determining uncertainty in calculations involves these rules:

- For multiplication (e.g., area $l \times b$), the relative error in the product is the sum of the relative errors in the individual quantities. The final

result is quoted with an uncertainty reflecting this.

- For subtraction, the number of significant figures can be reduced. The uncertainty combines based on the smallest number of decimal places, not necessarily the number of significant figures in the original data.

166. Explain the concept of dimensions of physical quantities. How are base quantities represented dimensionally?

Ans :

The dimensions of a physical quantity describe its nature by expressing it in terms of fundamental or base quantities.

- The dimensions are the powers to which base quantities are raised.
- The seven base quantities are represented by specific symbols in square brackets : Length [L], Mass [M], Time [T], Electric Current [A], Thermodynamic Temperature [K], Luminous Intensity [cd], and Amount of Substance [mol].

167. What are dimensional formulae and dimensional equations? Illustrate with the dimensional equation for force.

Ans :

These are tools for dimensional analysis:

- **Dimensional Formula** : An expression showing how and which base quantities represent the dimensions of a physical quantity. For example, the dimensional formula for force is $[MLT^{-2}]$.
- **Dimensional Equation** : An equation obtained by equating a physical quantity with its dimensional formula. For force [F], the dimensional equation is $[F] = [MLT^{-2}]$. This shows force has one dimension in mass, one in length, and -2 in time.

168. Discuss the dimensional representation of common mechanical quantities like volume, speed, and acceleration.

Ans :

In mechanics, physical quantities are typically expressed in terms of dimensions of Length [L], Mass [M], and Time [T].

- **Volume** : Being length $\times$ breadth $\times$ height, its dimensions are $[L] \times [L] \times [L] = [L^3]$. It has zero dimensions in mass $[M^0]$ and time $[T^0]$.
- **Speed/Velocity** : Being length/time, its dimensions are $[L]/[T] = [LT^{-1}]$.
- **Acceleration** : Being velocity/time, its dimensions are $[LT^{-1}]/[T] = [LT^{-2}]$.

- 169.** Explain the principle of homogeneity of dimensions and its application in checking equation consistency.

Ans :

The principle of homogeneity of dimensions is a fundamental concept in dimensional analysis. It states that magnitudes of physical quantities can be added together or subtracted from one another only if they have the same dimensions. For example, velocity cannot be added to force. This principle is used to check the correctness of an equation : if the dimensions of all terms on both sides of the equation are not the same, the equation is dimensionally inconsistent and therefore wrong.

- 170.** Discuss the primary applications and two main limitations of dimensional analysis.

Ans :

Dimensional analysis is a valuable tool in physics with several applications and limitations.

Applications:

- Checking the dimensional consistency or homogeneity of mathematical expressions.
- Deducing relations among different physical quantities.

Limitations:

- Dimensionless constants involved in relations cannot be obtained by this method.
- It does not distinguish between physical quantities having the same dimensions.

- 171.** Elaborate on how dimensional analysis can deduce relationships between physical quantities, using the time period of a simple pendulum as an example.

Ans :

Dimensional analysis can deduce relations if the dependence of a quantity on others is known as a product type. For a simple pendulum, if time period T depends on length (l), mass (m), and gravity (g), we write $T = k \cdot l^x m^y g^z$.

- By equating dimensions on both sides : $[L^0 M^0 T^1] = [L]^x [M]^y [L T^{-2}]^z$.
- This yields $x + z = 0, y = 0, -2z = 1$, giving $x = 1/2, y = 0, z = -1/2$.
- So, $T = k\sqrt{l/g}$. The dimensionless constant 'k' (actually 2π) cannot be found this way.

- 172.** What can be inferred if an equation passes the dimensional consistency test? What if it fails?

Ans :

Dimensional consistency is a critical test for physical equations:

- **If an equation fails the consistency test :** It means the dimensions of all terms are not

the same, proving the equation is wrong. A dimensionally inconsistent equation must be incorrect.

- **If an equation passes the consistency test :** It is not necessarily proven right. Dimensional consistency does not guarantee the equation is exact, as it doesn't account for dimensionless constants or functions.

- 173.** Why must arguments of special functions like trigonometric or logarithmic functions be dimensionless? Explain with an example.

Ans :

The arguments of special functions such as trigonometric (e.g., $\sin \theta$), logarithmic (e.g., $\ln x$), and exponential functions (e.g., e^x) must be dimensionless quantities. This is because these functions are often defined by infinite series expansions (e.g., $e^x = 1 + x + x^2/2! + \dots$). If 'x' had dimensions, terms in the series would have different dimensions, violating the principle of homogeneity that only like quantities can be added. An angle, for example, is a ratio of two lengths and is dimensionless.

- 174.** Explain the purpose and general guidelines for using SI prefixes for multiples and sub-multiples.

Ans :

SI prefixes are used to simplify the notation and numerical computation for both small and large physical quantities.

- They represent powers of 10 by which a unit is multiplied. For example, 'kilo' (k) means 10^3 and 'milli' (m) means 10^{-3} .
- Common prefixes like centi (c), mega (M), micro (μ), and nano (n) help in conveniently expressing measurements without writing many zeros, thus improving clarity and reducing errors in scientific communication.

- 175.** Describe how to calculate the total surface area and volume of a cube to appropriate significant figures if its side is measured.

Ans :

To find the surface area and volume of a cube with a measured side, arithmetic calculations are performed, and then results are rounded to appropriate significant figures.

- **Surface Area :** Calculated as $6 \times (\text{side})^2$.
- **Volume :** Calculated as $(\text{side})^3$.
- **Significant Figures :** Both results should be rounded off to the same number of significant figures as present in the measured length of the side. For example, if the side is 7.203 m (4

significant figures), the area and volume should also be reported to 4 significant figures.

- 176.** If the density of a substance is calculated from its mass and volume, how are significant figures handled in the result?

Ans :

When calculating density (mass/volume), the rule for multiplication/division concerning significant figures applies.

- The number of significant figures in the measured mass and volume are first determined.
- The calculated density should then be expressed to the same number of significant figures as the measurement with the least number of significant figures. For example, if mass has 3 significant figures and volume has 2, density is reported to 2 significant figures.

- 177.** Explain the term “relative error” and how it differs for measurements of different magnitudes but similar absolute accuracy.

Ans :

The relative error of a measured value indicates its accuracy relative to its size. It is the ratio of the absolute error to the measured value, often expressed as a percentage.

- For a given absolute accuracy (e.g., $\pm 0.01\text{g}$), the relative error depends on the magnitude of the measurement itself.
- For example, an accuracy of $\pm 0.01\text{g}$ in a 1.02g measurement gives a relative error of $\approx \pm 1\%$, while for 9.89g it's $\approx \pm 0.1\%$.

- 178.** What is the dimensional formula of pressure if it is defined as force per unit area?

Ans :

Pressure (P) is defined as Force (F) per unit Area (A).

- The dimensional formula for Force [F] is $[MLT^{-2}]$.
- The dimensional formula for Area [A] is $[L^2]$.

Therefore, the dimensional formula for Pressure [P] is $[F]/[A] = [MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$. This represents one dimension in mass, negative one in length, and negative two in time.

- 179.** Why is it important to use internationally accepted unit symbols like ‘m’ for metre or ‘s’ for second?

Ans :

Using internationally accepted unit symbols is vital for global scientific and technical communication.

- Standard symbols like ‘m’ for metre and ‘s’ for second ensure clarity and prevent misunderstandings when data is shared across different countries and disciplines.
- They contribute to the reproducibility of experiments and the reliable exchange of technical information in commerce and industry, forming a common language for measurements.

- 180.** Describe how to express the measurement 123000mm in scientific notation to clearly indicate three significant figures.

Ans :

The number 123000mm has three significant figures if the trailing zeros are not significant. To clearly indicate three significant figures and avoid ambiguity, scientific notation is used.

- The measurement can be written as $1.23 \times 10^5\text{mm}$.
- In this notation, the base number (1.23) explicitly shows the three significant figures (1, 2, and 3).

- 181.** Explain the significance of dimensional analysis in deducing the relationships between physical quantities, particularly when the exact form is unknown.

Ans :

Dimensional analysis is a powerful tool for deducing the form of relationships between interdependent physical quantities, even if the exact constants are unknown.

- By assuming a product-type dependence, it allows us to equate the dimensions of the involved quantities.
- Solving the resulting dimensional equations helps determine the exponents in the relationship, thus revealing how one quantity depends on others, which is crucial for theoretical development.

- 182.** What considerations are important when choosing a unit for a specific physical quantity?

Ans :

When choosing a unit for a physical quantity, several considerations are important:

- **International Acceptance** : The unit should ideally be part of an internationally accepted system like SI for universal understanding.
- **Proper Standardisation** : It must be based on a well-defined and reproducible reference standard.
- **Convenience** : The size of the unit should be convenient for the typical magnitudes of the quantity being measured in a particular context.

This is why multiples and sub-multiples (using prefixes) are also used.

FIVE MARKS QUESTIONS

183. Answer the following :

- Define a physical quantity.
- What is measurement of a physical quantity ?
- What is need of measurement ?

Ans :

- A quantity which can be measured directly or indirectly is called a physical quantity. Some of the examples of physical quantities are length, mass, time, area, velocity, force etc.
- Measurement of a physical quantity is the comparison of the quantity with another quantity of same kind which is taken as a reference quantity.

When we say that length of a room is 5 metres it means that it is five times conventionally chosen unit of length metre (m). Therefore, measurement of the length of a room is just in comparison with another length. Hence measurement of a physical quantity is just a comparison.

- In physics, we try to express many natural phenomena in terms of relationships among the quantities involved. These relations may be stated in sentences, but these are better expressed in terms of mathematical equations. To verify these equations or laws of physics it becomes necessary to measure the physical quantities involved. Hence we need measurement of physical quantities in order to express physical laws mathematically and to verify the laws.

184. Define and explain unit of a physical quantity.

Ans :

Unit of a physical quantity is defined as the reference standard of measurement of that quantity.

Measurement involves comparison of the quantity to be measured with a reference standard quantity. This reference standard of measurement is what we call a unit.

To determine the magnitude of a physical quantity completely, we should know

- the unit (u) in which it is measured and

- number of times that unit is contained in the quantity. i.e., numerical value (n).

If the length of a room is 5 metre, then unit is metre and numerical value is 5. Here metre is taken as a standard of measurement. This statement implies that one metre is contained five times in the length of the room. In general, if n is the numerical value when unit is u for a physical quantity Q , then

$$Q = nu$$

Since Q is always same for a given quantity, therefore, the bigger the unit, the smaller is the numerical value.

$$n_1 u_1 = n_2 u_2$$

- 185.** (a) Name the various prefixes and their symbols for expressing power of 10.
(b) What is the necessity of such like prefixes ?

Ans :

- The various prefixes are given below :

Power of 10	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	m
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

- Sometimes it is inconvenient to use a given unit for different measurements because in some cases the numerical value of quantity may be very large or very small. In that case it is convenient to express the result in the powers of 10. Example. Size of an atom is $\frac{1}{10,000,000,000}$ metre.

It can be conveniently written as 10^{-10} m. On the other hand, velocity of light is 3,000,000,00 ms^{-1} . It can be written as $3 \times 10^8 \text{ms}^{-1}$.

- 186.** (a) What is the concept of length in physics ?
 (b) What is the SI unit of length ? Define 'metre' in different ways. Which : is the most accurate ?

Ans :

- (a) The distance between two points in space is called length.
 The concept of length in physics is ultimately connected with the concept of distance in everyday life. The distance between two objects is nothing but an interval of space separating the two.

- (b) In SI, the unit of length is metre.

One metre is the distance between the lines marked on platinum-iridium rod kept at constant temperature of 273.16 K at one bar pressure in the International Bureau of Weights and Measures at Sevres, near Paris in France.

All other 'metres' had to be calibrated against this metre. This makes it very inconvenient. So in 1960, the Eleventh General Conference of Weights and Measures decided to redefine metre by adopting atomic standards.

On atomic standards, one metre is defined as to be equal to 1,650,763.3 wavelengths in vacuum of the radiation emitted due to transition between the levels $2p^{10}$ and $5d^5$ of the isotope of Krypton having mass number 86.

This definition is considered to be more accurate because wavelength of light emitted by a source is easily reproducible and does not change with time.

- 187.** Discuss uses of dimensional equation with suitable examples.

Ans :

Following are three uses of dimensional equation :

- (a) To check the correctness of physical relation
 The principle of homogeneity is used to check the accuracy of relation. If the powers of M , L and T are the same on both sides of the equation is correct otherwise not.

Example. Check the accuracy of

$$FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Dimensions formula of

$$\begin{aligned} FS &= [M^1 L^1 T^{-2}] \times [L] \\ &= [M^1 L^2 T^{-2}] \end{aligned}$$

Dimensional formula of

$$\begin{aligned} \frac{1}{2}mv^2 &= [M^1][L^2 T^{-2}] \\ &= [M^1 L^2 T^{-2}] \end{aligned}$$

Dimensional formula of

$$\frac{1}{2}mu^2 = [M^1 L^2 T^{-2}]$$

(1/2 is a dimensionless quantity. The relation is dimensionally correct because dimensions of all terms on both sides are the same).

- (b) To derive the relation between physical quantities

Following steps are involved :

- (i) Explore the possible factors upon which the physical quantity depends.
 (ii) Write the dimensions in terms of M , L and T
 (iii) Equate the powers of M , L and T on both sides to get values of unknown powers. Substituting these values in dimensional equation, we get the relation.

Example : Deduce an expression for the time period of a simple pendulum dimensionally.

The time period T of a simple pendulum may depend on mass (m) of the bob, length (l) of the pendulum and acceleration due to gravity (g).

Let $T \propto m^a l^b g^c$

or $T = k m^a l^b g^c \quad \dots(i)$

Dimension of $T = [T]$

Dimension of $m = [M]$

Dimension of $l = [L]$

Dimension of $g = [LT^{-2}]$

Writing the dimensions, we get

$$[T] = [M]^a [L]^b [LT^{-2}]^c$$

or $[M^0 L^0 T^1] = [M^a L^{b+c} T^{-2c}]$

Equating the powers of M , L and T on both sides, we get

$$a = 0, b + c = 0, -2c = 1$$

or $b = \frac{1}{2}, c = -\frac{1}{2}$

So eq (i) becomes

$$T = km^0 l^{1/2} g^{-1/2}$$

$$T = k\sqrt{\frac{l}{g}}$$

Actually

$$k = 2\pi$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- (c) Conversion of units from one system to other system

If n_1 and n_2 are numerical values of a quantity and u_1 and u_2 are units, then the physical quantity,

Q in one system,

$$Q = n_1 \times u_1 = n_1 u_1 \quad \dots(i)$$

and in the other

$$Q = n_2 \times u_2 = n_2 u_2 \quad \dots(ii)$$

Comparing (i) and (ii), we have

$$n_1 u_1 = n_2 u_2$$

Suppose the dimensional formula of a physical of the physical quantity is $[M^a L^b T^c]$ one system in which the units of mass, length and time are M_1 , L_1 and T_1 respectively, and n_1 be the numerical value of the physical quantity in the second system in which M_2 , L_2 and T_2 are the units of mass, length and time respectively. Then the size u_1 of the units of the physical quantity in one system is $[M_1^a L_1^b T_1^c]$ and the size u_2 in the second system is $[M_2^a L_2^b T_2^c]$. The physical quantity expressed in one system is $n_1 [M_1^a L_1^b T_1^c]$ and in the second system is $n_2 [M_2^a L_2^b T_2^c]$. So that

$$n_2 [M_2^a L_2^b T_2^c] = n_1 [M_1^a L_1^b T_1^c] \quad \dots(iii)$$

as from equation (iii), $n_2 u_2 = n_1 u_1$

$$\text{or} \quad n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c \quad \dots(iv)$$

Note. Before using equation (iii), the physical quantity must be expressed in absolute units.

Illustration. Convert a joule into ergs.

The joule and erg are the units of work in S.I. and C.G.S. system respectively.

The dimensional formula of work is $[M^1 L^2 T^{-2}]$

$$a = 1, b = 2, c = -2$$

S.I Units	C.G.S. Units
$n_2 = ?$	$n_2 = ?$
$M_1 = 1 \text{ kg}$	$M_2 = 1 \text{ g}$
$L_1 = 1 \text{ m}$	$L_2 = 1 \text{ cm}$
$T_1 = 1 \text{ s}$	$T_2 = 1 \text{ s}$

Putting these values in equation (4), we have

$$\begin{aligned} n_2 &= 1 \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[\frac{1 \text{ metre}}{1 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} \\ &= \left[\frac{1000}{1} \right]^1 \left[\frac{100}{1} \right]^2 \left[\frac{1}{1} \right]^{-2} = 10^7 \end{aligned}$$

$$1 \text{ joule} = 10^7 \text{ erg}$$

$$(1 \text{ kg} = 1000 \text{ g}, 1 \text{ m} = 100 \text{ cm})$$

188. What do you mean by significant figures ? Give examples.

Ans :

The significant figures are the numbers of digits

used to express the physical quantity with precision.

The number of significant digits in a physical quantity depends on the accuracy of the measuring instrument. If significant digits are more, the percentage error is less and vice versa.

Rules for significant figures :

- All non-zero digits are significant.
e.g. 54695 has 5 significant.
2.62456 has 6 significant.
- All zeroes occurring between non-zero digits are significant.
e.g. 540059 has 6 significant figures.
2.0059 has 5 significant figures.
- All zeroes to the right of decimal point and to the left of a non-zero digits are not significant.
e.g. 0.0545 has 3 significant figures.
000545 has 3 significant figures.
- All zeroes to the right of a non-zero digit in a number written with a decimal point are significant.
e.g. 5.4500 has 5 significant figures.
0.04500 has 4 significant figures. 5.00 has 3 significant figures.
- (a) All zeroes to the right of the last non-zero digits are not significant.
e.g. 2020 has 3 significant figures.
7890 has 3 significant figures.
 $2.64 \times 10^{24} \text{ kg}$ has 3 significant figures.
(b) All zeroes to the right of last non-zero digits are significant, if they come from a measurement.
e.g. 2020 m has 4 significant figures
or 20200 cm has 4 significant figures
or 2.020 km has 4 significant figures
and 0.082400 N m⁻² has 5 significant figures.

Numbers	Significant Figures
502	3
50.2	3
5.02	3
0.502	3
0.0502	3
0.05020	4
502.0	4
50.20	4

- 189.** State and explain the rules for finding the significant figures in the sum, difference, product and quotient of two numbers.

Ans :

The results of different observations are added, subtracted, multiplied or divided, to get desired results. But different numbers do not have the same precision in measurement. In such cases, the final result cannot be more precise than the least precise number involved in the calculation. In general, following rules are observed in reaching the final result :

Rule 1. (for addition or subtraction)

During addition and subtraction, the number of decimal places in the result should be equal to the smallest number of decimal places of terms in the operation.

e.g. (i) $8.211 + 1.1 = 9.311$

1.1 has one decimal place,

So, Answer = 9.3

(ii) $25.692 - 13.56 = 12.132$

13.56 has two decimal places,

so, Answer = 12.13

(iii) $12.425 - 6.0 = 6.425$

6.0 has one decimal place,

so, Answer = 6.4

Rule 2. (for product or quotient of numbers)

During multiplication and division, the number of significant figures in the final result is same as the smallest number of significant figures in any of the factors.

e.g. $4.1029 \times 1.30 = 5.337$

As the least precise number is 1.30 and has 3 significant figures.

Hence, Answer = 5.34

(rounding off to 3 significant figures)

As 3.52 has least significant figure i.e. 3, so rounding off the above result to three significant figures, we get answer = 2.00

- 190.** Discuss the applications of dimensional analysis in physics. What are its key uses?

Ans :

Dimensional analysis is a powerful technique with several important applications in physics:

- **Checking Dimensional Consistency** : It is used to verify the correctness of equations by ensuring that the dimensions of all terms are consistent

(principle of homogeneity). If an equation is dimensionally inconsistent, it must be wrong.

- **Deducing Relations Among Physical Quantities** : It can help in deriving the form of an equation relating different physical quantities if the dependence of one quantity on others is known as a product type.
- **Converting Units** : Although not explicitly detailed for this application, dimensional analysis underpins the logic for converting quantities from one system of units to another by ensuring dimensional equivalence.
- **Understanding Physical Behavior** : It provides insights into the nature of physical quantities and their interrelationships.

- 191.** Elaborate on the limitations of dimensional analysis. Why can it not determine dimensionless constants or certain functions?

Ans :

While dimensional analysis is highly useful, it has inherent limitations:

- **Dimensionless Constants** : It cannot determine the value of dimensionless constants of proportionality in an equation. For instance, in $T = k\sqrt{l/g}$, the constant 'k' (which is 2π) cannot be found dimensionally.
- **Nature of Functions** : It cannot derive relations involving trigonometric, logarithmic, or exponential functions, as these functions themselves and their arguments must be dimensionless.
- **Sum/Difference of Quantities** : It cannot be used to derive expressions involving the sum or difference of multiple terms if these terms are not known beforehand.
- **Distinguishing Quantities** : It does not distinguish between physical quantities that happen to have the same dimensions (e.g., work and torque).
- **Sufficiency** : A dimensionally correct equation is not necessarily a physically correct equation.

- 192.** Describe the method for deducing relationships among physical quantities using dimensional analysis, including its assumptions.

Ans :

Dimensional analysis can be used to deduce the form of a relationship among physical quantities based on certain assumptions:

- **Assumption of Dependence** : One must know which physical quantities the target quantity depends on (typically up to three independent variables).

- **Product-Type Relation** : The relationship is assumed to be a product of these quantities raised to unknown powers. For example, if T depends on l , m , and g , assume $T = k \cdot l^x m^y g^z$
- **Method:**
 1. Write the dimensional formula for each quantity involved.
 2. Substitute these into the assumed product-type equation.
 3. Equate the powers of the fundamental dimensions (M , L , T) on both sides of the equation.
 4. Solve the resulting system of linear equations for the unknown exponents (x , y , z).

This gives the form of the relation, though the dimensionless constant 'k' remains undetermined.

NUMERICAL PROBLEMS

- 193.** Find the value of 10 J in a system which has 100 cm, 10 g and 30 s as fundamental unit.

Ans :

The physical quantity associated with joule is energy.

Dimensional formula of energy : $[M^1 L^2 T^{-2}]$

So $a = 1$, $b = 2$, $c = -2$

S.I Units	New system
$M_1 = 1 \text{ kg}$	$M_2 = 100 \text{ g}$
$L_1 = 1 \text{ m}$	$L_2 = 10 \text{ cm}$
$T_1 = 1 \text{ s}$	$T_2 = 30 \text{ s}$
$n_1 = 10$	$n_2 = ?$

$$\begin{aligned}
 \text{Since } n_2 &= n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c \\
 &= 10 \left(\frac{1 \text{ kg}}{100 \text{ g}} \right)^1 \left(\frac{1 \text{ m}}{10 \text{ cm}} \right)^2 \left(\frac{1 \text{ s}}{30 \text{ s}} \right)^{-2} \\
 &= 10 \left(\frac{1000 \text{ g}}{100 \text{ g}} \right) \left(\frac{100 \text{ cm}}{10 \text{ cm}} \right) \left(\frac{1 \text{ s}}{30 \text{ s}} \right)^{-2} \\
 &= 10 \times 10 \times 100 \times 30 \times 30 \\
 &= 9 \times 10^6 \\
 10 \text{ J} &= 9 \times 10^6 \text{ new units}
 \end{aligned}$$

- 194.** The velocity v of sound waves, through a medium may be assumed to depend on (i) density of the medium, d , and (ii) its modulus of elasticity, E . Modulus of elasticity is a ratio of stress to strain and stress is force per unit area. Deduce by the

method of dimensions the formula for the velocity of sound.

Ans :

The velocity of sound can be written as :

$$v = k d^x E^y \quad \dots(i)$$

Here k is numerical constant. Taking dimensions, we have

$$[LT^{-1}] = \left[\frac{M}{L^3} \right]^x \left[\frac{MLT^{-2}}{L^2} \right]^y$$

$$\begin{aligned}
 \text{or } [M^0 L T^{-1}] &= [M^x L^{-3x}] [M^y L^{-y} T^{-2y}] \\
 &= [M^{x+y} L^{-3x-y} T^{-2y}]
 \end{aligned}$$

Equating the powers of L , M and T on both sides, we have

$$\begin{aligned}
 x + y &= 0 & -3x + y &= 1 & -2y &= -1 \\
 x + \frac{1}{2} &= 0 & -3x - \frac{1}{2} &= 1 & \text{or } y &= \frac{1}{2} \\
 x &= -\frac{1}{2} & y &= \frac{1}{2}
 \end{aligned}$$

Substituting the value of x and y in (i), we get

$$v = k d^{-1/2} E^{1/2}$$

$$\text{or } v = k \sqrt{\frac{E}{d}}$$

- 195.** The time of oscillation of t of a small drop of a liquid under surface tension depends upon the density d , the radius r and the surface tension s . Prove dimensionally that $t \propto \sqrt{d r^3 / s}$.

Ans :

Let $t = k d^x r^y r^z$, where k is constant

Dimensions of $t = [T]$

Dimensions of $d = [ML^{-3}]$

Dimensions of $r = [L]$

Dimensions of $s = [MT^{-2}]$, k is dimensionless.

$$[T] = [ML^{-3}]^x [L]^y [MT^{-2}]^z$$

$$[T] = [L^{(y-3x)} M^{x+z} T^{-2z}]$$

Comparing powers of M , L and T

$$\begin{aligned}
 y - 3x &= 0 & x + z &= 0 & -2z &= 1 \\
 y &= \frac{3}{2} & x &= \frac{1}{2} & z &= -\frac{1}{2}
 \end{aligned}$$

$$t = k \cdot d^{1/2} \cdot r^{3/2} \cdot s^{-1/2}$$

$$= k \sqrt{\frac{r^3 d}{s}}$$

$$t \propto \sqrt{\frac{r^3 d}{s}}$$

- 196.** Assuming that mass M of the largest stone that can be moved by a flowing river depends on, v the velocity of the river, ρ the density of water and ' g ' the acceleration due to gravity, show that mass M varies as the sixth power of velocity.

Ans :

Let $M = kv^a \rho^b g^c$, where k is a constant. Writing dimensions of all physical quantities on both sides, we get

$$[M^1 L^0 T^0] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$$

$$= [L^{a-3b+c} M^b T^{-a-2c}]$$

$$\text{or } [M^1 L^0 T^0] = [M^b L^{a-3b+c} T^{-a-2c}]$$

Comparing the powers of M , L and T on both sides, we get

$$b = 1 \quad \left| \begin{array}{l} -3b + c = 0 \\ -2c - 3b + c = 0 \\ -3b - c = 0 \\ -3 \times 1 - c = 0 \end{array} \right| \begin{array}{l} -a - 2c = 0 \\ a = -2c \\ a = (-2)(-3) \\ a = 6 \\ c = -3 \end{array}$$

$$M = kv^6 \rho g^{-3}$$

$$\text{or } M \propto v^6$$

that is, M varies as the sixth power of the velocity.

- 197.** If a composite physical quantity in terms of moment of inertia I , force F , velocity v , work W and length L is defined as $Q = \frac{IFv^3}{WL^3}$, find dimensions of Q and identify it.

Ans :

$$Q = \frac{IFv^3}{WL^3}$$

$$= \frac{[ML^2][MLT^{-2}][LT^{-1}]^3}{[ML^2 T^{-2}][L^3]}$$

$$\text{or } Q = [MT^{-2}]$$

This is the dimensional formula of surface tension, force constant, surface energy, hence the physical quantity Q can be any one of these.

- 198.** Given : mass of sun = 2.0×10^{30} kg, radius of sun = 7.0×10^8 m.

In what range do you expect the mass density of sun to be in the range of densities of solids or liquids or gases? Give an explanation of the result arrived at.

Ans :

$$M = 2.0 \times 10^{30} \text{ kg}$$

$$R = 7.0 \times 10^8 \text{ m}$$

Density of sun

$$= \frac{\text{mass}}{\text{volume}}$$

$$= \frac{2.0 \times 10^{30}}{\frac{4}{3}\pi(7.0 \times 10^8)^3} \text{ kg m}^{-3}$$

$$= \frac{2.0 \times 10^{30} \times 3 \times 7}{4 \times 22 \times 7 \times 7 \times 7 \times 10^{24}} \text{ kg m}^{-3}$$

$$= \frac{21}{44 \times 49 \times 7} \times 10^6 \text{ kg m}^{-3}$$

$$= 0.00139146 \times 10^6 \text{ kg m}^{-3}$$

$$= 1.39 \times 10^3 \text{ kg m}^{-3}$$

So, the mass density of the sun is in the range of densities of solids and not gases.

- 199.** Check the correctness of the relation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ dimensionally and numerically.

Ans :

$$\text{Given } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Dimensional formula of

$$c = [LT^{-1}]$$

Dimensional formula of

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{[M^1 L^1 T^{-2} A^{-2}][M^{-1} L^{-3} T^4 A^2]}}$$

$$= \frac{1}{\sqrt{L^{-2} T^2}} = \frac{1}{L^{-1} T}$$

$$= [LT^{-1}]$$

$$\text{LHS} = \text{RHS (Dimensionally)}$$

The relation is correct numerically.

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^9}}}$$

$$= 3 \times 10^8 \text{ m s}^{-1}$$

This is numerical value of c .

The relation is correct numerically.

Hence, the given relation is correct dimensionally as well as numerically.

CASE BASED QUESTIONS

- 200.** An old British engineering manual from the 1950s specifies the dimensions of a steel beam. The length is listed as 15 feet, the mass as 250 pounds, and the cross-sectional area in square inches. A modern engineer in India needs to verify if this beam can be used in a new construction project where all specifications are required in SI units. To do this, she must accurately convert these imperial measurements.

Questions:

1. The FPS system used in the manual had which base units for length, mass, and time?
2. Why was the SI system developed and internationally adopted, moving away from systems like CGS, FPS, and MKS?
3. If the engineer converts 'feet' to 'metres' and 'pounds' to 'kilograms', which system's base units for length and mass is she now aligning with, which forms the basis of SI?

Ans :

1. In the FPS system, the base units for length, mass, and time were foot, pound and second respectively.
2. The SI system was developed and adopted to provide a standard, internationally accepted system for measurement in scientific, technical, industrial, and commercial work, overcoming the confusion of different countries using different systems.
3. She is aligning with the MKS system's base units (metre, kilogram), which is the foundation for the SI units.

- 201.** A student is tasked with determining the density of a small, irregularly shaped metal object. Using a standard laboratory balance, she measures its mass to be 15.32 g. She then uses a beaker and water displacement method to find its volume, recording it as 5.4 cm³. She needs to calculate the density and report it with the appropriate precision.

Questions:

1. What are significant figures in a reported measurement?
2. Why is it superfluous or misleading to report a result with more digits than the significant digits?
3. How many significant figures are present in the measurement of mass (15.32 g) and volume (5.4 cm³)?
4. Calculate the density of the metal object and report the result to the correct number of significant figures.

Ans :

1. The reliable digits plus the first uncertain digit are known as significant digits or significant figures.
2. Reporting a result with more digits than significant digits is superfluous and misleading because it would give a wrong idea about the precision of the measurement.
3. Mass (15.32 g) has four significant figures. Volume (5.4 cm³) has two significant figures.

$$\begin{aligned}
 4. \quad \text{Density} &= \frac{\text{Mass}}{\text{Volume}} \\
 &= 15.32 \text{ g} / 5.4 \text{ cm}^3 \\
 &= 2.837037... \text{ g/cm}^3.
 \end{aligned}$$

The result should be rounded to two significant figures (the least number of significant figures in the input values).

Therefore, the density is 2.8g/cm³.

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- 202.** In 1999, NASA's Mars Climate Orbiter was lost during its mission. Investigations revealed that one engineering team used metric units (SI units, specifically newton-seconds for impulse) while another used Imperial units (pound-force seconds for impulse) for key spacecraft operations. This discrepancy in unit systems for a physical quantity (impulse) led to a navigational error, causing the orbiter to disintegrate in Mars' upper atmosphere.

Questions:

1. What is a "system of units"?
2. What were the base units for length, mass, and time in the CGS system?
3. The loss of the Mars Orbiter highlights the importance of using a standardized system of units in collaborative scientific and technical projects. Which system is currently internationally accepted for such work?
4. If a force was calculated as 100 pound-force and another as 100 newtons, are these forces equivalent? What does this imply about comparing physical quantities expressed in different unit systems?

Ans :

1. A complete set of units, both base units and derived units, is known as the system of units.
2. In the CGS system, the base units for length, mass, and time were centimetre, gram, and second, respectively.
3. The SI (Système Internationale d' Unites) is the internationally accepted system for measurement in scientific, technical, industrial, and commercial work.
4. No, 100 pound-force and 100 newtons are not equivalent. This implies that the result of a measurement of a physical quantity must be expressed by a number accompanied by a unit; without the unit, the number is ambiguous, and quantities cannot be compared or combined correctly if their units are different and not properly converted.

203. An astronomer measures the distance to a nearby star as 308,567,758,000,000,000 meters. For easier communication and to clearly indicate the precision of this measurement, she needs to express this in scientific notation with an appropriate number of significant figures. Let's assume the measurement is precise enough to warrant four significant figures.

Questions:

1. What is the primary purpose of using scientific notation for reporting measurements?
2. Express the given distance in scientific notation, assuming it has four significant figures
3. What is meant by the "order of magnitude" of a physical quantity?
4. If the diameter of the Earth is 1.274×10^7 m (order of magnitude 7) and the distance to the star is of the order of 10^{17} m, how many orders of magnitude larger is the star's distance compared to Earth's diameter?

Ans :

1. The best way to report every measurement, especially to remove ambiguities in determining the number of significant figures and to handle very large or small numbers, is in scientific notation (in the power of 10).
2. The distance is 308,567,758,000,000,000 m. With four significant figures, this becomes 3.086×10^{17} m.
3. The order of magnitude is the exponent (or power) 'b' of 10 when a number is expressed approximately as 10^b , after rounding the coefficient 'a' (in $a \times 10^b$) to 1 (for $a \leq 5$) or 10 (for $5 < a \leq 10$).
4. The star's distance is of the order of 10^{17} m (order of magnitude 17), and Earth's diameter is of the order of 10^7 m (order of magnitude 7). The difference in orders of magnitude is $17 - 7 = 10$. Thus, the star's distance is 10 orders of magnitude larger than Earth's diameter.

204. A surveyor measures a rectangular plot of land. The length is measured as 125.5 m and the width as 82.3 m. He needs to calculate the area of the plot and report it with the correct number of significant figures, reflecting the precision of his measurements.

Questions:

1. When multiplying or dividing measured quantities, how many significant figures should the final result retain?
2. How many significant figures are in the length measurement (125.5 m) and the width measurement (82.3 m)?
3. Calculate the area of the rectangular plot and report it with the appropriate number of significant figures.

4. If the surveyor instead measured the width as 82.30 m (implying higher precision), how would this affect the number of significant figures in the calculated area, assuming length is still 125.5 m?

Ans :

1. In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.
2. Length (125.5 m) has four significant figures. Width (82.3 m) has three significant figures.
3. Area = Length $\times$ Width

$$= 125.5 \text{ m} \times 82.3 \text{ m}$$

$$= 10328.65 \text{ m}^2.$$

The least number of significant figures in the measurements is three. So, the area should be reported to three significant figures as $1.03 \times 10^4 \text{ m}^2$ or 10300 m^2 (if using trailing zeros without a decimal is understood in context, though scientific notation is clearer). Using the book's example logic for density, 10300 m^2 implies ambiguity, so $1.03 \times 10^4 \text{ m}^2$ is best.

4. If the width was 82.30 m (4 significant figures) and length was 125.5 m (4 significant figures), then the result for the area (10328.65 m^2) would be reported with four significant figures, as $1.033 \times 10^4 \text{ m}^2$ (rounding 10328.65 to 10330).

205. A coach is analyzing a sprinter's performance. The sprinter covers a distance of 100.0 m in 9.87 s. The coach wants to calculate the average speed and needs to determine its dimensions and express it in derived SI units.

Questions:

1. What are derived units?
2. Speed is defined as distance/time. If distance is a length [L] and time is [T], what are the dimensions of speed?
3. If length is measured in meters (m) and time in seconds (s) (both SI base units), what is the derived SI unit for speed?
4. Calculate the sprinter's average speed to the correct number of significant figures using the given data.

Ans :

1. Units obtained for derived quantities, which can be expressed as combinations of base units, are called derived units.
2. The dimensions of speed are $[L]/[T]$.
3. The derived SI unit for speed would be meters per second (m/s or ms^{-1}).

4. Average speed = Distance / Time
 $= 100.0\text{m}/9.87\text{s}$
 $= 10.1317122\dots\text{m/s}.$

Distance (100.0 m) has 4 significant figures.
 Time (9.87 s) has 3 significant figures. The result should have 3 significant figures.
 Average speed = 10.1m/s.

- 206.** A student remembers a formula for kinetic energy (K) as $K = \frac{1}{2}mv^3 + ma$, where ' m ' is mass, ' v ' is velocity, and ' a ' is acceleration. Before using it, she decides to check its dimensional consistency.

Questions:

- What is the basic principle for adding or subtracting physical quantities in an equation based on their dimensions?
- Determine the dimensions of kinetic energy (K), mv^3 , and ma . (Energy has dimensions $[ML^2 T^{-2}]$).
- Based on the dimensional consistency, can the formula $K = \frac{1}{2}mv^3 + ma$ be correct? Explain why or why not.
- The dimensional consistency does not guarantee correct equations. Provide an example of a dimensionally correct but physically incorrect formula for kinetic energy from the options given in Example 1.4 and explain why dimensional analysis alone couldn't rule it out.

Ans :

- Physical quantities can be added together or subtracted from one another only if they have the same dimensions.
- Dimensions:
 Kinetic Energy $[K] = [ML^2 T^{-2}]$
 Mass $[m] = [M]$
 Velocity $[v] = [LT^{-1}]$
 Acceleration $[a] = [LT^{-2}]$
 $[mv^3] = [M][LT^{-1}]^3 = [ML^3 T^{-3}]$
 $[ma] = [M][LT^{-2}] = [MLT^{-2}]$
- The term mv^3 has dimensions $[ML^3 T^{-3}]$ and the term ma has dimensions $[MLT^{-2}]$. Since these two terms have different dimensions, they cannot be added together. Furthermore, neither of these terms has the dimensions of kinetic energy ($[ML^2 T^{-2}]$).
 Therefore, the formula $K = \frac{1}{2}mv^3 + ma$ is dimensionally incorrect and thus cannot be correct. (The factor $1/2$ is dimensionless and does not affect dimensional analysis).
- The formula $K = (\frac{3}{16})mv^2$ is dimensionally correct because
 $[mv^2] = [M][LT^{-1}]^2 = [ML^2 T^{-2}]$,

which are the dimensions of kinetic energy. However, the correct formula is $K = \frac{1}{2}mv^2$. Dimensional analysis cannot determine the value of dimensionless constants like $3/16$ or $1/2$.

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- 207.** An apprentice clock maker is trying to design a pendulum. He needs to understand how its period of oscillation (T) might depend on other factors. He hypothesizes that the period (T) depends on the length of the pendulum (l), the mass of the bob (m), and the acceleration due to gravity (g). He wants to use dimensional analysis to find a possible relationship.

Questions:

- If the period T is assumed to be a product of powers of l , m , and g , such as $T = k \cdot l^x m^y g^z$, where k is a dimensionless constant, what are the dimensions of T , l , m , and g ?
- Set up the dimensional equation by equating the dimensions of both sides.
- By equating the powers of M , L , and T on both sides, determine the values of x , y , and z to find the expression for T .
- What important piece of information about the pendulum's period formula cannot be determined through this dimensional analysis?

Ans :

- Dimensions:
 Period $[T] = [T]$
 Length $[l] = [L]$
 Mass $[m] = [M]$
 Acceleration due to gravity $[g] = [LT^{-2}]$
- The dimensional equation is:
 $[L^0 M^0 T^1] = [L]^x [M]^y [LT^{-2}]^z$
 $[L^0 M^0 T^1] = L^{x+z} M^y T^{-2z}$
- Equating the dimensions on both sides:
 For M : $y = 0$
 For T : $-2z = 1$
 $z = -1/2$
 For L : $x + z = 0$
 $x - 1/2 = 0$
 $x = 1/2$
 So, $T = k \cdot l^{1/2} m^0 g^{-1/2} = k\sqrt{l/g}$.

4. The value of the dimensionless constant k (which is actually 2π for a simple pendulum) cannot be obtained by the method of dimensions.
- 208.** A meteorologist is tracking changes in atmospheric pressure using a barometer that measures pressure in “bar”. On Monday, the reading was 1.012 bar. On Tuesday, a storm approached, and the reading dropped to 0.985 bar. The meteorologist needs to convert these readings to Pascals (Pa), the SI unit of pressure, for a report. (Use the conversion $1\text{bar} = 10^5\text{Pa}$).

Questions:

1. Is “bar” an SI unit? What is the SI unit of pressure?
2. If a measured quantity is 0.985 bar, how many significant figures does it have?
3. Convert the pressure reading of 0.985 bar to Pascals (Pa), maintaining the correct number of significant figures based on the conversion factor being exact for this context or defined.
4. If the uncertainty in the barometer reading is ± 0.001 bar, what is the uncertainty in the pressure reading in Pascals when the reading is 0.985 bar?

Ans :

1. “Bar” is a unit retained for general use, though outside SI. The SI unit of pressure is Pascal (Pa), which is N/m^2 (derived unit). The base units in SI are metre, kilogram, second, ampere, kelvin, mole, and candela.
2. The measurement 0.985 bar has three significant figures. (Zeroes to the left of the first non-zero digit are not significant if the number is less than 1 and they are on the right of the decimal point but to the left of the first non-zero digit. This rule applies to leading zeros like 0.02308, not 0.985. Here all non-zero digits are significant).
3. $0.985\text{bar} = 0.985 \times 10^5\text{Pa}$. Assuming 10^5 is an exact conversion factor, the result has three significant figures.
So, 98500 Pa or $9.85 \times 10^4\text{Pa}$.
4. Uncertainty in bar $= \pm 0.001$ bar.
Uncertainty in Pascals

$$= \pm 0.001 \times 10^5\text{Pa} = \pm 100\text{Pa}.$$

So the pressure is $98500 \pm 100\text{Pa}$.

- 209.** Until 2019, the kilogram was defined by a physical artifact – a cylinder of platinum-iridium alloy called the International Prototype Kilogram (IPK) stored in France. However, this definition had limitations, such as the potential for the IPK’s mass to change over time. The definition of the kilogram was revised

in 2018 by the General Conference on Weights and Measures.

Questions:

1. What is a “unit” in the context of measurement?
2. Why is it important for the definitions of base units, like the kilogram, to be stable and reproducible?
3. How is the kilogram now defined in terms of fundamental constants?
4. “With progress in technology, the measuring techniques get improved leading to measurements with greater precision. The definitions of base units are revised to keep up with this progress.” How does defining the kilogram in terms of fundamental constants, rather than a physical artifact, align with this statement?

Ans :

1. A unit is a certain basic, arbitrarily chosen, internationally accepted reference standard with which the measurement of any physical quantity involves comparison.
2. It is important for definitions of base units to be stable and reproducible to ensure consistency and accuracy in measurements worldwide, as all other derived units and physical constants depend on these base units. The definitions are revised to keep up with progress in technology and improved measuring techniques.
3. The kilogram (kg) is now defined by taking the fixed numerical value of the Planck constant (h) to be $6.62607015 \times 10^{-34}$ when expressed in the unit J s (which is equal to $\text{kg m}^2\text{s}^{-1}$), where the meter and second are defined in terms of the speed of light (c) and the caesium frequency (ΔV_{Cs}).
4. Defining the kilogram in terms of fundamental constants (like the Planck constant) makes its definition independent of any physical artifact that could degrade or change. These constants are believed to be universal and unchanging, thus providing a more stable and precisely realizable definition as measurement techniques improve. This aligns with revising definitions to keep up with technological progress for greater precision.

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CHAPTER 2

Motion in a Straight Line

SUMMARY

1. INTRODUCTION

Kinematics is a branch of physics that deals with the motion of objects without considering the forces or causes that bring about the motion. It focuses on parameters like displacement, velocity, acceleration, and time. In simple terms, kinematics helps us describe how an object moves, such as its speed or direction, without worrying about why it moves.

Dynamics, on the other hand, is the study of motion along with the forces that cause or change that motion. It explains why an object moves the way it does by considering the effects of forces and mass. Dynamics forms the foundation for understanding the laws of motion as proposed by Newton.

Together, kinematics and dynamics form the complete study of mechanics, which is essential for analyzing any physical motion in real life.

2. REST

A body is considered to be at rest if it does not undergo any change in its position over time with respect to its immediate surroundings. In other words, if the location of the object remains constant when observed from a fixed reference point, the object is said to be at rest.

3. MOTION

A body is considered to be at rest if it does not undergo any change in its position over time with respect to its immediate surroundings. In other words, if the location of the object remains constant when observed from a fixed reference point, the object is said to be at rest.

3.1 Type of motion according to the path

- (i) Rectilinear or translatory motion is that motion in which the particle is moving along a straight line.

- (ii) Circular or rotatory motion is that motion in which the particle is moving in a circle.
- (iii) Oscillatory or vibratory motion is that motion in which the particle moves to and fro or back and forth repeatedly about a fixed point.

3.2 Type of motion of a body according to the coordinate

One dimensional motion requires one co-ordinate to locate the position of the body w.r.t. t . e.g. a ball rolling straight on the ground.

Two dimensional motion requires two co-ordinate to locate the position of the body w.r.t. t . e.g. motion of a billiard ball on the board.

Three dimensional motion requires three co-ordinates to locate the position of the body w.r.t. t . e.g. a kite flying in the sky.

4. DISTANCE AND DISPLACEMENT

Distance is the actual path traversed by an object in the given interval of time.

Displacement is the change in the position of the object along a particular direction in the given time interval.

5. SPEED

The speed is the rate at which a moving body describes its path. Speed is purely a scalar quantity and hence it has magnitude only.

6. VELOCITY

Velocity is the time rate of change of displacement.

$$\text{i.e. } v = \frac{\text{Displacement}}{\text{Time}}$$

Its unit is m s^{-1} in SI and dimensional formula is $[M^0 L^1 T^{-1}]$

6.1 Average Velocity

If an object is moving with variable velocity, the average velocity is the ratio of total displacement to the total time

$$\begin{aligned} \text{i.e. } v_{av} &= \frac{\text{Total displacement}}{\text{Total time}} \\ &= \frac{\Delta x}{\Delta t} \\ \text{i.e. } v_{av} &= \frac{x' - x}{t' - t} \end{aligned}$$

6.2 Instantaneous Velocity

Instantaneous velocity of an object is the limiting value of the average velocity of the object in a small time interval around that instant, when the time interval approaches zero.

$$\begin{aligned} \text{i.e. } v &= \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \\ \text{i.e. } v &= \frac{dx}{dt} \end{aligned}$$

7. UNIFORM MOTION

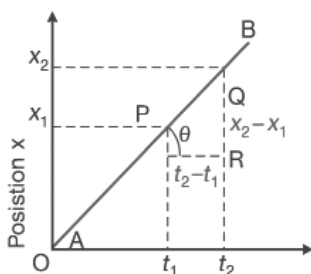
A body is said to be in uniform motion if its velocity is uniform.

8. POSITION-TIME GRAPH (UNIFORM MOTION)

The position-time graph of an object in uniform motion in one dimensional motion is a straight line inclined to the time axis as shown in Figure.

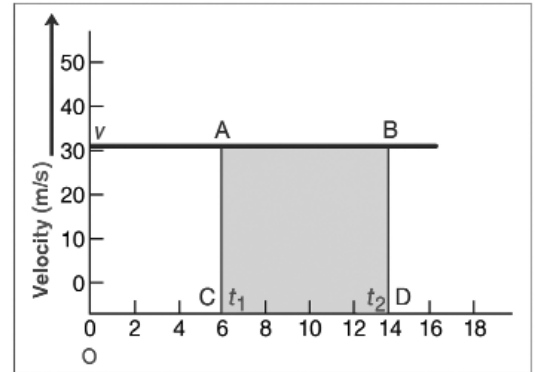
Slope of position time graph

$$= \frac{BR}{PR} = \frac{x_2 - x_1}{t_2 - t_1}$$



9. VELOCITY - TIME GRAPH (UNIFORM MOTION)

When a body is moving with uniform velocity the displacement of the body in the given time interval is equal to the area under the $v - t$ graph as shown in figure.



10. NON-UNIFORM MOTION

An object is said to be in non-uniform motion if it undergoes equal displacements in unequal intervals of time, however small these intervals may be.

11. RELATIVE VELOCITY IN ONE DIMENSIONAL MOTION

Relative velocity of one object w.r.t. another is the velocity with which one object moves w.r.t. another object i.e. it is the time rate of change of relative position of one object w.r.t. another.

12. ACCELERATED MOTION

An object having non-uniform motion, its velocity is different at different instants and the motion is said to be accelerated motion. If the change in velocity in each unit of time is constant, the acceleration is said to be constant acceleration, and such a motion is called uniformly accelerated motion. But if the change in velocity is not constant, the acceleration is called variable acceleration and the motion is called non-uniformly accelerated motion.

Uniformly accelerated motion

The motion of an object is said to be uniformly accelerated, if the same change in its velocity takes place in each unit of time.

Average acceleration

Average acceleration is defined as the time rate of change of velocity.

$$\text{i.e. } \text{acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\text{i.e. } a = \frac{v' - v}{t' - t} = \frac{\Delta v}{\Delta t}$$

Its unit in SI is m s^{-2} and dimensional formula is $[M^1 L^1 T^{-2}]$

Instantaneous acceleration

The acceleration of the object at a given instant of time is called instantaneous acceleration and is defined as the limiting value of average acceleration in a small time interval around the given instant, when the time interval tends to zero.

$$\text{i.e. } a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Formulae of uniformly accelerated motion

(i) Velocity - time relation

$$v = u + at$$

(ii) Position-time relation

$$x = x_0 + ut + \frac{1}{2}at^2$$

$$\text{or } x - x_0 = ut + \frac{1}{2}at^2$$

$$\text{or } s = ut + \frac{1}{2}at^2$$

(iii) Velocity-displacement relation

$$v^2 - u^2 = 2as$$

$$\text{or } v^2 - u^2 = 2a(x - x_0)$$

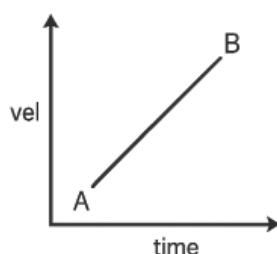
(iv) Displacement in n th second of motion

$$S_{\text{nth}} = s_n - s_{n-1}$$

$$= u + \frac{1}{2}a(2n - 1)$$

Velocity-time graph for uniformly accelerated motion

$v - t$ graph for a uniformly accelerated motion is a straight line inclined to the time axis as shown in figure.



Area under velocity-time graph (uniformly accelerated motion)

The area under velocity-time graph gives the distance travelled by the object in the given time interval.

NCERT TEXT-BOOK QUESTIONS

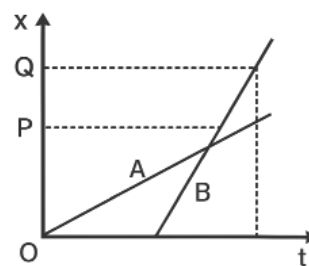
- In which of the following examples of motion can the body be considered approximately a point object :
 (a) a railway carriage moving without jerks between two stations,
 (b) a monkey sitting on top of a man cycling smoothly on a circular track,
 (c) a spinning cricket ball that turns sharply on hitting the ground,
 (d) a tumbling beaker that has slipped off the edge of a table ?

Ans :

In (a), (b) the body can be considered approximately a point object because the motion of the object involves changes of position by distance much greater than its size.

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- The position-time ($x - t$) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in figure. Choose the correct entries in the brackets below :
 (a) (A/B) lives closer to the school than (B/A).
 (b) (A/B) starts from the school earlier than (B/A).
 (c) (A/B) walks faster than (B/A).
 (d) A and B reach home at the (same/ different) time.
 (e) (A/B) overtakes (B/A) on the road (once/ twice)



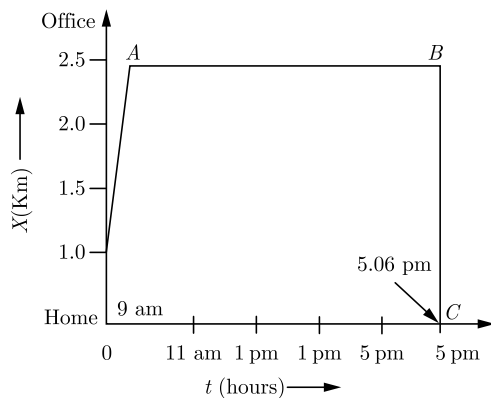
Ans :

- A lives closer to the school than B.
- A starts from the school earlier than B.
- B walks faster than A.
- A and B reach home at the same time.
- B overtakes A on the road once.

3. A woman starts from her home at 9.00 a.m., walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office upto 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1} . Choose suitable scales and plot the $x-t$ graph of her motion.

Ans :

5 km distance is covered in 1 hour, so 2.5 km distance is covered in $\frac{2.5}{5} = 0.5$ hour and 25 km distance is covered in 1 hour so 2.5 km distance is covered in $1/10$ hour i.e. 6 minutes. Moreover, as the velocity becomes zero at A, there must be a 'speeding down'. Therefore the $(x-t)$ graph must be curved. Similarly, the curves at O, B, C must be slightly curved.



4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1s. Plot the $x-t$ graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13m away from the start.

Ans :

Effective increase in position co-ordinates in

$$8 \text{ s} = 5 \text{ m} - 3 \text{ m} = 2 \text{ m}$$

2 metre distance is travelled by the drunkard in 8 s.

8 metre distance is travelled by the drunkard in

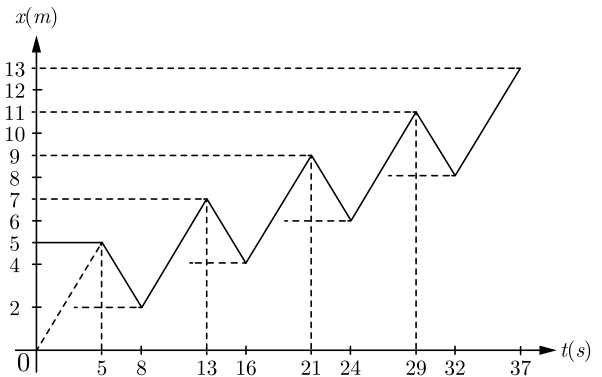
$$= \frac{8}{2} \times 8 = 32 \text{ s}$$

Now after 32 s, when the drunkard takes next 5 steps forward there will be the position of the pit in to which he falls.

Total time taken by drunkard to fall in the pit

$$= 32 + 5 = 37 \text{ s}$$

The $(x-t)$ plot is shown in figure.



5. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

Ans :

$$\begin{aligned} \text{Given,} \quad u &= 126 \text{ km h}^{-1} = \frac{126 \times 5}{18} \\ &= 35 \text{ m s}^{-1} \end{aligned}$$

$$v = 0, \quad s = 200 \text{ m}$$

$$\begin{aligned} \text{Using} \quad v^2 - u^2 &= 2as, \\ (0)^2 - (35)^2 &= 2 \times a \times 200 \end{aligned}$$

$$\text{or} \quad \frac{-35 \times 35}{2 \times 200} = a$$

$$\text{or} \quad a = -\frac{49}{16} = -3.06 \text{ m s}^{-2}$$

$$\text{Using,} \quad v = u + at$$

$$0 = 35 - \frac{49}{16} \times t$$

$$\text{or} \quad t = \frac{35 \times 16}{49} = 11.4 \text{ s}$$

6. A player throws a ball upwards with an initial speed of 29.4 m s^{-1} .

- What is the direction of acceleration during the upward motion of the ball ?
- What are the velocity and acceleration of the ball at the highest point of its motion ?
- Choose $x = 0$ and $t = 0$ to be the location and time of the ball at its highest point, vertically downwards direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.
- To what height does the ball rise and after how long does the ball return to the player's hands ? (Take $g = 9.8 \text{ m s}^{-2}$, and neglect air resistance).

Ans :

- Direction of acceleration during the upward motion of ball is vertically downward.

- (b) Velocity at the highest point of its motion = 0.
Acceleration of the ball at the highest point of its motion = 9.8 m s^{-2} vertically downward.
- (c) $x \geq 0$ (upward and downward motions)
 $v \leq 0$ (Upward motion) and $v \geq 0$ (downward motion) $a > 0$ throughout.
(Magnitude of -ve velocity decreases)
- (d) $v^2 - u^2 = 2gh$, where h is the maximum height attained by the ball.

$$(0)^2 - (29.4)^2 = 2(-9.8)h$$

or
$$h = \frac{29.4 \times 29.4}{2 \times 9.8} = 44.1 \text{ m}$$

Let 't' be the time taken by the ball at the highest point.

$$v' = v + a(t' - t)$$

$$0 = 29.4 + (-9.8) \times (t' - t)$$

or
$$(t' - t) = \frac{29.4}{9.8} = 3 \text{ s}$$

$$\text{Total time} = (3 + 3) = 6 \text{ s}$$

7. Read each statement below carefully and state with reasons and examples, if it is true or false : A particle in one dimensional motion :

- (a) with zero speed at an instant may have non-zero acceleration at that instant.,
(b) with zero speed may have non-zero
(c) with constant speed must have zero acceleration.
(d) with positive value of acceleration must be speeding up.

Ans :

- (a) True

When a body is projected upward at the highest point, velocity = 0, acceleration $\neq 0$.

- (b) False. In this case magnitude of velocity = speed, which is not zero.

- (c) True. Velocity will be constant acceleration = 0 (If the particle rebounds instantly with the same speed, it implies infinite acceleration which is non-physical, speed does not change).

- (d) False. (true only when the chosen positive direction is along the direction of motion).

8. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed time graph of its motion between $t = 0$ to 12 s.

Ans :

Time of ball

$$v^2 - u^2 = 2a(x - x_0)$$

$$v^2 - 0 = 2g[h - 0]$$

$$= 2 \times 9.8 \times 90 = 19.6 \times 90$$

$$v^2 = 1764$$

$$v = \sqrt{1764} = 42 \text{ m s}^{-1}$$

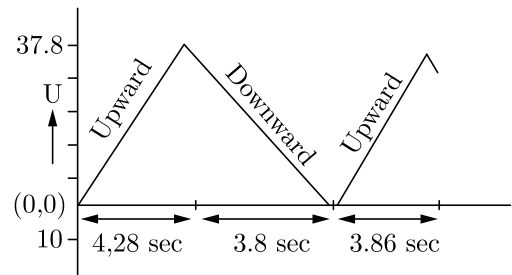
Velocity on striking the ground,

$$v = 42 \text{ m s}^{-1}$$

$$v = u + at$$

$$42 = 0 + 9.8t$$

$$t = \frac{42}{9.8} = \frac{42 \times 10}{98} = 4.28 \text{ s}$$



After impinging against the ground, velocity

$$= 42 - 42 \times \frac{1}{10}$$

Using,

$$v = u + at = 37.8 \text{ m s}^{-1}$$

$$0 = 37.8 - 9.8 \times t$$

Time of ascent $t = \frac{37.8}{9.8} = 3.86 \text{ s}$

$$\text{Time of descent} = 3.86 \text{ s}$$

Time spent up-to two rebounds

$$= 4.28 + 3.86 + 3.86 = 12 \text{ s}$$

Height to which the ball rises after first bound

$$(0)^2 - (37.8)^2 = 2(-9.8)[x' - 0]$$

$$x' = \frac{(37.8)^2}{2 \times 9.8} = 72.9 \text{ m}$$

The graph is as shown in the figure.

9.

Explain clearly with examples, the distinction between :

- (a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval.
(b) magnitude of average velocity over an interval of time, and the average speed over the same interval [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? (For simplicity, consider one dimensional motion only).

Ans :

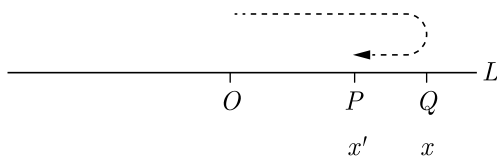
- (a) Displacement is the difference of final position co-ordinate and initial co-ordinate. It is a vector quantity. When a body changes its position, the straight line joining the initial and final positions of a body in the direction from initial to final positions is called its displacement, irrespective of the nature of the actual path taken by the body.

Distance is the total length of the path travelled by a body in a certain interval of time.

Examples

(1) One dimensional case

Suppose a point object starting from the point O at $t = 0$ has position co-ordinates x and x' at t and t' respectively, when the particles is at Q and P respectively. The particle travels to Q first then moves back to P [figure].



Displacement in time t' , $OP = x'$

But the actual distance travelled

$$= OQ + QP$$

$$= x + x'$$

(2) Motion in a circle

Suppose an object starting from a O , completes one revolution

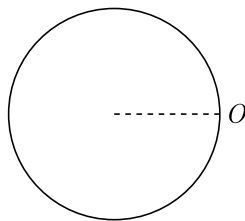
Then, displacement = 0

But the actual distance travelled

$$= \text{circumference of circle}$$

(b) Average velocity

$$= \frac{\text{Total displacement in given time interval}}{\text{Length of time interval}}$$



Average speed

$$= \frac{\text{Total distance travelled in a given time interval}}{\text{Length of time interval}}$$

Since magnitude of displacement and distance may not be equal, therefore magnitude of average velocity may not be equal to average speed.

Examples :

In example (1) of part (a), magnitude of average velocity between O and P

$$= \frac{OP}{t'} \quad \dots(1)$$

$$\text{Average speed} = \frac{OQ + QP}{t'} \quad \dots(2)$$

These two quantities are not equal.

10. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1} . Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1} . What is the

(i) 0 to 30 min

(ii) 0 to 50 min

(iii) 0 to 40 min?

(a) magnitude of average velocity and

(b) average speed of the man over the interval of time

Ans :

- (i) Net displacement in first 30 minutes $\left(\frac{1}{2} \text{ h}\right)$
 $= 2.5 \text{ km}$

(a) Magnitude of average velocity

= net displacement/total time taken

$$= \frac{2.5 \text{ km}}{1/2 \text{ h}} = 5 \text{ km h}^{-1}$$

(b) Average speed

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{2.5 \text{ km}}{1/2 \text{ h}} = 5 \text{ km h}^{-1}$$

- (ii) Total distance travelled in 50 minutes

Distance travelled in 30 min $\left(\frac{1}{2} \text{ h}\right)$ while going to the market = 2.5 km

Distance travelled in 20 min $\left(\frac{1}{3} \text{ h}\right)$ while walking back

$$= 7.5 \times \frac{1}{3} = 2.5 \text{ km}$$

Total distance travelled in 50 min $\left(\frac{50}{60} \text{ h}\right)$

$$= (2.5 + 2.5) = 5.0 \text{ km}$$

(a) Magnitude of average velocity

= displacement/total time taken

$$= \frac{(2.5 - 2.5)}{50/60} = 0$$

(b) Average speed

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{5 \times 60}{50} = 6 \text{ km h}^{-1}$$

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- (iii) Distance travelled in 30 min. while going to market = 2.5 km
 Distance travelled in 10 min (1/6 hr) while waling back

$$= 7.5 \times \frac{1}{6} = \frac{5}{4} \text{ km}$$

$$\text{Total distance} = 2.5 + \frac{5}{4} = \frac{15}{4} \text{ km}$$

$$\text{Displacement} = 2.5 - \frac{5}{4} = \frac{5}{4}$$

- (a) Magnitude of average velocity

$$\begin{aligned} &= \frac{\text{Net displacement}}{\text{Total time taken}} \\ &= \frac{5 \times 60}{4 \times 40} = \frac{15}{8} \text{ km h}^{-1} \\ &= 1\frac{7}{8} \text{ km h}^{-1} \end{aligned}$$

- (b) Average speed

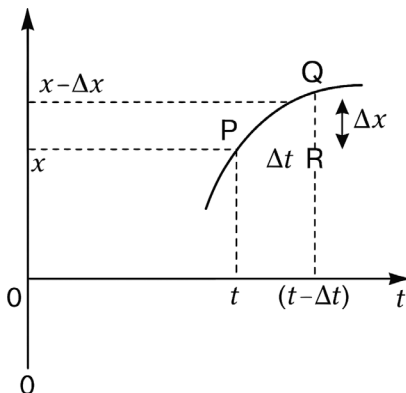
$$\begin{aligned} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{15 \times 60}{4 \times 40} = \frac{45}{8} \\ &= 5\frac{5}{8} \text{ km h}^{-1} \end{aligned}$$

It is clear from this solution that the magnitude of average velocity and average speed are in general, not equal.

11. In Q.9 and Q.10, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans :

Instantaneous speed is the speed of the object at any particular instant of time.



Consider a small displacement Δx in a time Δt between time interval t and $(t + \Delta t)$.

Average velocity in the interval

$$\Delta t, v_{av} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity at instant t

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \dots(1)$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} \\ &= \frac{\text{arc } PQ}{\Delta t} \end{aligned}$$

In the limit $\Delta t \rightarrow 0$, the gives instantaneous speed
 Instantaneous speed

$$\begin{aligned} &= \lim_{\Delta t \rightarrow 0} \frac{\text{arc } PQ}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{PQ}{\Delta t} \quad \dots(2) \\ &(\text{arc } PQ \approx \text{length of line } PQ) \end{aligned}$$

From ΔPQR ,

$$PQ^2 = PR^2 + QR^2$$

As $PQ \rightarrow \Delta t \rightarrow 0$, $PQ \rightarrow QR$

$$PQ \rightarrow \Delta x$$

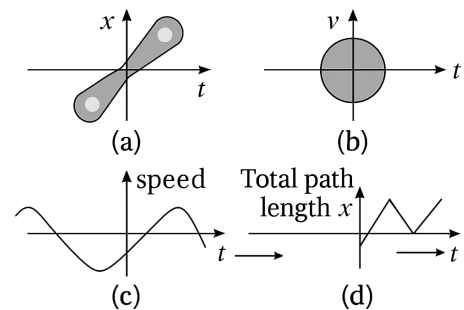
From (2),

$$\text{Instantaneous speed} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \quad \dots(3)$$

Hence, instantaneous speed

= Magnitude of velocity.

Look at the graphs (a) to (d) Figure carefully and state with reasons, which of these cannot possibly represent one dimensional motion of a particle.



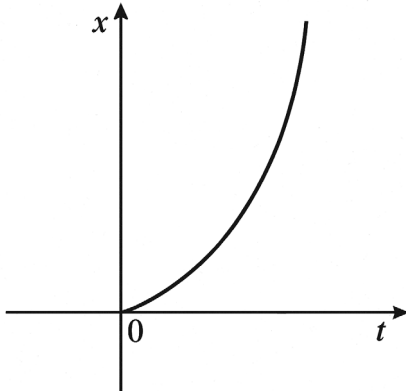
Ans :

- A particle cannot have two different positions at the same time.
- A particle cannot have velocity in opposite directions at the same time.
- Speed is always non-negative, so this is impossible.
- Total path length of a particle can never decrease with time.

Therefore all the four graphs are impossible.

It may be noted that the arrows in the graphs are meaningless.

13. Figure shows $x-t$ plot of one dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.



Ans :

No Wrong, $x-t$ plot does not show the trajectory of a particle.

For $t < 0$, $x = 0$

There is no motion at all.

For $t > 0$, the velocity is non-uniform but the motion may be along a straight line.

Context. A body is dropped from a tower ($x = 0$) at $t = 0$

Since the motion is uniformly accelerated, the displacement varies parabolically with time.

This graph can be of a particle being dropped from a certain height.

14. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1} . If the muzzle speed of the bullet is 150 m s^{-1} , with what speed does the bullet hit the thief's car ? (Note : Obtain the speed which is relevant for damaging the thief's car.)

Ans :

$$\begin{aligned} \text{Speed of the police van} &= 30 \text{ km h}^{-1} \\ &= \frac{25}{3} \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} \text{Speed of the thief's car} &= 192 \text{ km h}^{-1} \\ &= \frac{160}{3} \text{ m s}^{-1} \end{aligned}$$

Relative velocity of car w.r.t. van

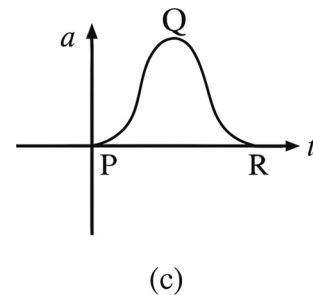
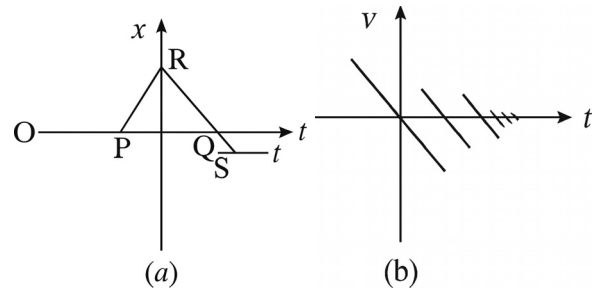
$$\begin{aligned} &= \frac{160}{3} - \frac{25}{3} \\ &= 45 \text{ m s}^{-1} \end{aligned}$$

Relative velocity of bullet w.r.t car

$$\begin{aligned} &= 150 - 45 \\ &= 105 \text{ m s}^{-1} \end{aligned}$$

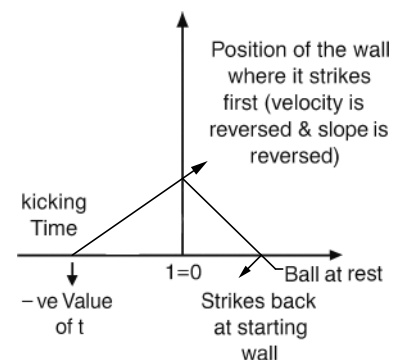
Muzzle speed of the Bullet $= 105 \text{ m s}^{-1}$

15. Suggest a suitable physical situation for each of the following graphs :



Ans :

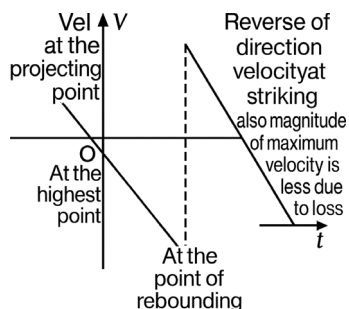
- (a) A ball at rest on a smooth floor is kicked, it rebounds from a wall with reduced speed and moves to the opposite wall which stops it.



In the graph $t = 0$ is taken when the ball strikes the wall first.

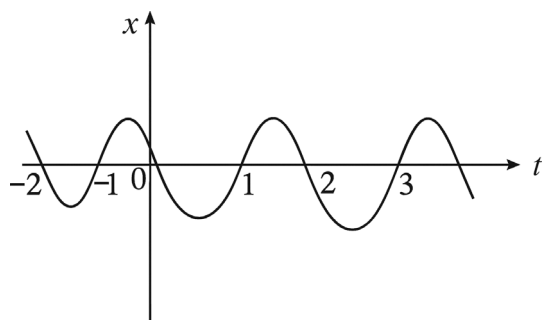
- (b) A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit.

Motion in a Straight Line



(c) A uniformly moving cricket ball turned back by hitting it with a bat for a very short time interval.

16. Figure gives the $(x-t)$ plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3\text{ s}$, 1.2 s , -1.2 s .



Ans :

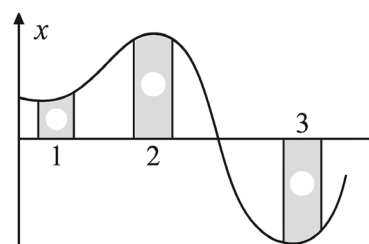
- (i) At $t = 0.3\text{ s}$
 $x < 0$, $v < 0$, and $a > 0$
- (ii) At $t = 1.2\text{ s}$
 $x > 0$, $v > 0$, and $a < 0$
- (iii) At $t = -1.2\text{ s}$
 $x < 0$, $v > 0$ and $a > 0$

Since time is negative

$\frac{\text{distance}}{\text{time}}$ is positive.

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17. Figure give the $x-t$ plot of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest and in which is it the least ? Give the sign of average velocity for each interval.



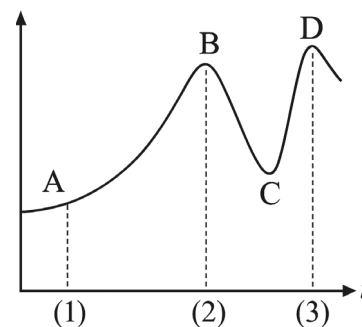
Ans :

Average speed is greatest in 3, because slope is greatest there. Sign of displacement is neglected.

Average speed is the least in 2, because slope is least there.

- (i) $v > 0$ in 1 (Distance increasing)
 (ii) $v > 0$ in 2 (Distance increasing)
 (iii) $v < 0$ in 3 (Distance decreasing)

18. Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D ?



Ans :

- (i) Average acceleration is greatest in magnitude in 2 because the change of velocity is more in the same time interval.
- (ii) Average speed is greatest in 3, because speed is higher than any other interval 1 or 2.
- (iii) $v > 0$ in 1, 2 and 3
 $a > 0$ in 1 and 3, $a < 0$ in 2
 $a = 0$ at the points A, B, C and D because there is instantaneous no change in speed.

OBJECTIVE QUESTIONS

- Which of the following refers to the change in position of an object with time?
(A) Acceleration
(B) Velocity
(C) Motion
(D) Displacement

Ans :

(C) Motion

Motion is fundamentally defined as the change in an object's position over a period of time. This concept is central to kinematics, which studies the description of motion.

- What is the term for the motion of objects along a straight line?
(A) Circular motion
(B) Rectilinear motion
(C) Rotational motion
(D) Projectile motion

Ans :

(B) Rectilinear motion

Rectilinear motion specifically describes the movement of an object in a straight line. This is the simplest form of motion to analyze in classical mechanics.

- When can an object in motion be treated as a point object?
(A) When its size is comparable to the distance it moves.
(B) When its size is much larger than the distance it moves.
(C) When its size is much smaller than the distance it moves.
(D) When it moves at a constant velocity.

Ans :

(C) When its size is much smaller than the distance it moves.

An object can be approximated as a point object if its dimensions are negligible compared to the distance it travels during the observation period, simplifying motion analysis.

- What branch of physics studies ways to describe motion without considering the causes of motion?
(A) Dynamics
(B) Statics
(C) Kinematics
(D) Mechanics

Ans :

(C) Kinematics

Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that cause them to move.

- The velocity at an instant is defined as the limit of the average velocity as the time interval approaches what value?
(A) Infinity
(B) One second
(C) Infinitesimally small
(D) A large value

Ans :

(C) Infinitesimally small

Instantaneous velocity is defined as the average velocity over an infinitesimally small time interval, mathematically expressed as the derivative of position with respect to time.

- In the language of calculus, the instantaneous velocity v is the differential coefficient of position x with respect to time t , denoted as:
(A) $\Delta x / \Delta t$
(B) dx / dt
(C) dt / dx
(D) $\Delta t / \Delta x$

Ans :

(B) dx / dt

Instantaneous velocity is precisely defined as the rate of change of position with respect to time, which in calculus notation is dx / dt .

- What does the slope of the tangent to a position-time graph at a specific point represent?
(A) Average acceleration
(B) Instantaneous velocity
(C) Average velocity
(D) Instantaneous acceleration

Ans :

(B) Instantaneous velocity

The slope of the tangent to the position-time graph at any instant gives the instantaneous velocity of the object at that particular moment in time.

- Which of the following statements about average speed and average velocity is incorrect?
(A) Average speed can be zero.
(B) Average velocity can be zero.
(C) Average speed is always greater than or equal to the magnitude of average velocity.
(D) Average speed is a scalar quantity.

Ans :

(A) Average speed can be zero.

Average speed is the total path length divided by the time interval. Since path length is always non-negative, average speed can only be zero if the object hasn't moved at all, but it cannot be negative. Average velocity can be zero if displacement is zero.

9. Which of the following correctly describes instantaneous speed?

- (A) It is always greater than the magnitude of instantaneous velocity.
- (B) It is always less than the magnitude of instantaneous velocity.
- (C) It is equal to the magnitude of instantaneous velocity.
- (D) It is the total distance divided by total time.

Ans :

(C) It is equal to the magnitude of instantaneous velocity.

Instantaneous speed is the magnitude of the instantaneous velocity. At any given instant, the speed is how fast an object is moving, irrespective of its direction.

10. Who first concluded that the rate of change of velocity with time is a constant for all objects in free fall?

- (A) Isaac Newton
- (B) Albert Einstein
- (C) Galileo Galilei
- (D) Aristotle

Ans :

(C) Galileo Galilei

Galileo Galilei, through his experiments with falling objects and objects on inclined planes, was the first to correctly determine that acceleration due to gravity is constant for all objects in free fall, neglecting air resistance.

11. Which of the following is NOT a characteristic of acceleration?

- (A) It can be positive.
- (B) It can be negative.
- (C) It can be zero.
- (D) It is always in the direction of motion.

Ans :

(D) It is always in the direction of motion.

Acceleration can be in the direction opposite to motion (deceleration), or even perpendicular to the direction of motion (as in uniform circular motion), not necessarily always in the direction of motion.

12. How does a position-time graph for positive acceleration differ from one for negative acceleration?

- (A) Positive acceleration graph curves upward; negative curves downward.
- (B) Positive acceleration graph curves downward; negative curves upward.
- (C) Positive acceleration graph is a straight line; negative is a curve.
- (D) Positive acceleration graph is a curve; negative is a straight line.

Ans :

(A) Positive acceleration graph curves upward; negative curves downward.

For positive acceleration, the position-time graph is a parabola opening upwards (curve steepens). For negative acceleration, it's a parabola opening downwards (curve flattens and then steepens downwards).

13. Assertion: The area under a velocity-time graph represents displacement.

Reason: For uniform motion, the velocity-time graph is a horizontal line, and the area (velocity $\times$ time) is displacement.

- (A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
- (B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
- (C) Assertion is true but Reason is false.
- (D) Assertion is false but Reason is true.

Ans :

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

The assertion is a fundamental concept in kinematics. The reason provides a simple case (uniform motion) that correctly demonstrates this principle, where displacement is indeed velocity multiplied by time, represented by the area of the rectangle under the v-t graph.

14. Consider the following statements regarding uniformly accelerated motion:

- 1. The velocity-time graph is a straight line.
- 2. The position-time graph is a parabola.
- 3. The acceleration-time graph is a horizontal line.
- 4. The average velocity is the arithmetic mean of initial and final velocities.

Which of the above statements are correct?

- (A) 1, 2, and 3 only
- (B) 1, 3, and 4 only
- (C) 1, 2, 3 and 4
- (D) 2 and 4 only

Ans :

(C) 1, 2, 3 and 4

All four statements are characteristic of uniformly accelerated motion. Constant acceleration means a straight line v - t graph, parabolic x - t graph, horizontal a - t graph, and the specific average velocity formula.

15. Match the physical quantity in Column I with its corresponding SI unit in Column II.

	Column I		Column II
1.	Velocity	P.	m/s^2
2.	Acceleration	Q.	m
3.	Displacement	R.	m/s
4.	Time	S.	s

- (A) 1-R, 2-P, 3-Q, 4-S
 (B) 1-P, 2-R, 3-Q, 4-S
 (C) 1-R, 2-Q, 3-P, 4-S
 (D) 1-S, 2-P, 3-Q, 4-R

Ans :

(A) 1-R, 2-P, 3-Q, 4-S

Velocity is measured in meters per second (m/s). Acceleration is measured in meters per second squared (m/s^2). Displacement is measured in meters (m). Time is measured in seconds (s).

16. Which of the following pairs correctly matches a kinematic equation with its condition of applicability?
 (A) $v = u + at$: Non-uniform acceleration
 (B) $s = ut + \frac{1}{2}at^2$: Uniform acceleration
 (C) $v^2 = u^2 + 2as$: Variable acceleration
 (D) $s = vt$: Uniform acceleration

Ans :

(B) $s = ut + \frac{1}{2}at^2$: Uniform acceleration

The equation $s = ut + \frac{1}{2}at^2$ is one of the standard kinematic equations derived specifically for motion with constant or uniform acceleration.

17. Identify the incorrect pair regarding graphical representation of motion:
 (A) Straight line x - t graph : Uniform velocity
 (B) Parabolic x - t graph : Uniform acceleration
 (C) Horizontal v - t graph : Object at rest
 (D) Sloped v - t graph : Uniform acceleration

Ans :

(C) Horizontal v - t graph : Object at rest

A horizontal v - t graph (slope = 0) indicates constant velocity. If this constant velocity is zero, then the object is at rest. However, if the constant velocity is non-zero, the object is in uniform motion. Thus,

“Object at rest” is not the only interpretation. The pair should ideally specify zero velocity for “Object at rest”.

18. Arrange the following steps in the correct sequence to determine instantaneous velocity graphically from a position-time graph:
 1. Draw a tangent to the curve at the specific instant.
 2. Plot the position-time graph.
 3. Calculate the slope of this tangent.
 4. Identify the point on the curve corresponding to the specific instant.
 (A) 2, 4, 1, 3
 (B) 1, 2, 4, 3
 (C) 2, 1, 4, 3
 (D) 4, 2, 1, 3

Ans :

(A) 2, 4, 1, 3

First, the position-time graph must be plotted. Then, the specific point in time is located on the graph. A tangent line is drawn to the curve at this point. Finally, the slope of this tangent line is calculated to find the instantaneous velocity.

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19. A car accelerates from rest at a constant rate of 2 m/s^2 . What is its velocity after 5 seconds?
 (A) 5 m/s
 (B) 10 m/s
 (C) 2 m/s
 (D) 7 m/s

Ans :

(B) 10 m/s

Using the kinematic equation $v = u + at$, where initial velocity $u = 0$, acceleration $a = 2 \text{ m/s}^2$, and time $t = 5 \text{ s}$. So, $v = 0 + (2 \times 5) = 10 \text{ m/s}$.

20. What is defined as the magnitude of instantaneous velocity?
 (A) Average speed
 (B) Instantaneous speed
 (C) Average acceleration
 (D) Instantaneous acceleration

Ans :

(B) Instantaneous speed

Instantaneous speed is precisely defined as the magnitude of the instantaneous velocity vector at a particular moment. It tells how fast an object is moving at that instant.

21. The SI unit of acceleration is:

(A) m/s
(B) m s²
(C) m/s²
(D) m²/s

Ans :

(C) m/s²

Acceleration is the rate of change of velocity with respect to time. Velocity is in m/s, and time is in s, so acceleration is in (m/s)/s = m/s².

22. For an object moving with constant acceleration, the average acceleration over any time interval is equal to:

(A) Zero
(B) The initial acceleration
(C) The final acceleration
(D) The constant value of acceleration

Ans :

(D) The constant value of acceleration

If an object's acceleration is constant, then its average acceleration over any time interval will be the same as this constant value.

23. What does the area under the velocity-time graph represent?

(A) Acceleration
(B) Displacement
(C) Speed
(D) Time

Ans :

(B) Displacement

The area under the velocity-time graph over a given time interval represents the displacement of the object during that interval.

24. The kinematic equation $v = v_0 + at$ relates which quantities?

(A) Final velocity, initial velocity, acceleration, and displacement
(B) Final velocity, initial velocity, acceleration, and time
(C) Displacement, initial velocity, acceleration, and time
(D) Final velocity, displacement, acceleration, and time

Ans :

(B) Final velocity, initial velocity, acceleration, and time

This equation directly relates the final velocity (v) of an object to its initial velocity (v_0), uniform acceleration (a), and the time interval (t) over which the acceleration occurs.

25. In the equation $x = x_0 + v_0 t + \frac{1}{2} at^2$, what does x_0 represent?

(A) Final position
(B) Initial velocity
(C) Initial position
(D) Displacement

Ans :

(C) Initial position

In this kinematic equation for uniformly accelerated motion, x_0 represents the initial position of the particle at time $t = 0$.

26. What is the acceleration of an object in free fall, neglecting air resistance, near the Earth's surface?

(A) Zero
(B) Approximately 9.8 m/s² downwards
(C) Approximately 9.8 m/s² upwards
(D) Variable depending on mass

Ans :

(B) Approximately 9.8 m/s² downwards

An object in free fall near the Earth's surface experiences a constant downward acceleration due to gravity, denoted by g , which is approximately 9.8 m/s².

27. Which of the following statements regarding free fall (neglecting air resistance) is incorrect?

(A) The acceleration is constant.
(B) The velocity is always downwards.
(C) The equations of uniformly accelerated motion apply.
(D) The distance fallen is proportional to the square of time.

Ans :

(B) The velocity is always downwards.

If an object is thrown upwards, its velocity is initially upwards, then becomes zero at the highest point, and then is downwards during its descent. So velocity is not always downwards in free fall.

28. Which of the following statements is true for an object thrown vertically upwards (neglecting air resistance)?

(A) Acceleration is zero at the highest point.
(B) Velocity is maximum at the highest point.
(C) Velocity is zero at the highest point.
(D) Acceleration changes direction at the highest point.

Ans :

(C) Velocity is zero at the highest point.

When an object is thrown vertically upwards, its velocity decreases as it rises, becomes momentarily zero at the highest point, and then increases in the downward direction.

- 29.** Galileo's law of odd numbers states that the distances traversed during equal intervals of time by a body falling from rest stand to one another in the ratio of:
- (A) 1:2:3:4...
 (B) 1:3:5:7...
 (C) 1:4:9:16...
 (D) 1:1:1:1...

Ans :

(B) 1:3:5:7...

Galileo's law of odd numbers describes that for a freely falling object starting from rest, the distances covered in successive equal time intervals are in the ratio of odd numbers (1, 3, 5, 7, etc.).

- 30.** Which of the following is NOT a kinematic equation for uniformly accelerated motion?
- (A) $v = v_0 + at$
 (B) $x = v_0 t + \frac{1}{2} at^2$ (assuming $x_0 = 0$)
 (C) $F = ma$
 (D) $v^2 = v_0^2 + 2ax$ (assuming $x_0 = 0$)

Ans :

(C) $F = ma$

$F = ma$ is Newton's second law of motion, which relates force, mass, and acceleration. While it involves acceleration, it's a dynamic equation, not purely kinematic.

- 31.** Stopping distance of a vehicle is proportional to the square of the initial velocity. If the initial velocity is tripled, the stopping distance (assuming same deceleration) becomes:
- (A) Three times
 (B) Six times
 (C) Nine times
 (D) One-third

Ans :

(C) Nine times

Since stopping distance d_s is proportional to v_0^2 , if v_0 is tripled, d_s will increase by a factor of $3^2 = 9$.

- 32.** Assertion: A particle momentarily at rest can have non-zero acceleration.
 Reason: When a ball is thrown upwards, at its highest point, its velocity is zero but its acceleration is g downwards.
- (A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
 (C) Assertion is true but Reason is false.
 (D) Assertion is false but Reason is true.

Ans :

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

The assertion is correct. The reason provides a valid example: a ball at the peak of its trajectory has zero instantaneous velocity but is still under the influence of gravity, hence accelerating downwards at g .

- 33.** Consider the following statements for one-dimensional motion:
1. An object with zero speed at an instant may have non-zero acceleration at that instant.
 2. An object with zero speed may have non-zero velocity.
 3. An object with constant speed must have zero acceleration.
 4. An object with positive value of acceleration must be speeding up.

Which of the above statements are true?

- (A) 1 and 3 only
 (B) 1, 3 and 4 only
 (C) 1, 2 and 3 only
 (D) All are true

Ans :

(A) 1 and 3 only

1. True (e.g., a ball at the top of its flight).
2. False (speed is the magnitude of velocity; if speed is zero, velocity must be zero).
3. True (constant speed in one dimension implies constant velocity if direction doesn't change, or zero velocity, hence zero acceleration).
4. False (if velocity is negative and acceleration is positive, the object is slowing down).

- 34.** Match the type of motion in Column I with the corresponding characteristic of its x-t graph in Column II.

	Column I		Column II
1.	Uniform motion	P.	Parabola
2.	Uniformly accelerated motion (starting from rest, positive acceleration)	Q.	Straight line inclined to time axis
3.	Object at rest	R.	Parabola opening upwards
4.	Uniformly decelerated motion (positive initial velocity)	S.	Horizontal straight line

- (A) 1-Q, 2-R, 3-S, 4-P
 (B) 1-S, 2-P, 3-Q, 4-R

- (C) 1-Q, 2-P, 3-S, 4-R
 (D) 1-R, 2-S, 3-P, 4-Q

Ans :

- (A) 1-Q, 2-R, 3-S, 4-P

1. Uniform motion (constant velocity) has a straight line x-t graph inclined to the time axis.
2. Uniformly accelerated motion (starting from rest, positive 'a') has an x-t graph that is a parabola opening upwards.
3. An object at rest has a horizontal straight line x-t graph (position doesn't change).
4. Uniformly decelerated motion (constant negative 'a') with positive initial velocity will have a parabolic x-t graph that opens downwards.

35. Which of the following physical situations correctly corresponds to the given x-t graph characteristic?
 (A) Ball dropped from a height : Straight line x-t graph
 (B) Car moving with constant velocity : Parabolic x-t graph
 (C) Stone thrown upwards, at its highest point : Slope of x-t graph is maximum
 (D) Object at rest : Horizontal line x-t graph

Ans :

- (D) Object at rest : Horizontal line x-t graph
 If an object is at rest, its position does not change with time, resulting in a horizontal line on an x-t graph.

36. Identify the incorrect pairing of a motion variable and its typical unit in a common system (not necessarily SI).
 (A) Speed : kilometers per hour
 (B) Acceleration : meters per second
 (C) Displacement : miles
 (D) Time : minutes

Ans :

- (B) Acceleration : meters per second
 The unit for acceleration is typically meters per second squared (m/s^2) or similar units of distance per time squared, not just distance per time.

37. Consider an object thrown vertically upwards. Arrange the following states of the object in the order they occur from the moment it is thrown until it returns to the thrower's hand.
 1. Velocity is zero.
 2. Velocity is maximum positive (upwards).
 3. Velocity is maximum negative (downwards).
 4. Acceleration is constant and downwards.
 (A) 2, 4, 1, 4, 3, 4
 (B) 2, 1, 3, 4

- (C) 4, 2, 1, 3
 (D) 2, 4, 1, 3

Ans :

- (A) 2, 4, 1, 4, 3, 4
 2. Initially, it has maximum positive (upward) velocity.
 4. Throughout its flight, its acceleration is constant and downwards (g).
 1. At the highest point, its velocity is momentarily zero.
 4. Acceleration is still constant and downwards.
 3. Just before being caught, it has maximum negative (downward) velocity.
 4. Acceleration is still constant and downwards.

38. A train starting from rest accelerates uniformly to a speed of 20 m/s in 10 seconds. What is the distance covered by the train during this time?
 (A) 50 m
 (B) 100 m
 (C) 200 m
 (D) 20 m

Ans :

- (B) 100 m
 First, find acceleration:
 $a = (v - u) / t = (20 - 0) / 10 = 2 \text{ m/s}^2$.
 Then use $s = ut + \frac{1}{2}at^2$
 $= 0 \times 10 + (1/2) \times 2 \times (10)^2 = 100 \text{ m}$.
 Alternatively, $s = ((u + v) / 2) t$
 $= ((0 + 20) / 2) \times 10 = 10 \times 10 = 100 \text{ m}$.

39. The rate of change of position with respect to time at a particular instant is termed:
 (A) Instantaneous acceleration
 (B) Average velocity
 (C) Instantaneous velocity
 (D) Average speed

Ans :

- (C) Instantaneous velocity
 This is the precise definition of instantaneous velocity. It describes how fast and in what direction an object is moving at a specific moment.

40. In uniformly accelerated motion, which quantity remains constant throughout the motion?
 (A) Velocity
 (B) Speed
 (C) Acceleration
 (D) Displacement

Ans :

- (C) Acceleration

The defining characteristic of uniformly accelerated motion is that the acceleration of the object does not change over time; it remains constant.

41. "Reaction time" in the context of driving refers to the time elapsed before a driver:
- (A) Completes the journey.
 - (B) Observes a hazard.
 - (C) Thinks and applies the brakes.
 - (D) Reaches maximum speed.

Ans :

(C) Thinks and applies the brakes.

Reaction time is the duration a person takes to observe a situation, process the information (think), and then initiate an action, such as applying brakes.

42. The kinematic equations of motion ($v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a(x - x_0)$) are applicable only when:
- (A) Velocity is constant.
 - (B) Acceleration is constant.
 - (C) Force is constant.
 - (D) Displacement is zero.

Ans :

(B) Acceleration is constant.

These fundamental equations are derived under the assumption that the acceleration of the object remains uniform throughout its motion.

43. If an object's velocity-time graph is a straight line parallel to the time axis, the object is:
- (A) At rest.
 - (B) Moving with uniform acceleration.
 - (C) Moving with uniform velocity.
 - (D) Moving with non-uniform acceleration.

Ans :

(C) Moving with uniform velocity.

A horizontal line on a v-t graph means the velocity is constant (which could be zero for an object at rest, or non-zero for uniform motion). The slope is zero, meaning zero acceleration.

44. The acceleration due to gravity, g , on Earth is approximately:
- (A) 9.8 m/s
 - (B) 9.8 m/s²
 - (C) 32 ft/s
 - (D) 32 ft/s²

Ans :

(B) 9.8 m/s²

The standard value for the acceleration due to gravity near the Earth's surface is approximately 9.8 meters per second squared.

45. If a particle is "speeding up", its acceleration is:
- (A) In the direction of velocity.
 - (B) Opposite to the direction of velocity.
 - (C) Perpendicular to the direction of velocity.
 - (D) Zero.

Ans :

(A) In the direction of velocity.

When an object is speeding up, its acceleration vector has a component in the same direction as its velocity vector.

46. Which of the following statements is incorrect regarding motion?
- (A) Displacement can be negative.
 - (B) Velocity can be negative.
 - (C) Speed can be negative.
 - (D) Acceleration can be negative.

Ans :

(C) Speed can be negative.

Speed is a scalar quantity representing the magnitude of velocity and is always non-negative. Displacement, velocity, and acceleration are vector quantities and can be negative depending on the chosen coordinate system.

47. Which statement accurately describes the motion of an object if its position-time graph is a straight line passing through the origin with a positive slope?
- (A) The object is at rest at the origin.
 - (B) The object is moving with uniform positive velocity starting from origin.
 - (C) The object is moving with uniform positive acceleration.
 - (D) The object is moving with non-uniform velocity.

Ans :

(B) The object is moving with uniform positive velocity starting from origin.

A straight line in an x-t graph indicates uniform velocity. A positive slope means positive velocity, and passing through the origin means it started from $x = 0$ at $t = 0$.

48. If a car travels east at 20 m/s and then turns around and travels west at 20 m/s, what can be said about its speed and velocity?
- (A) Speed and velocity are unchanged.
 - (B) Speed is unchanged, velocity has changed.
 - (C) Speed has changed, velocity is unchanged.
 - (D) Both speed and velocity have changed.

Ans :

(B) Speed is unchanged, velocity has changed.

The speed (20 m/s) remains the same in both cases. However, velocity is a vector quantity; since the

direction of motion changed from east to west, the velocity has changed.

49. All of the following are vector quantities EXCEPT:
 (A) Displacement
 (B) Speed
 (C) Velocity
 (D) Acceleration

Ans :

(B) Speed

Displacement, velocity, and acceleration are vector quantities as they have both magnitude and direction. Speed is a scalar quantity, representing only the magnitude of velocity.

50. What is the primary difference between distance and displacement?
 (A) Distance is a vector, displacement is a scalar.
 (B) Distance is always positive, displacement can be negative.
 (C) Distance is path dependent, displacement is path independent.
 (D) Distance and displacement are always equal.

Ans :

(C) Distance is path dependent, displacement is path independent.

Distance is the total path length covered, while displacement is the straight-line separation between the initial and final points, regardless of the path taken. Distance can be positive or zero, displacement can be positive, negative or zero.

51. Assertion: The slope of a displacement-time graph gives velocity.
 Reason: Velocity is defined as the rate of change of displacement.
 (A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
 (C) Assertion is true but Reason is false.
 (D) Assertion is false but Reason is true.

Ans :

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

The slope of a graph represents the rate of change of the y-axis quantity with respect to the x-axis quantity. Since velocity is the rate of change of displacement with time, the slope of the displacement-time graph indeed gives velocity.

52. Regarding the motion of a freely falling body (neglecting air resistance):
 1. Its velocity increases uniformly with time.

2. Its acceleration is directed upwards.
 3. The distance covered in the first second is half the magnitude of g .
 4. Its speed upon returning to the starting point (if thrown up) is equal to its initial speed.

Which statements are correct?

- (A) 1 and 4 only
 (B) 1, 3, and 4 only
 (C) 2 and 3 only
 (D) All are correct

Ans :

(B) 1, 3, and 4 only

1. True, velocity changes by g every second (e.g., $v = gt$ if starting from rest).
 2. False, acceleration due to gravity is always directed downwards.
 3. True, using $s = ut + \frac{1}{2}at^2$, with $u = 0, t = 1s, a = g$, gives $s = \frac{1}{2}g$.
 4. True, due to conservation of energy or symmetry of motion.

53. Match the graphical feature in Column I with the physical interpretation in Column II for a velocity-time graph.

	Column I		Column II
1.	Slope of the graph	P.	Displacement
2.	Area under the graph	Q.	Change in velocity
3.	Intersection with time axis	R.	Instantaneous acceleration
4.	Vertical intercept	S.	Initial velocity

- (A) 1-R, 2-P, 3-Q (instant when velocity is zero), 4-S
 (B) 1-P, 2-R, 3-S, 4-Q
 (C) 1-R, 2-Q, 3-P, 4-S
 (D) 1-S, 2-P, 3-Q, 4-R

Ans :

(A) 1-R, 2-P, 3-Q (instant when velocity is zero), 4-S

1. The slope of a v-t graph gives instantaneous acceleration.
 2. The area under a v-t graph gives displacement.
 3. Intersection with the time axis ($v = 0$) indicates an instant when the velocity is zero.
 4. The vertical intercept (value of v at $t = 0$) gives the initial velocity.

54. Which pair correctly describes a physical quantity and its nature?
 (A) Acceleration : Scalar
 (B) Velocity : Scalar

- (C) Speed : Vector
(D) Displacement : Vector

Ans :

(D) Displacement : Vector

Displacement has both magnitude (how far) and direction, making it a vector quantity. Acceleration and velocity are also vectors. Speed is a scalar.

55. Identify the incorrect statement and its reason:
(A) Statement: Speed is the magnitude of velocity.
Reason: True, by definition.
(B) Statement: A body can have constant speed and varying velocity. Reason: True, e.g., uniform circular motion.
(C) Statement: A body can have constant velocity and varying speed. Reason: False, if velocity is constant, its magnitude (speed) must also be constant.
(D) Statement: If acceleration is zero, velocity must be zero. Reason: False, if acceleration is zero, velocity is constant (which may or may not be zero).

Ans :

(D) Statement: If acceleration is zero, velocity must be zero. Reason: False, if acceleration is zero, velocity is constant (which may or may not be zero). The statement "If acceleration is zero, velocity must be zero" is incorrect. Zero acceleration means velocity is constant, but this constant value can be non-zero (uniform motion) or zero (at rest). The reason given is correct in identifying the statement as false.

56. A ball is dropped from a tall building. Consider the following quantities related to its motion just before it hits the ground (neglecting air resistance):
1. Magnitude of instantaneous velocity.
2. Magnitude of displacement from the release point.
3. Magnitude of instantaneous acceleration.
Arrange these in increasing order of their dependence on time t (from the moment of release).
(A) 3, 1, 2
(B) 1, 2, 3
(C) 3, 2, 1
(D) 2, 1, 3

Ans :

(A) 3, 1, 2

1. Magnitude of instantaneous velocity ($v = gt$) is proportional to t^1 .
2. Magnitude of displacement ($s = \frac{1}{2}gt^2$) is proportional to t^2 .

3. Magnitude of instantaneous acceleration (g) is constant, so it has t^0 dependence.

Thus, the order of increasing dependence on time is 3, 1, 2.

57. A car moving at 20 m/s brakes and comes to a stop in 5 s. Assuming uniform deceleration, what is the magnitude of the car's deceleration?
(A) 2 m/s²
(B) 4 m/s²
(C) 5 m/s²
(D) 10 m/s²

Ans :

(B) 4 m/s²

Using $v = u + at$, where $v = 0$ (final velocity), $u = 20$ m/s (initial velocity), $t = 5$ s.

So, $0 = 20 + a(5)$, which gives $5a = -20$, so $a = -4$ m/s². The magnitude of deceleration is 4 m/s².

58. The path length traversed by an object between two points is:
(A) Always equal to the magnitude of the displacement.
(B) Always greater than the magnitude of the displacement.
(C) Always less than the magnitude of the displacement.
(D) Greater than or equal to the magnitude of the displacement.

Ans :

(D) Greater than or equal to the magnitude of the displacement.

Path length is the total distance covered along the actual path. Displacement is the shortest distance between initial and final points. Path length is equal to the magnitude of displacement only if the motion is in a straight line without changing direction.

59. If an object's position does not change with time relative to a chosen origin, the object is said to be:
(A) In uniform motion
(B) In accelerated motion
(C) At rest
(D) In decelerated motion

Ans :

(C) At rest

If the position of an object remains unchanged with respect to time and a reference point (origin), the object is defined as being at rest.

60. For motion in a straight line, the position to the right of the origin is generally taken as:
(A) Negative
(B) Positive

- (C) Zero
(D) Dependent on velocity

Ans :

(B) Positive

By convention, in a one-dimensional coordinate system, positions to the right of the origin are considered positive, and positions to the left are considered negative.

ONE MARK QUESTIONS

61. What is particle kinematics ?

Ans :

It is a branch of mechanics which deals with the study of motion of the objects without taking into account the factors which cause the motion.

62. What is particle dynamics ?

Ans :

It is a branch of mechanics which deals with the study of motion of objects taking into account the factors which cause motion.

63. Define relative velocity.

Ans :

Relative velocity of one body with respect to other, when both are in motion, is the rate at which one body changes its position with respect to the other.

64. A particle starts from rest and covers a distance on a straight path in direct proportion to the square of time elapsed. What can one conclude about its acceleration ?

Ans :

Acceleration of the particle is uniform.

65. Give an example where displacement of a particle is equal to the distance covered by it.

Ans :

A body moving along a straight line with uniform velocity, covers distance equal to the displacement.

66. Define instantaneous velocity ?

Ans :

The velocity of a body at any instant of time is called instantaneous velocity and is given by

$$v = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$$

67. Can you think of any physical phenomenon involving the earth in which the earth can be treated as a point object ?

Ans :

When earth revolves around the sun, earth can be considered as point object.

68. Can a body moving with a constant speed have a varying velocity ?

Ans :

Yes if speed of body is constant and its direction is varying, its velocity will vary e.g. a body moving under circular motion.

69. Can a body have a constant velocity but a varying speed ?

Ans :

No, because if velocity of a body is constant, its speed cannot vary.

70. Under what condition the relation is correct ?

$$\Delta x = v \Delta t$$

Ans :

When body moves with uniform velocity.

71. What is the acceleration of a body at the highest point, when it is projected upward ?

Ans :

$$9.8 \text{ ms}^{-2}$$

72. Is it possible that the average velocity of a particle be zero, when its average speed is not zero. ?

Ans :

Yes, if the particle returns to the starting point.

73. Can the speed of a body be negative ?

Ans :

No, because speed is the rate of change of distance and distance covered by a particle can be positive or zero but never negative.

74. Can a body be simultaneously, at rest and in motion?

Ans :

Yes, Passengers travelling in a moving bus are at rest w.r.t. each other and are in motion w.r.t. a person standing outside the road.

- 75.** What is the nature of displacement time graph of a particle moving with uniform velocity ?

Ans :

Graph is a straight line passing through the origin.

- 76.** The displacement time graph for two particles 1 and 2 are straight lines inclined at angles 30° and 60° with time axis. What is the ratio of the velocities $v_1 : v_2$?

Ans :

Velocity is the slope of $x-t$ graph

$$\frac{v_1}{v_2} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1}{3}$$

- 77.** What is the nature of displacement time graph of a particle moving with constant acceleration ?

Ans :

A parabola

- 78.** What does the tangent at a point to the velocity time graph represent ?

Ans :

It represents the acceleration of the body at that instant.

- 79.** What does the tangent at a point to the displacement time graph represent ?

Ans :

It represents the velocity of the body at that instant.

- 80.** Is it possible, that the direction of acceleration and velocity be opposite to each other, in case, a body is moving ?

Ans :

Yes, a body projected in upward direction has velocity directed upwards and acceleration directed downwards.

- 81.** Is it possible for a body to be accelerated, if its speed is constant ?

Ans :

Yes, a body moving with uniform speed along a circular path has acceleration towards the centre of the circle which is called centripetal acceleration.

- 82.** What is meant by the one dimensional motion ?

Ans :

The motion is said to be one dimensional, if it takes place on a straight line. Only one space coordinate is required to describe one dimensional motion.

- 83.** What causes variation in velocity ?

Ans :

1. Change in magnitude only.
2. Change in direction only.
3. Change in magnitude and direction of velocity of the body.

- 84.** Can a particle accelerate, if its velocity is constant? Explain.

Ans :

No. Since acceleration is the rate of change of velocity, and if velocity is not changing with time, the acceleration of the body will be zero.

- 85.** What does speedometer of car measure ?

Ans :

Speedometer of car measures the instantaneous speed.

- 86.** What does the tangent at a point to the velocity-time graph represent ?

Ans :

It represents the acceleration of the body at that instant.

- 87.** What does the tangent at a point to the displacement-time graph represent ?

Ans :

It represents the velocity of the body at that instant.

- 88.** A ball is thrown straight up. What is its velocity and acceleration at the top ?

Ans :

Velocity = zero, acceleration = $+g$.

- 89.** What determines the direction of motion of an object : velocity or acceleration ?

Ans :

Velocity.

- 90.** A body is moving with uniform velocity of 10 ms^{-1} for last 10 s, what is its acceleration ?

Ans :

Zero, because velocity is uniform.

- 91.** What does slope of $(v-t)$ graph represent ?

Ans :

Acceleration.

92. A ball is thrown straight up. What is its velocity and acceleration at the highest point ?

Ans :

At highest point,

$$\text{velocity} = 0$$

and $\text{acceleration} = 9.8 \text{ m s}^{-2}$ (downwards).

93. Why does time occur twice in a unit of acceleration?

Ans :

$$\begin{aligned} \text{Acceleration} &= \frac{\text{Change in velocity}}{\text{Time}} \\ &= \frac{\text{Distance travelled/time}}{\text{Time}} \\ &= \frac{\text{Distance travelled}}{(\text{Time})^2} \end{aligned}$$

Thus we find that time occurs twice in the unit of acceleration.

94. What is the acceleration of a particle moving with a uniform velocity ?

Ans :

zero.

95. What is the nature of acceleration-time graph when the bod moves with constant acceleration ?

Ans :

Straight line parallel to time axis.

96. If the instantaneous velocity of a particle is zero will its instantaneous acceleration be necessarily zero ?

Ans :

No.

97. Can the velocity of an object be in a direction other than the direction of acceleration ?

Ans :

Yes.

98. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same initial speed. Which one will rise to the greater height ?

Ans :

Both will rise to the same height, because for a body moving with given initial speed and acceleration, the distance travelled is independent of the mass of the body.

99. Which of the two; velocity and acceleration, gives the direction of motion of the body ? Explain.

Ans :

Velocity gives the direction of motion of the body. When we throw a body vertically upwards, its velocity and direction of motion are upwards, but its acceleration is vertically downwards.

TWO MARKS QUESTIONS

100. If both the magnitude and direction of acceleration of a body is constant, does it mean that the path of the body be necessarily be a straight line ? :

Ans :

No, it is not necessary for a particle to move in a straight line if its acceleration is constant e.g. the path of a particle in case of angular projection is a parabolic path and acceleration (acceleration due to gravity g) is constant.

101. Can a body have zero velocity and still be accelerated?

Ans :

When a body is thrown upwards has zero velocity at the highest point where it is temporarily comes to rest. But at this point its acceleration is not zero and is equal to acceleration due to gravity which acts vertically downwards.

102. Show that absolute rest and motion are not possible.

Ans :

A body is said to be at absolute rest if it does not change its position with respect to a fixed point in the universe and is in motion if it changes its position with respect to a fixed point in universe. Since no fixed point has been found until now, hence absolute rest and absolute motion are meaningless.

103. A particle moving along a straight line path reverses its direction of motion twice. What can we say about the nature of its acceleration ?

Ans :

The particle must be having an acceleration that it is directed opposite to the displacement. This is the case with the oscillating particle. As the particle crosses the mean position towards either side, its motion is retarded. But when it reverses the direction of motion at the extreme position,

the particle is accelerated till it reaches the mean position.

- 104.** Why the direction of motion of a particle is given by its velocity and not by acceleration ?

Ans :

Because the direction of velocity is always along the direction of motion of particle but acceleration cannot indicate it. When the velocity of a particle is decreasing i.e. during retardation, the direction of motion is opposite to the direction of acceleration. Also if the particle is undergoing circular motion, the direction of acceleration is perpendicular to the direction of motion of the particle.

- 105.** A particle moving along a straight line reverses its direction of motion once. What can we say about the nature of its acceleration ?

Ans :

The motion of the particle must be under retardation, if it reverses direction of motion. After that, the particle will accelerate.

- 106.** When is an object in motion treated as a point object? Explain with an example.

Ans :

An object in motion is treated as a point object when its size is much smaller than the distance it moves in a reasonable duration. For example, when studying the revolution of the Earth around the Sun, the Earth can be treated as a point object due to the vast orbital distance.

- 107.** Differentiate between path length and displacement for an object in motion.

Ans :

Path length is the total distance covered by an object during its motion, regardless of direction; it is a scalar quantity. Displacement is the shortest distance between the initial and final positions of the object; it is a vector quantity having both magnitude and direction.

- 108.** Explain the term “rectilinear motion” with a suitable example.

Ans :

Rectilinear motion refers to the movement of an object along a straight line. For instance, a car moving on a straight highway without any turns or a train moving along a straight track are examples of rectilinear motion. This is the simplest form of motion.

- 109.** Define average velocity and state its SI unit.

Ans :

Average velocity is defined as the change in position or displacement (Δx) divided by the time interval (Δt) during which the displacement occurs. Its SI unit is meters per second (m/s). It gives a measure of how fast an object has been moving over a given interval.

- 110.** How does instantaneous velocity differ from average velocity?

Ans :

Instantaneous velocity is the velocity of an object at a specific instant in time, found by limiting the time interval to be infinitesimally small. Average velocity, however, describes the overall velocity over a finite duration, being the total displacement divided by the total time taken.

- 111.** What is instantaneous speed? How is it related to instantaneous velocity?

Ans :

Instantaneous speed is the speed of an object at a particular moment in time. It is defined as the magnitude of the instantaneous velocity at that instant. For example, if velocity is -5 m/s , speed is 5 m/s .

- 112.** Can the average speed of an object be greater than the magnitude of its average velocity? Explain.

Ans :

Yes, the average speed can be greater than the magnitude of the average velocity. This occurs when the object changes its direction of motion, making the total path length (used for average speed) greater than the magnitude of the displacement (used for average velocity).

- 113.** How is instantaneous velocity determined from a position-time graph?

Ans :

Instantaneous velocity at any point on a position-time graph is determined by the slope of the tangent to the curve at that specific point. This slope represents the rate of change of position with respect to time at that instant.

- 114.** Define acceleration. What is its SI unit?

Ans :

Acceleration is defined as the rate of change of velocity with time. It describes how quickly an object's velocity is changing, either in magnitude

or direction, or both. The SI unit of acceleration is meters per second squared (ms^{-2}).

- 115.** Distinguish between positive and negative acceleration in rectilinear motion.

Ans :

In rectilinear motion, positive acceleration generally means the velocity is increasing in the chosen positive direction, or decreasing in the negative direction. Negative acceleration (retardation) means the velocity is decreasing in the positive direction or increasing in the negative direction. The sign depends on the chosen coordinate system.

- 116.** How can you determine average acceleration from a velocity-time graph?

Ans :

Average acceleration over a time interval is determined by the slope of the straight line connecting the two points on the velocity-time graph corresponding to the start and end of that interval. It represents the average rate of change of velocity.

- 117.** Explain what a position-time graph curving upwards indicates about an object's acceleration.

Ans :

A position-time graph that curves upward indicates that the object has a positive acceleration. This means the object's velocity is increasing at a constant rate if the curve is parabolic, or simply that the velocity is increasing if the curve is generally upward.

- 118.** What does a position-time graph curving downwards indicate about acceleration?

Ans :

A position-time graph that curves downwards indicates that the object has a negative acceleration. This implies that the object's velocity is decreasing if moving in the positive direction, or its speed is increasing if moving in the negative direction, depending on the initial velocity.

- 119.** Describe the position-time graph for an object moving with zero acceleration.

Ans :

For an object moving with zero acceleration, its velocity is constant. The position-time graph for such an object is a straight line. The slope of this straight line represents the constant velocity of the object. If the velocity is zero, the line is horizontal.

- 120.** What physical quantity is represented by the area under a velocity-time graph? Explain.

Ans :

The area under a velocity-time graph represents the displacement of the object over a given time interval. For constant velocity, this area is a rectangle ($v \times t$). For varying velocity, it's the integral of velocity with respect to time, which gives displacement.

- 121.** Write down the three fundamental kinematic equations for uniformly accelerated motion.

Ans :

The three fundamental kinematic equations for uniformly accelerated motion are:

- $v = v_0 + at$
- $x = v_0 t + \frac{1}{2} at^2$ (assuming $x_0 = 0$)
- $v^2 = v_0^2 + 2ax$ (assuming $x_0 = 0$)

Here, v is final velocity, v_0 is initial velocity, a is acceleration, t is time, and x is displacement.

- 122.** How are the kinematic equations modified if the initial position of the particle is x_0 instead of zero?

Ans :

If the initial position is x_0 , the displacement x in the standard equations is replaced by $(x - x_0)$. The modified equations become:

- $v = v_0 + at$ (no change)
- $x - x_0 = v_0 t + \frac{1}{2} at^2$
- $v^2 = v_0^2 + 2a(x - x_0)$

- 123.** Describe the motion of an object under "free fall". What assumptions are made?

Ans :

An object is in free fall when it is released near the Earth's surface and accelerates downwards solely under the influence of gravity. It is assumed that air resistance is negligible and the acceleration due to gravity (g) is constant.

- 124.** What is the significance of Galileo's law of odd numbers in the context of free fall?

Ans :

Galileo's law of odd numbers states that the distances traversed by a freely falling body (starting from rest) during equal successive intervals of time are in the ratio of odd numbers (1:3:5:7...). This law highlights the nature of uniformly accelerated motion under gravity.

- 125.** Define stopping distance for a vehicle and provide the formula relating it to initial velocity and deceleration.

Ans :

Stopping distance is the distance a vehicle travels from the moment brakes are applied until it comes to a complete stop. The formula for stopping distance (d_s) is $d_s = \frac{v_0^2}{2a}$, where v_0 is the initial velocity and 'a' is the (negative) deceleration.

- 126.** If the initial velocity of a vehicle is doubled, how does its stopping distance change, assuming constant deceleration?

Ans :

If the initial velocity (v_0) of a vehicle is doubled, its stopping distance (d_s) increases by a factor of four. This is because stopping distance is proportional to the square of the initial velocity ($d_s \propto v_0^2$) when deceleration is constant.

- 127.** What is reaction time, and how can it be simply measured using a ruler?

Ans :

Reaction time is the duration a person takes to observe, think, and act in response to a situation. It can be measured by having a friend drop a ruler vertically; the distance 'd' the ruler falls before being caught is used in $t_r = \sqrt{\frac{2d}{g}}$ to estimate reaction time t_r .

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- 128.** Explain the meaning of "uniform motion" in a straight line.

Ans :

Uniform motion in a straight line means an object is moving with constant velocity. This implies that both its speed and direction of motion remain unchanged over time. Consequently, the acceleration of the object is zero during uniform motion.

- 129.** Can an object have a constant speed but still be accelerating? Explain with an example.

Ans :

Yes, an object can have a constant speed and still be accelerating if its direction of motion is changing. A common example is uniform circular motion, where an object moves in a circle at a constant speed, but its velocity vector changes direction, resulting in centripetal acceleration.

- 130.** What does it mean if the acceleration of a particle is in the direction opposite to its velocity?

Ans :

If the acceleration of a particle is in the direction opposite to its velocity, the particle is slowing down, or decelerating. This means the magnitude of its velocity is decreasing over time. This is independent of the chosen positive direction.

- 131.** Is it true that a positive value of acceleration always means an object is speeding up? Explain.

Ans :

No, a positive value of acceleration does not always mean an object is speeding up. If the velocity is negative and acceleration is positive, the object is actually slowing down. Speeding up occurs when velocity and acceleration have the same sign.

- 132.** Describe the velocity and acceleration of a ball thrown vertically upwards at its highest point.

Ans :

At its highest point, a ball thrown vertically upwards momentarily has zero velocity. However, its acceleration is not zero; it is equal to the acceleration due to gravity (g), acting downwards.

- 133.** Why are the kinematic equations of motion (e.g., $v = v_0 + at$) not always applicable?

Ans :

The standard kinematic equations of motion are strictly applicable only for motion with constant acceleration. If the acceleration of an object is varying with time, these equations cannot be used directly to describe its motion accurately.

- 134.** What is the displacement of a particle moving along the x-axis from $x = 4m$ to $x = -2m$?

Ans :

The displacement (Δx) is the final position minus the initial position.

$$\Delta x = x_f - x_i = (-2m) - (4m) = -6m.$$

The displacement is 6m in the negative x-direction.

- 135.** An object moves with a constant velocity of 10 m/s for 5 seconds. What is its acceleration?

Ans :

If an object moves with constant velocity, its velocity does not change over time. Therefore, its acceleration is zero. In this case, the acceleration of the object is $0m/s^2$.

- 136.** How does the position-time graph of an object moving with increasing velocity (positive acceleration) look?

Ans :

The position-time graph of an object moving with increasing velocity (positive acceleration) is a curve that bends upwards. The slope of the tangent to the curve at any point, representing instantaneous velocity, continuously increases with time.

- 137.** Explain the difference between the origin of a reference frame and the initial position of an object.

Ans :

The origin is a fixed reference point (usually zero) in a coordinate system, chosen for convenience. The initial position of an object is its location at the beginning of the time interval being considered, which may or may not coincide with the origin.

THREE MARKS QUESTIONS

- 138.** What do you mean by

- (a) Mechanics
- (b) Statics
- (c) Kinematics
- (d) Dynamics.

Ans :

- (a) **Mechanics :** It is that branch of physics which deals with motion of objects under the effect of forces in equilibrium.
- (b) **Statics :** Statics is the study of objects at rest. This requires the idea of forces in equilibrium, and time does not play an important role.
- (c) **Kinematics :** It is the study of motion of objects without taking into consideration, the cause of motion.
- (d) **Dynamics :** It is the study of motion of objects taking into consideration, the cause of motion. In the understanding of dynamics or motion, the measurement of time is essential.

- 139.** The distance travelled by a body varies directly proportional to the square of time. What type of motion this body has ?

Ans :

Given $x \propto t^2$
 $x = kt^2$

where k is constant of proportionality

$$v = \frac{dx}{dt} = \frac{d}{dt}(kt^2) = 2kt$$

And

$$a = \frac{dv}{dt} = \frac{d}{dt}(2kt) = 2k$$

or

$$a = \text{constant}$$

the body has uniform accelerated motion.

- 140.** Define motion. Give examples.

Ans :

It is the change of position of an object in the course of time.

The motion may be one dimensional (e.g. in a straight line), two dimensional (e.g., in one plane) or in three dimensions. (e.g. in space)

- (i) A train running on a railway track or a bus on a road, are examples of motion in one dimension. (Here, the widths and side-ways motions of train and bus are neglected).
- (ii) A boat sailing on a lake over distances which are small as compared to the earth's radius ; a carom coin or a billiard ball in motion ; these are examples of motion in two dimensions over a plane. Objects moving on the surface of the earth over distances comparable to the earth's radius are examples of motion in two dimensions, but not over a plane.
- (iii) Finally, the most general motions, we can consider, are of objects moving in space, involving all the three dimensions, e.g., the motion of gas molecule in space.

- 141.** Explain how far is it correct to consider an object as a "point" object.

Ans :

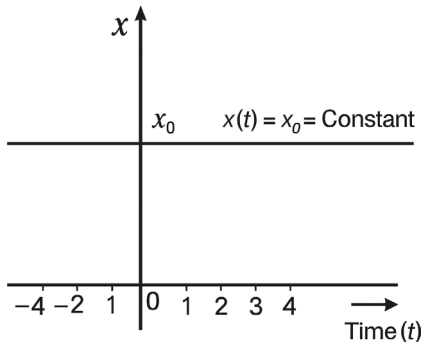
In general, whenever the motion of an object involves changes of position by distances much greater than its size, we can without much error neglect its size and think of it as a point-like object.

In many cases, the size of the object is much smaller than the distance it moves in a reasonable amount of time. For example, a train whose length is somewhat less than a kilometre may be on a journey of a thousand kilometres or more. In describing its journey over many hours, we may ignore its length and simply think of it as "dot" or a small point moving along the railway line. Similarly, a football in flight from one end of the field to the other can, to a good approximation, be treated as a "Point" or object of negligible size.

- 142.** Represent graphically the motion of a particle which is stationary.

Ans :

Position co-ordinate $x(t)$ is a constant, say x_0 and does not change with time at all. There is no motion at all, and the graph displaying $x(t)$ against t is straight line parallel to the time axis. Such a graph describes a bus standing still on a road, a train waiting at a station.

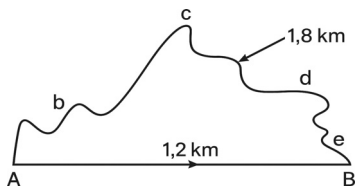


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- 143.** What is difference between distance and displacement?

Ans :

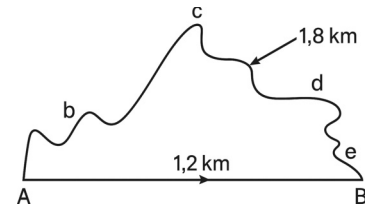
Let a person starts from A and reaches B via $AabcdeB$, he travels a distance as said above but actually he has gone from A to B . This shortest displacement from A to B is called displacement. Displacement is a vector quantity but the distance is a scalar quantity.



- 144.** What is the speed and velocity of a particle ? Are they numerically equal ?

Ans :

No. These may or may not be equal. Speed is the distance covered by a body in a unit time whereas velocity is the rate of change of displacement with time. In figure, let a person starting from A reaches B in 0.5 hour.



$$\text{Speed of person} = \frac{\text{Distance}}{\text{Time}} = \frac{1.8}{0.5} = 3.6 \text{ km h}^{-1}$$

$$\text{Velocity of person} = \frac{\text{Displacement}}{\text{Time}} = \frac{1.2}{0.5} = 2.4 \text{ km h}^{-1}$$

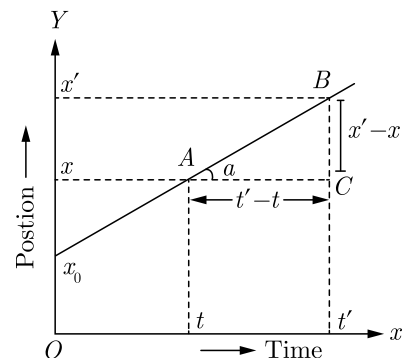
- 145.** Show that for a particle having ; uniform motion, the velocity of the particle is equal to the slope of position time graph with time axis ?

Ans :

For uniform motion, the position-time graph is a straight line inclined to time axis as shown in Figure.

Take two points A and B on $x-t$ graph at times t and t' respectively. If x and x' are position coordinates for these instants, then

$$\begin{aligned} \text{Velocity, } v &= \frac{x' - x}{t' - t} = \frac{BC}{AC} \\ &= \tan a \\ &= \text{Slope of } x-t \text{ graph} \end{aligned}$$



Hence, the slope of position-time graph for uniform motion is equal to velocity of the particle.

- 146.** Define speed. Distinguish between velocity and speed for a body in one dimensional motion. Can speed be negative ?

Ans :

Speed of a body is the distance travelled by it per unit time.

Speed is purely as scalar quantity and hence it has magnitude only. Speed is always either positive or zero, but never negative.

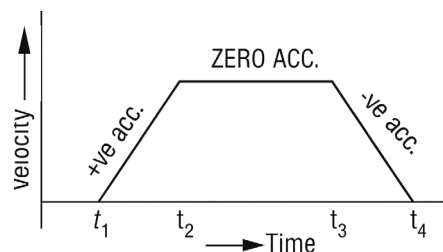
Velocity is a vector quantity and has magnitude and direction. Velocity can be negative. So we can say :
S.I. unit of speed is ms^{-1} .

Speed $|\vec{v}| \geq 0$

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- 147.** If the distance covered by a moving object varies directly on time, what conclusions could you draw about the motion and the forces.

Ans :

Given $x \propto t$

or $x = kt$

where k is a constant of proportionality

Since $v = \frac{dx}{dt}$
 $v = \frac{d}{dt}(kt) = k$

i.e. Speed is constant, thus no net force is acting on the body in the direction of motion.

- 148.** If the distance travelled by a moving object varies directly as the cube of time, how does the acceleration of the body depend on time ?

Ans :

Given $x \propto t^3$

so $x = kt^3$

where k is constant of proportionality

Since $v = \frac{dx}{dt}$
 $v = \frac{d}{dt}(kt^3) = 3kt^2$

And $a = \frac{dv}{dt} = 3k \frac{d}{dt}(t^2) = 6kt$

or $a \propto t$

i.e. acceleration varies directly as the time.

- 149.** What is meant by positive and negative accelerations?

Ans :

Positive and negative accelerations. In one dimension, an acceleration can be either positive or negative; these correspond to velocity (not speed) increasing or decreasing steadily. Suppose a bus has constant acceleration from t_1 to t_2 , then also from t_3 to t_4 , as shown in the velocity-time graph.

The accelerations in the two periods of changing velocity are :

$$t_1 \text{ to } t_2 : \quad \text{acceleration} = \frac{v}{t_2 - t_1} > 0 ;$$

$$t_3 \text{ to } t_4 : \quad \text{acceleration} = \frac{-v}{t_4 - t_3} < 0$$

- 150.** Does speeding up and slowing down refer to acceleration and decelerating respectively ?

Ans :

Speeding up and slowing down. We know that the acceleration could be either positive or negative in relation to a definite direction being chosen for increasing position coordinate. Negative acceleration is sometimes called deceleration (or retardation). However, positive and negative accelerations do not by themselves always correspond to “speeding up” and “slowing down”. It all depends on the value of the time (t' - t) in the formula

$$a = \frac{v' - v}{t' - t}$$

or $v' - v = a(t' - t)$

or $v' = v + a(t' - t) \quad \dots(1)$

If at $t = 0, v = u$

And at $t' = t, v' = v$

Equation (1) becomes

$$v = u + at$$

Thus, velocity depends on the value of t .

For all times before the time at which v vanishes, there is a slowing down, i.e. the speed keeps decreasing as time increases. However, after this time, there is a “speeding up” i.e. the velocity keeps increasing in magnitude. Whether it becomes large and positive or large and negative is determined by the sign of the acceleration.

- 151.** Derive the expression for distance travelled by an object in n th second of its motion.

Ans :

The distance travelled by a particle in n th second is the difference between the distance travelled by it in ' n ' seconds and $(n - 1)$ seconds.

i.e. $s_{nth} = S_n - S_{n-1}$

or $s_{nth} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2 \right]$
 $\left(S = ut + \frac{1}{2}at^2 \right)$

$$\begin{aligned} s_{nth} &= un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 + 1 - 2n) \right] \\ &= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{1}{2}a + an \\ &= u - \frac{a}{2} + an \\ s_{nth} &= u + \frac{a}{2}(2n-1) \end{aligned}$$

152. How is displacement calculated from a velocity versus time graph when velocity is not constant?

Ans :

When velocity is not constant, the displacement of an object is represented by the area under its velocity-time (v-t) graph between the initial and final time instants.

- This area calculation might involve dividing the area into simpler geometric shapes (like rectangles and triangles if acceleration is constant).
- More generally, for a continuously varying velocity, the displacement is the definite integral of the velocity function with respect to time over the given interval.

153. What information is needed to fully describe the rectilinear motion of a particle?

Ans :

To fully describe the rectilinear motion of a particle, several pieces of information are generally needed:

- The **initial position** (x_0) of the particle.
- The **initial velocity** (v_0) of the particle.
- The **acceleration** ('a') of the particle as a function of time (or if it's constant).

With these, one can determine the particle's position and velocity at any subsequent time 't' using kinematic equations or calculus.

154. Why is it important to state that realistic changes in velocity and acceleration are continuous?

Ans :

Stating that realistic changes in velocity and acceleration are continuous is important because discontinuous, or abrupt, changes would imply infinite forces or infinite power, which are not physically feasible.

- Continuous changes mean that velocity doesn't jump from one value to another instantly, and neither does acceleration.

- This smoothness in change reflects the way physical interactions actually occur in nature.

155. Describe the process of obtaining instantaneous velocity numerically if position data at different instants is available.

Ans :

Instantaneous velocity can be obtained numerically from position data at different instants by calculating average velocity over progressively smaller time intervals.

- Calculate $\Delta x/\Delta t$ for decreasing values of Δt around the instant of interest.
- As Δt approaches zero, the value of $\Delta x/\Delta t$ approaches a limiting value.
- This limiting value is the numerical approximation of the instantaneous velocity at that specific time.

156. What are the key assumptions made when deriving the standard kinematic equations for uniform acceleration?

Ans :

When deriving the standard kinematic equations, several key assumptions are made:

- The **acceleration 'a' is constant** throughout the motion in both magnitude and direction. This is the most crucial assumption.
- The motion is along a **straight line** (one-dimensional or rectilinear motion).
- Time 't' is measured from an initial instant usually denoted as $t = 0$.
- The object is treated as a **point particle**, or all parts of the object move identically.

157. Explain the term "average velocity" and how it is calculated for an object.

Ans :

Average velocity provides a measure of an object's overall displacement rate over a specific time interval.

- It is defined as the ratio of the total displacement (Δx) of the object to the total time interval (Δt) during which that displacement occurred.
- Mathematically, $\bar{v} = \Delta x/\Delta t = (x_f - x_i)/(t_f - t_i)$.
- It is a vector quantity, having the same direction as the displacement.

158. Explain instantaneous acceleration and how it is represented on a velocity-time graph.

Ans :

Instantaneous acceleration is the rate at which an object's velocity is changing at a specific instant

in time. It is defined as the limit of the average acceleration as the time interval (Δt) approaches zero ($a = dv/dt$).

On a velocity-time (v-t) graph:

- Instantaneous acceleration at any point is represented by the slope of the tangent to the v-t curve at that specific instant.

159. How do position-time graphs illustrate positive, negative, and zero acceleration?

Ans :

Position-time (x-t) graphs for an object can illustrate its acceleration through the curvature of the graph.

- **Positive acceleration:** The x-t graph curves upward, indicating that the velocity (slope) is increasing over time.
- **Negative acceleration:** The x-t graph curves downward, indicating that the velocity (slope) is decreasing over time (if positive) or becoming more negative.
- **Zero acceleration:** The x-t graph is a straight line (which can be horizontal or inclined), indicating constant velocity.

160. Derive the first kinematic equation, $v = v_0 + at$, for uniform acceleration.

Ans :

The first kinematic equation, $v = v_0 + at$, describes the final velocity of an object undergoing uniform acceleration. It can be derived from the definition of average acceleration.

- Average acceleration ($\bar{a}$) is defined as the change in velocity divided by the time interval: $\bar{a} = (v - v_0) / (t - 0)$.
- For uniform acceleration, the average acceleration ' $\bar{a}$ ' is equal to the constant acceleration 'a'.
- So, $a = (v - v_0) / t$.
- Rearranging this equation gives: $at = v - v_0$, which leads to $v = v_0 + at$.

161. Derive the kinematic equation $x = v_0 t + \frac{1}{2} at^2$ using a velocity-time graph for uniform acceleration.

Ans :

The kinematic equation for displacement, $x = v_0 t + \frac{1}{2} at^2$, can be derived from the area under the velocity-time graph for an object moving with uniform acceleration.

- The v-t graph is a straight line with initial velocity v_0 and final velocity $v = v_0 + at$.
- The displacement 'x' is the area under this graph between $t = 0$ and t . This area forms a trapezium (or a rectangle and a triangle).

- Area = Area of rectangle OACD + Area of triangle ABC = $v_0 t + \frac{1}{2} (v - v_0) t$.
- Since $v - v_0 = at$, substituting this gives

$$x = v_0 t + \frac{1}{2} (at) t = v_0 t + \frac{1}{2} at^2.$$

FIVE MARKS QUESTIONS

162. Answer the following question :

- Discuss the terms origin, unit and sense of passage of time.
- Discuss origin, unit and direction for position measurement of an object in motion along a straight line.

Ans :

(a) Origin

In any process involving change, we must be able to assign times to the various events that occur. For this purpose, some instant of time must be chosen as the origin and be assigned the value zero of time. This is called origin for representation of time.

Unit

The unit of time may be second, minute or hour as per convenience.

Sense of Passage of time

The events that occurred before the zero point of time will be assigned a negative number of units of time, while those that occur after zero point of time will carry a positive number of time units.

The choice of the origin of time and the unit of time are both matters of convenience. This is like dating historical events in years before or since the birth of Christ - B.C. or A.D.

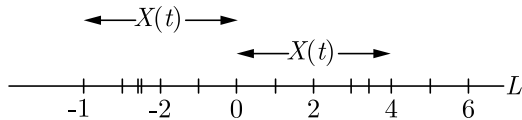
(b) Origin, unit and direction for position measurement of an object in one dimensional motion

Let us denote by L the straight line on which an object is moving. At each time, we want to describe its position by giving a number. For this, some point O on L must be chosen as the origin and some unit of length i.e. metre, (m) or kilometre (km) or centimetre (cm) depending on our convenience, must be chosen. Finally, one half of L to the right of O must be called the positive side and the other the negative side.

Position Co-ordinate

At each time t , the position of our moving object is given by a real number written as $x(t)$, positive or negative or zero. This is called the position co-ordinate.

This notation expresses the fact that the position $x(t)$ is a function of t . In figure, $x(t) = -2.5$ units and $x(t') = +3.5$ units at instants t, t' respectively.



- 163.** Define displacement of a particle in linear motion in one dimension. Does it depend upon the origin ? Can actual distance travelled by object in the time interval t to t' be greater than or equal to the magnitude of the displacement ?

Ans :

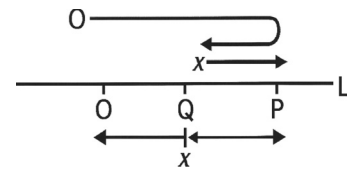
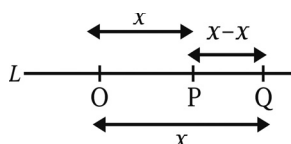
The displacement of an object between two instants of time is defined as the difference of the position coordinates of the object at final and initial instants of time in a given direction.

If x and x' are the position co-ordinates at instants t and t' respectively, then displacement in time t to $t' = x' - x$. Displacement is also a length and is measured in metres.

Observations

It must be obvious that

- displacement in a given interval of time can be positive, negative or zero for the position x' in relation to x
 - If x' is greater than x then displacement is +ve
 - If $x' = x$, then displacement = 0
 - If $x' < x$, then displacement is -ve.
- It is not dependent on the choice of the origin O on the line L made for measuring the position co-ordinate.
- The actual distance travelled by the object in the time interval t to t' is either equal to or greater than the magnitude of the displacement as shown in Figures, respectively.



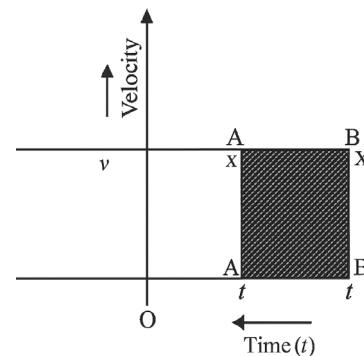
- (iv) Displacement unit of length.

In both the cases, magnitude of displacement is PQ but distances travelled are $(OP + PQ)$ and PQ respectively.

- 164.** How does velocity-time graph; give a geometrical way of calculating : the distance covered from time t to time t' ?

Ans :

The area under the velocity curve and time axis between times t and t' gives the distance covered by the object between time t and t' as shown below:



Suppose, $v =$ uniform velocity since velocity is uniform, the velocity-time graph is a straight line parallel to time axis.

By definition of velocity,

$$v = \frac{x' - x}{t' - t}$$

$$x' - x = v(t' - t) \quad \dots(i)$$

where t, t' , correspond to the points A' and B' as shown.

Now, in the graph, $AA' = BB' = v$

$$A'B' = (t' - t)$$

Putting the values of v and $(t' - t)$ in equation (i)

$$x' - x = AA' \times A'B'$$

or $x' - x =$ area of rectangle $AA'B'B$

$\therefore$ the displacement of the object between t and t' is equal to the area under the velocity curve and time axis between times t and t' .

- 165.** Derive the formula to find; position at t' when the initial position ; at instant t is known as object is ; moving with uniform velocity.

or

Show that $x = x_0 + vt$ or $s = vt$.

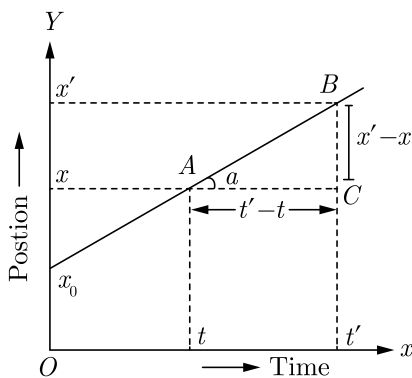
Ans :

For uniform motion, position-time graph of an object in one dimensional motion is a straight line inclined to time axis as shown in figure.

Take two points A and B , when particle has covered distance x and x' in time t and t' respectively.

velocity, $v = \text{slope of } x-t \text{ graph} = \tan a$

or
$$v = \frac{BC}{AC} = \frac{x' - x}{t' - t}$$



or
$$x' - x = v(t' - t) \quad \dots(1)$$

If at $t = 0, x = x_0$

And at $t' = t, x' = x$

Equation (1) becomes

$$x - x_0 = v(t - 0)$$

or
$$x - x_0 = vt \quad \dots(2)$$

or
$$x = x_0 + vt \quad \dots(3)$$

Writing $x - x_0 = s$, eq. (2) becomes

$$s = vt$$

- 166.** Define Relative Velocity. Show that the relative velocity of an object '2' relative to another object '1' moving in one dimension is $(v_2 - v_1)$.

Ans :

It is defined as the velocity of an object relative to another object which may be stationary or moving.

Expression for Relative Velocity : Consider two objects 1 and 2 each moving uniformly with velocities v_1 and v_2 in one direction. Let x_{10} and x_{20} be the positions of two particles at $t = 0$, then their positions at time t will be given by

$$x_1 = x_{10} + v_1 t$$

$$x_2 = x_{20} + v_2 t$$

$$x_2 - x_1 = x_{20} - x_{10} + (v_2 - v_1)t \quad \dots(1)$$

Let

$$x_{20} - x_{10} = x_0$$

(distance between two objects at $t = 0$)

$$x_2 - x_1 = x$$

(distance between two objects at $t = t$)

So eq. (1) becomes

$$x = x_0 + (v_2 - v_1)t$$

or

$$\frac{x - x_0}{t} = v_2 - v_1$$

L.H.S. gives the change in relative displacement per unit time,

It gives relative velocity of object 2 with respect to object 1.

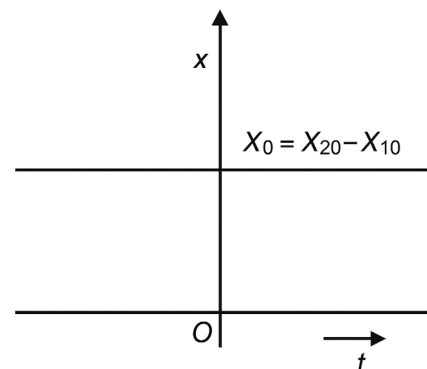
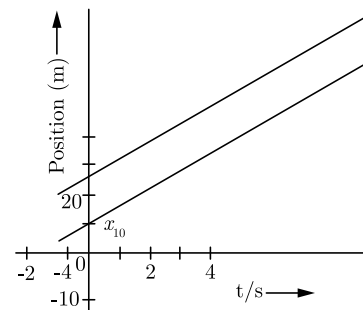
Relative velocity of object 2 w.r.t. object 1

$$= v_2 - v_1 = v_2 + (-v_1)$$

= Velocity of object 2 + reverse of velocity of object 1.

Following points may further be noted :

$v_1 = v_2$ or $(v_2 - v_1 = 0)$, the two objects stay a constant distance apart, so their position-time graphs are parallel straight lines, as shown in figure. However, the graph for relative displacement x with time will be parallel to t -axis as shown in figure.



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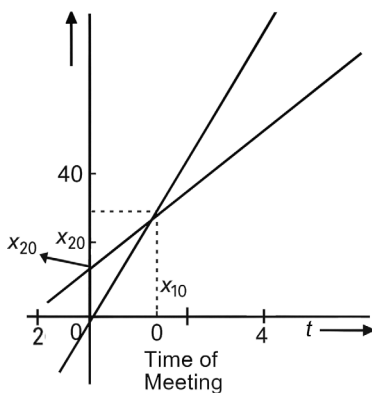
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- (ii) If however, $(v_2 - v_1)$ is non-zero, one graph is steeper than the other, therefore we see graphically that they must meet once at a common point.

For instance, suppose $x_{10} = 10 \text{ m}$, $x_{20} = 20 \text{ m}$, $v_1 = 20 \text{ ms}^{-1}$, $v_2 = 10 \text{ ms}^{-1}$, then the time at which they meet is $t = 1 \text{ s}$, at which time they are both at position $x_1 = x_2 = 30 \text{ m}$. Thus, object 1 overtakes object 2 at this time, as shown in figure. Before this time $x_2 - x_1$ is positive, after this time, it is negative.

- (iii) If v_1 and v_2 are of opposite signs, then the relative velocity $(v_2 - v_1)$ is greater than both v_1 and v_2 in magnitude e.g. when two fast trains cross each other in the opposite direction, each seems to go very fast indeed relative to the other.



- 167.** Define non-uniform motion, average velocity, instantaneous velocity, variable velocity, instantaneous acceleration and uniform acceleration. What is the physical meaning of acceleration ?

Ans :

Non-uniform motion

The motion of an object is said to be non-uniform if it covers unequal displacements in equal intervals of time, however small the time intervals may be.

Hence the motion is non-uniform if the velocity at different instants are not equal.

Example of non-uniform motion. When we travel in any vehicle or even walk or run, we start from a condition of rest ; then we “pick up speed” for some time ; we may then travel at constant velocity for a while, then possibly “slow down” and come to a stop. The periods of “picking up speed” and “slowing down” are periods of changing velocity.

Therefore, it is an example of non-uniform motion, and this leads to idea of acceleration.

Average velocity

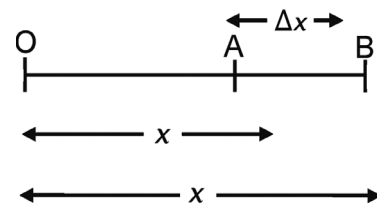
The average velocity during a certain time interval is equal to the average displacement per unit time during that time interval.

Suppose at time t , the particle is at position A , with $OA = x$. At a later time t' , it is at B with $OB = x'$. The average velocity between A and B is defined by

$$v_{av} = \frac{x' - x}{t' - t} = \frac{\Delta x}{\Delta t},$$

where $x' - x = \Delta x$ is the displacement of the particle and $(t' - t) = \Delta t$, is the elapsed time.

The average velocity is a vector since the ratio of a vector to a scalar is itself a vector. Its direction is the same as that of the displacement vector.



Instantaneous velocity

Instantaneous velocity is the limiting value of the fraction $(\Delta x / \Delta t)$ when the denominator approaches zero. This can be written in the form

Instantaneous velocity,

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Thus, we obtain the instantaneous velocity by finding the time derivative of the displacement.

Variable Velocity

If the displacements of a particle are equal in a particular direction, in equal intervals of time, however small the intervals may be, it is said to move with a variable velocity.

Units of velocity. In cgs system, velocity is expressed in centimetre per second (cm s^{-1}).

In SI units, velocity is expressed in metre per second (m s^{-1})

Acceleration

It is defined as rate of change of velocity w.r.t. time.

Average acceleration

It is measured by the ratio of change in velocity to the time interval.

If v and v' are the velocities at instants t and t' , then acceleration (a) is given by

$$a = \frac{v' - v}{t' - t} = \frac{\Delta v}{\Delta t},$$

where

$$\Delta v = v' - v,$$

$$\Delta t = t' - t$$

Acceleration may be uniform or non-uniform.

Instantaneous acceleration

It is acceleration at any instant and is defined as the limiting value of the average acceleration when the time interval Δt becomes very small. That is

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Thus, we obtain the instantaneous acceleration by finding the time derivative of the velocity. Its direction is the same as that of the limiting direction of the vector change in velocity.

Uniform acceleration

If the velocity changes in magnitude of an object are equal in equal intervals in time, it is said to be moving with a uniform acceleration.

It is given by

$$a = \frac{v' - v}{t' - t}$$

Variable acceleration

If an object moves in such a way, that its velocity changes in magnitude are unequal in equal intervals of time, it is said to be moving with a variable acceleration.

Units of acceleration

In the SI units, the acceleration is expressed in metre per second per second (m s^{-2}).

But the acceleration may also be expressed in other units, such as km h^{-2} or m s^{-2} .

Physical meaning of acceleration :

It tells us by how much the velocity is changing in each unit of time.

168. Derive the following relations : for non-uniformly accelerated motion. :

- (i) Velocity-time relation
- (ii) Displacement-time relation and
- (iii) $v^2 - u^2 = 2as$

Ans :

- (i) **Velocity-time relation** ($v = u + at$) : Let v , v' be the velocities at instants t and t' respectively. Then average acceleration is given by

$$a = \frac{v' - v}{t' - t}$$

or

$$v' - v = at$$

$$\text{or} \quad v' = v + at \quad \dots(i)$$

$$\text{If at} \quad t = 0, \quad v = u$$

$$\text{and at} \quad t' = t, \quad v' = v$$

Equation (i) becomes

$$v = u + at$$

This is the required result.

(ii) **Displacement-time relation** ($s = ut + \frac{1}{2}at^2$) :

Let x = position of an object at time t

x' = position of an object at time t'

So, Average velocity

$$v_{av} = \frac{x' - x}{t' - t} \quad \dots(ii)$$

If v = velocity of the object at time t

v' = velocity of the object at time t'

$$\text{Then} \quad v_{av} = \frac{v + v'}{2} \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$\frac{x' - x}{t' - t} = \frac{v + v'}{2}$$

$$\text{Since} \quad v' = v + a(t' - t)$$

$$\frac{x' - x}{t' - t} = \frac{v + v + a(t' - t)}{2}$$

$$\text{or} \quad x' - x = v(t' - t) + \frac{1}{2}a(t' - t)^2 \quad \dots(iii)$$

$$\text{If at} \quad t = 0, \quad x = x_0 \quad \text{and} \quad v = u$$

$$\text{And at} \quad t' = t, \quad x' = x$$

So, Eq. (iii) becomes

$$x - x_0 = u(t - 0) + \frac{1}{2}a(t - 0)^2$$

$$\text{or} \quad x - x_0 = ut + \frac{1}{2}at^2$$

$$\text{or} \quad x = x_0 + ut + \frac{1}{2}at^2 \quad \dots(iv)$$

Writing $x - x_0 = s$ (distance travelled) in eq.

(iv), we get

$$s = ut + \frac{1}{2}at^2$$

This is the required relation.

(iii) $v^2 - u^2 = 2as$:

If x , x' and v , v' be positions and velocities of a particle at time t and t' respectively, then

$$v_{av} = \frac{x' - x}{t' - t} \quad \dots(1)$$

$$\text{Also} \quad v_{av} = \frac{v + v'}{2} \quad \dots(2)$$

From eq. (1) and (2), we have

$$\frac{x' - x}{t' - t} = \frac{v + v'}{2}$$

$$\text{or} \quad x' - x = \frac{v + v'}{2}(t' - t) \quad \dots(3)$$

Also, acceleration,

$$a = \frac{v' - v}{t' - t}$$

$$t' - t = \frac{v' - v}{a} \quad \dots(4)$$

Putting eq. (4) in (3), we get

$$x' - x = \frac{v + v'}{2} \cdot \frac{v' - v}{a}$$

$$\text{or } x' - x = \frac{v'^2 - v^2}{2a}$$

$$\text{or } v'^2 - v^2 = 2a(x' - x) \quad \dots(5)$$

If at $x = x_0$, $v = u$

And at $x' = x$, $v' = v$

eq. (5) becomes

$$v^2 - u^2 = 2a(x - x_0)$$

Writing

$$x - x_0 = s$$

(distance travelled), we get

$$v^2 - u^2 = 2as$$

This is the required relation.

169. Derive the equations of motion by calculus.

Ans :

(i) $v = u + at$

Let v be the velocity at any instant and it changes dv

by dv in time dt , then $\frac{dv}{dt} = a$, the acceleration

$$\text{or } dv = a dt$$

$$\text{or } \int_u^v dv = a \int_0^t dt$$

$$[v]_u^v = a[t]_0^t$$

$$\text{or } v - u = a(t - 0)$$

$$v = u + at$$

(ii) $s = s_0 + ut + \frac{1}{2}at^2$

Now consider an object moving in straight line with uniform acceleration a . Let ds be the displacement of the object in time interval dt . The instantaneous velocity is given by

$$v = \frac{ds}{dt} \text{ or } ds = v dt$$

$$= (u + at) dt$$

$$\text{or } \int_{s_0}^s ds = u \int_0^t dt = a \int_0^t t dt,$$

where S_0 is the displacement at time zero.

$$[s]_{s_0}^s = [t]_0^t + a \left(\frac{t^2}{2} \right)_0^t$$

$$\text{or } s - s_0 = u(t - 0) + \frac{1}{2}a(t^2 - 0)$$

$$\text{or } s = s_0 + ut + \frac{1}{2}at^2$$

$$(iii) v^2 - u^2 = 2as$$

$$\text{We know, } a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt}$$

$$= \frac{ds}{dt} \times \frac{dv}{ds}$$

$$\text{or } a = v \frac{dv}{ds}$$

$$\text{or } a ds = v dv$$

$$\text{or } a \int_0^s ds = \int_u^v v dv$$

$$\text{or } a[s]_0^s = \left(\frac{v^{1+1}}{1+1} \right)_u^v$$

$$\text{or } a(s - 0) = \frac{v^2 - u^2}{2}$$

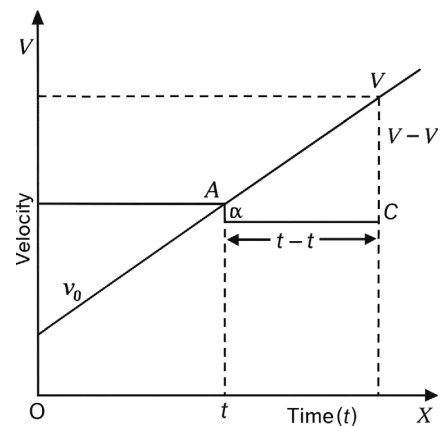
$$\text{or } 2as = v^2 - u^2$$

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170. Show that the slope of velocity time graph for uniformly accelerated motion is equal to the acceleration of the body.

Ans :

For uniformly accelerated motion, velocity time graph of a particle in one dimensional motion is a straight line inclined to time axis as shown in figure.



Let us consider two points A and B , when the velocity of the particle is v and v' in time t and t' respectively.

So, acceleration,

$$a = \frac{v' - v}{t' - t} = \frac{BC}{AC}$$

$$\text{or } a = \tan \alpha$$

$$\text{or } a = \text{slope of } v-t \text{ graph}$$

Hence, slope of $v-t$ graph is equal to acceleration of the body.

171. Show that the distance covered by a particle having uniform acceleration in time interval $(t' - t)$ is the area under v - t graph for time t' and t .

Ans :

We know that for uniformly accelerated motion, the velocity time graph is a straight line inclined to time axis as shown in Figure.

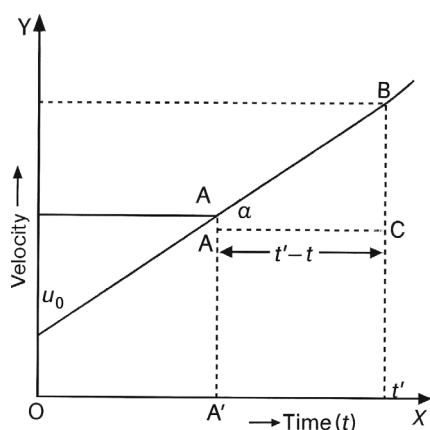
Let v and v' be velocities of the particle at time t and t' respectively at point A and B .

Area of trapezium $AA'B'B$

$$\begin{aligned} &= \frac{1}{2}(AA' + BB')A'B' \\ &= \frac{1}{2}(v + v')(t' - t) \\ &= \frac{v + v'}{2}(t' - t) \\ &= v_{av}(t' - t) \end{aligned}$$

= Average velocity $\times$ time taken

= Distance travelled by the particle
in time interval $(t' - t)$



Hence, the distance travelled by a particle having uniform acceleration is the area under velocity time graph.

NUMERICAL PROBLEMS

172. Two parallel rail tracks run north-south. Train A moves north with a speed of 54 kmh^{-1} and Train B moves south with a speed of 90 kmh^{-1} . What is the
- relative velocity of B with respect to A ?
 - relative velocity of ground with respect to B ?
 - velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 kmh^{-1} with respect to the train A) as observed by a man standing on the ground?

Ans :

Choose the positive direction of x -axis to be from south to north. Then,

$$v_A = +54 \text{ kmh}^{-1} = 15 \text{ ms}^{-1}$$

$$v_B = -90 \text{ kmh}^{-1} = -25 \text{ ms}^{-1}$$

- Relative velocity of B with respect to $A = v_B - v_A = -40 \text{ ms}^{-1}$ i.e. the train B appears to A to move with a speed of 40 ms^{-1} from north to south.
- Relative velocity of ground with respect to $B = 0 - v_B = 25 \text{ ms}^{-1}$
- Let the velocity of the monkey with respect to ground be v_M . Relative velocity of the monkey with respect to $A = v_M - v_A = -18 \text{ kmh}^{-1} = 5 \text{ ms}^{-1}$. Therefore,

$$v_M = (15 - 5) \text{ ms}^{-1} = 10 \text{ ms}^{-1}$$

173. A car travels a distance from A to B at a speed of 60 kmh^{-1} and returns to A at a speed of 30 kmh^{-1} . What is the average speed for the whole journey? What is the average velocity of the car?

Ans :

Let the distance from A to $B = S \text{ km}$

Then from A to B ,

$$t_1 = \frac{S}{60} \text{ hr}$$

Then from B to A ,

$$t_2 = \frac{S}{30} \text{ hr}$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{2S}{\left(\frac{S}{60} + \frac{S}{30}\right)} \\ &= \frac{2S}{\left(\frac{S}{60} + \frac{2S}{60}\right)} \\ &= \frac{2 \times 60 \times 30}{(60 + 30)} \\ &= 90 \text{ km hr}^{-1} \end{aligned}$$

Since the car returns to its initial position,

$$\begin{aligned} \text{Net displacement} &= \text{distance between the final} \\ &\quad \text{and initial position} \\ &= 0 \end{aligned}$$

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time taken}}$$

Average velocity of the car = 0

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- 174.** A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 kmh^{-1} and the other has the speed of 27 kmh^{-1} . The bird starts moving from first car towards the other and is moving with a speed of 36 kmh^{-1} and when the two cars were separated by 36 km . What is the total distance covered by the bird ? What is the total displacement of the bird ?

Ans :

$$\begin{aligned}\text{Relative speed of the cars} &= 27 - (-18) \\ &= 45 \text{ kmh}^{-1}\end{aligned}$$

Time required to meet the cars

$$\begin{aligned}t &= \frac{\text{distance}}{\text{velocity}} \\ &= \frac{36}{45} = 0.8 \text{ h}\end{aligned}$$

Distance covered by the bird

$$\begin{aligned}s &= vt \\ &= 36 \times 0.8 = 28.8 \text{ km}\end{aligned}$$

- 175.** A body is dropped from rest at a height of 150 m and simultaneously another body is dropped from rest from a point 100 m above the ground. What is their difference in height after they have fallen (a) 2 s (b) 3 s ? How does the difference in height vary with time

Ans :

Since both the bodies are falling under gravity and have same acceleration i.e. ' g ' therefore, they will cover equal distances in equal times, whether 2 s or 3 s . Thus the difference in their height will remain the same as in the beginning i.e. $= 50 \text{ m}$.

Thus the difference in height is independent of the time and remains constant.

- 176.** A balloon is ascending at the rate of 14 ms^{-1} at a height of 98 m above ground when a packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ?

Ans :

The initial velocity of the packet is the same as the velocity of the balloon i.e.

$$u = -14 \text{ ms}^{-1}$$

Negative sign indicates that the initial velocity is opposite to the direction of fall of the packet.

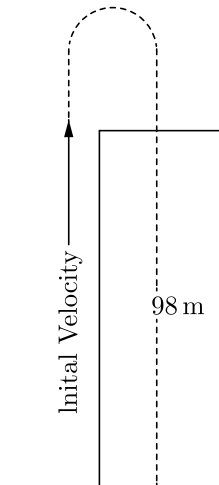
$$x - x_0 = s = 98 \text{ m}, a = g = 9.8 \text{ m/s}^2$$

Now
$$s = ut + \frac{1}{2}at^2$$

or
$$98 = -14 \times t + \frac{1}{2} \times 9.8t^2$$

or
$$4.9t^2 - 14t - 98 = 0$$

or
$$7t^2 - 20t - 140 = 0$$



or

$$\begin{aligned}t &= \frac{20 \pm \sqrt{400 + 3920}}{14} \\ &= \frac{20 + 65.7}{14} \\ &\quad (\text{Neglecting negative sign}) \\ &= \frac{85.7}{14} = 6.12 \text{ s}\end{aligned}$$

If v is the velocity with which the packet hits the ground, then

$$\begin{aligned}v &= u + at \\ &= -14 + 9.8 \times 6.12 \\ &= -14 + 60.01 \\ &= 46.01 \text{ ms}^{-1}\end{aligned}$$

- 177.** A car A is travelling on a straight level road with a uniform speed of 60 kmh^{-1} . It is followed by another car B which is moving with 70 kmh^{-1} , when the distance between them is 2.5 km the car B is given a deceleration of 20 kmh^{-2} . After what distance and time will B catch up with A ?

Ans :

Suppose the car B catches up the car A in t hour.

This distance travelled by the car A moving with a velocity of 60 kmh^{-1} in time t hour is given by $s_1 = 60t \text{ km}$.

The distance travelled by the car B moving with an initial velocity $u = 70 \text{ kmh}^{-1}$ and decelerated at the rate of 20 kmh^{-2} in time t hours.

$$x = x_0 + ut + \frac{1}{2}at^2 \quad \dots(i)$$

$$\begin{aligned}x - x_0 (= s_2) &= 70t + \frac{1}{2}(-20)t^2 \\ s_2 &= 70t - 10t^2 \quad \dots(ii)\end{aligned}$$

But $(x_2 - x_0) = 2.5 \text{ km}$

$$70t - 10t^2 - 60t = 2.5$$

$$10t^2 - 10t + 2.5 = 0$$

$$t = \frac{1}{2}h = 30 \text{ minutes}$$

From equation (ii),

$$\begin{aligned} s_2 &= 70\left(\frac{1}{2}\right) - 10\left(\frac{1}{2}\right)^2 \\ &= 32.5 \text{ km} \end{aligned}$$

- 178.** A bus is beginning to move with an acceleration of 1 ms^{-2} . A man who wants to board it can run at 9 ms^{-1} . Show that he can catch it in 8 sec., if he is 40m behind when it starts.

Ans :

Total distance moved by the man

$$= 9 \times 8 = 72 \text{ m}$$

Distance moved by the bus must be

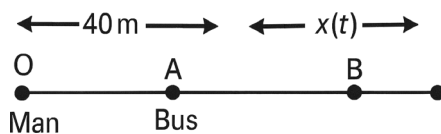
$$= 72 - 40 = 32 \text{ m}$$

Using, for the bus,

$$\begin{aligned} x &= x_0 + ut + \frac{1}{2}at^2 \\ x &= 0 + 0 \times 8 + \frac{1}{2} \times 1 \times 8 \times 8 \\ &= 32 \end{aligned}$$

Which is true.

The man will catch the bus in 8 seconds.



- 179.** A body travels 200 cm in first two seconds and 220 cm in next four seconds. What will be the velocity at the end of the seventh second from the start ?

Ans :

For first two seconds, we have

$$s = ut + \frac{1}{2}at^2$$

$$\text{or } 2 = u(2) + \frac{1}{2}a(2)^2$$

$$\text{or } 2 = 2u + 2a \quad \dots(i)$$

After four seconds, i.e. after time $t = 6 \text{ s}$,

$$\text{distance covered} = 200 + 220 = 420 \text{ cm} = 4.2 \text{ m}$$

$$4.2 = u(6) + \frac{1}{2}a(6)^2$$

$$\text{or } 4.2 = 6u + 18a \quad \dots(ii)$$

Solving eq. (i) and (ii), we get

$$u = 1.15 \text{ ms}^{-1} \text{ and}$$

$$a = -0.15 \text{ ms}^{-2}$$

Now velocity after 7 s

$$\begin{aligned} v &= u + at \\ &= 1.15 + (-0.15)7 \\ &= 0.01 \text{ ms}^{-1} \end{aligned}$$

- 180.** A truck starts from rest with an acceleration of 1.5 ms^{-2} while a car 150 metre behind starts from rest with an acceleration of 2 ms^{-2} . How long will it take before both the truck and car side by side, and how much distance is travelled by each ?

Ans :

Let x be the distance travelled by the truck when truck and car are side by side. The distance travelled by the car will be $(x + 150)$ as the car is 150 metre behind the truck. Applying the formula

$$s = ut + \left(\frac{1}{2}\right)at^2,$$

$$\text{We have } x = \frac{1}{2} \times (1.5)t^2 \quad \dots(i)$$

$$\text{and } (x + 150) = \frac{1}{2} \times (2)t^2 \quad \dots(ii)$$

Here t is the common time.

From eqs. (1) and (2), we have

$$\frac{x + 150}{x} = \frac{2}{1.5}$$

Solving we get, $x = 450$ metre (truck)

and $x + 150 = 600$ metre (car).

Substituting the value of x in eq. (1), we get

$$\begin{aligned} 450 &= \frac{1}{2}(1.5)t^2 \\ t &= \sqrt{\left(\frac{450 \times 2}{1.5}\right)} \\ &= \sqrt{600} = 24.5 \text{ sec} \end{aligned}$$

- 181.** The driver of fast train moving at the speed v_1 sights a goods train a distance d ahead of him on the same track moving in the same direction with a slower speed v_2 . He puts on the brakes and gives his train a constant retardation a . Show that

if $d > \frac{(v_1 - v_2)^2}{2a}$, there will be no collision

if $d < \frac{(v_1 - v_2)^2}{2a}$, there is a collision.

Ans :

There will be no collision if the speed of fast train is reduced to goods train i.e.

$$v = v_2$$

Initial relative velocity $v_1 = v_2$

If s is the distance covered by fast train in time t , then

$$v^2 = u^2 + 2as$$

Here

$$0 = (v_1 - v_2)^2 - 2as$$

or

$$s = \frac{(v_1 - v_2)^2}{2a}$$

For no collision, we must have

$$d > s$$

or

$$d > \frac{(v_1 - v_2)^2}{2a}$$

For collision to take place

$$d < \frac{(v_1 - v_2)^2}{2a}$$

- 182.** A stone is dropped from the top of a tall cliff and n seconds later another stone is thrown vertically downwards with a velocity u m/s. How far below the top of the cliff the second stone will overtake the first?

Ans :

Let the second stone overtakes the first after n second at depth h .

The distance covered by first stone in t seconds and by second stone in $(t - n)$ seconds are equal to h .

$$h = \frac{1}{2}gt^2 \quad \dots(i)$$

(since $u = 0$ for first stone)

Also

$$h = u(t - n) + \frac{1}{2}g(t - n)^2$$

[Since for second stone $u = u$ and time $= (t - n)$]
Equating (i) and (ii)

$$\frac{1}{2}gt^2 = u(t - n) + \frac{1}{2}g(t - n)^2$$

Solving for t ,

$$\text{We get} \quad t = \frac{\left(\frac{1}{2}gn - u\right)n}{(gn - u)}$$

Substituting this value in (i), we get

$$h = \frac{1}{2}g \frac{\left(\frac{1}{2}gn - u\right)^2 n^2}{(gn - u)^2}$$

This is also the distance traversed by second stone for overtaking the first stone below the top of tower.

- 183.** A body starts from rest and moving with uniform acceleration of 4 m/s^2 covers half of its total path during the last second of its motion. Find the time taken and the total distance covered.

Ans :

Let the body be in motion for n seconds. If S_n is the distance covered by the body in n seconds, then

from

$$s = v_0 t + \frac{1}{2}at^2$$

$$s_n = \frac{1}{2}an^2 \quad (v_0 = 0)$$

If $S_{n \text{ th}}$ is the distance covered in the last second of its motion, then

$$\begin{aligned} S_{n \text{ th}} &= v_0 + \frac{a}{2}(2n - 1) \\ &= \frac{a}{2}(2n - 1) \end{aligned}$$

According to the given condition,

$$S_{n \text{ th}} = \frac{1}{2}S_n$$

$$\text{or} \quad \frac{a}{2}(2n - 1) = \frac{1}{2}\left(\frac{1}{2}an^2\right)$$

$$\text{or} \quad 2(2n - 1) = n^2$$

$$\text{or} \quad n^2 - 4n + 2 = 0$$

$$\begin{aligned} \text{or} \quad n &= \frac{4 \pm \sqrt{(4)^2 - 4(1)(2)}}{2} \\ &= \frac{4 \pm \sqrt{8}}{2} \end{aligned}$$

Taking only the positive sign,

$$n = \frac{4 + \sqrt{8}}{2} = 3.414 \text{ s}$$

Also

$$\begin{aligned} s &= \frac{1}{2}an^2 = \frac{1}{2} \times (4)(3.414)^2 \\ &= 23.312 \text{ m} \quad (\because a = 4 \text{ m/s}^2) \end{aligned}$$

CASE BASED QUESTIONS

- 184.** A person travels from their home to work. In the first half of the journey, covering a distance of 15 km, they maintain an average speed of 30 km/h. Due to traffic in the second half, which is also 15 km, their average speed drops to 15 km/h. The entire journey is along a straight road.

Questions:

1. Is speed a scalar or a vector quantity?
2. How is average speed for an entire journey calculated when different parts of the journey are covered at different speeds?
3. Calculate the time taken for the first half of the journey and the second half of the journey.
4. What is the average speed of the person for the entire journey from home to work?

Ans :

1. Speed is a scalar quantity.
2. Average speed is calculated as the total path length divided by the total time interval.
3. Time for the first half (t_1)

$$= \text{Distance} / \text{Speed}$$

$$= 15 \text{ km} / 30 \text{ km/h} = 0.5 \text{ h.}$$

Time for the second half (t_2)

$$= \text{Distance} / \text{Speed}$$

$$= 15 \text{ km} / 15 \text{ km/h} = 1.0 \text{ h.}$$

4. Total distance = 15 km + 15 km = 30 km.

$$\text{Total time} = t_1 + t_2$$

$$= 0.5 \text{ h} + 1.0 \text{ h} = 1.5 \text{ h.}$$

$$\text{Average speed} = \text{Total distance} / \text{Total time}$$

$$= 30 \text{ km} / 1.5 \text{ h} = 20 \text{ km/h.}$$

- 185.** A train starts from rest and accelerates uniformly along a straight track. Its velocity-time (v-t) graph is a straight line passing through the origin with a positive slope. After 10 seconds, its velocity is 15 m/s.

Questions:

1. What does the slope of a velocity-time graph represent?
2. If the acceleration is constant, what is the relationship between average acceleration and instantaneous acceleration?
3. Calculate the average acceleration of the train during the first 10 seconds.
4. If the train continues to accelerate at this rate, what will its velocity be at $t = 20\text{s}$? What does the area under the v-t graph from $t=0$ s to $t=10\text{s}$ represent?

Ans :

1. The slope of the velocity-time graph at a particular time represents the instantaneous acceleration at that time. For uniform acceleration, this slope is constant.
2. If the acceleration is constant, the average acceleration equals the constant value of acceleration during any interval.
3. Average acceleration

$$\begin{aligned} \bar{a} &= \frac{\Delta v}{\Delta t} \\ &= \frac{15\text{m/s} - 0\text{m/s}}{10\text{s} - 0\text{s}} \\ &= 1.5\text{m/s}^2. \end{aligned}$$

4. Using $v = v_0 + at$, with $v_0 = 0$, $a = 1.5\text{m/s}^2$.
At $t = 20\text{s}$,

$$v = 0 + (1.5\text{m/s}^2)(20\text{s}) = 30\text{m/s.}$$

The area under the velocity-time graph represents the displacement over a given time interval.

- 186.** Ravi is standing on the rooftop of a tall building and accidentally drops a small rubber ball from rest. Since there is no significant air resistance acting on it, the ball begins to fall freely under the influence

of gravity. In this situation, the only force acting on the ball is the Earth's gravitational pull, which causes it to accelerate downward at approximately 9.8 m/s^2 .

Questions:

1. What is meant by "free fall"?
2. What is the direction of acceleration during the free fall of the ball? Is this acceleration constant or variable?
3. Using the kinematic equations, what will be the velocity of the ball after 2 seconds of fall?
4. How far will the ball have fallen in these 2 seconds?

Ans :

1. If air resistance is neglected, an object released near the surface of the Earth and accelerated downward under the influence of gravity is said to be in free fall.
2. The acceleration due to gravity is always downward. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant.
3. Using $v = v_0 + at$.
Here $v_0 = 0$, $a = -g = -9.8\text{m/s}^2$ (if upward is positive) or $a = g = 9.8\text{m/s}^2$ (if downward is positive). Taking downward as positive,

$$v = 0 + (9.8\text{m/s}^2)(2\text{s}) = 19.6\text{m/s.}$$

The equations use $a = -g$. So,

$$\begin{aligned} v &= 0 - gt \\ &= -9.8 \times 2 = -19.6\text{m/s.} \end{aligned}$$

4. Using $y = v_0 t + \frac{1}{2} at^2$.

Taking $u_0 = 0$ and $a = -g$:

$$\begin{aligned} y &= 0 \times t - \frac{1}{2} gt^2 \\ &= -\frac{1}{2} (9.8\text{m/s}^2) (2\text{s})^2 \\ &= -\frac{1}{2} (9.8) (4) = -19.6\text{m.} \end{aligned}$$

The distance fallen is 19.6 m.

- 187.** Two friends, Aryan and Kabir, are participating in a school science experiment. Aryan rolls a toy car (Object P) along a straight track at a constant speed of 10 m/s. He keeps it moving for 5 seconds without changing its speed. At the same time, Kabir starts another toy car (Object Q) from rest on the same track. His toy car gradually picks up speed because it accelerates uniformly at 2 m/s^2 for the same duration of 5 seconds. Both cars move along the same straight path during the experiment.

Questions:

1. What is the acceleration of object P?

- Describe the shape of the position-time (x-t) graph for object P and object Q.
- Calculate the distance covered by object P in 5 seconds.
- Calculate the final velocity and distance covered by object Q in 5 seconds.

Ans :

- For uniform motion (constant velocity), acceleration is zero. So, the acceleration of object P is 0m/s^2 .
- For object P (uniform motion), the x-t graph is a straight line inclined to the time axis. For object Q (uniform acceleration), the x-t graph is a parabola.

- Distance for P = velocity $\times$ time
 $= 10\text{m/s} \times 5\text{s} = 50\text{m}$.

- For Q: Final velocity
 $v = v_0 + at$
 $= 0 + (2\text{m/s}^2)(5\text{s}) = 10\text{m/s}$.

$$\begin{aligned}\text{Distance } x &= v_0 t + \frac{1}{2}at^2 \\ &= 0 \times 5 + \frac{1}{2}(2\text{m/s}^2)(5\text{s})^2 \\ &= \frac{1}{2}(2)(25) = 25\text{m}.\end{aligned}$$

- 188.** An object is dropped from rest and falls freely under gravity. Galileo's law of odd numbers states that "The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, 1:3:5:7...]. Let the equal interval of time be τ .

Questions:

- For an object in free fall from rest, how is the distance y traversed related to time t ?
- If y_0 is the distance traversed in the first time interval τ , what is the distance traversed in the first 2τ seconds, according to the formula for free fall?
- Using the formula $y = -\frac{1}{2}gt^2$, what is the distance traversed in the second interval τ (i.e., between $t = \tau$ and $t = 2\tau$)?
- Show that the distance traversed in the second interval τ is $3y_0$, where y_0 is the distance traversed in the first interval τ .

Ans :

- For an object in free fall from rest, the distance y traversed is given by $y = -\frac{1}{2}gt^2$ (where the negative sign indicates downward displacement if upward is positive, or magnitude $y = \frac{1}{2}gt^2$).

- Distance traversed in the first 2τ seconds is.

$$\begin{aligned}y(2\tau) &= \frac{1}{2}g(2\tau)^2 \\ &= \frac{1}{2}g(4\tau^2) = 4 \times \left(\frac{1}{2}g\tau^2\right)\end{aligned}$$

If $y_0 = \frac{1}{2}g\tau^2$ (magnitude), then distance is $4y_0$.

- Distance in the first interval ($t = 0$ to $t = \tau$):

$$d_1 = \frac{1}{2}g\tau^2.$$

Distance from $t = 0$ to $t = 2\tau$:

$$d_{\text{total}} = \frac{1}{2}g(2\tau)^2 = 2g\tau^2.$$

Distance traversed in the second interval (from $t = \tau$ to $t = 2\tau$) is

$$\begin{aligned}d_2 &= d_{\text{total}} - d_1 \\ &= 2g\tau^2 - \frac{1}{2}g\tau^2 = \frac{3}{2}g\tau^2.\end{aligned}$$

- Given $y_0 = \frac{1}{2}g\tau^2$ (distance in first interval). The distance traversed in the second interval is.

$$\begin{aligned}d_2 &= \frac{3}{2}g\tau^2 \\ &= 3 \times \left(\frac{1}{2}g\tau^2\right) = 3y_0\end{aligned}$$

This aligns with the 1:3 ratio for the first two intervals.

- 189.** A car is moving along a straight highway at a speed of 126 km/h. The driver applies brakes, and the car is brought to a stop within a distance of 200 m. Assume the retardation (deceleration) of the car is uniform.

Questions:

- Which kinematic equation relates initial velocity, final velocity, acceleration, and displacement?
- When a car is retarding, is its acceleration positive or negative (assuming the initial velocity is in the positive direction)?
- Convert the initial speed of 126 km/h to m/s.
- Calculate the retardation of the car.

Ans :

- The kinematic equation $v^2 = v_0^2 + 2ax$ relates initial velocity (v_0), final velocity (v), acceleration (a), and displacement (x).
- If the initial velocity is in the positive direction, retardation (slowing down) implies the acceleration is in the opposite direction, so it is negative.

- Initial speed

$$\begin{aligned}v_0 &= 126\text{km/h} \\ &= 126 \times \frac{1000\text{m}}{3600\text{s}} \\ &= 35\text{m/s}.\end{aligned}$$

- Here $v_0 = 35\text{m/s}$, $v = 0\text{m/s}$ (car stops), $x = 200\text{m}$.

$$\text{Using } v^2 = v_0^2 + 2ax:$$

$$0^2 = (35\text{m/s})^2 + 2 \times a \times (200\text{m})$$

$$0 = 1225 + 400a$$

$$a = -1225/400 = -3.0625\text{m/s}^2.$$

The retardation is 3.0625m/s^2 .

- 190.** To measure her reaction time, a student asks her friend to drop a ruler vertically through the gap between her thumb and forefinger. She catches it after it has travelled a distance d . In one trial, the distance d was found to be 21.0 cm . (Assume $g = 9.8\text{m/s}^2$)

Questions:

1. What is reaction time in the context of this experiment?
2. The ruler drops under free fall. What is its initial velocity (v_0) and acceleration (a)?
3. Which kinematic equation can be used to relate distance (d), initial velocity (v_0), acceleration (a), and reaction time (t_r)?
4. Estimate the student's reaction time based on the distance $d = 21.0\text{ cm}$.

Ans :

1. Reaction time is the time a person takes to observe, think, and act. In this experiment, it's the time elapsed from when the friend drops the ruler to when the student catches it.
2. The ruler drops under free fall, so its initial velocity $v_0 = 0$.
Its acceleration $a = g = 9.8\text{m/s}^2$ (taking downward as positive, or $a = -g$ if using the text's convention in Example 2.7).
3. The kinematic equation $d = v_0 t_r + \frac{1}{2} a t_r^2$ can be used. Since $v_0 = 0$, this simplifies to $d = \frac{1}{2} a t_r^2$. (uses $d = -\frac{1}{2} g t_r^2$ taking 'd' as negative displacement, which leads to the same magnitude for t_r).
4. Given $d = 21.0\text{cm} = 0.21\text{m}$, and $g = 9.8\text{m/s}^2$.

Using $d = \frac{1}{2} g t_r^2$:

$$\begin{aligned} t_r &= \sqrt{\frac{2d}{g}} \\ &= \sqrt{\frac{2 \times 0.21\text{m}}{9.8\text{m/s}^2}} = \sqrt{\frac{0.42}{9.8}} \\ &\approx \sqrt{0.042857} \approx 0.207\text{s}. \\ t_r &\approx 0.2\text{s}. \end{aligned}$$

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- 191.** An object moves in a straight line. Its velocity-time (v - t) graph shows that initially, for t_1 seconds, its velocity is positive and decreasing (negative acceleration). Then, at time t_1 , its velocity becomes zero. For $t > t_1$, its velocity becomes increasingly negative, meaning it moves in the opposite direction with the same magnitude of negative acceleration.

Questions:

1. If an object is moving in the positive direction with negative acceleration, is its speed increasing or decreasing?
2. What happens to the object's direction of motion at time t_1 ?
3. If the negative acceleration has a magnitude of ' a ', write the equation for velocity $v(t)$ for $0 \leq t \leq t_1$ if its initial velocity at $t = 0$ was v_0 .
4. Describe the motion of the object between t_1 and t_2 (where $t_2 > t_1$) if it continues with the same negative acceleration. What would its position-time graph look like qualitatively from $t = 0$ to t_2 ?

Ans :

1. If an object is moving in the positive direction with negative acceleration, its speed is decreasing.
2. At time t_1 , the object's velocity is zero, meaning it momentarily stops and then reverses its direction of motion because the acceleration is still negative.
3. $v(t) = v_0 - at$ for $0 \leq t \leq t_1$.
4. Between t_1 and t_2 , the object moves in the negative direction with increasing speed (since acceleration is negative and velocity is also becoming more negative). The position-time graph would initially curve upwards (positive velocity, decreasing slope magnitude), reach a maximum point at t_1 , and then curve downwards (negative velocity, increasing slope magnitude). The curve would be parabolic due to constant acceleration.

- 192.** The position of an object moving along the x -axis is given by the equation $x = a + bt^2$, where $a = 8.5\text{m}$ and $b = 2.5\text{m/s}^2$, and t is measured in seconds.

Questions:

1. What are the physical quantities represented by ' a ' and ' b ' in this equation, based on their units and the structure of kinematic equations?
2. What is the initial position of the object (at $t=0\text{s}$) ?
3. Calculate the instantaneous velocity of the object as a function of time using differentiation

$$v = \frac{dx}{dt}.$$

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4. What is the object's acceleration? Is this motion uniformly accelerated?

Ans :

1. 'a' represents the initial position (x_0) since its unit is meters. 'b' is related to acceleration; comparing $x = x_0 + v_0 t + \frac{1}{2} a t^2$ with $x = a + b t^2$, if $v_0 = 0$, then $b = \frac{1}{2} a_{acc}$, so $2b$ is the acceleration. The unit of 'b' is m/s^2 , confirming it relates to acceleration.
2. At $t=0\text{s}$, $x = a + b(0)^2 = a = 8.5\text{m}$. So, the initial position is 8.5 m.

$$\begin{aligned} 3. \quad v &= \frac{dx}{dt} \\ &= \frac{d}{dt}(a + b t^2) \\ &= \frac{d}{dt}(a) + \frac{d}{dt}(b t^2) \\ &= 0 + 2b t = 2b t. \end{aligned}$$

So,

$$v(t) = 2 \times (2.5\text{m/s}^2) \times t = 5.0t \text{ m/s}.$$

4. Acceleration.

$$\begin{aligned} a_{acc} &= \frac{dv}{dt} = \frac{d}{dt}(2b t) \\ &= 2b = 2 \times 2.5\text{m/s}^2 \\ &= 5.0\text{m/s}^2 \end{aligned}$$

Since the acceleration 5.0m/s^2 is a constant, the motion is uniformly accelerated.

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CHAPTER 3

Motion in a Plane

NODIA

SUMMARY

1. INTRODUCTION

The concept of direction was limited in one dimensional motion and only positive or negative values were used. But to describe motion in two or three dimensions, the concept of vectors has to be introduced. Now let us discuss about some important properties of vectors.

2. SCALARS

Scalars are physical quantities that have only magnitude and no direction, such as mass, length, time, temperature, real numbers, speed, work, and power.

3. VECTORS

Vectors are physical quantities that have both magnitude and direction, such as displacement, velocity, momentum, force, and acceleration. A vector is fully defined by its numerical value (including units) along with its direction in space

3.1 Representation of a vector

A vector is represented by a letter with an arrow on it e.g. $\vec{A}$. It may also be denoted by a bold face letter e.g. $\mathbf{A}$. The magnitude is represented by $|\vec{A}|$ (called mod $\vec{A}$) or A .

3.2 Importance of vectors

- Many laws of physics are expressed in compact form by using vectors.
- In many laws of physics, the derivation involves can be greatly simplified by using vectors.

Multiplying a vector by a real number either leaves the direction unchanged or reverses it, depending on whether the number is positive or negative

respectively, while the length of the vector is scaled by the magnitude of the number.

Triangle law of vector addition. If two vectors are represented both in magnitude and direction by two sides of a triangle taken in order, then their resultant is represented both in magnitude and direction by the third side of the triangle taken in the opposite direction.

4. LAWS OF VECTORS

The law of vectors are as follows :

4.1 Triangle Law of Vector Addition :

The Triangle Law of Vector Addition states that if two vectors are represented in both magnitude and direction as two sides of a triangle taken in order, then the third side of the triangle, taken in the reverse order, represents the resultant vector in both magnitude and direction.

4.2 Parallelogram law method :

If two vectors are represented in both magnitude and direction by two adjacent sides of a parallelogram starting from the same point, then their resultant is given in both magnitude and direction by the diagonal of the parallelogram drawn from that point.

4.3 Polygon law of vectors

“If a number of vectors are represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented completely by the closing side of the polygon taken in the opposite order.”

4.4 Resolution of a vector

The splitting up of a single vector into a number of vectors is called resolution of a vector and each vector thus obtained is called a component vector.

Resultant of two vectors $\vec{P}$ and $\vec{Q}$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\text{and } \tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

5. MULTIPLICATION OF VECTOR

Scalar product or dot product of two vectors $\vec{A}$ and $\vec{B}$ is represented by $\vec{A} \cdot \vec{B}$ is a scalar quantity and is given by the product of magnitude of $\vec{A}$ and $\vec{B}$ and cosine of the smaller angle between them.

$$\text{i.e. } \vec{A} \cdot \vec{B} = AB\cos\theta$$

Vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is a single vector whose magnitude is $AB\sin\theta$ and direction is perpendicular to the plane determined by two vectors

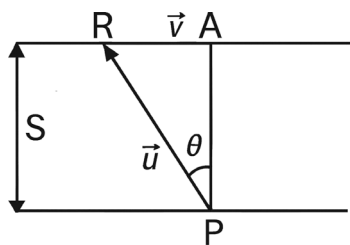
$$\text{i.e. } \vec{A} \times \vec{B} = AB\sin\theta \hat{n}$$

6. MOTION OF A BOAT IN A RIVER

(a) To cross the river in the shortest path.

The boat has to move along PR , and

$$\sin\theta = \frac{v}{u}$$



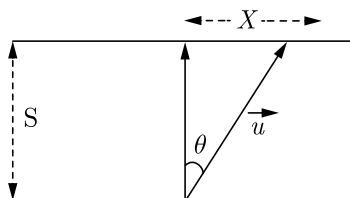
Also, time to cross the river

$$t = \frac{s}{\sqrt{u^2 - v^2}}$$

(b) To cross the river in shortest time.

The boat has to move at right angle to the flow of river and

$$t = \frac{s}{u}$$



7. MOTION IN A PLANE AND SPACE

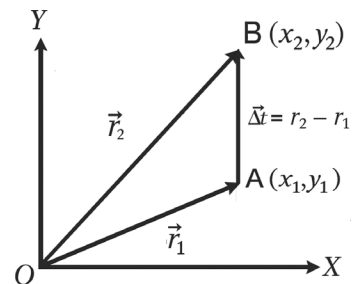
The motion of a body that occurs in two dimensions is known as motion in a plane, while the motion

that occurs in three dimensions is referred to as motion in space.

7.1 Displacement in two dimensions

If a particle is moving in xy having position vectors $\vec{r}_1$ and $\vec{r}_2$ in time t_1 and t_2 respectively as shown in Figure then using triangle law of vectors

$$\vec{OB} = \vec{OA} + \vec{AB}$$



$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \dots(i)$$

where $\Delta\vec{r}$ is displacement vector

$$\text{and } \vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

$$\text{and } \vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} \quad \dots(ii)$$

7.2 Velocity in two dimensions

If $\Delta\vec{r}$ is the displacement of a particle in time interval Δt , (Figure) then average velocity

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \quad \dots(iii)$$

$$\text{or } \vec{v}_{av} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{\Delta t}$$

$$= \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

When particle is moving with uniform velocity the above equation becomes

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\text{or } \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\text{or } |\vec{v}| = v = \sqrt{v_x^2 + v_y^2}$$

Similarly, acceleration in two dimensions is given by

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\text{or } |\vec{a}| = a = \sqrt{a_x^2 + a_y^2}$$

8. PROJECTILE

An object thrown into space and then moving under the action of earth's gravity is called a projectile.

Trajectory

The path followed by a projectile is called its trajectory.

Motion of projectile:

If a projectile is projected horizontally or in a direction making an angle θ with the horizontal, its path is a parabola.

8.1 Time of flight:

It is the time for which the projectile remains in air above the horizontal plane from the point of projection. It is given by

$$T = \frac{2u \sin \theta}{g}$$

8.2 Maximum height:

It is the greatest height to which a projectile rises above the point of projection. It is given by

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

8.3 Horizontal range:

It is the distance covered by projectile along horizontal between the point of projection to the point on ground, where the projectile returns again. It is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Angle of projection for maximum range:

It is given by

$$R_{\max} = \frac{u^2}{g}$$

when

$$\theta = 45^\circ$$

Two angles of projection:

There are two angles θ and $(90^\circ - \theta)$ for which the projectile covers the same horizontal range.

9. CIRCULAR MOTION :

If a particle moves in a circular path with a point as the centre such that it describes a definite angle in a definite time then its motion is called circular motion.

9.1 Angular velocity :

The angle which the particle describes in unit time is called angular velocity. If the particle makes n revolutions round the circle in a second then its angular velocity, $\omega = 2\pi n$ radians s^{-1} .

9.2 Centripetal acceleration :

The change in the direction of velocity vector as the particle moves round the circle produces an acceleration. This acceleration is always directed towards the centre. That is, it is always directed perpendicular to the velocity. It is called centripetal acceleration.

NCERT TEXT-BOOK QUESTIONS

1. State for each of the following physical quantities, if it is a scalar or a vector :
Volume, mass, speed, acceleration, density, number of moles, velocity, angular velocity, displacement and angular frequency.

Ans :

Scalars : Volume, mass, speed, density, number of moles, angular frequency.

Vectors : Acceleration, velocity, displacement, angular velocity.

2. Pick out the two scalar quantities in the following list :
Force, angular momentum, work, current, linear momentum, electric-field, average velocity, magnetic moment, relative velocity.

Ans :

Work, current.

3. Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction and charge.

Ans :

Impulse.

4. State with reasons, whether the following algebraic operations with scalars and vectors are meaningful:
 - (a) Adding any two scalars
 - (b) Adding a scalar to a vector of the same dimensions
 - (c) Multiplying any vector by any scalar
 - (d) Multiplying any two scalars
 - (e) Adding any two vectors
 - (f) Adding a component of a vector to the same vector.

Ans :

- (a) Adding any two scalars is not meaningful because they may not be of the same dimensions.

- (b) Adding a scalar to a vector of the same dimensions is not meaningful because scalar has no direction whereas vector has a direction.
- (c) Multiplying any vector by any scalar is meaningful e.g.

$$m(\vec{A}) = (m\vec{A})$$

- (d) Multiplying any two scalars is meaningful.
- (e) Adding any two vectors is not meaningful because the vectors must be of the same nature.
- (f) Adding a component of a vector to the same vector is meaningful.

5. Read each statement below carefully and state with reasons, if it is true or false :

- (a) the magnitude of a vector is always a scalar
- (b) each component of a vector is always a scalar
- (c) the total path length is always equal to the magnitude of the displacement vector of a particle
- (d) the average speed of a particle (defined a total path length divided by the time taken to cover the path) is either greater than or equal to the magnitude of average velocity of the particle over the same interval of time.
- (e) three vectors not lying in a plane can never add up to give null vector.

Ans :

- (a) True, because, the magnitude of a vector is completely specified simply by its numerical value, which is a scalar.
- (b) False, because each component of a vector is always a vector.
- (c) False, because displacement has in general a magnitude which is not equal to the actual path length.
- (d) True, because path length of actual path is either greater than or equal to displacement.
- (e) True, because the resultant of two vectors is in the same plane as that of two vectors.

6. Establish the following vector inequalities geometrically or otherwise :

- (i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
- (ii) $|\vec{a} + \vec{b}| \geq ||\vec{a}| - |\vec{b}||$
- (iii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$
- (iv) $|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$

When does the equality sign in above cases apply ?

Ans :

- (i) Consider two vectors $\vec{a}$ and $\vec{b}$ such that

$$\vec{a} + \vec{b} = \vec{R}, \vec{a} - \vec{b} = \vec{R}'$$

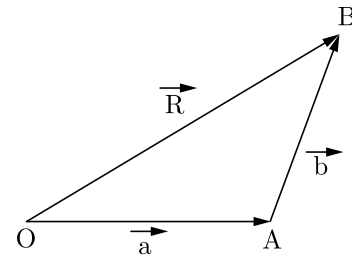


Figure : (a)

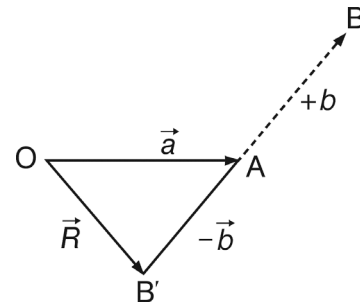


Figure : (b)

Then geometrically, the resultant of $\vec{a}$, $\vec{b}$ is $\vec{R}$ and the resultant of $\vec{a}$, $-\vec{b}$ is $\vec{R}'$.

In Figure (a),

$$|\vec{a}| = OA, |\vec{b}| = AB,$$

$$|\vec{R}| = OB = |\vec{a} + \vec{b}|$$

In a Δ , third side is always less than the sum of the other two sides,

$$OB < OA + AB$$

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \quad \dots(2)$$

Moreover, if the two vectors $\vec{a}$ and $\vec{b}$ are in the same direction, then

$$OB = OA + AB$$

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \quad \dots(2)$$

Combining (1) and (2), we get

$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

$$(ii) a^2 + b^2 + 2ab \cos \theta \geq a^2 + b^2 - 2ab \quad \dots(3)$$

(for $0 \leq \theta \leq \pi$)

$$\text{L.H.S. } |\vec{a} + \vec{b}|^2$$

$$\text{R.H.S. } = (a - b)^2 = (|\vec{a}| - |\vec{b}|)^2$$

$$\text{From (3), } |\vec{a} + \vec{b}|^2 \geq (|\vec{a}| - |\vec{b}|)^2$$

$$|\vec{a} + \vec{b}| \geq (|\vec{a}| - |\vec{b}|)$$

$$(iii) (\vec{a} - \vec{b})^2 = a^2 + b^2 - 2ab \cos \theta$$

$$< a^2 + b^2 + 2ab$$

$$(\because \cos \theta \leq 1 \text{ for } 0 \leq \theta \leq \pi)$$

$$\begin{aligned} \text{or } |\vec{a} - \vec{b}|^2 &\leq (|\vec{a}| + |\vec{b}|)^2 \\ |\vec{a} - \vec{b}| &= a^2 + b^2 - 2ab\cos\theta \\ &\geq a^2 + b^2 - 2ab \\ (\because \cos\theta \leq 1 \text{ for } 0 \leq \theta < \pi) \end{aligned}$$

$$\text{i.e. } |\vec{a} - \vec{b}|^2 \geq (\vec{a} - \vec{b})^2$$

$$\text{i.e. } |\vec{a} - \vec{b}|^2 \geq (|\vec{a}| - |\vec{b}|)^2$$

$$|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$$

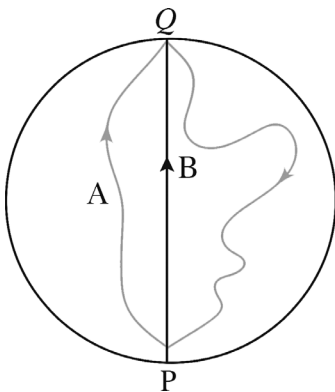
7. Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, which of the following statements are correct :

- (i) $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ must each be a null vector,
- (ii) The magnitude of $(\vec{a} + \vec{c})$ equals the magnitude of $(\vec{b} + \vec{d})$,
- (iii) The magnitude of $\vec{a}$ can never be greater than the sum of magnitude of $\vec{b}$, $\vec{c}$ and $\vec{d}$.
- (iv) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$ if they are collinear.

Ans :

- (i) $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ must each be a vector is not a correct statement.
- (ii) correct (iii) correct (iv) correct.

8. Three girls skating on a circular ice ground of radius 200 m start from a point P the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement vector for each girl? For which girl is this equal to the actual length of path skated?

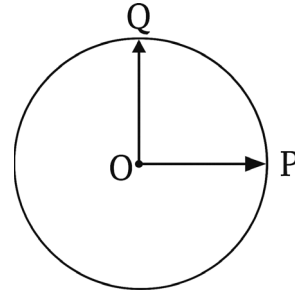


Ans :

Magnitude of the displacement vector for each girl = diameter = $2 \times 200 = 400$ m.

This is equal to the actual length of the path skated for girl B.

9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown. If the round trip takes 10 min, what is the (a) net displacement (b) average velocity and (c) average speed of the cyclist?



Ans :

- (a) Net displacement = zero
- (b) Average velocity = $\frac{\text{Net displacement}}{\text{Total time taken}} = \text{zero}$
- (c) Total distance travelled

$$\begin{aligned} &= OP + PQ + QO \\ &= OP + QO + PR \\ &= OP + OQ + \frac{1}{4}(2\pi r) \\ &= \left(1 + 1 + \frac{1}{4} \times 2 \times \frac{22}{7} \times 1\right) \text{ km} \\ &= \frac{25}{7} \text{ km} \end{aligned}$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total path length}}{\text{Total time taken}} \\ &= \frac{25/7}{10/60} \text{ kmh}^{-1} \\ &= 21.4 \text{ kmh}^{-1} \end{aligned}$$

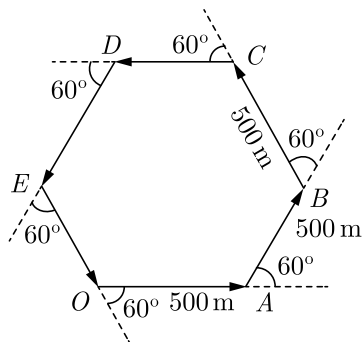
10. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Ans :

Suppose the motorist starts from the point A and moves in the direction AB then along BC.

- (i) Displacement at third turn : x -component of displacement of the motorist at the third turn

$$\begin{aligned} S_x &= 500 + 500 \cos 60^\circ + 500 \cos 120^\circ \\ &= 500 + 500 \times \frac{1}{2} + 500 \left(-\frac{1}{2}\right) \\ &= 500 \text{ m} \end{aligned}$$



y -component of displacement of the motorist at the third turn,

$$\begin{aligned} S_y &= 0 + 500 \sin 60^\circ + 500 \sin 120^\circ \\ &= 500 \times \frac{\sqrt{3}}{2} + 500 \frac{\sqrt{3}}{2} \\ &= 500\sqrt{3} \end{aligned}$$

Magnitude of resultant displacement

$$\begin{aligned} &\sqrt{S_x^2 + S_y^2} \\ &= \sqrt{(500)^2 + (500\sqrt{3})^2} \\ &= 500 \times \sqrt{4} = 500 \times 2 \\ &= 1000 \text{ km} \end{aligned}$$

If β is the angle which resultant displacement vector makes with x -axis, then

$$\begin{aligned} \tan \beta &= \frac{S_y}{S_x} = \frac{500\sqrt{3}}{500} = \sqrt{3} \\ \beta &= 60^\circ \end{aligned}$$

Hence the resultant displacement makes an angle of 60° with the direction of x -axis.

- (ii) Displacement at sixth turn: At the sixth turn, motorist is again at point O . The total displacement is O and again the direction of motorist is OA .

$$\text{Distance travelled} = 6 \times 500 \text{ m} = 3 \text{ km}$$

- (iii) Displacement at eighth turn : x -component of displacement after 8 turns,

$$\begin{aligned} S_x &= 500 \sin 300^\circ + 500 \sin 600^\circ = 0 \\ &= 500 \times 1 + 500 \left(+\frac{1}{2} \right) = 750 \end{aligned}$$

y -component,

$$\begin{aligned} S_y &= 500 \sin 300^\circ + 500 \sin 600^\circ \\ &= 500 \times 0 + 500 \times \sqrt{\frac{3}{2}} = 250\sqrt{3} \\ S &= \sqrt{S_x^2 + S_y^2} \\ &= \sqrt{(750)^2 + (250\sqrt{3})^2} \\ &= 866 \text{ m} \end{aligned}$$

If β' is the angle which the resultant makes with initial direction, then

$$\begin{aligned} \tan \beta' &= \frac{S_y}{S_x} = \frac{250\sqrt{3}}{750} = \frac{1}{\sqrt{3}} \\ \beta' &= 30^\circ \end{aligned}$$

Actual length of path after 8th turn

$$= 8 \times 500 \text{ m} = 4 \text{ km}$$

11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (a) the average speed of the taxi (b) the magnitude of average velocity ? Are the two equal ?

Ans :

- (a) Average speed of taxi

$$\begin{aligned} &= \frac{\text{Total path}}{\text{Time}} \\ &= \frac{23 \times 60}{28} \text{ kmh}^{-1} \\ &= 49.3 \text{ kmh}^{-1} \end{aligned}$$

- (b) Average velocity = $\frac{\text{Net displacement}}{\text{Total time taken}}$

$$\begin{aligned} &= \frac{10 \times 60}{28} = \frac{150}{7} \text{ kmh}^{-1} \\ &= 21.4 \text{ kmh}^{-1} \end{aligned}$$

No, they are not always equal. The average speed equals average velocity's magnitude only for a straight path.

12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ?

Ans :

$$\begin{aligned} h &= \frac{u^2 \sin^2 \theta}{2g} \\ 25 &= \frac{40 \times 40 \times \sin^2 \theta}{2 \times 9.8} \end{aligned}$$

$$\text{or } \sin \theta = \sqrt{\frac{4.9}{16}} = \frac{\sqrt{4.9}}{4}$$

$$\text{and } \cos \theta = \sqrt{1 - \sin^2 \theta} = \frac{\sqrt{11.1}}{4}$$

$$\begin{aligned} R &= \frac{u^2 \sin 2\theta}{g} \\ &= \frac{u^2 \times 2 \sin \theta \cos \theta}{g} \\ &= \frac{40 \times 40 \times \sqrt{4.9} \times \sqrt{11.1}}{9 \times 8 \times 4 \times 4} \\ &= 150.5 \text{ m} \end{aligned}$$

- 13.** A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

Ans :

Let the velocity of projection be u

The maximum horizontal range,

$$\begin{aligned} R_{\max} &= \frac{u^2}{g} \\ \frac{u^2}{g} &= 100 \end{aligned} \quad \dots(i)$$

The maximum height attained,

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

For the maximum vertical range, the angle of projection $\theta = 90^\circ$

$$h = \frac{u^2 \sin^2 90^\circ}{2g} = \frac{u^2}{2g} \quad \dots(2)$$

From (i) and (2),

$$\begin{aligned} H_{\max} &= \frac{1}{2} R = \frac{1}{2} \times 100 \text{ m} \\ &= 50 \text{ m} \end{aligned}$$

- 14.** A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

Ans :

Given $r = 80 \text{ cm} = 0.80 \text{ m}$, $v = \frac{14}{25}$

$$\begin{aligned} \omega &= 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{25} \\ &= \frac{88}{25} \text{ radian s}^{-1} \end{aligned}$$

$$\begin{aligned} v &= r \omega = \frac{80}{100} \times \frac{88}{25} \\ &= \frac{4 \times 88}{125} \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{Acceleration} &= \frac{v^2}{r} \\ &= \frac{4 \times 4 \times 88 \times 88 \times 100}{125 \times 125 \times 80} = 9.9 \text{ ms}^{-2} \end{aligned}$$

It acts along the radius towards the centre of the circle.

- 15.** An aircraft executes a horizontal loop of radius 1.00 km with steady speed of 900 kmh^{-1} . Compare its centripetal acceleration with the acceleration due to gravity.

Ans :

$$\begin{aligned} R &= 1 \text{ km} = 1000 \text{ m} \\ v &= 90 \text{ kmh}^{-1} = 250 \text{ ms}^{-1} \end{aligned}$$

Centripetal acceleration

$$\begin{aligned} &= \frac{v^2}{r} = \frac{250 \times 250}{1000} \\ &= 62.5 \text{ ms}^{-2} \end{aligned}$$

Centripetal acceleration/acceleration due to gravity

$$= \frac{62.5}{9.8} = 6.4 \text{ ms}^{-2}$$

- 16.** Read each statement below carefully and state, with reasons, if it is true or false :

- The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.
- The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
- The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Ans :

- False. It is true only for uniform circular motion.
- True. Because while leaving the circular path, the particle moves tangentially to the circular path.
- True. Because it again comes to its original position.

- 17.** The position of a particle is given by

$$\vec{r} = (3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}) \text{ m}$$

where t is in seconds and coefficients have the proper units for $\vec{r}$ to be in metres.

- Find the $\vec{v}$ and $\vec{a}$ of the particle.
- What is the magnitude and direction of velocity of the particle at $t = 2 \text{ s}$?

Ans :

Given $\vec{r} = (3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}) \text{ m}$

$$(a) \quad \vec{v} = \frac{d\vec{r}}{dt} = (3.0\hat{i} - 4.0t\hat{j}) - 0\hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 0 - 4.0\hat{j} = -4.0\hat{j}$$

$$(b) \quad \text{For } t = 2 \text{ s}$$

$$\vec{v} = (3.0\hat{i} - 4.0 \times 2\hat{j})$$

$$= (3.0\hat{i} - 8.0\hat{j}) = (3\hat{i} + 8\hat{j})$$

$$\vec{v} = \sqrt{3^2 + 8^2} = \sqrt{9 + 64}$$

$$\vec{v} = \sqrt{73} = 8.54 \text{ ms}^{-1}$$

$$\text{and } \tan \theta = \frac{8}{3} = 2.7$$

$$\theta = \tan^{-1} 2.7 = 69.7^\circ = 70^\circ$$

18. A particle starts from the origin at $t = 0$ s with a velocity of $10.0 \hat{j} \text{ ms}^{-1}$ and moves in x - y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j}) \text{ ms}^{-2}$
- (a) At what time is the x -coordinate of the particle 16 m ? What is the y -coordinate of the particle at that time ?
- (b) What is the speed of the particle at that time ?

Ans :

- (a) The position of the particle is given by

$$\begin{aligned}\vec{r}(t) &= \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\ &= 10.0\hat{i}t + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})t^2 \\ &= 10\hat{i}t + 4\hat{i}t^2 + 1\hat{j}t^2 \\ &= (10t + 4t^2)\hat{i} + 1t^2\hat{j}\end{aligned}$$

$$x(t) = 10t + 4t^2$$

and $y(t) = 1t^2\hat{j}$

- (a) Given $x(t) = 16 \text{ m}$, $t = ?$

So $16 = 10t + 4t^2$

or $2t^2 + 5t - 8 = 0$

or $t = 1.1 \text{ s}$

At $t = 1.1 \text{ s}$, $y = 1(1.1)^2 = 1.21 \text{ m}$

(b) $\vec{v} = \frac{d\vec{r}}{dt} = (10 + 8t)\hat{i} + 2.2\hat{j}$
 $= 18.8\hat{i} + 2.2\hat{j}$

$$\begin{aligned}\text{Speed} &= |\vec{v}| = \sqrt{(18.8)^2 + (2.2)^2} \\ &= 18.9 \text{ ms}^{-1}\end{aligned}$$

19. $\hat{i}$ and $\hat{j}$ are unit vectors along x -axis and y -axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

Ans :

Magnitude of the vector $\hat{i} + \hat{j}$,

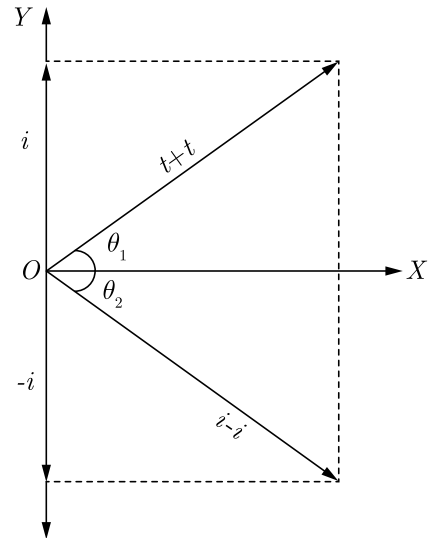
$$\begin{aligned}|\hat{i} + \hat{j}| &= \sqrt{(1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos \theta} \\ &= \sqrt{2}\end{aligned}$$

$$\tan \theta = \frac{|\hat{i}|}{|\hat{j}|} = 1$$

or $\theta = 45^\circ$ with direction of the x -axis.

Magnitude of the vector $(\hat{i} - \hat{j})$

$$|\hat{i} - \hat{j}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$



$$\tan \theta_2 = -\frac{|\hat{i}|}{|\hat{j}|} = -1$$

or $\theta_2 = -45^\circ$ with the direction of x -axis.

Component of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the direction $(\hat{i} + \hat{j})$

$$= \frac{(2+3)}{\sqrt{2}} = \frac{5}{\sqrt{2}} \quad \left[\because A \cos \theta = \frac{|\vec{A}|}{|\vec{B}|} \right]$$

Component of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the direction $(\hat{i} - \hat{j})$

$$= \frac{(2-3)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

20. For any arbitrary motion in space, which of the following relations are true :

- (i) $\vec{v}_{\text{average}} = (1/2)\{\vec{v}(t) + \vec{v}(t_2)\}$
 (ii) $\vec{v}_{\text{average}} = \{\vec{r}(t_1) - \vec{r}(t_2)\}/(t_2 - t_1)$
 (iii) $\vec{v}(t) = \vec{v}(0) = a\vec{t}$
 (iv) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \left(\frac{1}{2}\right)a\vec{t}^2$
 (v) $\vec{a}_{\text{average}} = \{v(t_2) - v(t_1)\}/(t_2 - t_1)$

Ans :

- (i) It is not true because it holds for uniform acceleration.
 (ii) True because it is a general result.
 (iii) It is not true. It holds for uniform acceleration.
 (iv) It is not true.
 (v) It is true because it is the general result.

21. Read each statement below carefully and state, with reasons and examples, if it is true or false : A scalar quantity is one that

- (a) is conserved in a process
 (b) can never take negative values
 (c) must be dimensionless

- (d) does not vary from one point to another in space.
 (e) has the same value for observers with different orientations of axes.

Ans :

- (a) False, because the scalar quantity may not be conserved, e.g., speed of body may be variable.
 (b) False, because scalar may have a -ve value, e.g., speed may be negative.
 (c) False, because a scalar may have dimension, e.g., speed has dimension $[L^1 T^{-1}]$.
 (d) False, scalar may be variable e.g. speed.
 (e) True, since a scalar does not have any orientation in space therefore, it has the same value with different orientations of axes.

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22. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30° , what is the speed of the aircraft ?

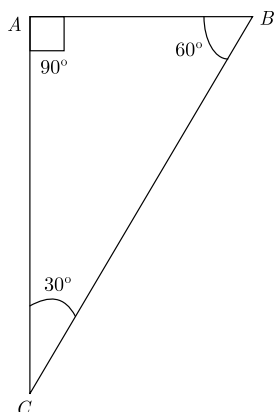
Ans :

The distance, $AC = 3400$ m

The distance, $AB = 10u$

where u is the speed of the aircraft.

$$\begin{aligned} \frac{10u}{\sin 30^\circ} &= \frac{3400}{\sin 60^\circ} \\ \text{or } u &= \frac{3400}{10} \times \frac{\sin 30^\circ}{\sin 60^\circ} \\ &= 340 \times \frac{1}{2} \times \frac{2}{\sqrt{3}} \\ u &= 196.3 \text{ ms}^{-1} \end{aligned}$$



OBJECTIVE QUESTIONS

1. Which of the following best defines a scalar quantity?
 (A) A quantity possessing both magnitude and a specified direction.
 (B) A quantity characterized by its magnitude only, specified by a number and unit.
 (C) A quantity that must adhere to the triangle law of vector addition.
 (D) A quantity whose value changes with the choice of coordinate system.

Ans :

(B) A quantity characterized by its magnitude only, specified by a number and unit.

A scalar quantity is completely specified by a single number, along with the proper unit, and has only magnitude, without any associated direction. Examples include mass, temperature, and time.

2. What characteristics define a vector quantity?
 (A) It is defined by its numerical magnitude and the appropriate physical unit.
 (B) It possesses both magnitude and direction and must obey the triangle law of addition.
 (C) It can be combined using ordinary algebraic rules for addition or subtraction.
 (D) It remains unchanged when displaced parallel to its original orientation.

Ans :

(B) It possesses both magnitude and direction and must obey the triangle law of addition.

A vector quantity is distinguished by having both a magnitude and a specific direction in space. Crucially, it must also conform to the triangle law (or parallelogram law) of vector addition.

3. Which of the following physical quantities is an example of a scalar?
 (A) Displacement
 (B) Velocity
 (C) Acceleration
 (D) Mass

Ans :

(D) Mass

Mass is a scalar quantity as it is fully described by its magnitude (e.g., 5 kg) and does not have an associated direction. Displacement, velocity, and acceleration are all vector quantities.

4. Identify the vector quantity from the following options:
- (A) Temperature
 - (B) Time
 - (C) Force
 - (D) Perimeter

Ans :

(C) Force

Force is a vector quantity because it has both magnitude (e.g., 10 Newtons) and a specific direction in which it acts. Temperature, time, and perimeter are scalar quantities.

5. What is the term for a vector representing the change in an object's position from an initial point to a final point?
- (A) Position vector
 - (B) Displacement vector
 - (C) Velocity vector
 - (D) Acceleration vector

Ans :

(B) Displacement vector

The displacement vector is defined as the straight line joining the initial and final positions of an object, indicating the net change in position, irrespective of the path taken.

6. Two vectors are considered equal if and only if:
- (A) They possess identical magnitudes, regardless of their respective directions.
 - (B) They point in the exact same direction, irrespective of their magnitudes.
 - (C) They have the same magnitude and the same direction.
 - (D) They originate from the same starting point in space.

Ans :

(C) They have the same magnitude and the same direction.

For two vectors to be equal, they must satisfy both conditions: they must have the same length (magnitude) and they must point in the exact same direction.

7. What happens to a vector when it is multiplied by a positive real number λ ?
- (A) Its direction reverses, and its magnitude changes by a factor of λ .
 - (B) Its direction remains the same, and its magnitude changes by a factor of λ .

- (C) Its direction rotates by an angle λ , and its magnitude remains unchanged.
- (D) Its direction becomes opposite, and its magnitude is scaled by $1/\lambda$.

Ans :

(B) Its direction remains the same, and its magnitude changes by a factor of λ .

Multiplying a vector by a positive real number λ scales its magnitude by λ . The direction of the vector remains unchanged. For example, $2\mathbf{A}$ has twice the magnitude of $\mathbf{A}$ but the same direction.

8. If a vector $\mathbf{A}$ is multiplied by a negative real number $-\lambda$, the resultant vector:
- (A) Retains its original direction, and its magnitude becomes $\lambda |\mathbf{A}|$.
 - (B) Has its direction reversed and its magnitude becomes $\lambda |\mathbf{A}|$.
 - (C) Retains its original direction, and its magnitude becomes $-\lambda |\mathbf{A}|$.
 - (D) Has its direction reversed, and its magnitude becomes $|\mathbf{A}|/\lambda$.

Ans :

(B) Has its direction reversed, and its magnitude becomes $\lambda |\mathbf{A}|$.

Multiplying a vector $\mathbf{A}$ by a negative number $-\lambda$ reverses the direction of $\mathbf{A}$ and changes its magnitude by a factor of λ . For example, $-2\mathbf{A}$ is in the opposite direction to $\mathbf{A}$ and has twice its magnitude.

9. Which law states that the order of addition of two vectors does not affect their sum (i.e., $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$)?
- (A) Associative law of vector addition
 - (B) Commutative law of vector addition
 - (C) Distributive law of vector addition
 - (D) Triangle law of vector addition

Ans :

(B) Commutative law of vector addition

The commutative law of vector addition states that the sum of two vectors is independent of the order in which they are added. Thus, $\mathbf{A} + \mathbf{B}$ is equivalent to $\mathbf{B} + \mathbf{A}$.

The law that states $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$ for vector addition is known as:

- (A) The commutative law of vectors
- (B) The associative law of vectors
- (C) The distributive law of vectors
- (D) The parallelogram law of vectors

Ans :

(B) The associative law of vectors

The associative law of vector addition implies that when adding three or more vectors, the way in which they are grouped for addition does not change the final resultant vector.

11. What is a null vector or a zero vector?

(A) A vector with unit magnitude and unspecified direction.

(B) A vector with zero magnitude and a specific direction.

(C) A vector with zero magnitude and no specific direction.

(D) A vector whose components are all equal to one.

Ans :

(C) A vector with zero magnitude and no specific direction.

A null vector, or zero vector, is defined as a vector that has zero magnitude. Since its magnitude is zero, its direction cannot be specified and is considered indeterminate.

12. How is the subtraction of vector **B** from vector **A** (i.e., $\mathbf{A} - \mathbf{B}$) defined in terms of vector addition?

(A) As the vector sum of **A** and **B** ($\mathbf{A} + \mathbf{B}$).

(B) As the vector sum of **A** and the negative of **B** ($\mathbf{A} + (-\mathbf{B})$).

(C) As the vector sum of **B** and the negative of **A** ($\mathbf{B} + (-\mathbf{A})$).

(D) As the vector difference between the magnitudes of **A** and **B**.

Ans :

(B) As the vector sum of **A** and the negative of **B** ($\mathbf{A} + (-\mathbf{B})$).

The subtraction of vector **B** from vector **A** is defined as the addition of vector **A** and the vector $-\mathbf{B}$ (which has the same magnitude as **B** but points in the opposite direction).

13. The process of splitting a vector into two or more vectors which, when added together, produce the original vector is called:

(A) Vector multiplication

(B) Vector differentiation

(C) Resolution of vectors

(D) Vector integration

Ans :

(C) Resolution of vectors

Resolution of a vector is the process of breaking it down into component vectors, typically along

mutually perpendicular axes. These component vectors, when added, yield the original vector.

14. What is a unit vector?

(A) A vector that always has a magnitude of zero.

(B) A vector having a unit magnitude and pointing in a specific direction.

(C) A vector whose direction is always along the positive x-axis.

(D) A vector that is used to define the units of physical quantities.

Ans :

(B) A vector having a unit magnitude and pointing in a specific direction.

A unit vector is a vector with a magnitude of exactly one. It is used primarily to specify a direction in space and has no dimensions or units associated with it.

What symbols are commonly used to denote the unit vectors along the x, y, and z-axes of a rectangular coordinate system, respectively?

(A) $\mathbf{a}, \mathbf{b}, \mathbf{c}$

(B) $\mathbf{x}, \mathbf{y}, \mathbf{z}$

(C) $\hat{i}, \hat{j}, \hat{k}$

(D) α, β, γ

Ans :

(C) $\hat{i}, \hat{j}, \hat{k}$

In a right-handed rectangular coordinate system, the unit vectors along the positive x, y, and z-axes are conventionally denoted by $\hat{i}, \hat{j}$, and $\hat{k}$, respectively. These vectors are mutually perpendicular.

16. If a vector **A** is represented in component form as $A_x \hat{i} + A_y \hat{j}$, what does A_x represent?

(A) The magnitude of vector **A** along the x-axis only.

(B) The x-component of the vector **A**, which is a scalar value.

(C) The unit vector in the direction of the x-axis.

(D) The angle that vector **A** makes with the y-axis.

Ans :

(B) The x-component of the vector **A**, which is a scalar value.

In the component form $A_x \hat{i} + A_y \hat{j}$, A_x and A_y are the scalar components of the vector **A** along the x and y-axes, respectively. $A_x \hat{i}$ is the vector component along the x-axis.

17. If a vector **A** has components A_x and A_y , its magnitude A is given by:

(A) $A_x + A_y$
 (B) $A_x^2 + A_y^2$
 (C) $\sqrt{A_x^2 + A_y^2}$
 (D) $|A_x - A_y|$

Ans :

(C) $\sqrt{A_x^2 + A_y^2}$

The magnitude of a vector A with components A_x and A_y in a plane is found using the Pythagorean theorem, which gives $A = \sqrt{A_x^2 + A_y^2}$.

18. The direction θ of a vector **A** with components A_x and A_y , with respect to the positive x-axis, is given by:

(A) $\tan \theta = \frac{A_x}{A_y}$
 (B) $\sin \theta = \frac{A_y}{A_x}$
 (C) $\cos \theta = \frac{A_x}{A}$
 (D) $\tan \theta = \frac{A_y}{A_x}$

Ans :

(D) $\tan \theta = \frac{A_y}{A_x}$

The angle θ that a vector **A** makes with the positive x-axis can be found using the tangent function, where $\tan \theta = \frac{A_y}{A_x}$.

Therefore, $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$.

19. When adding two vectors $\mathbf{A} = A_x \hat{i} + A_y \hat{j}$ and $\mathbf{B} = B_x \hat{i} + B_y \hat{j}$ analytically, the x-component of the resultant vector **R** is:

(A) $A_x - B_x$
 (B) $A_x + B_x$
 (C) $A_y + B_x$
 (D) $A_x \times B_x$

Ans :

(B) $A_x + B_x$

When adding vectors analytically, their corresponding components are added. Therefore, the x-component of the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is $R_x = A_x + B_x$.

20. The position vector **r** of a particle P located in a plane with reference to the origin is typically given by:

(A) $r = x\hat{i} - y\hat{j}$
 (B) $r = x\hat{j} + y\hat{i}$

(C) $r = x\hat{i} + y\hat{j}$
 (D) $r = \sqrt{x^2 + y^2}$

Ans :

(C) $r = x\hat{i} + y\hat{j}$

The position vector **r** of a particle P with coordinates (x, y) in a plane, relative to the origin, is expressed as $\mathbf{r} = x\hat{i} + y\hat{j}$, where x and y are the components along the x and y-axes.

21. If a particle moves from position **r** to **r'**, its displacement $\Delta \mathbf{r}$ is:

(A) $\mathbf{r} + \mathbf{r}'$
 (B) $\mathbf{r}' - \mathbf{r}$
 (C) $\mathbf{r} - \mathbf{r}'$
 (D) $|\mathbf{r}'| - |\mathbf{r}|$

Ans :

(B) $\mathbf{r}' - \mathbf{r}$

The displacement $\Delta \mathbf{r}$ of a particle is the change in its position vector. If the initial position vector is **r** and the final position vector is **r'**, then $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$.

22. The average velocity ($\bar{\mathbf{v}}$) of an object is defined as:

(A) The product of its displacement and the time interval.
 (B) The ratio of the total path length to the time interval.
 (C) The ratio of its displacement and the corresponding time interval.
 (D) The rate of change of its speed with respect to time.

Ans :

(C) The ratio of its displacement and the corresponding time interval.

Average velocity is a vector quantity defined as the displacement of an object divided by the time interval over which that displacement occurred ($\bar{\mathbf{v}} = \Delta \mathbf{r} / \Delta t$).

23. Instantaneous velocity **v** is defined as:

(A) The average velocity over a very long time interval.
 (B) The limit of the average velocity as the time interval approaches zero.
 (C) The total displacement divided by the total time of motion.
 (D) The magnitude of the displacement per unit time.

Ans :

(B) The limit of the average velocity as the time interval approaches zero.

Instantaneous velocity is the velocity of an object at a specific instant in time. It is mathematically defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small, or

$$\mathbf{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- 24.** The direction of the instantaneous velocity vector at any point on the path of an object is:
- Perpendicular to the path at that specific point.
 - Tangential to the path at that point and in the direction of motion.
 - Always directed towards the origin of the coordinate system.
 - Opposite to the direction of the object's acceleration.

Ans :

(B) Tangential to the path at that point and in the direction of motion.

The instantaneous velocity vector at any point on an object's trajectory is always directed along the tangent to the path at that point, indicating the direction of motion at that instant.

- 25.** If the velocity of an object is given by $\mathbf{v} = v_x \hat{i} + v_y \hat{j}$, then v_x is equal to:
- $\frac{dy}{dt}$
 - $\frac{dx}{dt}$
 - $\frac{d|\mathbf{v}|}{dt}$
 - $\frac{d\theta}{dt}$

Ans :

(B) $\frac{dx}{dt}$

For a velocity vector $\mathbf{v} = v_x \hat{i} + v_y \hat{j}$, the component v_x represents the rate of change of the x-coordinate with respect to time, so $v_x = \frac{dx}{dt}$. Similarly, $v_y = \frac{dy}{dt}$.

- 26.** The average acceleration $\bar{\mathbf{a}}$ of an object is defined as:
- The rate of change of its speed during a time interval.
 - The change in velocity divided by the time interval.
 - The product of its velocity and the time interval.
 - The velocity divided by the displacement during an interval.

Ans :

(B) The change in velocity divided by the time interval.

Average acceleration is a vector quantity defined as the change in velocity ($\Delta \mathbf{v}$) divided by the time interval (Δt) over which that change occurs ($\bar{\mathbf{a}} = \Delta \mathbf{v} / \Delta t$).

- 27.** Instantaneous acceleration $\mathbf{a}$ is defined as:
- The average acceleration over a significantly large time interval.
 - The total change in velocity divided by the total duration of motion.
 - The limit of the average acceleration as the time interval approaches zero.
 - The rate of change of the magnitude of velocity.

Ans :

(C) The limit of the average acceleration as the time interval approaches zero.

Instantaneous acceleration is the acceleration of an object at a specific instant in time. It's defined as the limit of the average acceleration as the time interval Δt becomes infinitesimally small, or $\mathbf{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$.

- 28.** For motion in two or three dimensions, the velocity and acceleration vectors:
- Must always be parallel to each other.
 - Must always be perpendicular to each other.
 - Can have any angle between 0° and 180° between them.
 - Must always be in opposite directions.

Ans :

(C) Can have any angle between 0° and 180° between them.

Unlike one-dimensional motion where velocity and acceleration are collinear, in two or three dimensions, the velocity and acceleration vectors can have any angle between 0° (speeding up in a straight line) and 180° (slowing down in a straight line) between them.

- 29.** If an object moves in the x-y plane with constant acceleration $\mathbf{a}$, and its initial velocity at $t=0$ is $\mathbf{v}_0$, its velocity $\mathbf{v}$ at time t is given by:
- $\mathbf{v} = \mathbf{v}_0 - \mathbf{a}t$
 - $\mathbf{v} = \mathbf{a}t - \mathbf{v}_0$
 - $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
 - $\mathbf{v} = \frac{\mathbf{v}_0 + \mathbf{a}t}{2}$

Ans :

(C) $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

For motion with constant acceleration $\mathbf{a}$, the velocity $\mathbf{v}$ at any time t is related to the initial velocity $\mathbf{v}_0$ by the vector equation $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$. This is a

direct extension of the one-dimensional kinematic equation.

- 30.** For an object starting from position $\mathbf{r}_0$ with initial velocity $\mathbf{v}_0$ and moving with constant acceleration $\mathbf{a}$, its position $\mathbf{r}$ at time t is:

- (A) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \mathbf{a} t^2$
- (B) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$
- (C) $\mathbf{r} = \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$
- (D) $\mathbf{r} = \mathbf{r}_0 + \mathbf{a} t + \frac{1}{2} \mathbf{v}_0 t^2$

Ans :

- (B) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$

This is the vector form of the kinematic equation for displacement under constant acceleration. The position $\mathbf{r}$ at time t depends on the initial position $\mathbf{r}_0$, initial velocity $\mathbf{v}_0$, acceleration $\mathbf{a}$, and time t .

- 31.** Motion in a plane (two dimensions) with constant acceleration can be treated as:

- (A) A single one-dimensional motion along the direction of acceleration.
- (B) Two separate simultaneous one-dimensional motions with constant accelerations along perpendicular directions.
- (C) Circular motion with constant angular acceleration around a fixed point.
- (D) A complex motion that cannot be simplified into one-dimensional components.

Ans :

- (B) Two separate simultaneous one-dimensional motions with constant accelerations along perpendicular directions.

A key principle in analyzing two-dimensional motion with constant acceleration is that the motion along two perpendicular axes (e.g., x and y) can be treated independently as two separate one-dimensional motions.

- 32.** An object that is in flight after being thrown or projected is called a:

- (A) Satellite
- (B) Comet
- (C) Projectile
- (D) Meteorite

Ans :

- (C) Projectile

A projectile is any object that, once launched into motion (projected), continues in motion by its own inertia and is influenced only by the downward force of gravity (ignoring air resistance).

- 33.** In the absence of air resistance, the acceleration acting on a projectile is:

- (A) Zero throughout its flight path.
- (B) Directed horizontally and constant in magnitude.
- (C) Directed vertically downward and equal to g (acceleration due to gravity).
- (D) Variable, depending on the projectile's velocity.

Ans :

- (C) Directed vertically downward and equal to g (acceleration due to gravity).

Once a projectile is launched, and neglecting air resistance, the only acceleration it experiences is the acceleration due to gravity, g , which acts vertically downwards.

- 34.** The horizontal component of a projectile's velocity, in the absence of air resistance:

- (A) Continuously increases throughout its flight path.
- (B) Continuously decreases until it reaches its maximum height.
- (C) Remains constant throughout its flight path.
- (D) Becomes zero at the peak of its trajectory.

Ans :

- (C) Remains constant throughout its flight path.
- Since there is no horizontal acceleration (assuming no air resistance), the horizontal component of the projectile's velocity ($v_{0x} = v_0 \cos \theta_0$) remains unchanged throughout its motion.

- 35.** The vertical component of a projectile's velocity:

- (A) Remains constant throughout its entire flight.
- (B) Changes continuously due to the force of gravity.
- (C) Is zero only at the point of launch.
- (D) Is maximum at the highest point of its trajectory.

Ans :

- (B) Changes continuously due to the force of gravity.
- The vertical component of a projectile's velocity is affected by the constant downward acceleration due to gravity. It decreases as the projectile rises, becomes zero at the maximum height, and increases as it falls.

- 36.** Which statement about the path of a projectile (in the absence of air resistance) is incorrect?

- (A) The path of the projectile is a parabola always.
- (B) The horizontal component of velocity is always constant.

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- (C) The vertical component of velocity is zero at maximum height.
 (D) The acceleration of the projectile is zero at its maximum height.

Ans :

(D) The acceleration of the projectile is zero at its maximum height.

The acceleration of a projectile (ignoring air resistance) is always g downwards, even at its maximum height where the vertical component of velocity is momentarily zero. The horizontal velocity is constant, and the path is parabolic.

37. Identify the correct statement regarding projectile motion, neglecting air resistance.
 (A) The time taken to reach maximum height is half the total time of flight.
 (B) The speed of the projectile is minimum at the point of launch.
 (C) The horizontal range is maximum when the projection angle is 90° .
 (D) The acceleration vector is always tangential to the trajectory path.

Ans :

(A) The time taken to reach maximum height is half the total time of flight.

Due to the symmetry of the parabolic path (when launched and landing at the same height), the time taken to reach the maximum height ($t_m = v_0 \sin \theta_0 / g$) is exactly half the total time of flight ($T_f = 2v_0 \sin \theta_0 / g$).

38. The time taken for a projectile launched with initial velocity v_0 at an angle θ_0 to reach its maximum height is given by:

- (A) $t_m = \frac{v_0 \cos \theta_0}{g}$
 (B) $t_m = \frac{v_0 \sin \theta_0}{g}$
 (C) $t_m = \frac{2v_0 \sin \theta_0}{g}$
 (D) $t_m = \frac{v_0 2 \sin^2 \theta_0}{2g}$

Ans :

(B) $t_m = \frac{v_0 \sin \theta_0}{g}$

At the maximum height, the vertical component of velocity (v_y) is zero. Using $v_y = v_{0y} - gt_m$, where $v_{0y} = v_0 \sin \theta_0$, we get $0 = v_0 \sin \theta_0 - gt_m$, which gives $t_m = \frac{v_0 \sin \theta_0}{g}$.

39. The total time of flight T_f for a projectile launched from and returning to the same horizontal level is:

- (A) $T_f = \frac{v_0 \sin \theta_0}{g}$
 (B) $T_f = \frac{v_0 \cos \theta_0}{g}$
 (C) $T_f = \frac{2v_0 \sin \theta_0}{g}$
 (D) $T_f = \frac{v_0 2 \sin 2\theta_0}{g}$

Ans :

(C) $T_f = \frac{2v_0 \sin \theta_0}{g}$

The total time of flight is twice the time taken to reach the maximum height, assuming the launch and landing points are at the same horizontal level.

Therefore, $T_f = 2t_m = \frac{2v_0 \sin \theta_0}{g}$.

40. The maximum height h_m reached by a projectile is given by the formula:

- (A) $h_m = \frac{v_0 2 \sin^2 \theta_0}{2g}$
 (B) $h_m = \frac{v_0 2 \cos^2 \theta_0}{2g}$
 (C) $h_m = \frac{v_0 \sin \theta_0}{g}$
 (D) $h_m = \frac{v_0 2 \sin 2\theta_0}{g}$

Ans :

(A) $h_m = \frac{v_0 2 \sin^2 \theta_0}{2g}$

The maximum height can be calculated using the kinematic equation $v_y^2 = v_{0y}^2 - 2gh_m$. At maximum height, $v_y = 0$ and $v_{0y} = v_0 \sin \theta_0$, leading to $h_m = \frac{(v_0 \sin \theta_0)^2}{2g}$.

41. The horizontal range R of a projectile launched from and returning to the same horizontal level is given by:

- (A) $R = \frac{v_0 2 \sin \theta_0}{g}$
 (B) $R = \frac{v_0 2 \cos \theta_0}{g}$
 (C) $R = \frac{v_0 2 \sin 2\theta_0}{g}$
 (D) $R = \frac{2v_0 2 \sin \theta_0 \cos \theta_0}{g^2}$

Ans :

(C) $R = \frac{v_0 2 \sin 2\theta_0}{g}$

The horizontal range is the horizontal distance traveled during the time of flight.

$$\begin{aligned}
 R &= v_{0x} T_f \\
 &= (v_0 \cos \theta_0) \left(\frac{2v_0 \sin \theta_0}{g} \right) \\
 &= \frac{v_0^2 (2 \sin \theta_0 \cos \theta_0)}{g} \\
 &= \frac{v_0^2 \sin 2\theta_0}{g}
 \end{aligned}$$

- 42.** For a given projection velocity, the horizontal range of a projectile is maximum when the angle of projection θ_0 is:

(A) 30°
 (B) 45°
 (C) 60°
 (D) 90°

Ans :

(B) 45°

The horizontal range

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

For R to be maximum, $\sin 2\theta_0$ must be maximum, which is 1. This occurs when $2\theta_0 = 90^\circ$, so $\theta_0 = 45^\circ$.

- 43.** The motion of an object following a circular path at a constant speed is known as:

(A) Uniform linear motion
 (B) Uniform circular motion
 (C) Non-uniform circular motion
 (D) Projectile motion

Ans :

(B) Uniform circular motion

Uniform circular motion is defined as the movement of an object along a circular path with a constant speed. Although the speed is constant, the velocity changes due to the continuous change in direction.

- 44.** Which of the following quantities is NOT constant in uniform circular motion?

(A) Speed of the object
 (B) Magnitude of acceleration
 (C) Velocity of the object
 (D) Kinetic energy of the object

Ans :

(C) Velocity of the object

In uniform circular motion, the speed is constant, and consequently, the magnitude of centripetal acceleration ($a_c = v^2/R$) and kinetic energy ($K = \frac{1}{2}mv^2$) are also constant. However, the velocity vector continuously changes direction.

- 45.** The acceleration of an object in uniform circular motion is always directed:

(A) Along the tangent to the circular path of the motion.
 (B) Away from the center of the circular path motion.
 (C) Towards the center of the circular path of motion.
 (D) In the direction of the object's instantaneous velocity vector.

Ans :

(C) Towards the center of the circular path of motion.

In uniform circular motion, the acceleration, known as centripetal acceleration, is always directed radially inward towards the center of the circle. This acceleration is responsible for changing the direction of the velocity.

- 46.** The magnitude of centripetal acceleration a_c for an object in uniform circular motion with speed v and radius R is:

(A) $a_c = vR$
 (B) $a_c = v2R$
 (C) $a_c = v/R$
 (D) $a_c = v^2/R$

Ans :

(D) $a_c = v^2/R$

The magnitude of centripetal acceleration required to keep an object moving in a circle of radius R with a constant speed v is given by the formula $a_c = v^2/R$.

- 47.** The angular speed ω is defined as:

(A) The rate of change of linear velocity of the object.
 (B) The rate of change of angular displacement of object.
 (C) The product of linear speed and radius of circle.
 (D) The linear displacement per unit angular displacement.

Ans :

(B) The rate of change of angular displacement of object.

Angular speed (ω) is a scalar quantity that describes how fast an object rotates or revolves relative to another point, i.e., the time rate of change of angular displacement ($\omega = \Delta\theta/\Delta t$).

48. The relationship between linear speed v and angular speed ω for an object in uniform circular motion of radius R is:

(A) $v = \omega/R$
 (B) $v = \omega R$
 (C) $a_c = \omega 2R$
 (D) $v = \omega/R^2$

Ans :

(B) $v = \omega R$

The linear speed v of a point on a rotating object is directly proportional to its angular speed ω and its distance R from the axis of rotation, given by the relation $v = \omega R$.

49. Centripetal acceleration a_c can also be expressed in terms of angular speed ω and radius R as:

(A) $a_c = \omega R^2$
 (B) $a_c = \omega 2R$
 (C) $a_c = \omega/R$
 (D) $a_c = \omega R$

Ans :

(B) $a_c = \omega 2R$

Since $v = \omega R$, substituting this into the formula for centripetal acceleration $a_c = v^2/R$ gives

$$\begin{aligned} a_c &= (\omega R)^2/R \\ &= \omega 2R^2/R \\ &= \omega 2R \end{aligned}$$

50. The time taken by an object to complete one full revolution in circular motion is known as its:

(A) Frequency
 (B) Angular speed
 (C) Time period
 (D) Centripetal time

Ans :

(C) Time period

The time period (t) of an object in circular motion is defined as the time it takes to complete one full revolution or cycle around the circular path.

51. Assertion (A): The displacement of an object can be zero even if the path length covered is non-zero. Reason (R): Displacement is the shortest distance between the initial and final positions, while path length is the total distance traversed.

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.

(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

Displacement is a vector from the initial to the final point. If an object returns to its starting point, its displacement is zero, even if it has traveled a considerable distance (path length). Reason R correctly defines both terms.

52.

Assertion (A): In uniform circular motion, the acceleration of the object is zero because its speed is constant.

Reason (R): In uniform circular motion, the direction of velocity changes continuously, leading to an acceleration.

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(D) A is false but R is true.

Assertion A is false; although speed is constant in uniform circular motion, the velocity is not (due to changing direction), so there is acceleration (centripetal acceleration). Reason R correctly states that the changing direction of velocity causes this acceleration.

53. Assertion (A): The horizontal range of a projectile is the same for angles of projection θ and $(90^\circ - \theta)$ for a given initial speed.

Reason (R): The horizontal range of a projectile is directly proportional to $\sin(2\theta)$

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The range formula is $R = (v_0^2 \sin(2\theta))/g$.

$$\begin{aligned} \text{Since, } \sin(2(90^\circ - \theta)) &= \sin(180^\circ - 2\theta) \\ &= \sin(2\theta) \end{aligned}$$

the ranges are equal. Reason R correctly states the proportionality.

- 54.** Consider the following statements about vectors:
1. A unit vector has a magnitude of one and no units.
 2. The sum of two vectors can be found using the parallelogram law.
 3. Multiplying a vector by -1 changes its magnitude but not its direction.
 4. The displacement vector depends on the path taken between two points.

Which of the statements are correct?

- (A) 1 and 2 only
(B) 1, 2, and 3 only
(C) 2 and 4 only
(D) 1, 3, and 4 only

Ans :

(A) 1 and 2 only
Statement 1 is correct. Statement 2 is correct (parallelogram law is equivalent to triangle law). Statement 3 is incorrect; multiplying by -1 reverses direction. Statement 4 is incorrect; displacement depends only on initial and final positions.

- 55.** Regarding projectile motion without air resistance, which of the following statements are true?
1. The horizontal acceleration is zero.
 2. The vertical acceleration is g downwards.
 3. The velocity at the maximum height is zero.
 4. The path of the projectile is always circular.
- Select the correct combination:

- (A) 1 and 2 only
(B) 1, 2, and 3 only
(C) 2 and 4 only
(D) 1, 3, and 4 only

Ans :

(A) 1 and 2 only
Statement 1 is true. Statement 2 is true. Statement 3 is false; only the vertical component of velocity is zero at maximum height, the horizontal component remains. Statement 4 is false; the path is parabolic.

- 56.** Which of the following statements concerning uniform circular motion are valid?
1. The kinetic energy of the particle is constant.
 2. The angular velocity is constant.
 3. The linear momentum of the particle is constant.
 4. The acceleration vector is constant.

Choose the correct option:

- (A) 1 and 2 only
(B) 2 and 3 only
(C) 1 and 4 only
(D) 1, 2, and 3 only

Ans :

(A) 1 and 2 only

Statement 1 is true (speed is constant). Statement 2 is true (uniform circular motion implies constant angular speed and fixed axis of rotation). Statement 3 is false (direction of linear momentum changes). Statement 4 is false (direction of centripetal acceleration changes).

- 57.** Match the physical quantity in Column I with its corresponding characteristic in Column II.

	Column I		Column II
1.	Scalar Quantity	P.	Obeys triangle law
2.	Vector Quantity	Q.	Magnitude only
3.	Unit Vector	R.	Rate of change of velocity
4.	Acceleration	S.	Magnitude is one

- (A) 1-Q, 2-P, 3-S, 4-R
(B) 1-P, 2-Q, 3-R, 4-S
(C) 1-Q, 2-S, 3-P, 4-R
(D) 1-S, 2-R, 3-Q, 4-P

Ans :

(A) 1-Q, 2-P, 3-S, 4-R

Scalar quantity has magnitude only (1-Q). Vector quantity obeys triangle law (2-P). Unit vector has magnitude one (3-S). Acceleration is the rate of change of velocity (4-R).

- 58.** Match the term related to projectile motion in Column I with its formula in Column II (v_0 is initial speed, θ_0 is projection angle, g is acceleration due to gravity).

	Column I		Column II
1.	Time of Flight	P.	$\frac{v_0 2 \sin^2 \theta_0}{2g}$
2.	Maximum Height	Q.	$\frac{v_0 2 \sin 2\theta_0}{g}$
3.	Horizontal Range	R.	$\frac{v_0 \sin \theta_0}{g}$
4.	Time to Max Height	S.	$\frac{2v_0 \sin \theta_0}{g}$

- (A) 1-S, 2-P, 3-Q, 4-R
(B) 1-P, 2-S, 3-R, 4-Q
(C) 1-S, 2-R, 3-Q, 4-P
(D) 1-Q, 2-P, 3-S, 4-R

Ans :

(A) 1-S, 2-P, 3-Q, 4-R

Time of Flight

$$T_f = \frac{2v_0 \sin \theta_0}{g} \text{ (1-S).}$$

Maximum Height

$$h_m = \frac{v_0^2 \sin^2 \theta_0}{2g} \text{ (2-P).}$$

Horizontal Range

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \text{ (3-Q).}$$

Time to Max Height

$$t_m = \frac{v_0 \sin \theta_0}{g} \text{ (4-R).}$$

59. Match the type of vector operation in Column I with its resulting property in Column II.

	Column I		Column II
1.	Addition of two vectors	P.	Results in a scalar quantity
2.	Multiplication by scalar	Q.	Changes magnitude and/or direction
3.	Resolution of a vector	R.	Resultant found by components
4.	Scalar (dot) product	S.	Splits vector into components

(A) 1-R, 2-Q, 3-S, 4-P

(B) 1-Q, 2-R, 3-P, 4-S

(C) 1-R, 2-S, 3-Q, 4-P

(D) 1-P, 2-Q, 3-S, 4-R

Ans :

(A) 1-R, 2-Q, 3-S, 4-P

Addition of two vectors results in a resultant found by components (1-R). Multiplication by a scalar changes magnitude and/or direction (2-Q). Resolution of a vector splits it into components (3-S). Scalar (dot) product results in a scalar quantity (4-P).

60. Rain is falling vertically with a speed of 30 ms^{-1} . A woman rides a bicycle with a speed of 10 ms^{-1} in the east to west direction. In which direction should she hold her umbrella to protect herself from the rain?

(A) At an angle $\tan^{-1} \frac{1}{3}$ with the vertical towards the west.(B) At an angle $\tan^{-1} \frac{1}{3}$ with the vertical towards the east.(C) At an angle $\tan^{-1}(3)$ with the vertical towards the west.(D) At an angle $\tan^{-1}(3)$ with the vertical towards the east.

Ans :

(B) At an angle $\tan^{-1} \frac{1}{3}$ with the vertical towards the east.

Let $\mathbf{v}_r$ be the velocity of rain and $\mathbf{v}_w$ be the velocity of the woman. The velocity of rain relative to the woman is $\mathbf{v}_{rw} = \mathbf{v}_r - \mathbf{v}_w$. If vertical is y-axis and west is x-axis, $\mathbf{v}_r = -30\hat{j}$ and $\mathbf{v}_w = -10\hat{i}$.

So,

$$\begin{aligned}\mathbf{v}_{rw} &= -30\hat{j} - (-10\hat{i}) \\ &= 10\hat{i} - 30\hat{j}\end{aligned}$$

The angle θ with the vertical is given by

$$\begin{aligned}\tan \theta &= \frac{\text{horizontal component}}{\text{vertical component}} \\ &= \frac{10}{30} = \frac{1}{3}\end{aligned}$$

She should hold the umbrella towards the east

ONE MARK QUESTIONS

61. What are scalar quantities ?

Ans :

Scalar quantities are those which have only magnitude but no direction e.g. mass, length, time, distance, speed, temperature are all scalar quantities.

62. What are vector quantities ?

Ans :

The physical quantities which are meaningful only if their magnitude as well as directions are given are called vectors or vector quantities. Displacement, velocity, acceleration, force, torque etc. are all vectors.

63. What is the resultant vector ?

Ans :

The resultant of two or more vectors is de-fined as that single vector which produces the same effect as is produced by individual vectors together.

64. What is equilibrant vector ?

Ans :

A single vector which balances two or more vectors acting on an object at the same time is called an equilibrant vector. It is equal and opposite to the resultant of various vectors acting on the body at the same time.

65. What is zero vector ?

Ans :

Zero vector is that vector which has a magnitude equal to zero and has an arbitrary direction. It is also called null vector and is represented by $\vec{0}$.

66. What is unit vector ?

Ans :

A vector whose magnitude is unity, is called a unit vector and has same direction as that of a given vector $\vec{a}$. A unit vector is written by a letter with cap or hat or caret over it. e.g. $\hat{a}$ is a unit vector of $\vec{a}$.

67. What is a negative vector ?

Ans :

A vector having the same magnitude as a vector $\vec{a}$ but the opposite direction, is defined as negative of the vector $\vec{a}$ and is denoted by $(-\vec{a})$.

68. What is the essential condition for addition of two vectors ?

Ans :

Vectors must be similar. A force can be added to force only. It cannot be added to velocity.

69. Can the sum of two vectors be a scalar ? Can it be numeric ?

Ans :

No, the sum of two vectors is always a vector. It cannot be numeric.

70. If $\vec{a}$ and $\vec{b}$ are two vectors, can $\vec{a} + \vec{b}$ be equal to zero ?

Ans :

$\vec{a} + \vec{b} = 0$ is possible only if $\vec{a}$ and $\vec{b}$ are opposite vectors.

71. The sum and difference of two vectors are equal in magnitude. What conclusion do you draw from this ?

Ans :

The two vectors have equal magnitude and are perpendicular to each other.

72. Can a vector change with time ? Give one example.

Ans :

Yes, a vector can change with time. For ex-ample,

velocity is a vector quantity and during a free fall, it changes with time.

73. Does vector addition hold for any two vectors ?

Ans :

No. The vector addition holds for two vectors of the same kind i.e. having same dimensions.

74. Why is zero vector needed ?

Ans :

Necessity of zero vector. It is required to express the result of subtraction of two equal vectors; e.g.,

$$(i) \quad \vec{A} - \vec{A} = \vec{0}$$

Now the right hand side of this equation has to be a vector whose magnitude is zero.
it is to be $\vec{0}$

$$(ii) \quad \text{It is required to multiply a vector by zero, i.e. } \vec{A} = \vec{0}$$

75. What is the physical meaning of zero vector ?

Ans :

Zero vector is used to represent vector when its magnitude is zero, e.g.,

(i) The position for the origin is $\vec{0}$.

(ii) Displacement of a stationary particle is $\vec{0}$.sl

(iii) Acceleration of a particle moving with uniform velocity is $\vec{0}$.

76. State the properties of vector addition.

Ans :

(i) Vector addition is commutative

$$\text{i.e.,} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

(ii) Vector addition is associative

$$\text{i.e.,} \quad \vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

(iii) Vector addition is distributive

$$\text{i.e.,} \quad N(\vec{A} + \vec{B}) = N\vec{A} + N\vec{B}$$

What is meant by angular displacement ?

Ans :

Angular displacement of the object moving around a circular path is defined as the angle traced out by the radius vector at the axis of the circular path in a given time.

$$\text{Angle (in radian)} = \frac{\text{Length of arc}}{\text{Radius of arc}}$$

78. What is angular velocity ?

Ans :

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

$$\omega = \frac{d\theta}{dt}$$

Its unit is radian per second and dimensions $[T^{-1}]$.

79. What is time period and frequency ? How are these related to each other ?

Ans :

Time period T is the time taken by an object to complete 1 revolution on its circular path. Frequency is the number of revolutions completed by an object in a unit time. Its unit is Hz.

$$T = \frac{1}{v} \text{ and } v = \frac{1}{T}$$

80. What is the relation between angular velocity ω , frequency v and time period T ?

Ans :

It is
$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi v$$

81. What is radial force or normal force ?

Ans :

Radial or normal force are the other names of centripetal force. Force acting on an object undergoing uniform circular motion is called centripetal force.

82. Can there be motion in two dimensions with an acceleration only in one dimension ?

Ans :

Yes, it is possible. During projectile motion, the acceleration acts only vertically downwards whereas the projectile follows a parabolic path.

83. What is an angular acceleration ? Give its unit and dimensions.

Ans :

Angular acceleration is the rate of change of angular velocity with time, $a = \frac{d\omega}{dt}$. Its unit is rad. s^{-2} and dimensions $[T^{-2}]$.

84. At what point of trajectory an : object thrown upward is the acceleration perpendicular to the velocity.

Ans :

At the highest point of trajectory, the acceleration is perpendicular to the velocity.

85. Name the physical quantities which remain constant for a particle moving along a circular path in a horizontal plane.

Ans :

Speed, kinetic energy and angular momentum.

86. Can a body have constant speed but variable velocity ?

Ans :

Yes, e.g. circular motion.

87. What do you mean by null vector ?

Ans :

A vector whose magnitude is zero and direction is arbitrary is called a zero vector or a null vector.

88. Define equal vectors.

Ans :

Two vectors are said to be equal vector if they have equal magnitude and direction.

89. Define negative vectors.

Ans :

Negative vector of a vector is another vector having same magnitude but drawn in opposite direction.

90. Define unit vector.

Ans :

A vector whose magnitude is unity and direction is same as that of a given vector.

or

A vector divided by its magnitude is called unit vector along the direction of the vectors.

91. Express the unit vector $\hat{A}$ in mathematical form.

Ans :

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

92. What is the resultant vector ?

Ans :

The resultant of two or more vectors is defined as that single vector which produces the same effect as is produced by individual vectors together.

- 93.** Can the magnitude of the resultant vector of the two given vectors is less than the magnitude of any of the given vectors ?

Ans :

Yes. This is possible if angle between the two vectors is obtuse ($> 90^\circ$).

- 94.** Can we multiply a vector by a real number ?

Ans :

Yes, When a vector $\vec{A}$ is multiplied by a real number n , we get another vector $n\vec{A}$ whose magnitude is n times and direction is same (if n is +ve) or opposite (if n is -ve).

- 95.** Pick out only vector quantity in the following list : Temperature, pressure, impulse, time, power, total length charge, coefficient of friction.

Ans :

Impulse.

- 96.** Is pressure a vector quantity ?

Ans :

Pressure is always taken to be normal to the plane of the area on which it is acting. Since this direction is unique, it has no significance and as such, pressure is not a vector.

- 97.** Vectors cannot be added algebraically. Why ?

Ans :

Vectors cannot be added algebraically as apart from magnitude, these possess directions.

- 98.** Under what conditions, the direction of sum and difference of two vectors will be the same ?

Ans :

This is possible when the two vectors are (i) unequal in magnitude and (ii) are in the same direction.

- 99.** Give an example of motion in two dimensions.

Ans :

Motion of earth around sun or projectile motion.

- 100.** Give an example of motion in three dimensions.

Ans :

A flying bird or a flying aeroplane.

- 101.** Can a body be accelerated without speeding up or slowing down ?

or

Is it possible to accelerate a body if its speed is constant ?

Ans :

Yes, by changing the direction of motion.
e.g. circular motion having constant speed.

- 102.** Can the speed of a body change if its velocity is constant ?

Ans :

No, because speed of a body is equal to magnitude of its velocity.

- 103.** Can the velocity of a particle vary even if speed is constant ?

Ans :

Yes, by changing the direction of motion and keeping speed constant.

- 104.** Define three dimensional motion.

Ans :

The motion of an object is said to be three dimensional if all the three coordinates required to specify the position of the object in space change w.r.t. time.

- 105.** Define uniform acceleration.

Ans :

The acceleration of a particle is said to be uniform if its velocity changes by equal amounts in equal intervals of time, however small the time interval may be.

- 106.** Is the rocket in flight an illustration of a projectile ?

Ans :

A rocket is propelled by the combustion of fuel and is not under the effect of gravity alone. As such it is not an example of a projectile.

- 107.** Keeping the angle of projection same, what is the effect on horizontal range of a projectile when its velocity is doubled ?

Ans :

As $R = \frac{v_0^2 \sin 2\theta}{g}$, $R \propto v_0^2$. On doubling v_0 , R becomes $(2)^2$ i.e. 4 times.

- 108.** Is the maximum height attained by projectile largest when horizontal range is maximum ?

Ans :

No. The horizontal range is maximum when $\theta = 45^\circ$ and the maximum height is largest when $\theta = 90^\circ$

TWO MARKS QUESTIONS

- 109.** What is meant by resolution of a vector ?

Ans :

Resolution of a vector is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the single vector has been splitted are called component vectors.

If the resolved vectors are perpendicular to each other, the resolved components are called rectangular components.

- 110.** What is meant by projectile and trajectory ?

Ans :

Projectile is the name given to a body thrown with some initial velocity and allowed to move without any propellation i.e., without being propelled by any engine or fuel. The path followed by projectile is called trajectory.

- 111.** What is the need for vectors ?

Ans :

To define certain physical quantities which have magnitude and direction, mathematical quantities called vectors can be used.

For example, if an object moves through a distance of 100 m from its position, then the quantity is not completely defined because the displacement may be in any direction. If we specify direction also, then it gives exact meaning and full physical information, e.g., if displacement is 100 m due north, it defines the direction of displacement also. Therefore, we associate vectors with those quantities which have direction apart from magnitude.

- 112.** Is a quantity having magnitude and direction always represents a vector ? Give examples.

Ans :

No, a quantity possessing both the magnitude and direction may not represent a vector. For example,

electric current flowing in a conductor or finite angular displacement etc., possess magnitude and direction but are not vector quantities. Because a quantity possessing magnitude and direction to be a vector, must obey the commutative law for addition.

- 113.** Can three vectors not in one plane give a zero resultant ? Can four vectors do ?

Ans :

Three vectors not in one plane cannot give a zero resultant. This is because, the resultant of any two vectors, which lies in their own plane, cannot cancel the effect of the third vector as the vector does not lie in the plane of the other two.

The resultant of four vectors not in one plane may be zero.

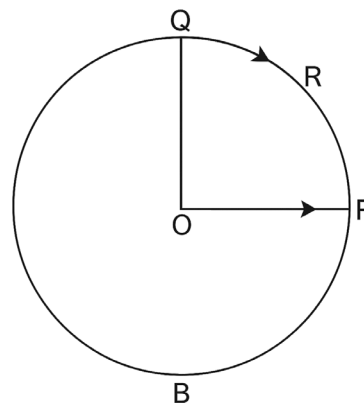
- 114.** A cyclist starts from centre O of a circular park of radius 1 km and moves along the path $OPRQO$ as shown in the Figure. If he maintains constant speed of 10 ms^{-1} , what is his acceleration at point R in magnitude and direction ?

Ans :

Centripetal acceleration

$$a = \frac{v^2}{R} = \frac{10 \times 10}{1} \\ = 100 \text{ ms}^{-2}$$

Direction of acceleration is towards O i.e. RO .



- 115.** A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have
- greatest speed
 - smallest speed? Explain.

Ans :

- The ball will have greatest speed just before it hits the ground, because as the ball fall, its speed goes on increasing till it hits the ground.

- (b) The speed of the ball will be minimum ($v = 0$) at the highest point.

116. Displacement vector is fundamentally a position? Comment on this statement.

Ans :

The displacement vector basically gives the position of a point like the position vector. The only difference between the two is that, the displacement vector gives the position of a point at a point other than the origin and position vector gives the position of a point w.r.t. origin. Since choice of origin is arbitrary, so the given statement is true.

- 117.** A person sitting in a moving train throws a ball vertically upwards. How does the ball appear to move to an observer
(a) inside the train
(b) outside the train ?

Ans :

- (a) The ball appears to first move vertically upwards and then fall vertically downwards.
(d) The ball appears to describe a parabolic path.

- 118.** A projectile is projected at an angle of 15° to the horizontal with speed v . If another projectile is projected with the same speed, then at what angle with the horizontal it must be projected so as to have the same range.

Ans :

To have the same range (with same velocity), the sum of the two angles of projection should be 90° . Since one of the angles is 15° , the other should be $90^\circ - 15^\circ = 75^\circ$.

- 119.** An aeroplane moving with a velocity v drops a packet from a height h . What is the time after which the packet reaches the ground ?

Ans :

The horizontal velocity only affects the range. The vertical component of initial velocity ($u_y = 0$), so we get

$$h = u_y t + \frac{1}{2} g t^2$$

or
$$h = 0 + \frac{1}{2} g t^2$$

or
$$t = \sqrt{\frac{2h}{g}}$$

- 120.** At what point on the projectile trajectory is the
(i) potential energy maximum
(ii) kinetic energy minimum and
(iii) total energy is maximum ?

Ans :

- (i) at the highest point on account of maximum height.
(ii) at the highest point (but not zero) on account of least velocity.
(iii) total energy is conserved all along the trajectory.

- 121.** Why the motion of oblique projectile becomes horizontal at the highest point ?

Ans :

At the highest point, the vertical component of velocity becomes zero. It is only horizontal component which is non-zero and hence the motion of the projectile is horizontal at highest point.

- 122.** Can a vector be multiplied with both dimensional and non-dimensional : scalars ?

Ans :

Yes. When a vector is multiplied with a dimensional scalar, the dimensions of the resultant vector is different with the dimensions of the scalar. e.g. when mass is multiplied with velocity vector, the dimensions of the resultant vector will be that of momentum, and when a vector is multiplied by non-dimensional scalar, the dimensions of the resultant scalar will be the same as the dimensions of the given vector.

THREE MARKS QUESTIONS

- 123.** What is the effect of multiplication of vectors by real number ?

Ans :

Multiplication of a vector by real number. The process of multiplying a vector by a real number gives another vector whose magnitude is equal to the product of the magnitude of the given vector and a real number, and whose direction is the same as that of the given vector if the given number is +ve and is opposite to the given vector if the number is negative.

e.g.

If $n = 2$, then

$$\vec{A} \times 2 = 2\vec{A} \quad \vec{A} \uparrow \times 2 = \uparrow 2\vec{A}$$

If $n = -2$, then

$$\vec{A} \times -2 = -2\vec{A} \quad \vec{A} \uparrow \times -2 = \downarrow 2\vec{A}$$

Physical examples

(i) Momentum, $\vec{p} = \vec{m}v$

(ii) Impulse, $\vec{I} = \vec{F} \cdot t$

- 124.** Define unit vector, negative vector, like and unlike vectors.

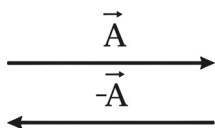
Ans :

- (i) Unit vector. A vector whose magnitude is unity, is called a unit vector and has the same direction as that of given vector $\vec{A}$. Unit vector is written by a letter A with cap or hat or caret over it. e.g. $\hat{A}$ is a unit vector of $\vec{A}$.

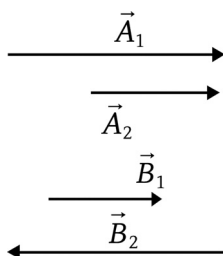
Any given vector $\vec{A}$ can be expressed as $\vec{A} = A\hat{A}$ where A is the magnitude of vector $\vec{A}$

By definition, $\hat{A} = \frac{\vec{A}}{A}$

- (ii) Negative vector. A vector having the same magnitude as a vector $\vec{A}$ but in the opposite direction, is defined as negative of the vector $\vec{A}$ and is denoted by $(-\vec{A})$.



- (iii) Like and Unlike vectors. Like vectors are parallel vectors having the same direction, while vectors having opposite directions are called unlike vectors. in Fig. 4.39 $\vec{A}_1, \vec{A}_2$ are like vectors but $\vec{B}_1, \vec{B}_2$ are unlike vectors.



- 125.** What is null (or zero) vector ? What are its characteristics and what is the physical meaning of null vector ?

Ans :

Null vector. A vector having zero magnitude and no specific direction is called a null vector or zero vector. It is obtained when a vector is either subtracted from itself or multiplied by zero.

i.e. $\vec{A} - \vec{A} = \vec{0} = 0$

$\vec{A} \times 0 = \vec{0} = 0$

Characteristics

1. Its magnitude is zero.
2. Its direction is unspecified.
3. Multiplication of non-zero number with zero vector is also zero.

i.e. $n \times \vec{0} = \vec{0} = 0$

4. Addition or subtraction of zero vector from any vector is again the same vector.

i.e. $\vec{A} + \vec{0} = \vec{A}$

$\vec{A} - \vec{0} = \vec{A}$

Physical meaning

1. It represents the displacement of a stationary object.
2. It represents the acceleration of a particle moving with uniform velocity.
3. It represents the position vector of origin.

- 126.** The magnitude of the resultant of vectors $\vec{P}$ and $\vec{Q}$ is given by $R^2 = P^2 + Q^2$. What is the angle between $\vec{P}$ and $\vec{Q}$?

Ans :

If θ is the angle between $\vec{P}$ and $\vec{Q}$, then the magnitude of resultant vector R is given by

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

Here $R^2 = P^2 + Q^2$

$$P^2 + Q^2 = P^2 + Q^2 + 2PQ \cos \theta$$

or $2PQ \cos \theta = 0$

or $\cos \theta = 0$

or $\theta = 90^\circ$

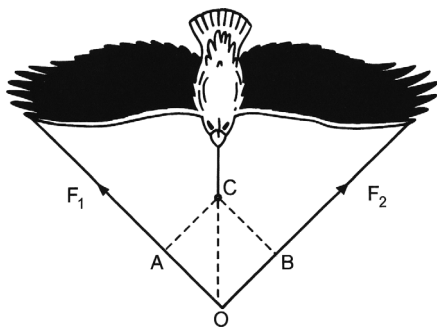
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- 127.** Is the flying of a bird an example of parallelogram law of addition of vectors ? Explain.

Ans :

Yes. When a bird flies, it pushes the air with its wings with forces $\vec{F}_1$ and $\vec{F}_2$ downwards and the line of action of two wings meet at O and the reactions of these forces is represented by $\vec{OA}$ and $\vec{OB}$ and the resultant of $\vec{OA}$ and $\vec{OB}$ is given by $\vec{OC}$ in accordance to the parallelogram law of vector i.e.

$$\vec{OA} + \vec{OB} = \vec{OC}$$



This resultant upward force $\vec{OC}$ is responsible for the flight of the bird.

- 128.** Show that the horizontal range is maximum when angle of projection is 45° .

Ans :

Since horizontal range R for a projectile fired at an angle θ with horizontal is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

For R to be maximum,

$$\sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$2\theta = 90^\circ$$

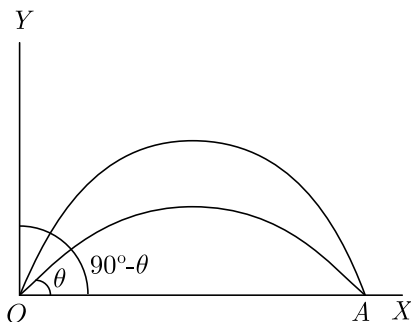
$$\theta = 45^\circ \text{ and } R_{\max} = \frac{u^2}{g}$$

- 129.** Show that there are two angles of projections for a projectile to have same horizontal range.

Ans :

Since horizontal range R for a projectile fired at an angle θ with horizontal is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad \dots(i)$$



Let R_1 be horizontal range for angle of projection $(90^\circ - \theta)$ with horizontal. So

$$R_1 = \frac{u^2 \sin 2(90^\circ - \theta)}{g}$$

$$= \frac{u^2 \sin(180^\circ - 2\theta)}{g}$$

$$= \frac{u^2 \sin 2\theta}{g}$$

$$[\sin(180^\circ - 2\theta) = \sin 2\theta]$$

$$R_1 = R \quad [\text{Using eq. (i)}]$$

Hence, horizontal range is same for angle of projection θ and $(90^\circ - \theta)$.

- 130.** The time of flight of a body projected with initial velocity u at an angle θ is T . What will be the time of flight if the body is projected with same velocity at an angle $(90^\circ - \theta)$?

Ans :

The time of flight of projectile, when projected at an angle θ with horizontal, is given by

$$T = \frac{2u \sin \theta}{g} \quad \dots(1)$$

If the angle of projection is $(90^\circ - \theta)$, the time of flight will be

$$T_1 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u}{g} \cos \theta \quad \dots(2)$$

From Eqs. (i), (2), we have

$$\frac{T_1}{T} = \cot \theta$$

$$\text{or } T_1 = T \cot \theta$$

- 131.** Show the maximum horizontal range is 4 times the maximum height attained by the projectile.

Ans :

For maximum horizontal range $\theta = 45^\circ$

$$\begin{aligned} R_{\max} &= \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 2(45^\circ)}{g} \\ &= \frac{u^2 \sin 90^\circ}{g} \end{aligned}$$

$$R_{\max} = \frac{u^2}{g}$$

For max. height,

$$\begin{aligned} h_{\max} &= \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45^\circ}{2g} \\ &= \frac{u^2}{4g} \end{aligned}$$

$$\frac{R_{\max}}{h_{\max}} = \frac{u^2}{g} \cdot \frac{4g}{u^2} = 4$$

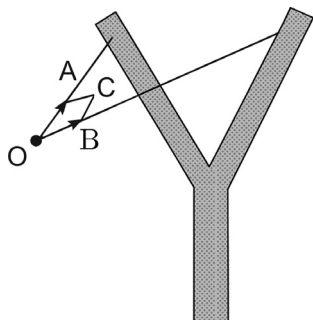
$$R_{\max} = 4 h_{\max}$$

- 132.** Explain the working a sling on the basis of a parallelogram law of addition of vector.

Ans :

A sling is a Y-shaped wooden or metallic frame, and a rubber band is attached to the two ends of the sling. The stone to be thrown is held at a point

O . On pulling the rubber band, the tensions are produced along OA and OB . On releasing the stone, it moves along OC under the effect of resultant tension with large speed.



FIVE MARKS QUESTIONS

133. (a) Distinguish between scalar and vector quantities. Give examples.
 (b) How is a vector represented graphically? How is it written?

Ans :

- (a) Scalar quantities

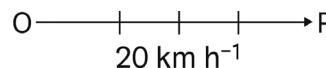
Basis	Scalar Quantity	Vector Quantity
Magnitude	Has only magnitude	Has both magnitude and direction
Direction	No direction	Has specific direction
Example	Speed, Mass, Time	Velocity, Force, Displacement
Representation	Represented by a number with unit	Represented by an arrow

- (b) Representation of a vector :

A vector is represented by an arrow. The length of the arrow, according to some convenient scale, represents the magnitude of the vector. The direction of arrow head gives the direction of vector.

Example. Let us represent a velocity of 20 kmh^{-1} due east. Let 1 cm represents 5 kmh^{-1} . Then, a four centimetre long line directed towards east will represent completely the velocity of 20 kmh^{-1} due east. In Figure, the arrow OP pointing east is a vector representing, on a scale, a velocity of 20 kmh^{-1} due east.

The end of the vector where the arrow head is marked is called the head or terminal point or terminus of the vector. The other end is called the tail or initial point or origin of the vector. In below figure P is the head and O is the tail of the vector $\overrightarrow{OP}$.



A vector quantity is denoted by putting an arrow over the letter representing the quantity. It may also be denoted by a bold face letter. For example, force vector may be represented by $\vec{F}$ or F .

The magnitude of $\vec{F}$ is written as $|\vec{F}|$ and read as modulus of $\vec{F}$. The magnitude of a vector may also be represented by an ordinary letter without an arrow head. Thus, F represents the magnitude of $\vec{F}$.

The magnitude of a vector is a scalar quantity.

134. Define position vector and displacement vector in two dimensions.

Ans :

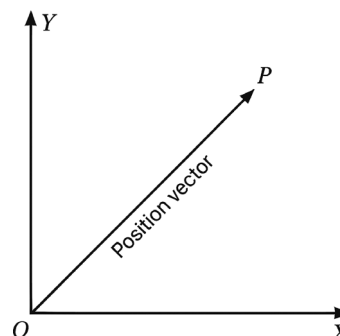
Position Vector

Position vector of a point object at any instant is the vector whose magnitude gives the distance of the point from the origin and whose direction gives the direction in which the point lies as seen from the origin.

Choose some point O in the plane as the origin. Then if at the time t the object is at the point P in the plane, we imagine a straight line from O to P , and call it the position vector of the object at time t . We write $\overrightarrow{OP}$ for this vector, and picture it as an arrow with tail at O and tip at P .

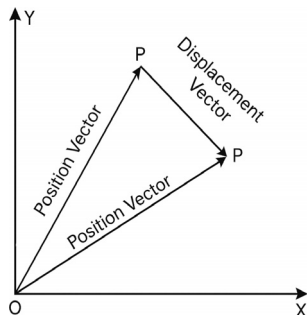
The vector $\overrightarrow{OP}$ gives two types of informations :

- Distance of the point P from the point O , which is distance OP .
- Direction in which P lies as seen from the point O .



Displacement Vector

It is a vector which tells by how much and in which direction an object has changed position from one instant to another instant.



Example. If an object changes its position from a point P to P' , then the vector with tail at P and tip at P' and $\vec{PP'}$ is the displacement vector.

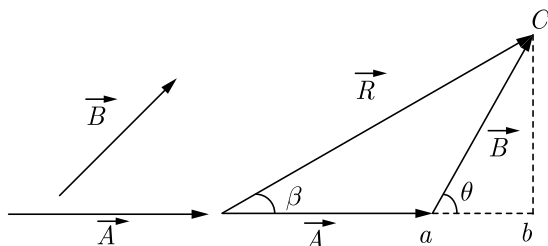
- 135.** What is resultant vector ? State and explain triangle law of vector addition.

Ans :

The resultant of two or more vectors is a single vector which produces exactly the same physical effect as is produced by the given vectors.

Statement. If two vectors are represented in magnitude and direction by two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the third side of triangle taken in the opposite direction.

We are given two vectors $\vec{A}$ and $\vec{B}$. Let vector $\vec{A}$ be represented by $\vec{oa}$. Shift the vector $\vec{B}$ parallel to itself until its tail coincides with the tip of $\vec{A}$, then complete the triangle to get $\vec{R}$ from the tail of $\vec{A}$ to the tip of $\vec{B}$ in its new position. The vector $\vec{R}$ is represented by sum of $\vec{oa}$ and $\vec{ac}$.



i.e. $\vec{oc} = \vec{oa} + \vec{ac}$

or $\vec{R} = \vec{A} + \vec{B}$

Magnitude of resultant R . To find the magnitude of the resultant, draw perpendicular cd on oa produced. Here, In right angled triangle adc ,

$$\frac{cd}{ac} = \sin \theta$$

$$cd = ac \sin \theta = B \sin \theta \quad \dots(1)$$

And $\frac{ad}{ac} = \cos \theta$

$$ad = ac \cos \theta = B \cos \theta \quad \dots(2)$$

Also, $cd^2 + ad^2 = ac^2$

In right angled $\triangle odc$,

$$\begin{aligned} oc^2 &= od^2 + cd^2 \\ &= (oa + ad)^2 + cd^2 \\ &= oa^2 + ad^2 + 2oa \cdot ad + cd^2 \end{aligned}$$

or $oc^2 = oa^2 + ac^2 + 2oa \cdot ad$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Determination of direction. Let the resultant $\vec{R}$ make $\angle \beta$ with the direction of vector $\vec{A}$. Then

$$\tan \beta = \frac{cd}{od} = \frac{cd}{oa + ad}$$

$$\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$$

- 136.** Answer the following question :

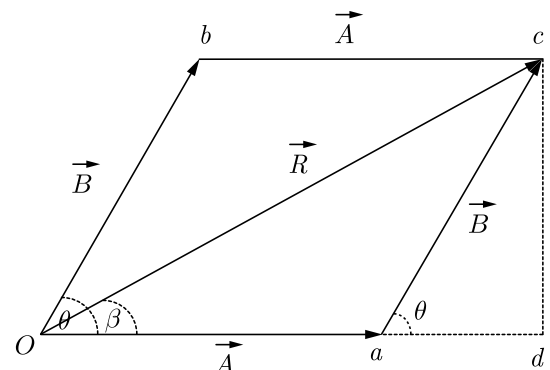
- State Parallelogram law of vector addition.
- Calculate the resultant of two vectors inclined at angle θ .

Discuss when $\theta = 0, \pi$ and $\frac{\pi}{2}$ radian.

Ans :

- If two vectors be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

- Let $\vec{A}$ and $\vec{B}$ be two vectors acting at an angle θ represented by the adjacent sides $\vec{oa}$ and $\vec{ob}$ of the parallelogram. To find the resultant complete the parallelogram $oacb$. By parallelogram law of vector, the diagonal $\vec{oc}$ the parallelogram represents the resultant of vectors $\vec{A}$ and $\vec{B}$.



i.e. $\vec{oc} = \vec{oa} + \vec{ob}$

or $\vec{R} = \vec{A} + \vec{B}$

To find magnitude and direction of $\vec{R}$. Draw perpendicular cd on oa produced. Proceeding as discussed in the case of triangle law of vector addition, the magnitude and direction of the resultant will be given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$

Special Cases

(i) When $\theta = 0$ i.e. two vectors are parallel.

$$\cos 0^\circ = 1$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos 0^\circ}$$

$$= \sqrt{A^2 + B^2 + 2AB}$$

$$= A + B \text{ (maximum)}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{B\sin 0^\circ}{A + B\cos 0^\circ}$$

$$\beta = 0 \quad [\sin 0^\circ = 0]$$

(ii) when $\theta = 180^\circ$ or π radian i.e. two vectors are anti-parallel.

$$R = \sqrt{A^2 + B^2 + 2AB\cos 180^\circ}$$

$$\cos 180^\circ = -1$$

$$= \sqrt{A^2 + B^2 - 2AB}$$

$$= A - B$$

or $B - A = A \sim B$ (Minimum)

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{B\sin 180^\circ}{A + B\cos 180^\circ}$$

$$= 0 \quad [\sin 180^\circ = 0]$$

$$\beta = 0$$

(iii) When $\theta = 90^\circ$ or $\pi/2$ radian i.e. two vectors are perpendicular.

$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ}$$

$$[\cos 90^\circ = 0]$$

$$R = \sqrt{A^2 + B^2}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{B\sin 90^\circ}{A + B\cos 90^\circ}$$

$$= \frac{B}{A} \quad [\sin 90^\circ = 1]$$

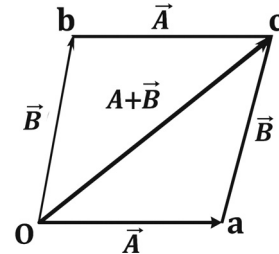
they are added. i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

From Figure,

$$\vec{ob} = \vec{ac} = \vec{B}$$

$$\vec{oa} = \vec{bc} = \vec{A}$$



From $\triangle obc$ $\vec{B} + \vec{A} = \vec{oc}$ (Using $\triangle$ law) ... (1)

From $\triangle oac$, $\vec{A} + \vec{B} = \vec{oc}$... (2)

From equation (1) and (2)

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

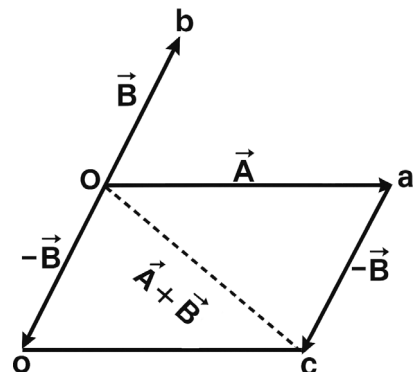
Hence vector addition is commutative.

(b) Subtraction of two vectors. To subtract a vector $\vec{B}$ from a vector $\vec{A}$, we use the rule for multiplying a vector by a negative number; first get $-\vec{B}$ and then add that to $\vec{A}$.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} + (-1)\vec{B}$$

The method of subtracting vectors is illustrated in the [Figure], in which $\vec{oa}$ and $\vec{ob}$ represent the vectors $\vec{A}$ and $\vec{B}$ respectively. Then $\vec{od} = -\vec{B}$. Applying Parallelogram law of vector addition, $\vec{oc}$ is resultant of $\vec{A}$ and $-\vec{B}$.

Hence $\vec{oc} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$



137. (a) Show that vector addition is commutative.
(b) What do you mean by vector subtraction?

Ans :

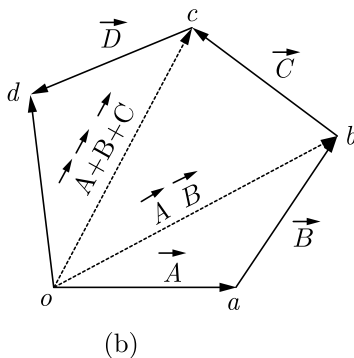
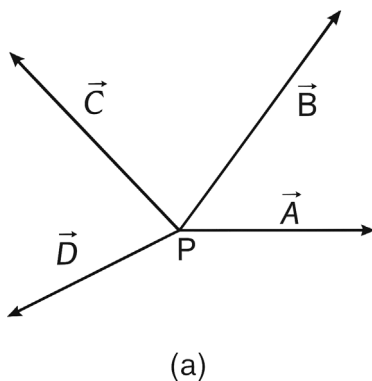
- (a) Vector addition is commutative if order of vectors remains the same in whatever order

- 138.** State Polygon law of vector addition and prove it using Triangle law of vector addition.

Ans :

“If a number of vectors are represented in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented completely by the closing side of the polygon taken in the opposite order”.

Let $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ be the vectors acting at a point P . To find the resultant vector $\vec{R}$ using geometrical method, represent $\vec{A}$ by $\vec{oa}$ place the tail of $\vec{B}$ on the head of $\vec{A}$, the tail of $\vec{C}$ on the head of $\vec{B}$ and the tail of $\vec{D}$ on the head of $\vec{C}$. The vector joining the tail of $\vec{A}$ to the head of $\vec{D}$ is the resultant vector as shown in the figure.



Proof. Consider triangle oab in fig. (b), the vectors $\vec{A}$ and $\vec{B}$ are represented by the sides oa and $\vec{ab}$ of the triangle taken in the same order. So its closing side $\vec{ob}$ represents their resultant.

such that $\vec{ob} = \vec{oa} + \vec{ab}$

or $\vec{ob} = \vec{A} + \vec{B}$

Now applying triangle law of vectors to the triangle obc , we have

$$\vec{oc} = \vec{ob} + \vec{bc}$$

or $\vec{oc} = \vec{A} + \vec{B} + \vec{C}$

Applying Δ law of vector to Δocd ,

$$\vec{od} = \vec{oc} + \vec{cd}$$

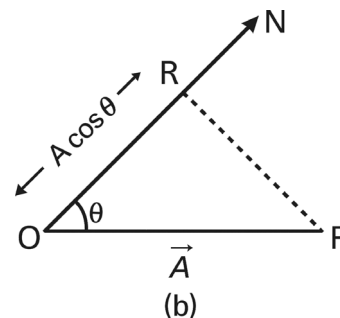
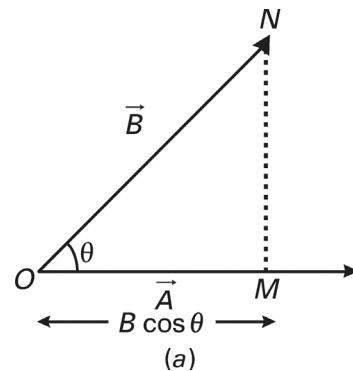
$$\vec{od} = \vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

- 139.** (a) Define and explain scalar product or dot product of two vectors. Give geometrical interpretation of dot product.
 (b) Give the characteristics of the dot product.
 (c) Give an example of scalar product.

Ans :

- (a) The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ represented by $\vec{A} \cdot \vec{B}$ is a scalar quantity given by the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and cosine of the smaller angle between them. Thus

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \dots(i)$$



Geometrical interpretation. Let $\vec{OP}$ and $\vec{ON}$ represent two vectors $\vec{A}$ and $\vec{B}$ inclined to each other at an angle θ . Draw NM perpendicular to OP Fig(a) and PR perpendicular to ON [Fig (b)]. From relation (i), we have

$$\vec{A} \cdot \vec{B} = AB \cos \theta = A(B \cos \theta)$$

But $B \cos \theta$ is the component of $\vec{B}$ along $\vec{OP}$ i.e. $B \cos \theta = OM$

$$\vec{A} \cdot \vec{B} = A(OM)$$

Again $\vec{A} \cdot \vec{B} = AB \cos \theta = B(A \cos \theta)$

But $A \cos \theta$ is the component of $\vec{A}$ along $\vec{ON}$ and is equal to OR .

$$\vec{A} \cdot \vec{B} = B(OR)$$

Hence, the scalar product of two vectors may also be defined as the product of the magnitude of one of the two vectors and the magnitude of the component of the other in the direction of first.

(b) Characteristics of scalar product

(i) Scalar product is commutative

i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Since $\vec{A} \cdot \vec{B} = AB \cos \theta$

or $\vec{A} \cdot \vec{B} = BA \cos \theta = \vec{B} \cdot \vec{A}$

or $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

(ii) Scalar Product of collinear vectors

(a) When the two vectors lie along the same straight or parallel lines and act in the same direction, then $\theta = 0^\circ$ and $\cos 0 = 1$

So $\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB$

Similarly,

$$\vec{A} \cdot \vec{A} = AA \cos 0^\circ = AA = A^2$$

Thus, scalar product of two like parallel vectors is equal to the product of their magnitudes.

(b) If the two vectors lie along parallel or same line but act in the opposite directions, then

$$\theta = 180^\circ \text{ and } \cos 180^\circ = -1$$

$$\vec{A} \cdot \vec{B} = AB \cos 180^\circ = -AB$$

Thus, dot product of two unlike parallel vectors is negative and is equal to the product of their magnitudes.

(iii) Scalar product of perpendicular vectors.

If two vectors are perpendicular to each other, then

$$\theta = 90^\circ \text{ and } \cos 90^\circ = 0$$

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ = AB(0) = 0$$

Thus, the condition of perpendicularity of two vectors is that their scalar product should be equal to zero.

(iv) Scalar product of unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$

$$\hat{i} \cdot \hat{j} = 1 \cdot 1 \cdot \cos 90^\circ = 0$$

Similarly, $\hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

And $\hat{i} \cdot \hat{i} = 1 \times 1 \times \cos 0 = 1$

Similarly, $\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

(v) Scalar product is distributive

i.e. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

(vi) Product of vectors $\vec{A}$ and $\vec{B}$ in terms of components

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\text{or } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

(c) Work is scalar

140. What is meant by vector product or cross product of two vectors ?

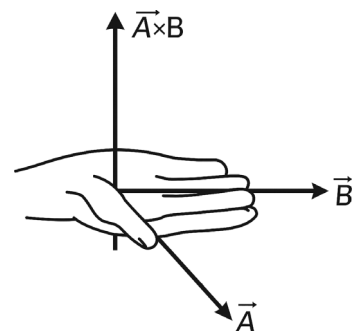
Ans :

The cross product of two vectors is a single vector whose magnitude is equal to the product of the magnitudes of the two vectors multiplied by the sine of the smaller angle between them. (The angle is taken as that between vectors diverging from or covering to a point). Its direction is perpendicular to the plane determined by two vectors.

i.e. $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

where $\hat{n}$ is a unit vector along the direction of $(\vec{A} \times \vec{B})$. The direction of the vector product is also given by the following two rules :

(i) Right Hand Palm Push Rule. Hold your right hand flat with the fingers in the plane of the palm and thumb sticking out so that the thumb points in the direction of first vector (say $\vec{A}$) and fingers in the direction of second vector (say $\vec{B}$), then the direction of the resultant is given by the push of the palm.

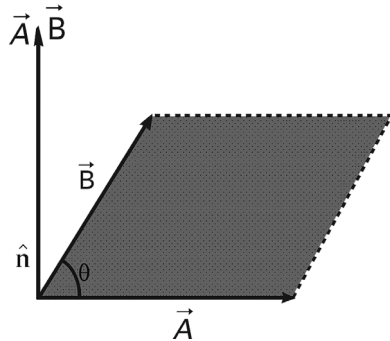


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141. What are the properties of cross product ?

Ans :

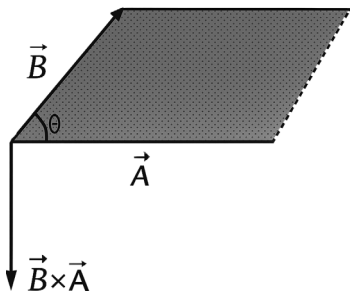
- (i) Vector Product is Anti-commutative,
i.e. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$



Proof : $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

$\hat{n}$ is perpendicular to $\vec{A}$ and $\vec{B}$ by right hand rule
it is $\perp$ and upward to plane containing $\vec{A}$ and $\vec{B}$
[Fig 4.52 (a)]

$$\vec{B} \times \vec{A} = AB \sin \theta \hat{n} \quad \dots(i)$$



where $\hat{n}$ is perpendicular and downward to plane
containing $\vec{A}$ and $\vec{B}$

But $\hat{n}' = -\hat{n}$

Substituting in (i)

$$\begin{aligned} \vec{B} \times \vec{A} &= AB \sin \theta (-\hat{n}) \\ &= -(AB \sin \theta \hat{n}) \\ &= -(\vec{A} \times \vec{B}) \end{aligned}$$

- (ii) Cross Product of Collinear Vectors. For collinear
(parallel) vectors, $\theta = 0^\circ$

Hence the condition for two vectors to be parallel is
that their cross product should be zero.

Again $\vec{A} \times \vec{A} = AA \sin 0^\circ = 0$ and also

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

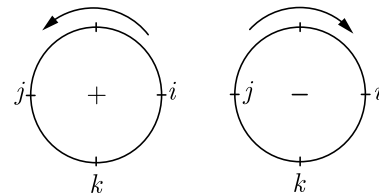
Cross Product of Perpendicular Vectors

Here $\theta = 90^\circ \vec{A} \times \vec{B} = AB \sin 90^\circ \hat{n} = AB \hat{n}$

Thus cross product of two perpendicular vectors
is equal to the product of their magnitudes and
pointing in the direction of unit vector $\hat{n}$. The
direction of unit vector is perpendicular to the plane
 $\vec{A}$ and $\vec{B}$ determined by right hand screw rule.

$$\begin{aligned} \text{Also, } \hat{i} \times \hat{j} &= \hat{k}; \hat{j} \times \hat{k} = \hat{i}; \hat{k} \times \hat{i} = \hat{j}; \\ \hat{j} \times \hat{i} &= -\hat{k}; \hat{k} \times \hat{j} = -\hat{i}; \hat{i} \times \hat{k} = -\hat{j} \end{aligned}$$

This can be remembered from the following rule:



- (iv) Cross product is distributive i.e.

$$\text{i.e. } \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

- (v) Cross-product of vector in terms of components
and base vector is

$$\begin{aligned} \vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ \vec{B} &= B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \\ \vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) \\ &\quad + \hat{k}(A_x B_y - A_y B_x) \end{aligned}$$

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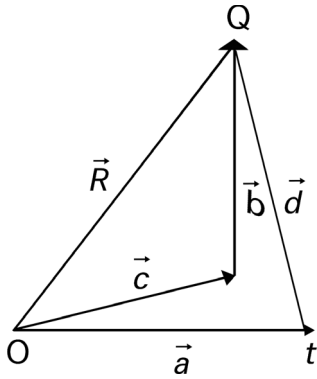
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142. What do you mean by resolution of a vector in
a plane ? Resolve a given vector $\vec{R}$ along any
direction.

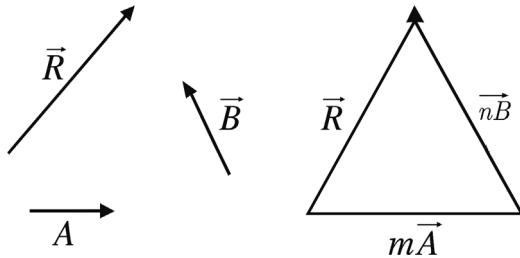
Ans :

The process of splitting of a vector into two or more
vectors is called resolution of a vector, and the parts
so obtained are called components of the vectors.

A single vector $\vec{R}$ represented by $\overrightarrow{OQ}$ can be
resolved to any two component vectors $\vec{a}$, $\vec{b}$ or $\vec{c}$,
 $\vec{d}$ etc. [Fig 4.54], according to triangle law of vector
addition.



Let a vector $\vec{R}$ is to be resolved into two components in the direction of $\vec{A}$ and $\vec{B}$ [Fig. 4.55]. Let us represent $\vec{R}$ by $\vec{OQ}$ i.e. $\vec{OQ} = \vec{R}$. Now draw two vectors $\vec{OP} \parallel \vec{A}$ and $\vec{PQ} \parallel \vec{B}$, so that we may get a ΔOPQ .



From Δ law of vector addition,

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

or $\vec{R} = m\vec{A} + n\vec{B}$

where m and n are real numbers and $m\vec{A}$ and $n\vec{B}$ are called components of $\vec{R}$ along $\vec{A}$ and $\vec{B}$ respectively.

- 143.** Resolve a given vector into three rectangular components (in space i.e. in three dimensions).

Ans :

When a vector is splitted into three component vectors at right angle to each other, the component vectors are called rectangular components of a vector in three dimensions.

Let a represent $\vec{A} = \vec{OR}$

be resolved into two components in two mutually perpendicular directions OX and OY which from Cartesian coordinate axis system. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along OX and OY respectively.

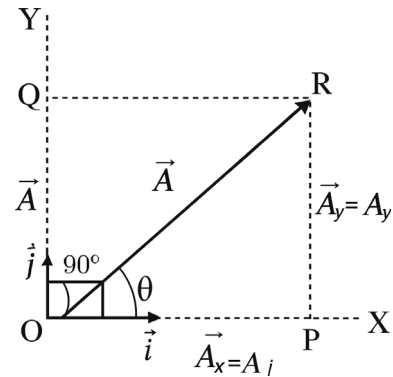
From R , draw $RP \perp OX$, $RQ \perp OY$,

Let $\vec{OP} = \vec{A}_x = A_x \hat{i}$

$$\vec{OQ} = \vec{A}_y = A_y \hat{j}$$

$$\vec{PR} = \vec{OQ} = \vec{A}_y = A_y \hat{j}$$

By triangle law of addition of vectors,



$$\vec{OP} + \vec{PR} = \vec{OR}$$

or $\vec{OR} = \vec{OP} + \vec{PR}$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Putting the values, we get

$$\vec{A} = A_x \hat{i} + A_y \hat{j} \quad \dots(1)$$

$\vec{OP}$, $\vec{OQ}$ are rectangular components of the given vector $\vec{A}$. From right angled triangle OPR ,

$$\frac{OP}{OR} = \cos \theta$$

$$OP = OR \cos \theta \text{ or } A_x = A \cos \theta \quad \dots(2)$$

$$\frac{PR}{OR} = \sin \theta$$

$$PR = OR \sin \theta \text{ or } A_y = A \sin \theta, \quad \dots(3)$$

where θ is the angle which $\vec{OR}$ makes with OX .

$$\vec{A} = A_x \hat{i} + A_y \hat{j},$$

$$\text{where } A_x = A \cos \theta \quad \dots(4)$$

$$A_y = A \sin \theta \quad \dots(5)$$

Relation between A ,

Squaring and adding (4) and (5), we get

$$\begin{aligned} A_x^2 + A_y^2 &= A^2 \cos^2 \theta + A^2 \sin^2 \theta \\ &= A^2 (\cos^2 \theta + \sin^2 \theta) = A^2 \\ (\because \cos^2 \theta + \sin^2 \theta &= 1) \end{aligned}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

i.e.

$$A = |\vec{A}| = (A_x^2 + A_y^2)^{1/2}$$

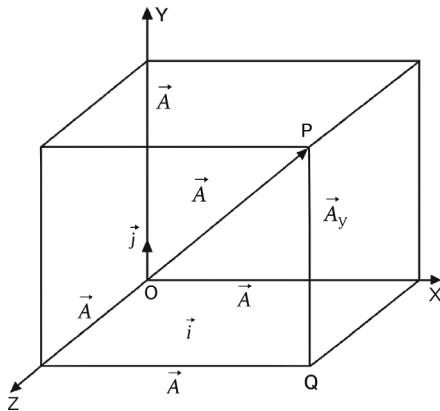
- 144.** Resolve a given vector into three rectangular components (in space i.e. in three dimensions).

Ans :

When a vector is splitted into three component vectors at right angle to each other, the component vectors are called rectangular components of a

vector in three dimensions.

Let us represent $\vec{A}$ by $\vec{OP}$ and let $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ be three rectangular components of $\vec{A}$ along X , Y and Z -axis respectively.



From polygon law of vector addition, we have

$$\vec{OP} = \vec{OL} + \vec{LQ} + \vec{QP}$$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\vec{A} = \vec{A}_x \hat{i} + \vec{A}_y \hat{j} + \vec{A}_z \hat{k} \quad \dots(i)$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x , y , z axis respectively.

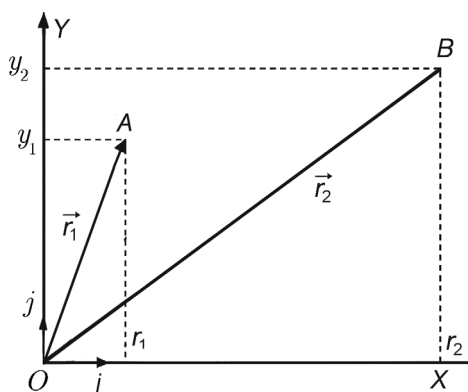
The magnitude of $\vec{A}$ will be

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- 145.** Show that the rectangular components of displacement vectors are the differences of the rectangular components of the position vectors at two instants.

Ans :

Suppose an object is moving in a plane.



Let $\vec{r}_1$ = Position vector at instant t_1

$\vec{r}_2$ = Position vector at instant t_2

Displacement vector from t_2 to t_1 ,

$$\vec{AB} = \vec{r}_2 - \vec{r}_1 \quad \dots(1)$$

Let x_1 , y_1 be the rectangular components of $\vec{r}_1$ along x -axis and y -axis respectively.

$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} \quad \dots(2)$$

Similarly, if $x_2 \hat{i}$, $y_2 \hat{j}$ be the rectangular components of $\vec{r}_2$, then

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} \quad \dots(3)$$

Putting the values in equation (1),

$$\vec{AB} = (x_2 \hat{i} + y_2 \hat{j}) - (x_1 \hat{i} + y_1 \hat{j})$$

$$\text{or } \vec{AB} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j}$$

x component of displacement = $x_2 - x_1$

y component of displacement = $y_2 - y_1$

Hence rectangular components of displacement vector are the difference of the rectangular components of $\vec{r}_2 - \vec{r}_1$.

- 146.** Find the displacement of a particle after certain time in two dimensions moving with uniform velocity.

or

Show that $\vec{r} = \vec{r}_0 + \vec{v}t$ and hence derive the relation $x = x_0 + v_x t$ and $y = y_0 + v_y t$

Ans :

A particle is said to be moving with uniform velocity, if it covers equal displacements in equal intervals of time

Let $\vec{r}_1$ and $\vec{r}_2$ be position vectors of a particle moving in two dimensional motion having uniform velocity $\vec{v}$ at instants t and t' respectively

$$\text{So } \vec{v} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\text{or } \vec{r}_2 - \vec{r}_1 = \vec{v}(t_2 - t_1)$$

$$\text{or } \vec{r}_2 = \vec{r}_1 + \vec{v}(t_2 - t_1) \quad \dots(1)$$

If at $t = 0$, $\vec{r}_1 = \vec{r}_0$

and at $t_2 = t$, $\vec{r}_2 = \vec{r}$

Then equation (1) becomes

$$\vec{r} = \vec{r}_0 + \vec{v}(t - 0)$$

$$\text{or } \vec{r} = \vec{r}_0 + \vec{v}t \quad \dots(2)$$

In terms of rectangular components,

$$\vec{r} = x \hat{i} + y \hat{j}$$

$$\vec{r}_0 = x_0 \hat{i} + y_0 \hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

So equation (2) becomes

$$x \hat{i} + y \hat{j} = x_0 \hat{i} + y_0 \hat{j} + (v_x \hat{i} + v_y \hat{j})t$$

$$\text{or } x \hat{i} + y \hat{j} = (x_0 + v_x t) \hat{i} + (y_0 + v_y t) \hat{j}$$

Equating the coefficient of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$x = x_0 + v_x t$$

$$y = y_0 + v_y t$$

- 147.** What is a projectile ? What do you understand by trajectory ? Show that motion of a projectile fired horizontally is a parabola.

Ans :

Projectile : Any object projected in space or air such that it moves under the effect of gravity alone is called a projectile.

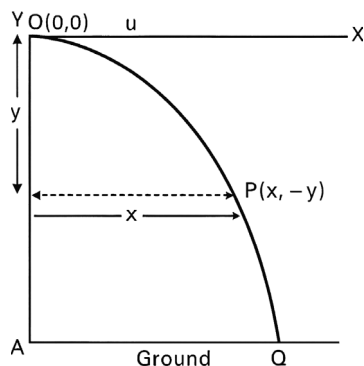
e.g. a bullet fired from a gun or a javelin thrown by an athlete etc.

Trajectory : The path followed by a projectile is called its trajectory.

Consider a projectile fired from point O with velocity u parallel to horizontal. Take O as the origin and horizontal of line OX (in the direction of velocity of projection of the projectile) as positive direction of X -axis and the vertical line OY in downward direction as negative direction of Y -axis as shown in Fig. 4.18.

At $t = 0$, $x_0 = 0$, $y_0 = 0$

Suppose at any time t , projectile reaches point P so that its positions along X and Y -axis are given by x and $-y$ respectively.



Motion along horizontal. If we neglect friction due to air, then velocity of the projectile along horizontal will remain constant.

Thus, initial velocity along horizontal,

$$u_x = u$$

Acceleration along horizontal, $a_x = 0$

Using $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

Put $x_0 = 0$, $u_x = u$ and $a_x = 0$

$$x = 0 + ut + \frac{1}{2}(0)t^2$$

or $x = ut$

or $t = \frac{x}{u}$... (1)

Motion along vertical Since projectile is fired parallel to horizontal, its initial velocity along vertical, $u_y = 0$

Acceleration along vertical, $a_y = -g$

The negative sign is due to the reason that acceleration due to gravity is along negative Y -axis.

Using $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

Put $y_0 = 0$, $u_y = 0$, $a_y = -g$ and $y_0 = -y$,

We have

$$-y = 0 + 0 \times t + \frac{1}{2}(-g)t^2$$

or $y = \frac{1}{2}gt^2$

Using equation (i), we get

$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \frac{g}{2u^2}x^2$$

This is an equation of parabola. Hence projectile follows a parabolic path.

- 148.** A body is projected with a velocity u in a direction making angle θ with the horizontal. Show that path of the projectile is a parabola. Find the maximum height reached, the time of flight and horizontal range.

Ans :

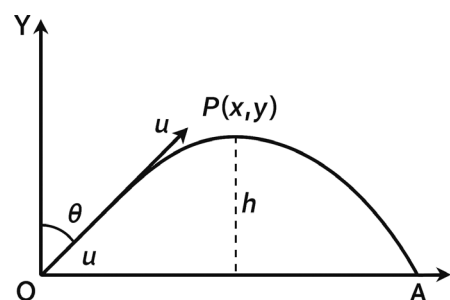
Let a body be projected from point O with velocity ' u ' at an angle θ with horizontal. Resolve velocity u into two rectangular components u_x and u_y along X -axis and Y -axis with ' O ' as origin.

At $t = 0$, $x_0 = 0$; $y_0 = 0$

The components $u_x = u \cos \theta$ and

$u_y = u \sin \theta$ act independently and simultaneously.

Let after time ' t ', the projectile reaches at point P .



Motion along horizontal. If we neglect friction due to air, then horizontal component of the velocity i.e. $u \cos \theta$ will remain constant.

Thus

initial velocity along horizontal,

$$u_x = u \cos \theta$$

acceleration along horizontal,

$$a_x = 0$$

Using $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

Put $x_0 = 0$, $u_x = u \cos \theta$ and $a_x = 0$

$$x = 0 + (u \cos \theta)t + \frac{1}{2}(0)t^2$$

or $x = (u \cos \theta)t$

or $t = \frac{x}{u \cos \theta} \quad \dots(1)$

Motion along vertical

Initial velocity along vertical, $u_y = u \sin \theta$

Acceleration along vertical, $a_y = -g$

Using $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

Put $y_0 = 0$, $u_y = u \sin \theta$ and $a_y = -g$

we have $y = 0 + (u \sin \theta)t + \frac{1}{2}(-g)t^2$

or $y = (u \sin \theta)t - \frac{1}{2}gt^2$

Equation of trajectory. The equation of the trajectory of the projectile can be obtained by substituting the value of t from (1) in above equation.

Therefore, we have

$$y = (u \sin \theta) \frac{x}{u \cos \theta} - \frac{1}{2}g \times \left(\frac{x}{u \cos \theta} \right)^2$$

or $y = x \tan \theta - \left(\frac{g}{2u^2 \cos^2 \theta} \right) x^2$

or $y = k_1 x + k_2 x^2 \quad \dots(2)$

where $k_1 = \tan \theta$ and $k_2 = -\left(\frac{g}{2u^2 \cos^2 \theta} \right)$

As equation (2) is equation of parabola, it follows that a projectile fired at some angle with the horizontal moves along a parabolic path.

(i) Time of flight. It is the time for which the projectile remains in air above the horizontal plane from the point of projection.

On reaching the highest point A , the vertical component of the velocity of the projectile must become zero i.e. $v_y = 0$

Using $v_y = u_y + a_y t$

Put $u_y = u \sin \theta$, $a_y = -g$, $t = \frac{T}{2}$

and $v_y = 0$, we have

$$0 = u \sin \theta + (-g) \frac{T}{2}$$

or $T = \frac{2u \sin \theta}{g}$

(ii) Maximum height attained (h). It is the greatest height of which a projectile rises above the point of projection.

Using $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

Put $y_0 = 0$, $u_y = u \sin \theta$, $a_y = -g$,

$$t = \frac{T}{2} = \frac{u \sin \theta}{g} \text{ and } y = h,$$

we have $h = 0 + (u \sin \theta) \times \frac{u \sin \theta}{g} + \frac{1}{2}(-g) \left(\frac{u \sin \theta}{g} \right)^2$

or $h = \frac{u^2 \sin^2 \theta}{2g}$

(iii) Horizontal range. It is the distance covered by projectile along horizontal between the point of projection to the point on ground, where the projectile returns again.

It is denoted by R . Obviously, range R is the horizontal distance covered by the projectile with uniform velocity $u \cos \theta$ in time equal to time of flight T . Therefore,

$$R = u \cos \theta + \frac{2u \sin \theta}{g} = \frac{u^2 (2 \sin \theta \cos \theta)}{g}$$

As $2 \sin \theta \cos \theta = \sin 2\theta$, we have

$$R = \frac{u^2 \sin 2\theta}{g} \quad \dots(iii)$$

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149. Derive the relation between linear velocity and angular velocity.

or

Show that $v = r\omega$.

Ans :

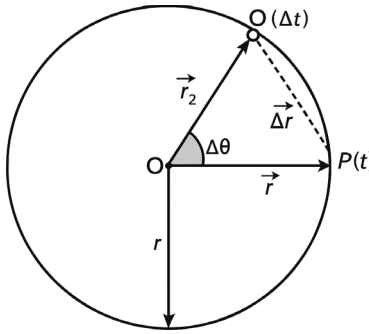
Consider a particle moving in a circular path having radius r . The positions P and Q are represented by position vector $\vec{r}_1$ and $\vec{r}_2$ respectively as shown in Figure.

The linear displacement of the particle in time interval Δt is given by

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

and $|\vec{r}_2| = |\vec{r}_1| = r$

Thus instantaneous velocity $\vec{v}$ of the particle at P is given by



$$\vec{v} = L \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$$

or $v = L \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \quad \dots(1)$

Also $\Delta \theta = \frac{\text{arc } PQ}{r} \left\{ \because \text{angle} \frac{\text{arc}}{\text{radius}} \right\}$

As $\Delta \theta \rightarrow 0$, arc $PQ \rightarrow$ chord $PQ = \Delta r$

$$\Delta \theta = \frac{\Delta r}{r}$$

or $\Delta r = r \Delta \theta \quad \dots(2)$

Putting eq. (2) in (1), we get

$$v = L \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

$$= r \omega$$

$$v = r \omega$$

- 150.** Derive relation between linear acceleration and angular velocity.

or

Show that $a = r \omega^2$

Ans :

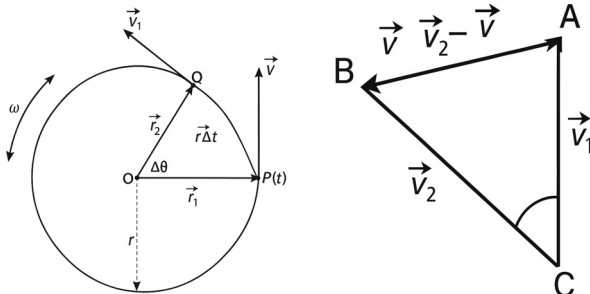
Consider a particle moving in a circular path of radius r having constant speed v . Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the particle at points P and Q respectively.

Let $\vec{OP} = \vec{r}_1$ and $\vec{OQ} = \vec{r}_2$ such that

$$|\vec{v}_1| = |\vec{v}_2| = v$$

Since motion of the particle is uniform

$$|\vec{r}_1| = |\vec{r}_2| = r$$



Draw $\vec{CA} \parallel \vec{v}_1$ and $\vec{CB} \parallel \vec{v}_2$ as shown in Figure. Complete the ΔCAB by drawing vector $\vec{AB}$.

Using Triangle law of vector addition in ΔABC , we have

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

The direction of $\Delta \vec{v}$ is towards the centre of the circle.

ΔPOQ and ΔABC are isosceles triangles having same vertex angle $\Delta \theta$. So they are similar

$$\frac{\Delta v}{v} = \frac{PQ}{OP}$$

or $\frac{\Delta v}{v} = \frac{v \Delta t}{r}$

or $\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$

Centripetal acceleration,

$$a = L \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

or $a = \frac{v^2}{r}$

Since $v = r \omega$,

$$a = \frac{r^2 \omega^2}{r}$$

or $a = r \omega^2$

- 151.** Explain how the motion of an object is described in three dimensional spaces.

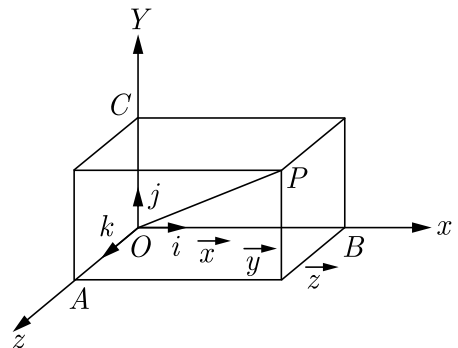
Ans :

Position vector in three dimensional spaces.

To describe the position vector $\vec{r}$ of a moving object, we choose an origin O and three units vector $\hat{i}$, $\hat{j}$, $\hat{k}$ at O which are mutually perpendicular. Then we write $\vec{r}$ as the sum as follows.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and call $x\hat{i}$, $y\hat{j}$, $z\hat{k}$ as the rectangular (cartesian) coordinates of the position vector $\vec{r}$. Geometrically, we have a rectangular parallelepiped describing the resolution.



Expressions for velocity and acceleration

For the velocity and acceleration vectors $\vec{v}$, $\vec{a}$, we have

Velocity, $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$,

where $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$

Acceleration,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k},$$

where $a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$

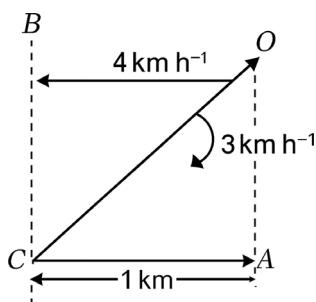
$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

$$a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

The only accelerated motions in three dimensions are those in which the object moves in a straight line in space with constant speed. Any other kind of motion definitely involves acceleration.

NUMERICAL QUESTIONS

152. A river one km wide is flowing at 3 km h^{-1} . A swimmer whose velocity in still water is 4 km h^{-1} can swim only for 20 minutes. What direction should he strike out in order to reach the opposite bank in 20 minutes? What total distance does he swim?



Ans :

Velocity of man in still water = 4 km h^{-1}

He can cross the river in 20 minutes.

Therefore, he has to strike perpendicular to the bank of the river

$$R = \sqrt{4^2 + 3^2}$$

or $R = 5 \text{ km h}^{-1}$

Total distance he will swim

$$= 5 \times \frac{20}{60}$$

$$= 1.67 \text{ km along OC.}$$

153. A man travels 30 m due north on a straight road and reaches a road junction. He then turns through 90° towards the east and moves 40 m straight. Find (a) what is the distance travelled and (b) what is the displacement?

Ans :

Displacement $\vec{OA} = 30 \text{ m}$ (along north)

Displacement $\vec{AB} = 40 \text{ m}$ (along east)

The total distance travelled

$$= 30 + 40 = 70 \text{ m}$$

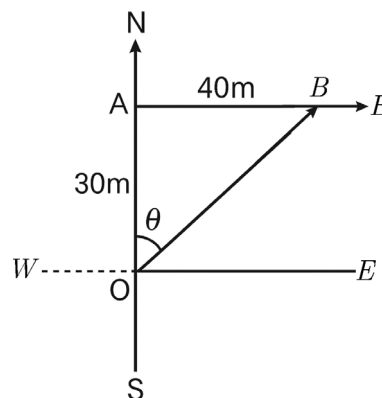
The resultant displacement

$$= \vec{OA} + \vec{AB} = \vec{OB}$$

or

$$OB = \sqrt{OA^2 + AB^2}$$

$$= \sqrt{30^2 + 40^2} = 50 \text{ m}$$



The displacement is 50 m along OB.

If $\angle AOB = \theta$

Then $\tan \theta = \frac{40}{30} = \frac{4}{3}$

or $\theta = \tan^{-1} 4/3$

The displacement is in a direction θ° east of north.

154. Show that for elevation which exceed or fall short of 45° by equal amount, the range is equal.

Ans :

We know that, horizontal range h for angle of projection θ is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

At angle $(45^\circ + \theta)$, the range is

$$R_1 = \frac{u^2 \sin [2(45^\circ + \theta)]}{g}$$

$$= \frac{u^2 \sin (90^\circ + 2\theta)}{g}$$

or $R_1 = \frac{u^2 \cos 2\theta}{g} \dots (1)$

At angle $(45^\circ - \theta)$, the range is

$$R_2 = \frac{u^2 \sin[2(45^\circ - \theta)]}{g}$$

$$= \frac{u^2 \sin(90^\circ - 2\theta)}{g}$$

or $R_2 = \frac{u^2 \cos 2\theta}{g} \quad \dots(ii)$

From eq. (i) and (ii), we have

$$R_2 = R_1$$

- 155.** A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 kmh^{-1} . At what angle of sight (w.r.t horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target.

Ans :

Given $u = 720 \text{ kmh}^{-1}$

$$= 720 \times \frac{5}{18} \text{ ms}^{-1}$$

$$= 200 \text{ ms}^{-1}$$

$$h = 1.5 \text{ km} = 1500 \text{ m}$$

$$g = 9.8 \text{ ms}^{-2}$$

Since horizontal range

$$R = u\sqrt{\frac{2h}{g}} \quad \dots(i)$$

If ϕ is the angle of sight, then

$$\tan \phi = \frac{h}{R}$$

Using Eq. (i), we get,

$$\tan \phi = \frac{h}{u\sqrt{\frac{2h}{g}}} = \frac{1}{u}\sqrt{\frac{gh}{2}}$$

$$= \frac{1}{200}\sqrt{\frac{9.8 \times 1500}{2}}$$

$$= \sqrt{\frac{7350}{200}}$$

$$= \frac{85.732}{200}$$

or $\tan \phi = 0.4287$

So $\phi = \tan^{-1}(0.4287) = 23^\circ 12'$

- 156.** An aeroplane is in level flight at 144 km per hour at an altitude of 1000 m. How far from a given target should it release a bomb so as to hit the target ?

Ans :

Here $u_x = 144 \text{ kmh}^{-1}$

$$= 144 \times \frac{5}{18} = 40 \text{ ms}^{-1}$$

$$\left[1 \text{ kmh}^{-1} = \frac{5}{18} \text{ ms}^{-1} \right]$$

$$y = 1000 \text{ m}$$

If 't' is the time taken by the bomb to reach the ground, then

Since $y = y_0 + \frac{1}{2}u_y t + \frac{1}{2}gt^2$

But $y_0 = 0$ and $u_y = 0$,

$$y = \frac{1}{2}gt^2$$

$$t^2 = \frac{2y}{g} = \frac{2 \times 1000}{9.8} = \frac{10000}{49}$$

or $t = \sqrt{\frac{10000}{49}} = \frac{100}{7} \text{ second}$

Horizontal distance,

$$x = u_x t = 40 \times \frac{100}{7} = 571.43 \text{ m}$$

Hence, the bomb should be released 571.43 m before the target.

- 157.** A bomb is dropped from an aircraft when it is directly above a target at a height of 1000 m. The aircraft is moving with a velocity of 500 km/hr. Will the bomb hit the target ? If not, by how much distance will the bomb miss the target ?

Ans :

Here height, $y = 1000 \text{ m}$

Speed of aircraft, $u_x = 500 \text{ km/hr}$

$$= \frac{500 \times 5}{18} = \frac{1250}{9} \text{ m/s}$$

As the bomb is dropped from aircraft when it is directly above the target, the bomb will not hit the target. In fact it will miss it by a horizontal distance

$$x = u_x t = u_x \sqrt{\frac{2y}{g}}$$

$$= \frac{1250}{9} \times \sqrt{\frac{2 \times 1000}{9.8}}$$

$$= 1985 \text{ m}$$

- 158.** An aeroplane flying horizontally with a speed of 72 kmh^{-1} releases a bomb at a height of 1000 m. Find the time taken by the bomb to reach the ground and also the magnitude and direction of the velocity with which it strikes the ground, given $g = 9.8 \text{ ms}^{-2}$.

Ans :

Here $u = 72 \text{ kmh}^{-1}$

$$= 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$

$$y = 1000 \text{ m}$$

$$g = 9.8 \text{ ms}^{-2}$$

Now $y = y_0 + \frac{1}{2}gt^2$

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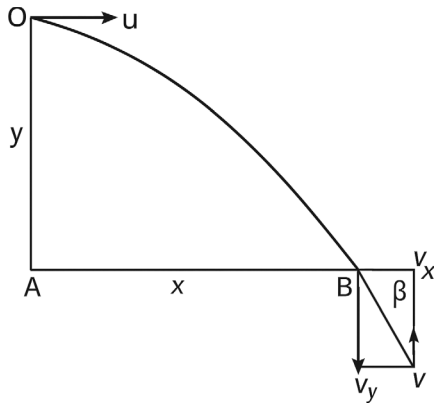
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$$t^2 = \frac{2y}{g} = \frac{2 \times 1000}{9.8} \quad [y_0 = 0]$$

$$= \frac{10000}{49}$$

or

$$t = \sqrt{\frac{10000}{49}} = \frac{100}{7} \text{ s}$$

To find velocity v , when body strikes the ground, we have

The horizontal component $u_x = u = 20 \text{ ms}^{-1}$ and the vertical component

$$v_y = gt = 9.8 \times \frac{100}{7}$$

$$= 140 \text{ ms}^{-1}$$

The resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$

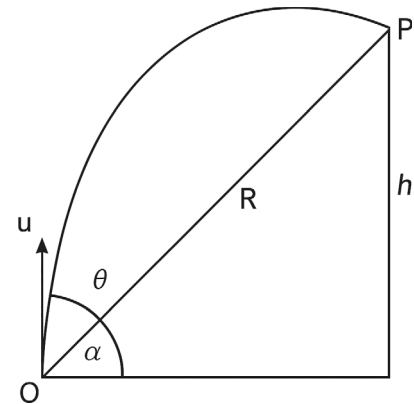
$$= \sqrt{(20)^2 + (140)^2}$$

$$= 141.4 \text{ ms}^{-1}$$

If the resultant velocity v is inclined at an angle β with horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{140}{20} = 7$$

$$\beta = \tan^{-1} 7 = 81^\circ 49'$$



From Fig. 4.68 $R = OP = h \operatorname{cosec} a$.

To hit the enemy, the range of shell should be

$$R = h \operatorname{cosec} a$$

But $R = \frac{2u^2 \cos \theta \sin(\theta - a)}{g \cos^2 a}$

$$h \operatorname{cosec} a = \frac{2u^2 \cos \theta (\sin \theta - a)}{g \cos^2 a}$$

$$u^2 = \frac{gh \cos^2 a \operatorname{cosec} a}{\cos \theta \sin(\theta - a)}$$

$$= \frac{gh \cos^2 a \operatorname{cosec} a}{\sin(2\theta - a) - \sin a}$$

u is minimum when $\sin(2\theta - a)$ is maximum i.e.

$$\sin(2\theta - a) = 1 \text{ or } 2\theta - a = \frac{\pi}{2}$$

In this situation

$$u_{\min}^2 = gh \frac{\cos^2 a \operatorname{cosec} a}{1 - \sin a}$$

$$= gh \frac{(1 - \sin^2 a) \operatorname{cosec} a}{1 - \sin a}$$

$$= gh(1 + \sin a) \operatorname{cosec} a$$

$$= gh(\operatorname{cosec} a + 1)$$

i.e. $u_{\min} = \sqrt{gh(1 + \operatorname{cosec} a)}$

160. An object is covering distance in direct proportion to t^3 , where t is the time elapsed.

- (a) What conclusions might you draw about the acceleration? Is it constant or increasing or decreasing or zero?
- (b) What might you conclude about the force acting on the object?

Ans :

Here, $x = kt^3$

where k is constant of proportionality.

(a) Now,

$$v = \frac{dx}{dt} = \frac{d}{dt}(kt^3) = 3kt^2$$

Also,

$$a = \frac{dv}{dt} = \frac{d}{dt}(3kt^2) = 6kt$$

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- 159.** The angular elevation of an enemies position on a hill of height ' h ' is a . Show that in order to hit a shell on it, the initial velocity ' u ' of projection should not be less than $\sqrt{gh(1 + \operatorname{cosec} a)}$ where g is acceleration due to gravity.

Ans :

Let the shell is fired at an angle θ with speed u . The point of projection is O and enemy's position is P .

or $a \propto t$

Therefore, acceleration of the moving object increases directly with time.

(b) Since force on the body, $F = ma$, it follows that

$$F \propto t$$

i.e. accelerating force is increasing uniformly with time.

- 161.** $\vec{A} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x , y and z axes respectively. Find $\vec{A} + \vec{B}$, $\vec{A} - \vec{B}$, $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$.

Ans :

$$(i) \quad \vec{A} + \vec{B} = (-2\hat{i} + 3\hat{j} - 4\hat{k}) + (3\hat{i} - 4\hat{j} + 5\hat{k}) \\ = (\hat{i} - \hat{j} + \hat{k})$$

$$(ii) \quad \vec{A} - \vec{B} = (-2\hat{i} + 3\hat{j} - 4\hat{k}) - (3\hat{i} - 4\hat{j} + 5\hat{k}) \\ = (-5\hat{i} + 7\hat{j} - 9\hat{k})$$

$$(iii) \quad \vec{A} \cdot \vec{B} = (-2\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (3\hat{i} - 4\hat{j} + 5\hat{k}) \\ = (-2)(3) + 3(-4) + (-4)(5) \\ = -6 - 12 - 20 = -38$$

$$(iv) \quad \vec{A} \times \vec{B} = \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -4 \\ 3 & -4 & 5 \end{vmatrix} \\ = \hat{i}(15 - 16) - \hat{j}(-10 + 12) + \hat{k}(8 - 9) \\ = -\hat{i} - 2\hat{j} - \hat{k}$$

- 162.** Find a unit vector that is perpendicular to both $\vec{A}$ and $\vec{B}$ where $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

Ans :

Let $\vec{C} = \vec{A} \times \vec{B}$. Clearly, $\vec{C}$ being the vector product of $\vec{A}$ and $\vec{B}$, is perpendicular to both $\vec{A}$ and $\vec{B}$.

Now

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} \\ = \hat{i}[1 \times 2 - (1)(-1)] - \hat{j}[2 \times 2 - 1 \times 1] \\ + \hat{k}[2(-1) - 1 \times 1]$$

$$\text{or} \quad \vec{C} = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

If $\hat{C}$ is the unit vector in the direction of $\vec{C}$

$$\vec{C} = C\hat{C} \text{ or } \hat{C} = \frac{\vec{C}}{C}$$

$$\text{Now,} \quad C = \sqrt{3^2 + (-3)^2 + (-3)^2} \\ = \sqrt{27} = 3\sqrt{3}$$

$\hat{C}$ (unit vector perpendicular to $\vec{A}$ and $\vec{B}$)

$$\text{or} \quad \hat{C} = \frac{\vec{C}}{C}$$

$$\text{or} \quad \hat{C} = \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}} \\ = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

- 163.** A ball is projected from O with an initial velocity 700 cm/s in a direction 37° above the horizontal. A ball B, 500 cm away from O on the line of the initial velocity of A, is released from rest at the instant A is projected. Find

(a) the height through which B falls. Before it is hit by A.

(b) the direction and magnitude of the velocity A at the time of impact.

[given $g = 10 \text{ m/s}^2$, $\sin 37^\circ = 0.6$ and $\cos 37^\circ = 0.8$]

Ans :

The situation is shown in Fig. 4.69

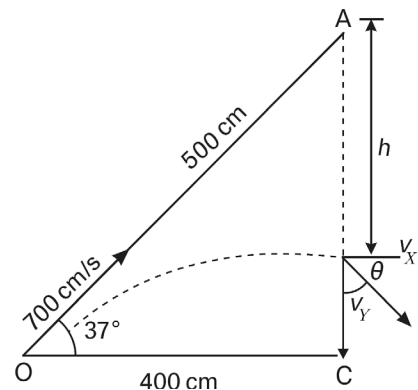
(a) Let the ball collide after t sec.

From Figure,

$$OC = OA \cos 37^\circ \\ = 500 \cos 37^\circ \\ = 500 \times 0.8 \\ = 400 \text{ cm} \quad \dots(1)$$

Horizontal velocity

$$y = 700 \times \cos 37^\circ \\ OC = 700 \times \cos 37^\circ \times t \\ = 700 \times 0.8 \times t = 560t$$



From eqs (1) and (2),

$$560t = 400$$

or $t = \left(\frac{5}{7}\right) \text{ sec}$

Now $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 1000 \times \left(\frac{5}{7}\right)^2$
 $= 255.1 \text{ cm}$

(b) Let at the time of impact, v_x and v_y be the horizontal and vertical velocities respectively, then

$$v_x = 700 \times \cos 37^\circ$$

$$= 700 \times 0.8 = 560 \text{ cm/s}$$

and $v_y = -700 \times \sin 37^\circ + 1000 \times \left(\frac{5}{7}\right)$
 $= -700 \times 0.6 + \left(\frac{5000}{7}\right)$
 $= -420 + 714.3$
 $= +294.3 \text{ cm/sec}$
 (downward)

Velocity of the ball at the time of collision

$$v = \sqrt{(v_x^2 + v_y^2)}$$

$$v = \sqrt{(560)^2 + (294.3)^2}$$

$$= 632.6 \text{ cm/sec}$$

Again $\tan \theta = \frac{v_y}{v_x} = \frac{294.3}{560}$

or $\theta = \tan^{-1}\left(\frac{294.3}{560}\right) = 27^\circ 43'$

CASE BASED QUESTIONS

- 164.** A motorboat is racing towards the north at 25 km/h. The water current in that region is 10 km/h in the direction of 60° east of south. The boat's captain needs to determine the actual direction and speed of the boat with respect to the ground to reach the destination accurately.

Questions:

1. What is a vector quantity? Is velocity a scalar or a vector?
2. When two vectors are added, does the order of addition matter for the resultant vector? Name this property.
3. If the velocity of the motorboat is $\mathbf{v}_b$ and the water current velocity is $\mathbf{v}_c$, how is the resultant velocity $\mathbf{R}$ of the boat represented in terms of $\mathbf{v}_b$ and $\mathbf{v}_c$?

Ans :

1. A vector quantity is a quantity that has both a magnitude and a direction and obeys the triangle law of addition or equivalently the parallelogram law of addition. Velocity is a vector quantity.

2. No, the order of addition does not matter for the resultant vector. This property is called the commutative law of vector addition ($\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$).
3. The resultant velocity $\mathbf{R}$ is the vector sum of $\mathbf{v}_b$ and $\mathbf{v}_c$, i.e., $\mathbf{R} = \mathbf{v}_b + \mathbf{v}_c$.

- 165.** An artillery cannon fires a projectile with an initial velocity v_0 at an angle θ_0 with the horizontal x-axis. The motion of the projectile is analyzed, assuming air resistance is negligible. The acceleration acting on the projectile after it's fired is solely due to gravity, directed vertically downward.

Questions:

1. What is a projectile?
2. What are the horizontal and vertical components of the initial velocity v_0 ?
3. What is the acceleration of the projectile in the horizontal (a_x) and vertical (a_y) directions after it is launched?
4. The path of the projectile is a parabola. How is this derived from the equations of motion for x and y coordinates?

Ans :

1. An object that is in flight after being thrown or projected is called a projectile.
2. The components of initial velocity v_0 are: $v_{0x} = v_0 \cos \theta_0$ (horizontal) and $v_{0y} = v_0 \sin \theta_0$ (vertical).
3. The acceleration in the horizontal direction $a_x = 0$. The acceleration in the vertical direction $a_y = -g$ (where g is the acceleration due to gravity).
4. The path of the projectile is derived by eliminating time (t) from the equations for x and y coordinates: $x = (v_0 \cos \theta_0)t$ and $y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$. This results in an equation of the form $y = ax + bx^2$, which is the equation of a parabola.

- 166.** An insect is trapped in a circular groove of radius 12 cm on a wheel. The wheel moves steadily, and the insect completes 7 revolutions in 100 s. The motion is an example of uniform circular motion.

Questions:

1. What is uniform circular motion?
2. Even though the speed is constant in uniform circular motion, why is there an acceleration? What is this acceleration called and in which direction does it act?
3. Calculate the angular speed (ω) of the insect's motion.
4. Is the acceleration vector of the insect a constant vector? What is its magnitude?

Ans :

1. When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion.
2. Even though the speed is constant, the velocity of the object is changing continuously in direction, so the object undergoes acceleration. This acceleration is called centripetal acceleration and it is always directed towards the centre of the circle.
3. The insect completes 7 revolutions in 100 s, so the time period $T = \frac{100}{7}$ s.

The angular speed

$$\begin{aligned}\omega &= 2\pi/T \\ &= 2\pi \times (7/100) \\ &= 14\pi/100 \approx 0.44\text{rad/s}.\end{aligned}$$

4. The acceleration vector is not a constant vector because its direction changes continuously, always pointing towards the centre of the circle. However, its magnitude is constant and is given by

$$\begin{aligned}a_c &= \omega^2 R \\ &= (0.44\text{s}^{-1})^2 \times (12\text{cm}) \\ &\approx 2.3\text{cm/s}^2.\end{aligned}$$

- 167.** A vector $\mathbf{A}$ lies in the x-y plane. Lines are drawn from the head of $\mathbf{A}$ perpendicular to the coordinate axes, giving vector components $\mathbf{A}_1$ along the x-axis and $\mathbf{A}_2$ along the y-axis such that $\mathbf{A}_1 + \mathbf{A}_2 = \mathbf{A}$. The angle $\mathbf{A}$ makes with the x-axis is θ .

Questions:

1. What is a unit vector? What are the unit vectors along the x- and y-axes denoted as?
2. How can the vector components $\mathbf{A}_1$ and $\mathbf{A}_2$ be expressed using the unit vectors and the scalar components A_x and A_y ?
3. How are the scalar components A_x and A_y related to the magnitude of $\mathbf{A}$ (denoted as A) and the angle θ ?
4. If A_x and A_y are given, how can the magnitude A and the direction θ of vector $\mathbf{A}$ be obtained?

Ans :

1. A unit vector is a vector of unit magnitude and points in a particular direction. Unit vectors along the x-, y- (and z-) axes are denoted by $\hat{i}$ (and $\hat{k}$) respectively.
2. The vector components can be expressed as $\mathbf{A}_1 = A_x \hat{i}$ and $\mathbf{A}_2 = A_y \hat{j}$.
3. The scalar components are related to the magnitude A and angle θ by $A_x = A \cos \theta$ and $A_y = A \sin \theta$.

4. If A_x and A_y are given, the magnitude A can be obtained using $A = \sqrt{A_x^2 + A_y^2}$.

The direction θ can be obtained using

$$\tan \theta = \frac{A_y}{A_x}.$$

- 168.** A particle starts from the origin ($r_0 = 0$) at $t = 0$ with an initial velocity $\mathbf{v}_0 = 5.0\hat{i}$ m/s. It moves in the x-y plane under the action of a force which produces a constant acceleration $\mathbf{a} = (3.0\hat{i} + 2.0\hat{j})$ m/s².

Questions:

1. What is the general vector equation for the position $\mathbf{r}$ of a particle at time t , if its initial position is $\mathbf{r}_0$, initial velocity is $\mathbf{v}_0$, and it moves with constant acceleration $\mathbf{a}$?
2. How can motion in a plane (two-dimensions) with constant acceleration be treated in terms of its components?
3. Using the given data, write the expressions for the x-coordinate $x(t)$ and y-coordinate $y(t)$ of the particle as functions of time.
4. If the x-coordinate of the particle is 84 m at a certain time t , what will be its y-coordinate at that same time?

Ans :

1. The general vector equation for position is

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2.$$

2. Motion in a plane (two-dimensions) with constant acceleration can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.
3. Given $\mathbf{r}_0 = 0$, $\mathbf{v}_0 = 5.0\hat{i}$ m/s (so $v_{0x} = 5.0$ m/s, $v_{0y} = 0$), and $\mathbf{a} = (3.0\hat{i} + 2.0\hat{j})$ m/s² (so $a_x = 3.0$ m/s², $a_y = 2.0$ m/s²).

$$\begin{aligned}x(t) &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ &= 0 + 5.0t + \frac{1}{2} (3.0) t^2 \\ &= 5.0t + 1.5t^2.\end{aligned}$$

$$\begin{aligned}y(t) &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\ &= 0 + 0 \cdot t + \frac{1}{2} (2.0) t^2 \\ &= 1.0t^2.\end{aligned}$$

4. Given $x(t) = 84$ m, so $5.0t + 1.5t^2 = 84$. Solving this quadratic equation gives $t = 6$ s (the positive solution). At $t = 6$ s, $y(6) = 1.0(6)^2 = 36.0$ m.

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- 169.** Rain is falling vertically with a speed of 35ms^{-1} . A wind starts blowing after some time with a speed of 12ms^{-1} in the east to west direction. A boy waiting at a bus stop needs to hold his umbrella at an angle to protect himself from the rain.

Questions:

1. If two vectors **A** and **B** are given, how is their sum **A + B** found using the graphical (triangle) method?
2. The velocity of rain $\mathbf{v}_r$ is vertical, and the velocity of wind $\mathbf{v}_w$ is horizontal. What is the angle between these two vectors?
3. The resultant velocity **R** of the raindrops will be the vector sum of $\mathbf{v}_r$ and $\mathbf{v}_w$. What will be the magnitude of this resultant velocity R?
4. In which direction (angle with the vertical) should the boy hold his umbrella to protect himself?

Ans :

1. To find the sum **A + B** using the triangle method, place vector **B** so that its tail is at the head of vector **A**. Then, join the tail of **A** to the head of **B**. This line represents the vector sum **R**.
2. Since $\mathbf{v}_r$ is vertical and $\mathbf{v}_w$ is horizontal, the angle between them is 90° .
3. The magnitude of R is

$$\begin{aligned} R &= \sqrt{v_r^2 + v_w^2} \\ &= \sqrt{35^2 + 12^2} \text{ms}^{-1} \\ &= \sqrt{1225 + 144} = \sqrt{1369} \\ &= 37\text{ms}^{-1}. \end{aligned}$$

4. The direction θ that R makes with the vertical is given by

$$\begin{aligned} \tan \theta &= \frac{v_w}{v_r} = \frac{12}{35} \\ &\approx 0.343. \end{aligned}$$

$$\text{So, } \theta = \tan^{-1}(0.343) \approx 19^\circ$$

The boy should hold his umbrella in the vertical plane at an angle of about 19° with the vertical towards the east (since wind is from east to west, the rain will appear to come from the east).

- 170.** The position of a particle is given by the vector $\mathbf{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$ meters, where t is in seconds.

Questions:

1. How is the instantaneous velocity vector $\mathbf{v}(t)$ of a particle obtained from its position vector?
2. Recall: How is the instantaneous acceleration vector $\mathbf{a}(t)$ of a particle obtained from its velocity vector $\mathbf{v}(t)$?

3. Find the velocity vector $\mathbf{v}(t)$ and acceleration vector $\mathbf{a}(t)$ for the given particle.
4. Determine the magnitude and direction (angle with the x-axis for the x-y component) of the velocity $\mathbf{v}(t)$ at $t = 1.0\text{s}$.

Ans :

1. The instantaneous velocity $\mathbf{v}(t)$ is the time derivative of the position vector $\mathbf{r}(t)$, i.e.,

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}.$$

2. The instantaneous acceleration $\mathbf{a}(t)$ is the time derivative of the velocity vector $\mathbf{v}(t)$, i.e.,

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt}.$$

3.
$$\mathbf{v}(t) = \frac{d}{dt}(3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k})$$
$$= 3.0\hat{i} + 4.0t\hat{j} \text{ m/s}.$$

$$\begin{aligned} \mathbf{a}(t) &= \frac{d}{dt}(3.0\hat{i} + 4.0t\hat{j}) \\ &= +4.0\hat{j} \text{ m/s}^2. \end{aligned}$$

4. At $t = 1.0\text{s}$,

$$\begin{aligned} \mathbf{v}(1.0) &= 3.0\hat{i} + 4.0(1.0)\hat{j} \\ &= 3.0\hat{i} + 4.0\hat{j} \text{ m/s}. \end{aligned}$$

Magnitude:

$$\begin{aligned} v &= \sqrt{3.0^2 + 4.0^2} \\ &= \sqrt{9 + 16} = \sqrt{25} = 5.0 \text{ m/s}. \end{aligned}$$

Direction (angle θ with x-axis):

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4.0}{3.0}\right) \\ &\approx 53^\circ \text{ with the x-axis}. \end{aligned}$$

- 171.** A projectile is launched with an initial velocity v_0 at an angle θ_0 to the horizontal. The horizontal distance it travels before returning to the same initial height is called its horizontal range R. The expression for R is given by

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}.$$

Questions:

1. For what angle of projection θ_0 is the horizontal range R maximum for a given initial velocity v_0 ?
2. What is the maximum horizontal range (R_m) in terms of v_0 and g ?
3. Galileo stated that “for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal”. If one angle is $45^\circ + \alpha$, what is the other angle according to this statement?
4. Prove Galileo’s statement using the formula for R.

Ans :

1. The range R is maximum when $\sin(2\theta_0)$ is maximum (i.e., 1), which occurs when $2\theta_0 = 90^\circ$, so $\theta_0 = 45^\circ$.
2. The maximum horizontal range is]

$$R_m = \frac{v_0^2}{g}.$$

3. If one angle is $45^\circ + \alpha$, the other angle that gives the same range is $45^\circ - \alpha$.
4. For $\theta_0 = 45^\circ + \alpha$, $2\theta_0 = 90^\circ + 2\alpha$.
So $\sin(2\theta_0) = \sin(90^\circ + 2\alpha) = \cos(2\alpha)$.
For $\theta_0 = 45^\circ - \alpha$, $2\theta_0 = 90^\circ - 2\alpha$.
So $\sin(2\theta_0) = \sin(90^\circ - 2\alpha) = \cos(2\alpha)$.
Since $\sin(2\theta_0)$ is the same for both angles, their ranges are equal:

$$R = \frac{v_0^2 \cos(2\alpha)}{g}.$$

- 172.** Riya and Neha are taking part in a school physics activity. Riya walks 10 meters east from a fixed point, represented by vector $\mathbf{A}$. At the same time, Neha walks 10 meters west from the same starting point, represented by vector $\mathbf{B}$. Although both cover equal distances, Neha's direction is opposite to Riya's. Vector $\mathbf{B}$ has the same magnitude as vector $\mathbf{A}$ but points in the opposite direction.

Questions:

1. What is a null vector or a zero vector?
2. What is the result of adding a vector $\mathbf{A}$ to its negative $-\mathbf{A}$?
3. Can the direction of a null vector be specified? Why or why not?
4. If an object starts at point P , moves to point P' , and then returns to point P , what is its total displacement vector?

Ans :

1. A null vector or a zero vector is a vector with zero magnitude, represented by $\mathbf{0}$.
2. The sum of $\mathbf{A}$ and $-\mathbf{A}$ is $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$ (a null vector).
3. Since the magnitude of a null vector is zero, its direction cannot be specified.
4. Since the initial and final positions coincide, the displacement is a "null vector".

- 173.** A displacement vector $\mathbf{D}$ has a magnitude of 10 m and points due east. This vector is then multiplied by a real number $\lambda = -2$.

Questions:

1. What happens to the magnitude of a vector when it is multiplied by a positive real number λ ?
2. What happens to the direction of a vector when it is multiplied by a negative real number $-\lambda$?

3. What is the magnitude of the new vector $\mathbf{D}' = -2\mathbf{D}$?
4. What is the direction of the new vector $\mathbf{D}'$?

Ans :

1. Multiplying a vector $\mathbf{A}$ with a positive number λ gives a vector whose magnitude is changed by the factor λ ($|\lambda\mathbf{A}| = \lambda|\mathbf{A}|$ if $\lambda > 0$) but the direction is the same as that of $\mathbf{A}$.
2. Multiplying a vector $\mathbf{A}$ by a negative number $-\lambda$ gives another vector whose direction is opposite to the direction of $\mathbf{A}$ and whose magnitude is λ times $|\mathbf{A}|$.
3. The magnitude of $\mathbf{D}'$ is
$$|-2| \times |\mathbf{D}| = 2 \times 10\text{m} = 20\text{m}.$$
4. Since $\lambda = -2$ is negative, the direction of $\mathbf{D}'$ is opposite to the direction of $\mathbf{D}$. If $\mathbf{D}$ points due east, then $\mathbf{D}'$ points due west.

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CHAPTER 4

Laws of Motion

SUMMARY

1. FORCE

Force is an external effort in the form of push or pull, which produces or tends to produce motion in a body at rest or stops or tends to stop a moving body or changes or tends to change the direction of motion of a body.

Force, $F = ma$

2. INERTIA

The inertia is the property of a body by virtue of which it opposes any change in a state of rest or of uniform motion in a straight line.

or

The inability of a body to change by itself its state of rest or state of uniform motion in a straight line is called inertia.

It is of three types.

- (i) Inertia of rest : The inability of a body to change by itself its state of rest is called inertia of rest.

For example, a person standing in a bus falls forward when bus suddenly starts moving forward.

- (ii) Inertia of motion : The inability of a body to change by itself its state of uniform motion in a straight line is called inertia of motion.

For example, when a moving bus suddenly stops, a person standing in it falls forward.

- (iii) Inertia of direction : The inability of a body to change by itself its direction of motion is called inertia of direction.

For example, when a bus takes a sharp turn, a person sitting in the bus experiences a force acting away from the centre of the curved path due to his tendency to move in the original direction.

3. MOMENTUM

The momentum of a body is defined quantity of motion and is measured as the product of its mass and velocity.

i.e., $\text{momentum} = \text{mass} \times \text{velocity}$

$$\vec{p} = m\vec{v}$$

It is a vector quantity having direction same as the direction of velocity. Its unit in SI is kg ms^{-1} and the dimensional formula is $[M^1 L^1 T^{-1}]$. Momentum is the quantity of motion contained in a body.

3.1 The Law of Conservation of Momentum

When two or more bodies interact with one another, their total momentum along a straight line remains constant, provided no external forces are acting.

e.g. Recoil of a gun, explosion of bomb.

4. NEWTON'S LAWS OF MOTION

Newton made a systematic study of motion and arrived at three laws of motion called Newton's laws of motion.

4.1 Newton's First Law of Motion

Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by external force to change that state. This is also called law of inertia.

4.2 Newton's Second Law of Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

i.e.
$$\vec{F} = \frac{d\vec{p}}{dt}$$

4.3 Newton's Third Law of Motion

To every action, there is an equal and opposite reaction.

i.e. $\vec{F}_{AB} = -\vec{F}_{BA}$

Since both Newton's 1st law and 2nd law of motion are contained in 2nd law, hence 2nd law of motion is called real law of motion.

5. IMPULSE

The change in momentum of a body is called impulse.

$$\text{Impulse} = \text{Force} \times \text{time}$$

6. APPARENT CHANGE IN WEIGHT OF A BODY IN A LIFT

(a) When the lift has an upward acceleration 'a', then apparent weight of man increases as upward force is given by

$$R_1 = m(g + a)$$

(b) When the lift has downward acceleration 'a', then apparent weight of man decreases as downward force is given by

$$R_2 = m(g - a)$$

7. CONNECTED MOTION

If two masses m_1 and m_2 ($m_1 > m_2$) are connected by an inextensible string which passes over a light and frictionless pulley, the acceleration of the system.

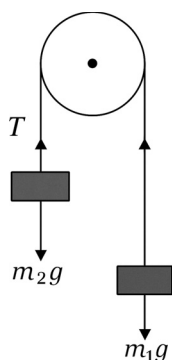
$$\alpha = \frac{(m_1 - m_2)}{(m_1 + m_2)}g$$

As $\alpha = \frac{(m_1 - m_2)}{(m_1 + m_2)}g < g$

So $\alpha < g$

If $m_1 = m_2 = m$ (say), then

$$\alpha = 0$$



Tension in the string,

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

If $m_1 = m_2 = m$ (say), then

$$T = mg$$

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8. CONCURRENT FORCES

A number of forces acting at the same point are called concurrent forces.

9. EQUILIBRIUM IN CONCURRENT FORCES

A number of concurrent forces are said to be in equilibrium if their resultant is zero.

10. FRICTION

Friction is the force opposing the relative motion between two bodies which are in contact with each other.

10.1 Static Friction

Static friction is the force of friction which exactly balances the applied force during a stationary state of the body.

10.2 Limiting Friction

Limiting friction is the maximum value of static friction which comes into play when a body is just going to start sliding over the surface of another body.

Laws of limiting friction :

1. Limiting force of friction between two bodies in contact is directly proportional to the normal reaction between them i.e.,

$$F \propto R$$

2. The direction of the force of limiting friction is always opposite to the direction in which one body is just starting its motion over the other.

3. Force of limiting friction is independent of the apparent area of contact so long as the normal reaction between the two bodies in contact remains the same.

4. The force of limiting friction between any two bodies in contact depends upon the nature of the surfaces in contact.

10.3 Dynamic or Kinetic Friction

Dynamic or kinetic friction comes into play when two bodies are in relative motion.

Kinetic friction is of two types :

- (i) Sliding friction. It comes into play when a solid body slides over the surface of another body.
- (ii) Rolling friction. It comes into play when a body rolls over the surface of another body.

10.4 Coefficient of Friction

The ratio of limiting friction to the normal reaction between any two given surfaces is a constant. This constant is called the coefficient of friction, μ . If F is the limiting friction and R the normal reaction, then

$$\frac{F}{R} = \mu \text{ called coefficient of friction}$$

$$F = \mu R$$

This coefficient is also called the coefficient of static friction.

Kinetic friction is always less than static friction.

10.5 Angle of Friction

It is the angle between the normal reaction and resultant of limiting friction and normal reaction.

Coefficient of friction is equal to the tangent of angle of friction.

10.6 Angle of Repose

It is the angle of inclined plane with the horizontal at which a body placed over it just begins to slide down.

Angle of friction = Angle of repose

Methods of increasing friction

- (i) By making both the surfaces rough.
- (i) By making both the surfaces smooth.

11. DYNAMICS OF CIRCULAR MOTION

In uniform circular motion, speed and magnitude of acceleration are constant, but as their directions always keep on changing, velocity and acceleration are not constants $\vec{v}$ and $\vec{a}_c$ are always perpendicular to each other.

11.1 Centripetal Force

A force which deflects a particle of its straight line path and makes it to move in a circular path is called centripetal force.

The centripetal force required by a particle moving in a circle of radius r with velocity v and having mass m is given by

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$\frac{4\pi^2 mr}{T^2} = 4\pi^2 m v^2 r$$

It always acts along the radius of the circle towards the centre of the circle.

Though the magnitude of centripetal force remains constant in uniform circular motion, its direction keeps on changing.

11.2 Centrifugal force

The outward radial force experienced by an object, when in circular motion is called centrifugal force. It is not a real force. Since we cannot identify the source of centrifugal force, so it is called fictitious or pseudo force. It acts along the radius and away from the centre of the circle.

Magnitude of centrifugal force.

$$F = \frac{mv^2}{r}$$

Which is same as that of the centripetal force. It is regarded as the reaction of centripetal force.

12. MOTION OF VEHICLE

12.1 On a Circular Level Road

The required centripetal force is provided by the friction between the tyres and the road, and maximum velocity of the vehicle is given by

$$v_{\max} = \sqrt{\mu rg}$$

12.2 On a Circular Banked Road

If the road is banked through an angle θ then maximum velocity of the vehicle is given by

$$v_{\max} = \sqrt{rg \tan \theta}$$

On the same basis, curved railway tracks are also banked. The level of outer rail is raised a little above the level of inner rail, while laying a curved railway track.

13. BENDING OF A CYCLIST

When a cyclist goes round a circular track, centripetal force is required. As force of friction is small and uncertain, dependence on it is not safe. To avoid dependence on the force of friction for obtaining centripetal force, the cyclist has to bend a little inward from his vertical position, while going round a curve.

If θ is the angle which the cyclist makes with the vertical while turning, then

$$\tan \theta = \frac{v^2}{rg}$$

Knowing v , r and g , we can find θ . Practically θ may be slightly less than the calculated value due to some contribution of the force of friction for providing centripetal force.

14. MOTION IN A VERTICAL CIRCLE

If a body of mass m is moving in a vertical circle,

- (i) The tension in the string at the lowest point is

$$T_1 = mg + \frac{mv_1^2}{r}$$

- (ii) The tension at the highest point

$$T_2 = \frac{mv_2^2}{r} - mg$$

- (iii) Velocity of the body at lowest point

$$v_2 = \sqrt{5gr}$$

- (iv) Velocity of the body at highest point

$$v_1 = \sqrt{5gr}$$

NCERT TEXT-BOOK QUESTIONS

(For simplicity of numerical calculations, take $g = 10 \text{ ms}^{-2}$).

- Give the magnitude and direction of net force on :
 - a drop of rain falling down with a constant speed,
 - a cork of mass 10 g floating on water,
 - a kite skilfully held stationary in the sky,
 - a car moving with a constant velocity of 30 km/h on a rough road,
 - a high speed electron in space far from all gravitational objects and free of electric and magnetic fields.

Ans :

- Since drop of rain falls with a constant speed due to inertia of motion, therefore, no force acts on the rain drop.
- Due to inertia, cork of mass 10 g is floating on water, therefore no force acts on the cork.
- The kite is held stationary in the sky due to its inertia of rest, therefore, no force acts on it.
- No net force. The car is moving with a constant velocity of 30 km/h on a rough road due to its inertia of motion.
- No net force because it is far away from all material agencies producing electromagnetic and gravitational forces.

- A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble.

- during its upward motion,
- during its downward motion,
- at the highest point where it is momentarily at rest. Do your answer alter if the pebble were thrown at an angle of say 45° with the horizontal direction ?

Ignore air resistance throughout.

Ans :

- Mass of pebble = 0.05 kg

$$g = -10 \text{ ms}^{-2}$$

$$\text{Net force} = mg = 0.05 \text{ kg} \times (-10 \text{ ms}^{-2})$$

$$= 0.5 \text{ N}$$

(acting vertically downwards)

- Net force = $mg = 0.05 \text{ kg} \times (-10 \text{ ms}^{-2})$

$$= 0.5 \text{ N}$$

(acting vertically downwards)

- The stone is at rest at the highest point.

$$\text{Therefore net force} = mg = 0.05 \text{ kg} \times (-10 \text{ ms}^{-2})$$

$$0.5 \text{ N}$$

(acting vertically downwards)

No, answers remain unaltered if pebble were thrown at angle of 45° with horizontal direction.

The motion of the pebble is being affected by the downward pull of gravity (neglecting effect of air) so that the velocity of the pebble at any instant has two components.

- the horizontal component which remains constant.
- the vertical component. But these two components are independent of each other. So, our answers do not alter.

- Given the magnitude and direction of the net force acting on a stone of mass 0.1 kg

- just after it is dropped from the window of a stationary train.
- just after it is dropped from the window of a train running at a constant velocity of 36 km h^{-1} .
- just after it is dropped from the window of a train accelerating 1 ms^{-2}
- lying on the floor of a train which is accelerating with 1 ms^{-2} , the stone being at rest relative to the train. Neglect air resistance through out.

Ans :

(a) Given, $m = 0.1 \text{ kg}$
 $a = g = 10 \text{ ms}^{-2}$
 Net force $F = mg = 0.1 \times 10 \text{ N}$
 $= 1 \text{ N}$, vertically downward.

(b) Net force $= 0.1 \text{ kg} \times 10 \text{ ms}^{-2}$
 $= 1 \text{ N}$, vertically downward.

(C) net force $= 0.1 \text{ kg} \times 10 \text{ ms}^{-2}$
 $= 1 \text{ N}$, vertically downward.
 Acceleration of train has no vertical downward component.

(d) Net force $= m \times a$
 $= 0.1 \text{ kg} \times 1 \text{ ms}^{-2}$
 $= 0.1 \text{ N}$,
 along the direction of the motion of the train

4. One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is :

(i) T (ii) $T - \frac{mv^2}{l}$

(iii) $T + \frac{mv^2}{l}$ (iv) 0

T is the tension in the string (Choose the correct alternative)

Ans :

(i) T is correct.
 The tension is acting towards the centre and along radius provides the necessary centripetal force i.e.

$$T = \frac{mv^2}{l}$$

5. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1} . How long does the body take to stop ?

Ans :

$$F = -50 \text{ N},$$

$$m = 20 \text{ kg}, u$$

$$15 \text{ ms}^{-1},$$

$$v = 0$$

Using $F = ma$,

$$50 = 20 \times a$$

Acceleration,

$$a = -\frac{15}{20} = -2.5 \text{ ms}^{-2}$$

Using $v = u + at$

$$0 = 15 - 2.5t \quad (v = 0 \text{ body stops})$$

$$2.5t = 15$$

$$t = \frac{15}{2.5} = 6 \text{ s}$$

6. A constant force acting on a body of mass 3 kg changes its speed from 2 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is magnitude and direction of the force ?

Ans :

$$m = 3 \text{ kg}, u = 2 \text{ ms}^{-1}, v = 3.5 \text{ ms}^{-1}, t = 25 \text{ s}$$

Using $v = u + at$

$$3.5 = 2 + a \times 25$$

$$1.5 = 25a$$

$$a = \frac{1.5}{25} = 0.06 \text{ ms}^{-2}$$

Using $F = ma$
 $= 3 \times 0.06 = 0.18 \text{ N}$

Along the direction of motion.

7. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Ans :

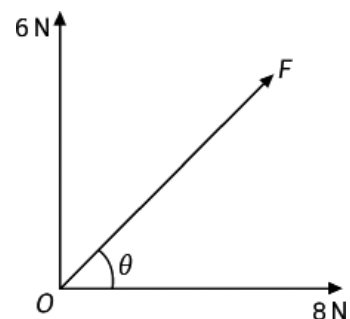
$$\text{Resultant force} = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{8^2 + 6^2}$$

(Given by || gm law of forces)

$$F = \sqrt{64 + 36}$$

$$= \sqrt{100} = 10 \text{ N}$$



If resultant makes an angle θ with 8 N,

$$\text{then } \tan \theta = \frac{6}{8} = \frac{3}{4}$$

$$\theta = \tan^{-1} \frac{3}{4} = 37^\circ \text{ with the direction of } 8 \text{ N force}$$

$$\text{Using } F = ma$$

$$10 \text{ N} = 5 \text{ kg} \times a$$

$$a = \frac{10}{5} = 2 \text{ ms}^{-2}$$

8. The driver of three-wheeler moving with a speed of 36 km h^{-1} sees a child standing in the middle of the road and brings his vehicle to rest in 4 s just in time to save the child. What is the average retarding force on the vehicle?

The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Ans :

$$\text{Speed, } u = 36 \text{ km h}^{-1} = \frac{36 \times 1000}{60 \times 60} \\ = 10 \text{ ms}^{-1}$$

$$v = 0 \text{ } t = 4 \text{ s}$$

Total mass

$$m = 400 \text{ kg} + 65 \text{ kg} = 465 \text{ kg}$$

$$\text{Using } v = 0, u + at$$

$$0 = 10 + a \times 4$$

$$a = -\frac{10}{4} = -2.5 \text{ ms}^{-2}$$

$$F = ma$$

$$= 465 \times (-2.5) = -1162.5 \text{ N}$$

Retarding force = 1162.5 N

9. A rocket with a lift off mass 20,000 kg is blasted upwards with an initial acceleration of 5 ms^{-2} . Calculate the internal thrust (force) of the blast.

Ans :

$$m = 20,000 \text{ kg}$$

$$g = 10 \text{ ms}^{-2}$$

$$W = mg = 20,000 \times 10$$

$$= 20.0 \times 10^4$$

$$N = 2.0 \times 10^5 \text{ N}$$

The resultant upward force acting on the rocket is $(F_1 - W)$ and a is the initial vertical acceleration, then from

$$F = ma$$

$$F_1 - W = ma$$

$$F_1 = W + ma$$

$$= 2.0 \times 10^5 + 20,000 \times 5$$

$$= (2.0 + 1) \times 10^5$$

$$= 3.0 \times 10^5 \text{ N}$$

10. A particle of mass 0.4 kg moving with a constant speed of 10 ms^{-1} to the north is subject to a constant force of 8 N directed towards the south for 30 s. Take the instant the force is applied to be $t = 0$, the position of the particle at that time to be $x = 0$, and predict its position at $t = -5 \text{ s}$, 25 s, 100 s.

Ans :

$$m = 0.4 \text{ kg},$$

$$u = 10 \text{ ms}^{-1} \text{ to the north}$$

$$F = 8 \text{ N towards the south, } (t = 30 \text{ s})$$

- (i) when $t = -5 \text{ s}$

$$s = ut$$

$$= 10 \text{ ms}^{-1} \times (-5 \text{ s}) = -50 \text{ m}$$

- (ii) when $t = 25 \text{ s}$

$$s = ut$$

$$= 10 \times 25 = 250 \text{ m}$$

$$a = \frac{F}{m} = \frac{-8 \text{ N}}{0.4} = -20 \text{ ms}^{-2}$$

Distance travelled under the influence of force

$$s = \frac{1}{2} at^2 = \frac{1}{2} (-20) \times (25)^2$$

$$= 6250 \text{ m}$$

Net distance travelled in 25 s

$$= -6250 + 250$$

$$= -6000 \text{ m} = -6 \text{ km}$$

- (iii) Out of 100 s. let s_1 be the distance covered in 30 s and s_2 be the distance covered in 70 sec.

$$s_1 = 10 \times 30 + \frac{1}{2} (-20) (30)^2$$

$$= -8700 \text{ m}$$

Velocity after 30 sec.

$$v = 10 + (-20) 30 = -590 \text{ ms}^{-1}$$

As no force acts in next 70 sec., therefore distance travelled in next 70 s.

$$s_2 = -590 (70) = -41300 \text{ m}$$

$$s = s_1 + s_2 = -41300 - 8700$$

$$= -50000 \text{ m} = -50 \text{ km}$$

Position at $t = 100 \text{ s}$ is 50 km.

11. A truck starts from rest and accelerates uniformly with 2.0 ms^{-2} . At $t = 10 \text{ s}$, a stone is thrown by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity and (b) acceleration of the stone at $t = 11 \text{ s}$? (Neglect air resistance).

Ans :

- (a) Since the truck starts from rest, so initial velocity of stone is zero and the stone will possess horizontal velocity due to motion of truck and vertical downward velocity due to gravity.

So, $u_x = 0$ and $u_y = 0$, $a = 2 \text{ ms}^{-2}$
 $t = 10 \text{ s}$

Horizontal velocity of stone

Using $v_x = u_x + at$
 $= 0 + 2 \times 10$
 $= 20 \text{ ms}^{-1}$

Vertical velocity of stone

Using $v_y = u + at$
 $= 0 + 10(1)$
 $= 10 \text{ ms}^{-1}$

Resultant velocity of stone,

$$|v| = \sqrt{20^2 + 10^2}$$

$$= 22.36 \text{ ms}^{-2}$$

$$\tan \theta = \frac{v_y}{v_x} = \frac{10}{20} = 0.5$$

$$\theta = \tan^{-1} 0.5 = 3^\circ \text{ with the horizontal direction.}$$

- (b) Verbal component of acceleration $= 10 \text{ ms}^{-2}$
 since velocity is constant,
 Horizontal component of acceleration $= 0$
 Acceleration of the stone at 11 s

$$= \sqrt{(10)^2 + 0^2} = 10 \text{ ms}^{-2}$$

12. A bob of mass 0.1 kg hung from ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 ms^{-1} . What is the trajectory of the bob if the string is cut when the bob is :

- (a) at one of its extreme positions,
 (b) at its mean position ?

Ans :

$$m = 0.1 \text{ kg.}$$

- (a) Since there is no horizontal component of the velocity at its extreme position, the only acceleration on it is in vertically downward direction (due to force of gravity).

Hence the bob falls vertically downwards.

- (b) The bob at its mean position is affected by the downward pull of gravity and its velocity in the horizontal direction. Therefore, it falls down along a parabolic path.

13. A man weighs 70 kg. He stands on a weighing scale in a lift which is moving ?

- (a) upwards with a uniform speed of 10 ms^{-1} .
 (b) downwards with a uniform acceleration of 5 ms^{-2} .
 (c) upwards with a uniform acceleration of 5 ms^{-2}
 (d) What would be the reading on the scale in each case ? What would be the reading if the lift mechanism failed and is hurled down freely under gravity ?

Ans :

$$\text{Mass of man} = 70 \text{ kg}$$

- (a) Since speed is uniform, therefore Acceleration,

$$a = 0$$

$$R = mg = 70 \times 10 = 700.0 \text{ N}$$

- (b) $a = 5 \text{ ms}^{-2}$

$$mg - R = ma$$

$$R = mg - ma$$

$$= m(g - a)$$

$$R = 70(10 - 5) = 350 \text{ N}$$

$$= 350 \text{ N, Reading is 35 kg}$$

$$R = mg + ma$$

$$= m(g + a)$$

$$= 70(10 + 5) = 350 \text{ N}$$

$$= 350. \text{ N, Reading is 35 kg}$$

- (c) $R = mg + ma$

$$= m(g + a)$$

$$= 70(10 + 5) = 70 \times 15$$

$$= 1050 \text{ N}$$

$$\text{Reading is 105 kg}$$

- (d) In case of free fall of the lift,

$$a = g$$

$$\text{then } N = m(g - g) = 0$$

Hence, under the free fall the scale would read zero .

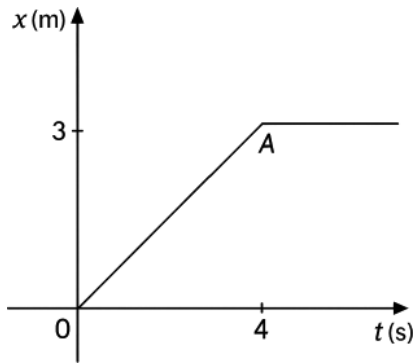
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14. The Figure shows the position-time graph of a particle of mass 4 kg. What is the :
 (a) Force on the particle for $t < 0$, $t > 4$ s, $0 < t < 4$ s ?
 (b) Impulse at $t = 0$ and $t = 4$ s ; (Consider one-dimensional motion only).



Ans :

- (a) At $t < 0$, Force on the particle = 0
 At $t > 4$ s, no force acts on the particle as velocity is constant and acceleration is zero.
 As $0 < t < 4$ s. In this case, the velocity is uniform and hence there is no acceleration. Therefore, force on the particle = 0.

- (b) At $t = 0$ impulse is zero

$$t = 4 \text{ s}, v = \frac{3}{4} \text{ ms}^{-1} \\ = 0.75 \text{ ms}^{-1}$$

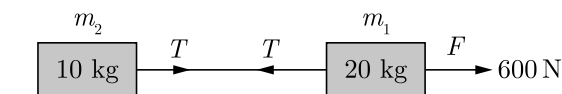
Impulse = Change in momentum

$$= m(v - u) \\ = 4\left(\frac{3}{4} - 0\right) = 3 \text{ kg ms}^{-1} \text{ at}$$

15. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string, a horizontal force $F = 600$ N is applied at (i) A, (ii) B along the direction of string. What is the tension in string in each case ?

Ans :

The tension depends upon the end on which pull is opened.



Suppose F, force applied on (20 kg mass) m_1 .

$$F - T = m_1 a$$

$$F - m_2 a = m_1 a$$

$$F = m_1 a + m_2 a = a(m_1 + m_2)$$

$$600 = a(20 + 10)$$

$$a = \frac{600}{30} = \frac{60}{3} \text{ ms}^{-2} = 20 \text{ ms}^{-2}$$

Tension T acts on the mass $m_2 (= 10 \text{ kg})$

$$T = m_2 a$$

$$= 10 \times 20 = 200 \text{ N if applied at the } 20 \text{ kg mass.}$$

If force is applied on 10 kg mass

$$T = m_1 a = 20 \times 20 = 400 \text{ N}$$

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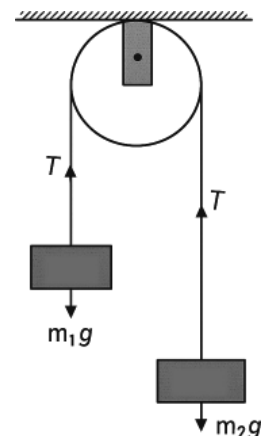
16. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.

Ans :

$$m_1 = 8 \text{ kg}, m_2 = 12 \text{ kg}$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} \\ = \frac{(12 - 8) \times 10}{12 + 8} \\ = \frac{4}{20} \times 10 = 2.0 \text{ ms}^{-2}$$

$$T = \frac{2m_1 m_2 g}{(m_1 + m_2)} \\ = \frac{2 \times 8 \times 8 \times 12 \times 10}{(8 + 12)} \\ = \frac{2 \times 8 \times 12 \times 10}{20} = 96 \text{ N}$$



17. A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the product must be emitted in opposite directions.

Ans :

Let the mass of the nucleus at rest be M Its velocity = 0

Total initial momentum of nucleus = $M \times 0 = 0$

Let it split into two nuclei each of mass m_1 and m_2
Let the velocity of one nucleus be v and that of the other is V

Final momentum of system = $m_1 v + m_2 V = 0$

Therefore total momentum before disintegration is equal to the total momentum after disintegration.

$$m_1 v + m_2 V = 0$$

or

$$m_2 V = -m_1 v$$

Therefore, the products after collision must move in opposite directions.

18. Two billiard balls each of mass 0.05 kg and moving in opposite directions with speed of 6 ms⁻¹ collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?

Ans :

Mass of each billiard ball = 0.05 kg

Speed of each ball before collision = 6 ms⁻¹

Speed of each ball after collision = -6 ms⁻¹

Initial momentum of each ball before collision

$$m_1 u_1 = 0.05 \times 6 = 0.3 \text{ kg ms}^{-1}$$

Final momentum of each ball

$$= m_1 u_2$$

$$= 0.05(-6)$$

$$= -0.3 \text{ kg ms}^{-1}$$

Impulse = Change in momentum

= Final momentum - Initial momentum

$$= (-0.3 - 0.3) \text{ kg ms}^{-1}$$

$$= -0.6 \text{ kg ms}^{-1}$$

The impulse imparted is 0.6 kg ms⁻¹ to each ball due to the other.

19. A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms⁻¹, what is the recoil of the gun ?

Ans :

Since $mv + MV = 0$

$$V = -\frac{mv}{M} = -\frac{.02 \times 80}{100} \\ = -.016 \text{ ms}^{-1}$$

Negative sign shows that the gun recoils.

20. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km h⁻¹. What is the impulse imparted to the ball ? (Mass of the ball is 0.15 kg).

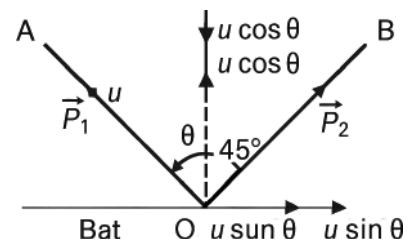
Ans :

Mass of the ball = 0.15 kg

Initial speed, $u = 54 \text{ km h}^{-1}$

$$= \frac{54 \times 1000}{60 \times 60} = 15 \text{ ms}^{-1}$$

Here $\theta = \frac{45^\circ}{2} = 22.5^\circ$



Impulse imparted to the ball

$$= mu \cos \theta - (-mu \cos \theta)$$

$$= 2mu \cos \theta$$

$$= 2 \times 0.15 \times 15 \times \cos 22.5^\circ$$

$$= 4.2 \text{ kg ms}^{-1}$$

21. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with speed of 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?

Ans :

$$\omega = 2\pi$$

$$v = 2\pi \frac{40}{60} = \frac{4\pi}{3}$$

Tension, $T = mR$

$$\omega^2 = 0.25 \times 1.5 \times \frac{16\pi^2}{9} = 6.6 \text{ N}$$

Since $T_{\max} = \frac{mv_{\max}^2}{R}$

$$\begin{aligned}\text{So } 200 &= \frac{0.25}{1.5} v_{\max}^2 \\ v_{\max}^2 &= 200 \times \frac{150}{25} = 1200 \\ v_{\max} &= 34.6 \text{ ms}^{-1}\end{aligned}$$

22. If in Q. 5.21 the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :

- the stone moves radially outwards.
- the stone flies off tangentially from the instant the string breaks.
- the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle ?

Ans :

- the stone flies off tangentially from the instant the string breaks.

23. Explain why :

- a horse cannot pull a cart and run in empty space ?
- passengers are thrown forward from their seats when a speeding bus stops suddenly ?
- it is easier to pull a lawn mover than to push it?
- a cricketer moves his hands backwards when holding a catch ?

Ans :

- In pulling the cart forward, the horse exerts force on the ground which gives reaction. The reaction of the earth may be resolved in two rectangular components. The horizontal component is responsible for the forward motion of the system as a whole. In the case of empty space the backward reaction is not possible. Hence the horse cannot pull the cart and run in empty space.
- When a speeding bus stops suddenly, the passengers are thrown forward because due to inertia of motion the passengers tend to remain in their state of motion even though the bus has come to rest.
- Because in case of pull the roller becomes lighter in weight and due to decrease in normal reaction the force of friction decreases. While in case of push, normal reaction and hence force of friction increases. Hence it is easier to pull than to push a lawn roller.
- By doing so the cricket player increases the time and hence reduces the force exerted by the ball on his hands.

OBJECTIVE QUESTIONS

1. Which of the following terms describes the property of a body to resist changes to its state of rest or of uniform motion?

- Momentum
- Inertia
- Force
- Acceleration

Ans :

- Inertia

Inertia is the inherent property of a body that makes it resist any change in its state of rest or uniform motion unless an external force compels it to change that state.

2. What is the primary assertion of Aristotle's view on motion, which was later found to be flawed?

- An external force is required to stop a body in motion.
- An external force is required to keep a body in motion.
- No external force is needed to keep a body in uniform motion.
- Frictional forces always enhance the motion of a body.

Ans :

- An external force is required to keep a body in motion.

Aristotle held the view that if a body is moving, something external is required to keep it moving. This idea is now understood to be incorrect due to the presence of forces like friction.

3. According to Galileo's experiments with inclined planes, what happens to an object moving on a frictionless horizontal plane?

- It accelerates uniformly.
- It decelerates uniformly.
- It moves with constant velocity.
- It eventually comes to rest due to air resistance.

Ans :

- It moves with constant velocity.

Galileo concluded from his experiments that an object moving on a frictionless horizontal plane must neither have acceleration nor retardation, meaning it should move with constant velocity.

4. Which scientist formulated the laws of motion, building upon Galileo's ideas?

- Aristotle
- Isaac Newton

- (C) Johannes Kepler
(D) Tycho Brahe

Ans :

- (B) Isaac Newton

Isaac Newton built on Galileo's ideas and laid the foundation of mechanics in terms of three laws of motion that are named after him.

5. Newton's first law of motion is also known as the law of:
- (A) Action-reaction
(B) Momentum
(C) Inertia
(D) Gravitation

Ans :

- (C) Inertia

Newton's first law of motion is essentially a restatement of Galileo's law of inertia, describing a body's resistance to change in its state of motion.

6. If the net external force on a body is zero, what can be said about its acceleration according to Newton's first law?
- (A) Its acceleration is constant but non-zero.
(B) Its acceleration is increasing.
(C) Its acceleration is zero.
(D) Its acceleration is decreasing.

Ans :

- (C) Its acceleration is zero.

Newton's first law of motion states that if the net external force on a body is zero, its acceleration is zero. The body will either remain at rest or continue to move with uniform velocity.

7. What is the SI unit of force?
- (A) Joule (J)
(B) Watt (W)
(C) Newton (N)
(D) Pascal (Pa)

Ans :

- (C) Newton (N)

The SI unit of force is the newton (N), which is defined as the force that causes an acceleration of 1 ms^{-2} to a mass of 1 kg.

8. Which of the following statements about Newton's first law of motion is incorrect?
- (A) It implies that a force is needed to change the velocity of a body.
(B) It applies to bodies both at rest and in uniform motion.

- (C) It suggests that a body in motion will naturally come to rest without any external force.
(D) It is also known as the law of inertia.

Ans :

- (C) It suggests that a body in motion will naturally come to rest without any external force.
Newton's first law states that a body in motion continues to move with a uniform velocity unless an external force acts on it. It does not naturally come to rest without an opposing force like friction.

Which of the following statements correctly describes the concept of momentum?

- (A) Momentum is the product of mass and acceleration.
(B) Momentum is a scalar quantity.
(C) Momentum is the product of mass and velocity.
(D) Momentum is the rate of change of velocity.

Ans :

- (C) Momentum is the product of mass and velocity.
Momentum of a body is defined as the product of its mass (m) and velocity (v). It is a vector quantity, meaning it has both magnitude and direction.

10. Why does a passenger in a bus get thrown backward when the bus suddenly starts?
- (A) Due to the force applied by the bus seat.
(B) Due to the inertia of the passenger's body.
(C) Due to the increased friction between the feet and the floor.
(D) Due to the air resistance pushing the passenger back.

Ans :

- (B) Due to the inertia of the passenger's body.

When the bus starts suddenly, the lower part of the passenger's body (feet in contact with the floor) moves with the bus. However, due to inertia, the upper part of the body tends to remain at rest, causing a relative backward motion.

11. Newton's second law of motion relates the net external force on a body to its:
- (A) Displacement
(B) Velocity
(C) Acceleration
(D) Position

Ans :

- (C) Acceleration

Newton's second law of motion states that the rate of change of momentum of a body is directly proportional to the applied force and takes place

in the direction in which the force acts. This is commonly expressed as $F = ma$.

- 12.** According to Newton's second law, force is directly proportional to the rate of change of:

(A) Energy
(B) Momentum
(C) Displacement
(D) Velocity

Ans :

(B) Momentum

Newton's second law of motion states that the rate of change of momentum of a body is directly proportional to the applied force and acts in the direction of the force.

- 13.** What is impulse defined as?

(A) The product of force and displacement.
(B) The product of mass and acceleration.
(C) The product of force and time duration.
(D) The rate of change of energy.

Ans :

(C) The product of force and time duration.

Impulse is defined as the product of force and the time duration for which the force acts. It is also equal to the change in momentum of the body.

- 14.** Newton's third law of motion is often stated as: "To every action, there is always an equal and opposite _____."

(A) Momentum
(B) Reaction
(C) Acceleration
(D) Displacement

Ans :

(B) Reaction

Newton's third law of motion states that to every action, there is always an equal and opposite reaction. This means forces always occur in pairs.

- 15.** If body A exerts a force on body B (F_{AB}), then according to Newton's third law, body B exerts a force on body A (F_{BA}) such that:

(A) $F_{AB} = F_{BA}$
(B) $F_{AB} = -F_{BA}$
(C) $F_{AB} > F_{BA}$
(D) $F_{AB} < F_{BA}$

Ans :

(B) $F_{AB} = -F_{BA}$

Newton's third law states that the force exerted by body A on body B is equal in magnitude and

opposite in direction to the force exerted by body B on body A.

- 16.** Action and reaction forces, as described by Newton's third law, act on:

(A) The same body.
(B) Different bodies.
(C) Only bodies in contact.
(D) Only bodies not in contact.

Ans :

(B) Different bodies.

A crucial aspect of Newton's third law is that action and reaction forces act on different bodies. Therefore, they do not cancel each other out when considering the motion of a single body.

- 17.** The law of conservation of momentum states that the total momentum of an isolated system of interacting particles:

(A) Always increases.
(B) Always decreases.
(C) Is conserved.
(D) Depends on the internal forces only.

Ans :

(C) Is conserved.

The law of conservation of momentum, derived from Newton's second and third laws, states that the total momentum of an isolated system (one with no net external force) remains constant.

- 18.** Which of the following statements about Newton's third law is incorrect?

(A) Forces always occur in pairs.
(B) Action is the cause and reaction is the effect.
(C) Action and reaction forces are equal in magnitude.
(D) Action and reaction forces act on different bodies.

Ans :

(B) Action is the cause and reaction is the effect.

Newton's third law does not imply a cause-effect relationship where action precedes reaction. The forces occur simultaneously as a mutual interaction between two bodies.

- 19.** Which statement correctly describes equilibrium of a particle?

(A) The particle has zero velocity.
(B) The net external force on the particle is zero.
(C) The particle has zero acceleration and zero velocity.
(D) Only one force acts on the particle.

Ans :

(B) The net external force on the particle is zero. Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero. This means the particle is either at rest or in uniform motion.

20. When a gun fires a bullet, the gun recoils. This is an example of:

(A) Newton's first law of motion.
 (B) Newton's second law of motion.
 (C) Newton's third law of motion.
 (D) The law of conservation of energy.

Ans :

(C) Newton's third law of motion.

The force exerted by the gun on the bullet (action) is accompanied by an equal and opposite force exerted by the bullet on the gun (reaction), causing the recoil. This also illustrates conservation of momentum.

21. Which of the following forces is NOT a contact force?

(A) Frictional force
 (B) Normal reaction force
 (C) Tension in a string
 (D) Gravitational force

Ans :

(D) Gravitational force

Gravitational force is a non-contact force that acts between objects with mass, even when they are separated by a distance. Friction, normal reaction, and tension are contact forces.

22. What is the primary difference between static friction and kinetic friction?

(A) Static friction is always greater than kinetic friction.
 (B) Static friction acts on moving objects, while kinetic friction acts on stationary objects.
 (C) Static friction opposes impending motion, while kinetic friction opposes actual relative motion.
 (D) Static friction depends on the area of contact, while kinetic friction does not.

Ans :

(C) Static friction opposes impending motion, while kinetic friction opposes actual relative motion. Static friction (f_s) is a self-adjusting force that opposes the tendency of motion between surfaces at rest, up to a maximum limit. Kinetic friction (f_k) acts when there is relative motion between the surfaces.

23. Assertion (A): A cricketer moves his hands backward while catching a fast-moving cricket ball.

Reason (R): Moving the hands backward increases the time of impact, thereby reducing the force experienced by the hands.

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

By increasing the time (Δt) over which the momentum of the ball is brought to zero, the cricketer reduces the average force ($F = \Delta p / \Delta t$) exerted on his hands, preventing injury.

24. Assertion (A): The centripetal force acting on a body in uniform circular motion does no work on the body.

Reason (R): In uniform circular motion, the centripetal force is always perpendicular to the instantaneous velocity (and displacement).

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

Work done is given by $W = F \cdot d \cos \theta$. Since the centripetal force is always perpendicular to the displacement (tangential to the circle), the angle θ is 90° , and $\cos 90^\circ = 0$. Thus, no work is done.

25. Assertion (A): It is easier to pull a lawn mower than to push it.

Reason (R): Pulling reduces the normal reaction and hence the frictional force, while pushing increases the normal reaction and friction.

(A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

When pulling, the upward component of the applied force reduces the normal reaction. When pushing, the downward component increases the normal reaction. Since friction is proportional to the normal reaction, pulling makes it easier.

26. Consider the following statements regarding Newton's laws of motion:

1. The first law defines inertia.
2. The second law provides a measure of force.
3. The third law implies that forces always exist in pairs.
4. Internal forces within a system can change the system's total momentum.

Which of the above statements are correct?

- (A) 1, 2, and 3 only
 (B) 1, 3, and 4 only
 (C) 2, 3, and 4 only
 (D) 1, 2, 3, and 4

Ans :

(A) 1, 2, and 3 only

Statement 1 is correct as the first law is the law of inertia. Statement 2 is correct as $F = ma$ gives a quantitative measure of force. Statement 3 is correct as action-reaction pairs always exist. Statement 4 is incorrect; only external forces can change the total momentum of a system.

27. A book is at rest on a table. Which of the following statements are true?

1. The net force on the book is zero.
2. The force of gravity on the book and the normal force by the table on the book are an action-reaction pair.
3. The weight of the book is equal in magnitude to the normal force exerted by the table.
4. If the table were frictionless and pushed, the book would remain at rest relative to the ground.

- (A) 1 and 3 only
 (B) 1, 3, and 4 only
 (C) 2 and 3 only
 (D) All statements are true.

Ans :

(A) 1 and 3 only

Statement 1 is true as the book is at rest. Statement 2 is false; the force of gravity on the book and the force of gravity by the book on the Earth form an action-reaction pair. The normal force on the book

by the table and the force by the book on the table form another pair. Statement 3 is true for a book at rest on a horizontal table. Statement 4 is false; due to inertia, the book would tend to stay in its original position if the table moved.

28. Which of the following statements about friction are correct?

1. Static friction is a self-adjusting force.
2. The maximum value of static friction is independent of the area of contact.
3. Kinetic friction is generally less than the maximum static friction.
4. Rolling friction is typically greater than sliding friction.

- (A) 1, 2, and 3 only
 (B) 1 and 3 only
 (C) 2, 3, and 4 only
 (D) 1, 2, 3, and 4

Ans :

(A) 1, 2, and 3 only

Statement 1 is correct. Statement 2 is correct; $(f_s)_{max} = \mu_s N$. Statement 3 is correct. Statement 4 is incorrect; rolling friction is generally much smaller than sliding friction.

29. Match the physical concepts in Column I with their descriptions in Column II.

	Column I		Column II
1.	Inertia	P.	Rate of change of momentum
2.	Force	Q.	Product of mass and velocity
3.	Momentum	R.	Resistance to change in state of motion
4.	Impulse	S.	Product of force and time duration

- (A) 1-R, 2-P, 3-Q, 4-S
 (B) 1-Q, 2-S, 3-P, 4-R
 (C) 1-R, 2-Q, 3-P, 4-S
 (D) 1-S, 2-P, 3-Q, 4-R

Ans :

(A) 1-R, 2-P, 3-Q, 4-S

Inertia is the resistance to change in the state of motion. Force is defined by Newton's second law as the rate of change of momentum. Momentum is the product of mass and velocity. Impulse is the product of force and the time duration over which it acts.

30. Match the type of force in Column I with an example in Column II.

	Column I		Column II
1.	Gravitational Force	P.	Force exerted by a compressed spring
2.	Frictional Force	Q.	Force holding a book on a table
3.	Normal Force	R.	Force attracting Earth to the Sun
4.	Spring Force	S.	Force opposing a sliding box

- (A) 1-R, 2-S, 3-Q, 4-P
 (B) 1-P, 2-Q, 3-R, 4-S
 (C) 1-R, 2-Q, 3-S, 4-P
 (D) 1-S, 2-R, 3-P, 4-Q

Ans :

- (A) 1-R, 2-S, 3-Q, 4-P

Gravitational force is exemplified by the Earth-Sun attraction. Frictional force opposes the motion of a sliding box. Normal force supports a book on a table. Spring force is exerted by a compressed or stretched spring.

31. Match the law of motion in Column I with its consequence or related concept in Column II.

	Column I		Column II
1.	Newton's First Law	P.	$F = ma$
2.	Newton's Second Law	Q.	Conservation of Momentum
3.	Newton's Third Law	R.	Definition of Inertia
4.	Combined Second and Third Law	S.	Action-Reaction Pair

- (A) 1-P, 2-R, 3-S, 4-Q
 (B) 1-R, 2-P, 3-S, 4-Q
 (C) 1-R, 2-S, 3-P, 4-Q
 (D) 1-Q, 2-P, 3-S, 4-R

Ans :

- (B) 1-R, 2-P, 3-S, 4-Q

Newton's First Law defines inertia. Newton's Second Law gives the relation $F = ma$. Newton's Third Law describes action-reaction pairs. The law of conservation of momentum is a consequence of Newton's second and third laws.

32. Identify the correct pair relating a force and its nature:

- (A) Tension in a string : Non-contact force
 (B) Force of friction : Always acts in the direction of motion
 (C) Weight of a body : Contact force due to Earth's surface
 (D) Centripetal force : Force required for circular motion

Ans :

- (D) Centripetal force : Force required for circular motion

Centripetal force is the name given to the net force directed towards the center, which is necessary for an object to move in a circular path. The other pairs are incorrect descriptions.

33. Which of the following pairs correctly matches a term with its definition?

- (A) Momentum : The quantity of matter in a body
 (B) Impulse : The change in velocity per unit time
 (C) Inertia : The force that opposes motion
 (D) Equilibrium : Net external force is zero

Ans :

- (D) Equilibrium : Net external force is zero

A particle or body is said to be in equilibrium when the net external force acting on it is zero. This results in zero acceleration, meaning the body is either at rest or moving with constant velocity.

34. Identify the incorrect pair:

- (A) Aristotle's Fallacy : An external force is needed to keep a body in motion.
 (B) Newton's First Law : If net external force is zero, acceleration is zero.
 (C) Newton's Second Law : Force is the product of mass and displacement.
 (D) Newton's Third Law : Forces always occur in equal and opposite pairs.

Ans :

- (C) Newton's Second Law : Force is the product of mass and displacement.

Newton's Second Law states that force is the product of mass and acceleration ($F = ma$), or more generally, the rate of change of momentum. Force is not the product of mass and displacement.

35. Which of the following pairs incorrectly matches a force with its characteristic?

- (A) Gravitational force : Acts between any two masses
 (B) Frictional force : Depends on the nature of surfaces in contact

- (C) Normal reaction : Always acts vertically downwards
- (D) Centripetal force : Always directed towards the center of the circular path

Ans :

(C) Normal reaction : Always acts vertically downwards

The normal reaction force acts perpendicular to the surface of contact. It is not always vertically downwards; for example, on an inclined plane, it is perpendicular to the incline.

- 36.** Arrange the following steps in solving problems in mechanics using free-body diagrams in the correct sequence:

1. Draw a separate diagram of the chosen system showing all forces on it.
2. Identify action-reaction pairs if multiple systems are chosen.
3. Choose a convenient part of the assembly as the system.
4. Include known information about forces and treat others as unknowns to be found using laws of motion.
5. Draw a schematic diagram of the assembly.

- (A) 5, 3, 1, 4, 2
 (B) 3, 5, 1, 2, 4
 (C) 5, 1, 3, 4, 2
 (D) 3, 1, 5, 4, 2

Ans :

- (A) 5, 3, 1, 4, 2

The systematic approach involves first drawing the overall setup, then isolating a system, drawing its free-body diagram with all forces, using knowns and unknowns with laws of motion, and finally considering action-reaction pairs if needed for other parts.

- 37.** Consider the historical development of ideas about motion. Arrange the following scientists/philosophers in chronological order of their significant contributions discussed in the chapter:

1. Isaac Newton
2. Galileo Galilei
3. Aristotle

- (A) 3, 2, 1
 (B) 2, 3, 1
 (C) 1, 2, 3
 (D) 3, 1, 2

Ans :

- (A) 3, 2, 1

Aristotle's ideas on motion were prevalent in ancient times. Galileo, in the seventeenth century, challenged Aristotelian mechanics and introduced the law of inertia. Newton, building on Galileo's work, formulated his laws of motion.

- 38.** A car of mass 1000 kg accelerates from rest to a speed of 20 m/s in 10 seconds. What is the average net force acting on the car?

- (A) 1000 N
 (B) 2000 N
 (C) 5000 N
 (D) 20000 N

Ans :

- (B) 2000 N

First, calculate the acceleration:

$$\begin{aligned} a &= (v - u) / t \\ &= (20 \text{ m/s} - 0 \text{ m/s}) / 10 \text{ s} \\ &= 2 \text{ m/s}^2 \end{aligned}$$

Then, use Newton's second law:

$$F = ma = 1000 \text{ kg} \times 2 \text{ m/s}^2 = 2000 \text{ N}$$

- 39.** The term 'impending motion' is most relevant to which type of force?

- (A) Kinetic friction
 (B) Static friction
 (C) Gravitational force
 (D) Centripetal force

Ans :

- (B) Static friction

Static friction is the force that opposes impending motion, meaning the motion that would occur if friction were absent when an external force is applied.

- 40.** The law of conservation of momentum is a direct consequence of which of Newton's laws?

- (A) First and Second Laws
 (B) Second and Third Laws
 (C) First and Third Laws
 (D) First Law only

Ans :

- (B) Second and Third Laws

The law of conservation of momentum can be derived from Newton's second law (rate of change of momentum is force) and Newton's third law (mutual forces between interacting particles are equal and opposite).

41. The coefficient of static friction (μ_s) is generally:
- (A) Greater than the coefficient of kinetic friction (μ_k).
 - (B) Less than the coefficient of kinetic friction (μ_k).
 - (C) Equal to the coefficient of kinetic friction (μ_k).
 - (D) Unrelated to the coefficient of kinetic friction (μ_k).

Ans :

- (A) Greater than the coefficient of static friction (μ_s).

Experimental observations show that the maximum value of static friction is typically greater than kinetic friction. It takes more force to start an object moving than to keep it moving.

42. When a body is in uniform circular motion, the force directed towards the center is called:
- (A) Centrifugal force
 - (B) Centripetal force
 - (C) Frictional force
 - (D) Gravitational force

Ans :

- (B) Centripetal force

The force required to keep an object moving in a circular path at constant speed is directed towards the center of the circle and is called the centripetal force.

43. What is the formula for the maximum speed a car can take on a banked road without friction to avoid slipping?
- (A) $v_{max} = \sqrt{\mu_s Rg}$
 - (B) $v_{max} = \sqrt{Rg \tan \theta}$
 - (C) $v_{max} = Rg \tan \theta$
 - (D) $v_{max} = \mu_s Rg$

Ans :

- (B) $v_{max} = \sqrt{Rg \tan \theta}$

For a banked road without friction (or the optimum speed when friction is present but not needed), the centripetal force is provided by the horizontal component of the normal reaction. This leads to $v_o = \sqrt{Rg \tan \theta}$.

44. Which of the following is an example of a self-adjusting force?
- (A) Gravitational force
 - (B) Kinetic friction
 - (C) Static friction
 - (D) Spring force

Ans :

- (C) Static friction

Static friction adjusts its magnitude to be equal and opposite to the applied parallel force, up to its maximum limiting value, to keep the object at rest.

45. The property of inertia is most closely related to a body's:
- (A) Velocity
 - (B) Acceleration
 - (C) Mass
 - (D) Weight

Ans :

- (C) Mass

Mass is a quantitative measure of inertia. A more massive body has greater inertia, meaning it resists changes in its state of motion more strongly.

46. Which of the following statements about circular motion is incorrect?
- (A) For uniform circular motion, the speed is constant.
 - (B) The acceleration in uniform circular motion is directed radially inward.
 - (C) The centripetal force is always provided by friction.
 - (D) The velocity vector in circular motion is always tangential to the path.

Ans :

- (C) The centripetal force is always provided by friction.

The centripetal force can be provided by various types of forces, such as tension in a string, gravitational force, or friction, depending on the situation. It's not always friction.

47. Which of the following statements accurately describes the conditions for translational equilibrium of a rigid body?
- (A) The net external torque on the body must be zero.
 - (B) The body must be at rest.
 - (C) The net external force on the body must be zero.
 - (D) The angular acceleration of the body must be zero.

Ans :

- (C) The net external force on the body must be zero.

For translational equilibrium, the vector sum of all external forces acting on the rigid body must be zero. This ensures that the linear acceleration of the center of mass is zero.

48. If a body is moving with constant velocity, what can be concluded about the net external force acting on it?
- (A) The net external force is constant and non-zero.
 (B) The net external force is zero.
 (C) The net external force is increasing.
 (D) The net external force is acting perpendicular to the velocity.

Ans :

(B) The net external force is zero.

According to Newton's first law, if a body is moving with constant velocity (which implies zero acceleration), the net external force acting on it must be zero.

49. A 'couple' in mechanics refers to:
- (A) Any two forces acting on a body.
 (B) Two equal forces acting on a body.
 (C) Two equal and opposite forces with different lines of action.
 (D) Two forces that cause only translational motion.

Ans :

(C) Two equal and opposite forces with different lines of action.

A couple consists of a pair of forces of equal magnitude but acting in opposite directions along different lines of action. A couple produces rotation without translation.

50. The principle of moments is primarily used to analyze the equilibrium of a:
- (A) Particle
 (B) Lever
 (C) Freely falling body
 (D) Projectile

Ans :

(B) Lever

The principle of moments (load arm $\times$ load = effort arm $\times$ effort for rotational equilibrium) is a fundamental concept in analyzing the mechanics of levers.

51. Where is the center of gravity of a uniform thin rod located?
- (A) At one of its ends.
 (B) At its geometric center.
 (C) Outside the rod.
 (D) It depends on the material of the rod.

Ans :

(B) At its geometric center.

Due to its uniform mass distribution and symmetry, the center of gravity (and center of mass, assuming uniform gravity) of a uniform thin rod is located at its geometric center.

52. The 'free-body diagram' of a system shows:
- (A) Only the internal forces acting within the system.
 (B) Only the external forces acting on the system.
 (C) Both internal and external forces acting on the system.
 (D) The velocities and accelerations of parts of the system.

Ans :

(B) Only the external forces acting on the system. A free-body diagram is a schematic representation of a chosen system, showing all the external forces exerted on that system by its environment. Internal forces are not shown.

53. What is the SI unit of momentum?

- (A) N s
 (B) N/s
 (C) J s
 (D) J/s

Ans :

(A) N s

Momentum ($p = mv$) has units of kgms^{-1} . Since force $F = ma$, $1\text{N} = 1\text{kgms}^{-2}$. Therefore, $1\text{Ns} = 1\text{kgms}^{-2} \times \text{s} = 1\text{kgms}^{-1}$.

Impulse, which is change in momentum, also has the unit Ns.

54. For a body in rotational equilibrium:

- (A) The net external force is zero.
 (B) The net external torque is zero.
 (C) The linear acceleration is zero.
 (D) The angular velocity is zero.

Ans :

(B) The net external torque is zero.

For a body to be in rotational equilibrium, the vector sum of all external torques acting on it must be zero. This ensures that its angular acceleration is zero.

55. Which law states that an external force is required to keep a body in motion, a view now considered flawed?

- (A) Newton's first law
 (B) Galileo's law of inertia
 (C) Aristotelian law of motion
 (D) Kepler's law of orbits

Ans :

(b) rolling friction is less than sliding friction.

71. Proper inflation of tyres saves fuel. This is because :

- (a) contact area is increased
- (b) contact area is decreased
- (c) normal reaction is reduced
- (d) normal reaction is increased

Ans :

(b) Contact area is decreased.

72. Aeroplanes jets etc. are streamlined to reduce

- (a) dynamic friction
- (b) sliding friction
- (c) rolling friction
- (d) fluid friction

Ans :

(d) Fluid friction

73. A horse pulls a cart harder during the first few steps because :

- (a) limiting friction is higher than kinetic friction
- (b) kinetic friction is higher than limiting friction.
- (c) sliding friction is higher than rolling friction.
- (d) frictional forces stops acting after few steps.

Ans :

(a) Limiting friction is higher than kinetic friction.

74. A box is placed on an inclined plane and has to be pushed down. The angle of inclination is :

- (a) equal angle of friction
- (b) more than angle of friction
- (c) equal to angle of repose
- (d) less than the angle of repose.

Ans :

(d) less than the angle of repose.

When the angle of inclination is equal to angle of repose, the body just begins to slide down the plane, so if a body placed on an inclined plane has to be pushed down, it means that the angle of inclination is less than the angle of repose.

75. A piece of stone tied at one end of a string is whirled in a circle in the horizontal plane with constant speed. Then

- (a) its kinetic energy will go on increasing
- (b) its kinetic energy will go on decreasing
- (c) kinetic energy will remain constant
- (d) centripetal force will do work on it.

Ans :

(c) kinetic energy will remain constant

kinetic energy $E = \frac{1}{2}mv^2$ is also constant

ONE MARK QUESTIONS

76. What is a force ?

Ans :

Force is the push or pull which produces or tries to produce motion in a body at rest or stops or tries to stop a body in motion or changes the direction of motion of the body. The force may produce a change in shape of the body.

77. What is inertia ?

Ans :

The inability of a body to change the position of rest or of uniform motion by itself is called inertia.

78. Are Newton's first law of motion and Galileo's law on inertia different ?

Ans :

No.

79. What is linear momentum ?

Ans :

Momentum or linear momentum is the quantity of motion contained in the body. It is measured by the product of mass and velocity.

80. What is the principle of conservation of momentum?

Ans :

In an isolated system, the vector sum of the linear momentum of all the bodies of the system is conserved and is not affected due to mutual action and reaction.

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81. What is the concept of inertial mass ?

Ans :

Mass is the measure of inertia of the body in linear motion. Such a mass which accounts for linear inertia of the body is called inertial mass of the body.

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- 62.** A shell in flight explodes into four unequal parts. Which of the following is conserved?

(a) Momentum and kinetic energy.
 (b) Momentum
 (c) Kinetic energy
 (d) Neither momentum nor kinetic energy

Ans :

(b) Momentum

The momentum remains conserved because the explosion is due to internal forces of the system.

- 63.** A man sitting in a train which is in motion is facing the engine. He tosses a coin up, the coin falls behind him. The train is :

(a) moving forward with acceleration
 (b) moving forward with uniform speed
 (c) moving backward with uniform speed
 (d) moving backward with acceleration

Ans :

(a) moving forward with acceleration

As the coin falls backward, it means that the train is moving forward with acceleration.

- 64.** A stretching force of 1 N is applied at one end of a spring balance and an equal stretching force is applied at the other end at the same time. The reading on the balance will be :

(a) 0 N
 (b) 1 N
 (c) 2 N
 (d) 0.5 N

Ans :

(b) 1 N

The reading on the balance will be 1 N because action and reaction are equal and opposite.

- 65.** A bird weights 2 kg and is inside a cage of 1 kg. If it starts flying then what is weight of bird and cage assembly ?

(a) 1.5 kg (b) 2.5 kg
 (c) 3 kg (d) 4 kg.

Ans :

(c) 3 kg

Total weight = weight of bird + weight of cage
 $= 2 + 1 = 3 \text{ kg}$

- 66.** A man weighs 80 kg. He stands on a weighing scale in the lift, which is moving upward with a uniform acceleration of 5 ms^{-2} . What would be the reading on the scale ? ($g = 10 \text{ m/s}^2$).

(a) Zero (b) 400 N
 (c) 800 N (d) 1200 N

Ans :

(d) 1200 N

$$R = m(g + a) \\ = 80(10 + 5) = 1200 \text{ N}$$

- 67.** Three forces start acting simultaneously on a particle moving with velocity v . These forces are represented in magnitude and direction by three sides of a triangle taken in the same order. The particle will now move with a velocity :

(a) less than $\vec{v}$ (b) more than $\vec{v}$
 (c) $\vec{v}$ only (d) cannot say

Ans :

(c) $\vec{v}$ only

The particle will move with a velocity $\vec{v}$ because resultant of these forces is zero.

- 68.** Air is thrown on the sail of a stationary boat by an electric fan kept on it. The boat will :

(a) remain stationary.
 (b) start moving in the direction in which air is blown.
 (c) start moving in the direction opposite to that in which air is blown.
 (d) start moving with uniform acceleration. Ans.
 (a) Because action and reaction acts on the boat.

Ans :

(a) remain stationary.

Because action and reaction acts on the boat.

- 69.** Brakes of very small contact area are not used although friction is independent of area because friction

(a) resists motion
 (b) causes wear and tear
 (c) depends upon nature of materials
 (d) operating in this case is sliding friction. Ans.
 (b) Due to wear and tear.

Ans :

(a) resists motion

Due to wear and tear.

- 70.** The wheels are made circular because

(a) less material is used
 (b) rolling friction is less than sliding friction
 (c) it is easier to inflate them
 (d) it is easier to deflate them

- 129.** Give an example when a body moving with uniform speed has acceleration ?

Ans :

A body moving in a circle, has an acceleration $\frac{v^2}{r}$ towards the centre of the circle

- 130.** What physical quantities remain constant for a particle moving along a circular path in a horizontal plane ?

Ans :

Speed, kinetic energy and angular momentum remain constant.

- 131.** What provides the centripetal force to a car taking turn on a level road ?

Ans :

A force of friction between the tyres of the car and the road, provides the centripetal force.

- 132.** Define centrifugal force.

Ans :

The outward radial force experienced by an object, when in circular motion, is called centrifugal force.

- 133.** What provides the centripetal force to a satellite revolving around the earth ?

Ans :

Gravitational force of attraction between the satellite and the earth provides the required centripetal force.

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- 134.** What is the relation between coefficient of friction and angle of repose ?

Ans :

$$\mu = \tan \alpha$$

- 135.** What is the unit of coefficient of friction ?

Ans :

It has no units.

- 136.** Out of static friction, limiting friction and dynamic friction, which is largest ?

Ans :

Limiting friction is largest of all.

TWO MARKS QUESTIONS

- 137.** Why do you say that first law of motion gives the definition of force ?

Ans :

According to first law of motion, a body continuous to be at rest or in uniform motion, unless an external agency brings change in these states. This is the definition of force according to which force is an agency which brings about a change in state of rest or of uniform motion of a body.

- 138.** Define momentum. Give its SI units. Is it a scalar or a vector quantity ?

Ans :

Momentum ($\vec{p}$)

Momentum of a body is defined as quantity of motion possessed by it.

It is measured as the product of mass and velocity.

If m = mass of body and

$\vec{v}$ = linear velocity of body,

then linear momentum

$$\vec{p} = m\vec{v}$$

SI unit of momentum is kg ms^{-1} .

Momentum of a body is a vector quantity. Its direction is the same as that of v .

- 139.** When a standing bus starts the passengers get backward push and when it comes to stop, they get a forward push. Why ?

Ans :

This is because of inertia. When the bus starts, the body of the passenger is still at rest and gets a backward push. Similarly due to inertia the body of the passenger is sharing the motion of the running bus, but when brakes are applied suddenly, the passenger is still moving and gets a forward push.

- 140.** Bodies of larger mass need large initial effort for putting them into motion. Why ?

Ans :

All bodies have an internal property that they themselves cannot change their state of rest or of uniform motion. The bodies of larger mass possess larger inertia of rest. Thus, to put them into motion, large initial efforts are needed.

- 82.** Why is it that an athlete always runs some distance before taking a leap or jump ?

Ans :

An athlete runs for some distance before taking a jump so that his forward momentum may help him in his muscular efforts to get high jump.

- 83.** If we shake branches of a fruit tree, the fruits fall. Explain.

Ans :

By shaking, the branches of the tree come into motion, while the fruits are still at rest due to inertia of rest. The result is that fruits are separated from branches and fall down due to their own weight.

- 84.** If a ball is thrown up in a moving train, it comes back to the same place, why ?

Ans :

The ball comes back to the same place because the ball in the train moves in the horizontal direction at the same speed as that of train (due to inertia of motion).

- 85.** What is the cause of friction ? In which case is it maximum ?

Ans :

It is force of attraction between the molecules of substances in contact. It is maximum when both the surfaces are extremely smooth.

- 86.** How is coefficient of friction altered when the weight of the body is doubled ?

Ans :

It remains constant. It depends only on the nature of the surfaces in contact.

- 87.** What is the direction of force of friction ?

Ans :

The direction of force of friction is always opposite to the direction of motion of the body.

- 88.** Can a horse pull a cart and run in empty space ?

Ans :

No, because reaction is must for the motion of system and in empty space, reaction is not available.

- 89.** Since action and reaction are always equal in magnitude and opposite in direction, how can anything . ever be accelerated ?

Ans :

No doubt, action and reaction are equal and opposite but they act on different bodies hence the motion becomes possible.

- 90.** A body is moving such that its linear momentum remains constant. Is the body in equilibrium ?

Ans :

Given $p = \text{Constant}$

$$\frac{dp}{dt} = 0$$

$$F = 0$$

So the body is in equilibrium

- 91.** It is difficult to move a cycle along a road with its brakes on. Why ?

Ans :

Because the brakes increases the force of friction between the rim of the wheels and the brake is too large to move the cycle.

- 92.** Carts with rubber wheels are easier to ply than those with iron wheels. Why ?

Ans :

Because, the force of friction between rubber wheel and the road is much smaller than the force of friction between iron wheels and road.

- 93.** Why are wheels made circular ?

Ans :

Since rolling friction is less than sliding friction and the circular wheels roll without sliding i.e. they convert sliding friction into rolling friction.

- 94.** What is centripetal force ?

Ans :

An external $\perp$ force required to make a body moving in circular path with uniform speed is called centripetal force.

- 95.** Why is a curved road banked : on the outer side ?

Ans :

Because the banked road provides an inward component which provides the required centripetal force.

- 96.** Is friction self-adjusting force ?

Ans :

Static friction is a self-adjusting force. Other forces of friction are not self-adjusting.

Ans :

(C) Aristotelian law of motion

Aristotle's view was that an external force is required to keep a body in motion. This was disproved by Galileo and Newton who showed that force is needed to change motion, not maintain uniform motion in the absence of friction.

56. Which of the following correctly describes the conditions for a body to be considered approximately a point object in motion analysis?

- (A) Its mass must be very small.
- (B) Its size is much smaller than the distance it moves.
- (C) It must be perfectly rigid.
- (D) It must be moving at a constant velocity.

Ans :

(B) Its size is much smaller than the distance it moves.

An object can be treated as a point object if its size is negligible compared to the distances involved in its motion or the effects being studied.

57. All of the following are examples of external forces EXCEPT:

- (A) The force of gravity on a falling apple by the Earth.
- (B) The force of friction on a sliding box by the floor.
- (C) The force exerted by the engine of a car on the car itself.
- (D) The force between two colliding billiard balls within the system of two balls.

Ans :

(D) The force between two colliding billiard balls within the system of two balls.

The forces between the two colliding billiard balls are internal forces if the system is defined as the two balls. The force of the car's engine on the car is also an internal force (the car moves due to the external friction from the road). The question is subtly worded. If "car itself" implies an internal part acting on another internal part, it's internal. However, the ultimate propulsive force from the road is external. The clearest example of internal force here is between the billiard balls.

58. Compare the inertia of a 10 kg iron ball and a 5 kg wooden ball.

- (A) The iron ball has greater inertia.
- (B) The wooden ball has greater inertia.
- (C) Both have the same inertia because inertia is independent of mass.

- (D) Both have the same inertia if their volumes are equal.

Ans :

(A) The iron ball has greater inertia.

Inertia is directly proportional to mass. Since the iron ball has a greater mass (10 kg) than the wooden ball (5 kg), it has greater inertia.

59. A cyclist is riding at a constant speed of 10 m/s on a level road. What is the net external force acting on the cyclist and bicycle system?

- (A) Equal to the air resistance.
- (B) Equal to the cyclist's pedaling force.
- (C) Zero.
- (D) Equal to the gravitational force.

Ans :

(C) Zero.

Since the cyclist is moving at a constant speed (constant velocity, as the road is level and implies straight motion for this context), the acceleration is zero. According to Newton's first law, if the acceleration is zero, the net external force must be zero. The pedaling force is balanced by air resistance and other frictional forces.

60. A block of mass 2 kg is placed on a frictionless horizontal surface. A horizontal force of 10 N is applied to it. What is the acceleration of the block?

- (A) 0.2 m/s²
- (B) 2 m/s²
- (C) 5 m/s²
- (D) 20 m/s²

Ans :

(C) 5 m/s²

According to Newton's second law, $F = ma$.

Therefore,

$$a = F/m = 10\text{N}/2\text{kg} = 5\text{m/s}^2.$$

Since the surface is frictionless, the applied force is the net horizontal force.

61. When a body is stationary :

- (a) there is no force acting on it.
- (b) the forces acting on it are not in contact with it.
- (c) the forces acting on it balance each other
- (d) the body is in vacuum.

Ans :

(c) the forces acting on it balance each other.

When forces acting on a body balance each other, the body is said to be stationary.

- 114.** Why an athlete runs some steps before taking a long jump ?

Ans :

An athlete runs for some distance before taking a long jump so that his forward momentum may help him in his muscular efforts to get high jump.

- 115.** Why do we slip on a rainy day ?

Ans :

On a rainy day, the ground becomes wet and the coefficient of friction between our feet and ground is reduced to a large amount and that cause us to slip.

- 116.** What is friction ?

Ans :

Friction may be defined as the force which acts when a body slides or tends to slide on the surface of another body and acts in tangential direction to the surface in contact and opposite

- 117.** What is the unit of coefficient of limiting direction of motion of body friction ?

Ans :

It has no unit.

- 118.** Define angle of friction.

Ans :

It is the angle which the resultant force of limiting friction and normal reaction makes with the normal reaction.

- 119.** Why are wheels of an automobile made circular ?

Ans :

Since rolling friction is less than sliding friction so wheels of automobile are made circular so that they roll in the road.

- 120.** Can we get off a frictionless horizontal surface by jumping ?

Ans :

No, as a frictionless surface is unable to provide reaction which is necessary for jumping.

- 121.** Name the two types of dynamic (kinetic) friction.

Ans :

(i) Sliding friction (ii) Rolling friction.

- 122.** What is the cause of friction ? In which case is it maximum ?

Ans :

It is the force of attraction between the molecules of substances in contact. It is maximum when both the surfaces are extremely smooth.

- 123.** How does coefficient of friction is altered when the weight of body is doubled ?

Ans :

It remains constant. It depends only on the nature of the surfaces in contact.

- 124.** Is friction self-adjusting force ?

Ans :

Static friction is a self-adjusting force. Other forces of friction are not self-adjusting.

- 125.** How friction is reduced in fast moving vehicle ?

Ans :

Fast moving vehicles are given streamlined shape to reduce the force of friction.

- 126.** Why are rockets given conical shape ?

Ans :

Conical shape of the rockets reduces atmospheric friction.

- 127.** What is the angular velocity of the hour hand of a clock ?

Ans :

Angle travelled by hour hand in 12 hour is given by

$$2\pi \text{ radian} = 12 \times 60 \times 60 \text{ s}$$

Angular velocity of hour hand a clock

$$\begin{aligned}\omega &= \frac{2\pi}{7} = \frac{2\pi}{12 \times 60 \times 60} \\ &= \frac{\pi}{1800} \text{ rad s}^{-1} \\ &= 0.001745 \text{ rad s}^{-1}\end{aligned}$$

- 128.** Why skidding takes place on a rainy day along a curved path ?

Ans :

The friction between road and tyres of the vehicle may not be sufficient to provide the necessary central force.

97. How is friction reduced in a fast moving vehicle ?

Ans :

Fast moving vehicles are given streamlined shape to reduce the force of friction.

98. How is inertia related to mass of a body ?

Ans :

Larger the mass of a body, more is its inertia.

99. If you jerk a piece of paper under a book quick enough, the book will not move. Why ?

Ans :

Due to inertia of rest.

100. If net force acting on a body is zero, then will the body remain necessarily in rest position ?

Ans :

No, the body may be moving uniformly along a straight line.

101. Calculate the mass of a body weighing 100 N. Take $g = 10 \text{ ms}^{-2}$.

Ans :

$$m = \frac{W}{g} = 10 \text{ kg}$$

102. If a body is not at rest, the net external force acting on it cannot be zero. Is it true or false ?

Ans :

False because if a body is moving uniformly along a straight line even then the net external force acting on it will be zero.

103. Define one newton.

Ans :

The force is said to be 1N, if it produces 0.1 ms^{-2} acceleration in 1 kg mass of a body.

104. What is the net force on a cork floating on water ?

Ans :

Zero.

105. Prove that if no external force is acting on a body, its momentum will remain unchanged.

Ans :

Since force $F = \frac{dp}{dt}$
If $F = 0$, then

$$\frac{dp}{dt} = 0$$

or $p = \text{constant}$

106. If two ends of a spring balance are pulled each by a force of 10 kg. wt. What will be the reading of the balance ?

Ans :

Reading of the balance will be 10 kg wt.

107. Action and reaction forces do not balance each other. Why ?

Ans :

Since action and reaction act on two different bodies and hence cannot cancel each other.

108. A meteorite burns in the atmosphere before it reaches the earth's surface. What happens to its momentum ?

Ans :

The momentum of the meteorite is transferred to the atmosphere in the form of heat energy.

109. What is impulsive force ?

Ans :

A force which acts for a small time and also varies with time is called impulsive force.

110. When a ball is thrown upwards, its momentum first decreases and then increases. Is conservation of linear momentum violated in this process ?

Ans :

No, the momentum conservation principle applies only when no external force acts on the system.

111. Explain why a horse cannot pull a cart and run in empty space.

Ans :

For the motion of the cart-horse system, reaction is necessary, which is not available in empty space.

112. Why a gun recoils back when it is being fired ?

Ans :

The gun recoils back when it is being fired to conserve the linear momentum.

113. On what factors inertia of a body depends ?

Ans :

Inertia depends upon the mass of the body. Larger the mass of a body, more is the inertia.

- 141.** Heavier the rifle, lesser the kick. Why ?

Ans :

The velocity of recoil of rifle is given by

$$\vec{V} = -\frac{m\vec{v}}{M}$$

–ve sign indicates that the direction of motion of rifle is opposite to the direction of motion of bullet. Thus for a given momentum.

$$V \propto \frac{1}{M}$$

So heavier the rifle, lesser the kick

- 142.** When a ball is thrown up, the magnitude of its momentum decreases. Does this violate the principle of conservation of momentum ?

Ans :

We know that principle of conservation of momentum holds in the absence of external force or forces. Since gravity plays the role of external force, the decrease in momentum does not violate the principle of conservation of momentum.

- 143.** Polishing beyond certain limit may increase friction between the surfaces. Explain.

Ans :

When surfaces are polished, the area of actual contact between them increases. If the surfaces are polished beyond certain limit, the area of actual contact increases to such an extent that the cohesive force between the surfaces becomes more than force of friction.

- 144.** Automobile tyres have generally irregular projections over their surfaces. Why ?

Ans :

Irregular tyre surfaces increase friction between the tyres and the road. This provides a firm grip between the tyres and the road and prevents slipping

- 145.** It is difficult to move a cycle along a road with its brakes on. Explain.

Ans :

When the brakes are on, a large frictional force comes into play between the rims of the cycle wheels and the brakes. This makes the motion of the cycle on the road difficult.

- 146.** Explain how friction helps in walking ?

Ans :

When we walk, we press our feet against the ground slantingly in the backward direction. It is the

reaction of the ground on our feet in the forward direction, arising due to friction between the two which helps us to walk.

- 147.** Is a large brake on a bicycle wheel more effective than small one ? Explain.

Ans :

No, both the large and the small brakes are equally effective. This is due to the reason that the force of limiting friction is independent of the area of surfaces in contact so long as the normal reaction remains the same.

- 148.** Why a person sitting inside a vehicle is thrown outwards, when vehicle rounds a curve suddenly ?

Ans :

Because due to inertia of direction a person inside the vehicle tends to maintain its direction of motion when vehicle rounds a curve suddenly.

- 149.** How do you account for function of mudguards in vehicles ?

Ans :

The wheels of moving vehicle throw out the mud tangentially due to directional inertia and mudguards over the wheels stop this mud on the clothes of the rider of a two wheeler.

THREE MARKS QUESTIONS

- 150.** Show that a heavier body having the same velocity as that of lighter body, possesses more momentum.

Ans :

Let M = Mass of heavier body

m = mass of lighter body

If v = velocity of each mass

momentum of heavier mass

$$p_M = Mv$$

momentum of lighter mass

$$p_m = mv$$

$$\frac{p_M}{p_m} = \frac{Mv}{mv}$$

$$\frac{p_M}{p_m} = \frac{M}{m}$$

Since $M > m$, $p_M > p_m$

It is because of this reason that momentum of a car

is much greater than that of a tennis ball if both are travelling with the same speed.

- 151.** A heavier and a lighter body ; have the same momentum, show that lighter body possesses more velocity.

Ans :

Let V = Velocity of mass
 v = velocity of mass m
 $p_M = MV$ and
 $p_m = mv$

If the momenta of bodies are the same,

i.e. $p_M = p_m$
 $MV = mv$
 $\frac{v}{V} = \frac{M}{m}$

Since $M > m$, then $v > V$

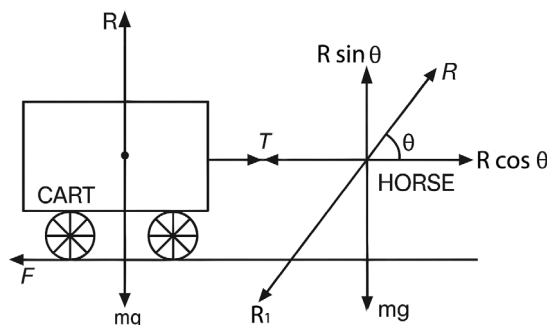
If tennis ball and car have the same momentum, then the tennis ball will possess very large velocity.

- 152.** If horse pulls the cart and the : cart pulls the horse then why do they : move at all ?

Ans :

The diagrammatic representation of various forces are as shown in the figure. Horse applies an action R_1 on the ground resulting in reaction R of the ground on the horse.

R can be resolved into two rectangular components $R \cos \theta = H$ in forward direction and $R \sin \theta$ vertically upwards to balance the weight of the horse.



Since horse and cart are moving forward with acceleration a , hence considering their motions one by one,

Motion of horse. $H - T = ma$
 Motion of cart. $T - F = ma$.

where T is the tension in the string and F the force of friction on the cart.

Adding them, we have

$$H - T + T - F = ma + Ma$$

$$H - F = a(m + M)$$

$$a = \frac{H - F}{m + M}$$

Acceleration will be there if $H > F$

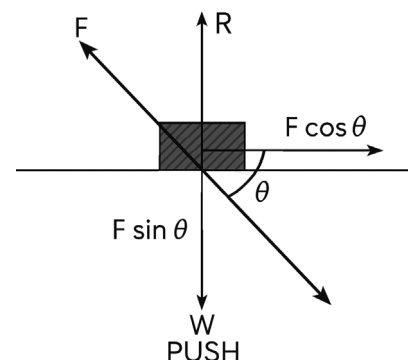
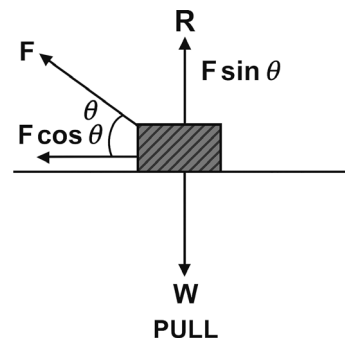
- 153.** Why is it easier to pull than to push a body ?

Ans :

When a body is placed on a horizontal plane,

$$R = F \quad \dots(1)$$

When a body is pulled by a force subtending an angle θ with horizontal, the force of pull can be resolved into two components $F \cos \theta$ horizontally and $F \sin \theta$ vertically upwards. In equilibrium,



$$R + F \sin \theta = W$$

$$R = W - F \sin \theta \quad \dots(2)$$

When the body is pushed by a force F subtending an angle θ with horizontal, F can be resolved into two components $F \cos \theta$ horizontally and $F \sin \theta$ vertically downwards. In equilibrium,

$$R = W + F \sin \theta \quad \dots(3)$$

Thus we find the normal reaction in case of push is much greater than that in case of pull. Also force

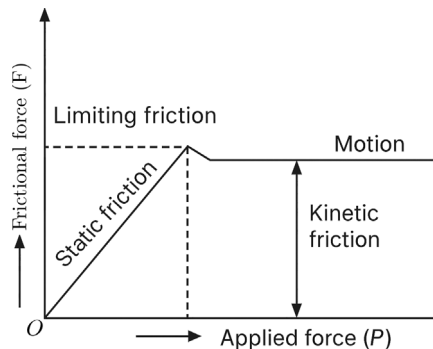
of friction is proportional to normal reaction. Hence force of friction in case of push will be greater than in case of pull.

It is easier to pull than to push

154. Why is limiting friction greater than kinetic friction?

Ans :

When a body is placed over another body, the molecular bonds are formed at the surfaces of actual contact. In order to move the body, we have to break these bonds. The maximum force required to break these bonds is called limiting friction and when the body starts moving on the other body, new bonds are formed at the point of actual contact. It is easy to break these new bonds. Hence kinetic friction is less than limiting friction. In other words, limiting friction is greater than kinetic friction as shown in Figure.



155. Define angle of friction and show that the coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction.

Ans :

Angle of friction (α)

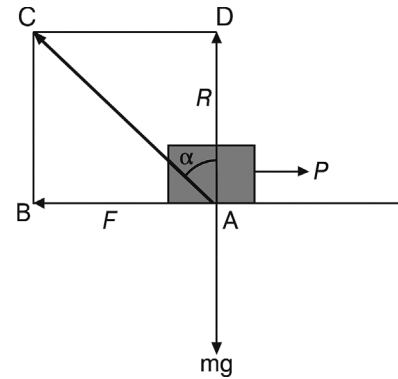
Angle of friction between two surfaces in contact is defined as the angle which the resultant of force of limiting friction F and normal reaction R makes with the direction of normal reaction R .

Figure shows that weight mg of the body is balanced by normal reaction R represented the force of limiting friction F . So angle represented by AC is the angle of friction.

From the figure, we find that

$$\tan \alpha = \frac{CD}{AD} = \frac{AB}{AD} = \frac{F}{R} = \mu$$

$$\mu = \tan \alpha$$



Thus coefficient of limiting friction between any two surfaces in contact is equal to the tangent of angle of θ friction between them.?

156. Derive an expression for acceleration of a body moving down an inclined plane.

Ans :

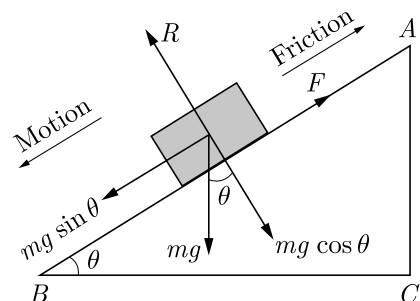
When a body is placed on an inclined plane whose angle of inclination is more than angle of repose, it moves down with acceleration a .

Various forces acting on the body are :

- (i) Weight mg of the body acting vertically downward.
- (ii) Reaction R acting perpendicular to the plane.
- (iii) Force of friction acting upwards (opposite to direction of motion.)

Weight mg can be resolved into two rectangular components ; $mg \cos \theta$ to plane and $mg \sin \theta$ parallel to plane. Let a be the acceleration in downward direction, then

$$mg \sin \theta - F = ma$$



But

$$\frac{F}{R} = \mu$$

$$F = \mu R = \mu mg \cos \theta$$

$$mg \sin \theta - \mu mg \cos \theta = ma$$

$$mg(\sin \theta - \mu \cos \theta) = ma$$

$$a = g(\sin \theta - \mu \cos \theta)$$

When one body is placed over another body, the actual area of contact is very small as compared to apparent area of contact. The cause of friction is due to atomic and molecular forces of attraction between the surfaces at the point of actual contact [Fig. 5.46]. The pressure at the point of contact is very high and a type of cold welding takes place. When one surface slides over the other, the bonds between the molecules are continuously broken and reform at the other points. The force required to break these bonds between the molecules is called friction.

Static friction :

Let a block be placed on a table. Apply a small force on it. If the block does not move, it means that the applied force is balanced by frictional force. Hence

The force of friction which comes into play between the surfaces of contact of two bodies before the actual motion starts is called static friction.

If the applied force is increased, the block may not still move which means that the value of static friction has also increased. But there is a limit beyond which the force of friction cannot increase. This maximum static friction is called limiting friction. Thus static friction is self-adjusting force upto limiting friction.

Limiting friction :

The maximum value of force of friction which comes into play when a body just begins to slide over the surface of another body is called limiting friction.

Dynamic or kinetic friction :

When the applied force exceeds the limiting friction, the motion starts, the force of friction is called kinetic friction.

The force of friction which comes into play between the surfaces of contact of two bodies, when one body is in steady motion over the surface of another body, is called kinetic friction.

body is just starting its motion over the other.

- Force of limiting friction is independent of the apparent area of contact so long as the normal reaction between the two bodies in contact remains the same.
- The force of limiting friction between any two bodies in contact depends upon the nature of the surfaces in contact.

Coefficient of Friction : According to first law of limiting friction,

$$F \propto R \text{ or } F = \mu R$$

where μ is a constant called coefficient of limiting friction between the two surfaces in contact.

$$\mu = \frac{F}{R}$$

Coefficient of limiting friction between two surfaces is defined as the ratio of the force of limiting friction and the normal reaction between them.

Coefficient of kinetic friction

The kinetic friction (F_k) is directly proportional to the normal reaction (R) between the two surfaces in contact.

i.e. $F_k \propto R$

$$F_k = \mu_k R$$

$$\mu_k = \frac{F_k}{R}$$

where μ_k is called coefficient of kinetic friction. μ_k is always less than, μ because $F_k < F$

Unit and dimensions of coefficient of friction (μ)

Since μ is the ratio of the same quantities so it has no units and no dimensions.

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- 175.** State laws of limiting friction. Define coefficient of limiting friction and coefficient of kinetic friction. What are units and dimensions of coefficient of friction ?

Ans :

Laws of limiting friction are :

- Limiting force of friction between two bodies in contact is directly proportional to the normal reaction between them i.e.,

$$F \propto R$$

- The direction of the force of limiting friction is always opposite to the direction in which one

- 176.** Define angle of repose and show that the coefficient of limiting friction is equal to tangent of angle of repose. Hence show that angle of repose is equal to angle of friction.

Ans :

Angle of repose (θ)

The minimum angle made by the inclined plane with the horizontal surface such that a body lying on the inclined plane just begins to slide down is called angle of repose.

Let a body of mass m be placed on an inclined plane having angle of repose θ as shown in Figure.

i.e. $\vec{F} \propto \frac{d\vec{p}}{dt}$

$$\vec{F} = k \frac{d\vec{p}}{dt}$$

But $\vec{p} = m\vec{v}$

$$\vec{F} = k \frac{d}{dt}(m\vec{v}) = km \frac{d\vec{v}}{dt}$$

$$\vec{F} = km \vec{a}$$

Standard units are selected in such a way that $k = 1$

$$\vec{F} = m\vec{a}$$

Again $\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z$

$$\vec{a} = \hat{i}a_x + \hat{j}a_y + \hat{k}a_z$$

$$\vec{F} = m[\hat{i}a_x + \hat{j}a_y + \hat{k}a_z]$$

$$\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$$

Thus, the second law helps us to measure the force.

- 161.** Define force and a unit of force. What are different units and how are these related to each other ?

Ans :

Force. It is a pull or push which changes or tends to change the state of rest or of uniform motion of a body or changes its direction.

Since $F = ma$

So, if $m = 1, a = 1$, then $F = 1$

Hence a unit of force is that which produces in a unit mass a unit acceleration.

Different units of force

Absolute units of force are newton (N) and dyne (dyne).

1N is that force which produces an acceleration of 1 m s^{-2} in a mass of 1 kg.

1 dyne is that much force which produces an acceleration of 1 cm s^{-2} in a mass of 1 kg.

1N	=	$1 \text{ kg} \times 1 \text{ ms}^{-2}$
1N	=	$1000 \times \text{g} \times 100 \text{ cm s}^{-2}$
1N	=	10^5 g cm s^{-2}
1N	=	10^5 dyne

Gravitational Units or Practical Units of Force

1 kg wt or 1 kg f is that force which produces an acceleration of 9.8 m s^{-2} in a mass of 1 kg. It is also equal to the force with which a mass of 1 kg is attracted by the earth towards its centre.

1 g wt or 1 g f is that force which produces an acceleration of 981 cm s^{-2} in a mass of 1 g. It is also

equal to the force with which a mass of 1 gram is attracted by the earth towards its centre.

$$1 \text{ kg } f = 9.8 \text{ N}$$

$$1 \text{ g } f = 981 \text{ dyne}$$

- 162.** Discuss consequences of Newton's second law of motion

Ans :

- 1. Newton's second law of motion leads to the concept of inertial mass.**

Consider two forces each of magnitude F acting on the masses m_1 and m_2 . Let a_1 and a_2 be the magnitudes of accelerations produced in the masses m_1 and m_2 respectively.

From Newton's second law of motion,

$$F = m_1 a_1 = m_2 a_2$$

$$\frac{a_2}{a_1} = \frac{m_1}{m_2}$$

If $m_1 > m_2$, then $a_2 > a_1$.

It means that acceleration produced in heavier mass is less than the acceleration of lighter mass. This can be summed up by the equation

$$a = \frac{F}{m}$$

If mass m is large, the acceleration produced by a given force F is small.

Thus mass m measures the resistance or the inertia offered by the object to the change in its velocity or acceleration produced by a force. 'm' is called inertial mass. It is an intrinsic property of body.

- 2. Determination of the force**

Newton's second law gives the force to be applied to produce a change in velocity $d\vec{v}$

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

If m and $d\vec{v}$ are known, force F acting can be determined.

- 3. Velocity in the body**

Using the relation,

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

If the force F is given, we can predict the change in velocity produced in the time interval dt .

- 4. No force is needed to move a body uniformly along a straight line.**

Using the relation,

$$\vec{F} = m\vec{a}$$

For a body having uniform motion along a straight line

$$\vec{a} = 0$$

$$\begin{aligned}\vec{a} &= 0 \\ \frac{d\vec{v}}{dt} &= 0 \\ \vec{v} &= \text{Constant}\end{aligned}$$

Thus, if no external force is acting on a body, the body will have uniform velocity i.e. a body at rest will remain at rest or a body with certain uniform velocity continuous to move with the same velocity. This is Newton's 1st law of motion. Hence first law can be derived from second law i.e. 1st law is contained in 2nd law.

- (ii) Third law from second law: Consider an isolated system (i.e. a system in which no external force is acting) consisting of two particles A and B interacting with each other. Let particle 1 exert a force $\vec{F}_{12}$ on particle 2 and particle 2 exert force $\vec{F}_{21}$ on particle 1 for a time dt.

From Newton's 2nd law of motion,

Total rate of change of momentum of particle 1 and 2 = $\vec{F}_{ext}$

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \vec{F}_{ext}$$

For an isolated system,

$$\begin{aligned}\vec{F}_{ext} &= 0 \\ \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) &= 0 \\ \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} &= 0 \\ \vec{F}_{12} + \vec{F}_{21} &= 0\end{aligned}$$

This is Newton's 3rd law of motion.

Hence Newton's third law can be derived from second law. Hence second law is the real law of motion.

- 167.** State the law of conservation of momentum. Derive it from Newton's 2nd law of motion.

Ans :

It states that in the absence of external force, the total linear momentum of the system remains constant.

Consider an isolated system consisting of n particles having momentum

$$\vec{p}, \vec{p}_2, \dots, \vec{p}_n$$

Total momentum of the system

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n \quad \dots(1)$$

Differentiating both sides w.r.t. t, we get

$$\begin{aligned}\frac{d\vec{p}}{dt} &= \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \dots + \frac{d\vec{p}_n}{dt} \\ &= \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n\end{aligned}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{ext}$$

(where $\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$ is the total external force acting on the system)

$$\vec{F}_{ext} = 0, \text{ then}$$

$$\frac{d\vec{p}}{dt} = 0$$

$$\vec{p} = \text{Constant}$$

Using eq. ... (1)

$$\vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n = \text{Constant}$$

This is law of conservation of momentum i.e. when no external force is acting on a system (isolated system), total momentum of the system is always constant.

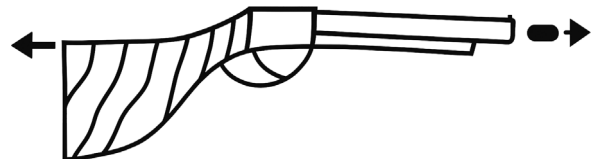
In other words, we can say that, in an isolated system, total initial momentum is always equal to total final momentum of the system.

- 168.** Explain (i) Recoil of a gun and (ii) Explosion of a bomb.

Ans :

- (i) Recoil of a gun :

Consider a gun of mass m_1 fires the bullet of mass m_2 which moves with a velocity $\vec{v}_2$ in the forward direction. Let $\vec{v}_1$ be the velocity of the recoil of a gun.



Since before firing both the gun and the bullet were at rest, therefore, the total initial momentum of the system (gun and bullet) = 0. The final momentum of the system (gun and bullet)

$$= m_1 \vec{v}_1 + m_2 \vec{v}_2$$

According to law of conservation of linear momentum,

Initial momentum of the system = Final momentum of the system

$$\text{i.e. } 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$m_1 \vec{v}_1 = - m_2 \vec{v}_2$$

$$\vec{v}_1 = - \frac{m_2}{m_1} \vec{v}_2$$

The negative sign indicates that the direction of the velocity of the recoil of the gun is opposite to that of the bullet. Hence, the gun acquires backward momentum.

- (iv) When lift is moving upward with acceleration a , then

$$R - mg = ma$$

$$R = mg + ma = m(g + a)$$

i.e., apparent weight of the person is more than the actual weight.

- (v) When lift is moving downward with acceleration a , then

$$mg - R = ma$$

$$R = mg - ma = m(g - a)$$

i.e., apparent weight of the person is less than the actual weight.

- (vi) If lift is moving down with $a = g$, then substituting this value in above equation, we have

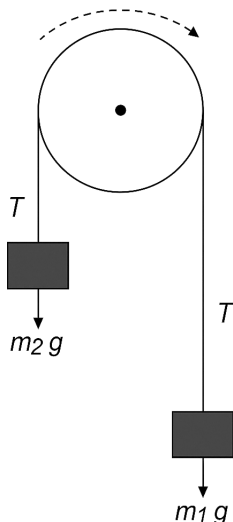
$$R = m(g - g) = 0$$

i.e., the body will appear weightless.

- 165.** Derive the expression for acceleration and tension in a string in a connected motion.

Ans :

Consider two bodies of masses m_1 and m_2 such that $m_1 > m_2$. Let these be tied at the ends of an inextensible string which passes over a light and frictionless pulley. Let the bigger body move downwards with an acceleration ' a '. The smaller body will also move upwards with the same acceleration ' a '. Since the pulley is light and frictionless, therefore, the tension in the string shall be the same on both the sides of the pulley.



On the bigger body, the various forces acting are :

- (i) Its weight m_1g acting downwards.

- (ii) The tension T in the string acting upwards. As the bigger body moves downwards with acceleration a , the net downward force on it is m_1a .

Clearly,

$$m_1g - T = m_1a \quad \dots(1)$$

On the smaller body, the various forces acting are :

- (i) Its weight m_2g acting downwards.
(ii) The tension T in the string acting upwards. As the smaller body moves upwards with acceleration a , the net upwards force on it is m_2a .

Clearly,

$$T - m_2g = m_2a \quad \dots(2)$$

Acceleration of system

Adding (1) and (2), we get

$$m_1g - m_2g = (m_1 + m_2)a$$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \quad \dots(3)$$

Since $m_1 > m_2$; $a < g$

i.e. the value of acceleration a is always less than acceleration due to gravity.

Tension in the string. Dividing (1) by (2),

$$\begin{aligned} \frac{m_1g - T}{T - m_2g} &= \frac{m_1a}{m_2a} \\ m_1m_2g - m_2T &= m_1T - m_1m_2g \\ 2m_2m_2g &= (m_1 + m_2)T \quad \dots(4) \end{aligned}$$

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

If $m_1 = m_2 = m$ (say)

From (3) $a = 0$

i.e., there will be no acceleration if masses are equal

From (4) $T = mg$

- 166.** Show that Newton's second law of motion is the real law.

Ans :

Since Newton's first and third laws of motion can be obtained from second law, hence 2nd law of motion is called the real law.

- (i) First law from second law From Newton's second law of motion,

$$\vec{F} = m\vec{a}$$

If external force $\vec{F} = 0$ then

$$m\vec{a} = 0$$

Since $m \neq 0$

157. What is the need of banking a circular road.?

or

Explain the need for banking of tracks.

Ans :

When an automobile taking a turn on a level road, centripetal force is provided by the friction between the road and the tyres and causes wear and tear of the tyre. To avoid this, the curved road is given an inclination sloping upwards the outer circumference. This reduces wearing out of the tyres because the horizontal component of the normal reaction provides the necessary centripetal force and it also allows a vehicle to move faster than the speed on a level road. Banking of the road prevents skidding-off the vehicle from the road, making driving safe.

FIVE MARKS QUESTIONS

158. Define the term inertia. Explain three types of inertia with examples.

Ans :

The inherent property of objects to remain at rest or in uniform motion unless acted upon by a force is called inertia.

Inertia may show itself in any of the following three forms :

(a) Inertia of rest. It is that property by virtue of which a body at rest tends to remain at rest.

Illustrative Examples :

(i) When a blanket is beaten with a stick, the dust particles fall off. It is due to the fact that the blanket is suddenly set in motion and dust particles tend to remain at rest due to inertia of rest.

(ii) When a high speed bullet hits a glass pane, it makes a clear circular hole in the glass. It is due to the fact that the particles of glass are unable to share the quick motion of the fast moving bullet.

(b) Inertia of motion. It is that property by virtue of which a body in state of uniform motion tends to keep on moving at the same rate.

Illustrative Examples :

(i) When a man jumps out of a moving bus, he falls forward. When his feet touch the ground, the lower part of body comes to rest while the upper part continues to move due to inertia of motion.

(ii) An athlete runs before jumping because in this way, the inertia of motion of his body at the time of jumping enables him to have a longer jump.

(c) Inertia of direction. It is the property of a body by virtue of which it tends to maintain its direction.

Illustrative examples :

(i) Consider a stone tied at the end of a string and whirled in a horizontal direction. If the string breaks in the process of rotation, the stone will fly off tangentially, along a straight line due to inertia of direction.

(ii) The sparks coming out of a grinding stone are tangential to the sum of rotating stone and fly in straight lines.

159. State and explain Newton's 1st law of motion.

Ans :

Newton's First Law of Motion : It states that every body continues in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Explanation : The state of rest and that of uniform motion are both examples of zero acceleration. The first law tells us that in order to change such a state of motion we need a force.

If a body is at rest, we will have to apply a force on it to make it move. e.g. if we drop an object it starts falling. Apparently the object moves without any force but it is the gravitational pull of the earth that makes the object fall.

If a body is moving with constant speed in a straight line, and if we want to change its speed we will have to apply force e.g. when bus is at rest or travelling with constant velocity, the motion is unaccelerated and force acting is zero. When the bus speeds up there should be a force acting in the direction of motion. When the bus is slowing down there must be force acting opposite to the direction of motion. These facts illustrate that due to friction, this motion is never perpetual. Even to change the direction of motion of a body, a force is required.

160. What is Newton's second law of motion ? How can the force be measured using second law of motion ?

Ans :

According to Newton's second law of motion, the rate of change of momentum of a body is directly proportional to the external force applied on the body and takes place in the direction of force.

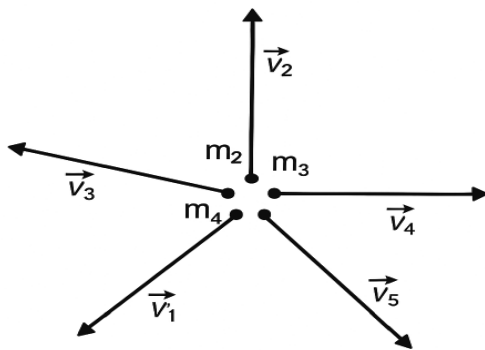
- (ii) Explosion of a bomb : Let a bomb initially at rest explodes into five fragments of masses m_1 , m_2 , m_3 , m_4 , and m_5 as shown in Fig. 5.43.

The different fragments fly off in the different directions with different velocities.

Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ and $\vec{v}_5$ be their respective velocities. Since initial momentum of the bomb being at rest is zero, therefore, according to the law of conservation of linear momentum, the final momentum (after explosion) of the bomb must be zero.

$$\text{i.e. } m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + m_5\vec{v}_5 = 0$$

$$\sum m\vec{v} = 0$$



If in the above equation, all the quantities except one are known, then the unknown quantity can be calculated.

Billions and billions of years ago when the big-bang occurred and the whole cosmos came into being, the initial momentum was zero. Now the galaxies appear to be going away, we can calculate their velocities and from them we find out their masses using the simple principle of conservation of momentum.

- 169.** What is an impulse ? How is it related to the change in momentum?

Ans :

Whenever a force acts on a body, its effectiveness depends upon the magnitude of force and time for which it acts. But if a force acts for a very short time, then we measure the total effect of the force instead of its magnitude and time for which it acts. This is called impulse. For example, if a ball strikes with a bat, the force acts for a very short time called impulsive force.

Thus impulse is defined as the product of the force and the time for which the force acts.

$$\text{Impulse} = \text{Force} \times \text{time}$$

If force $\vec{F}$ acts on a body for time dt , then impulse of this force

$$d\vec{I} = \vec{F} dt$$

If force acts for time t , then

$$\vec{I} = \int d\vec{I} = \int_0^t \vec{F} dt \quad \dots(1)$$

Impulse momentum theorem

From Newton's 2nd law of motion, we have

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} dt = d\vec{p} \quad \dots(2)$$

If at $t = 0$, $\vec{p} = \vec{p}_1$

and at $t = t$, $\vec{p} = \vec{p}_2$

Then integrating equation (2) within the above limits, we get

$$\int_0^t \vec{F} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

If $\vec{F}_{av}$ is the average force (constant) during the impact, then

$$\vec{I} = \int_0^t \vec{F}_{av} dt = \vec{p}_2 - \vec{p}_1$$

$$\vec{I} = \vec{F}_{av} t$$

Thus, the product of average force and the time of impact is equal to the change in momentum produced during the impact.

In SI, the unit of impulse is Ns or kg ms⁻¹.
Dimensional formula

The dimensional formula of impulse is [MLT⁻¹]

- 170.** Illustrate some applications of impulse.

Ans :

Some applications of impulse are as follows :

1. A cricket player lowers his hands while catching a ball. Because by lowering his hands, time taken to catch the ball increases, hence lesser force is required to stop the ball ($I = Ft$) and hence the player saves himself from getting hurt.
2. Cars, buses, bogies of trains etc. are provided with spring system and shockers. The spring system and shockers increase the time interval of jerks, which decreases the force acting on the passengers during the jerks.
3. China wares are wrapped in straw or paper pieces before packing. This is done to increase the time of jerks during transportation. This decreases the possibility of action of greater force and saves the material

from breaking.

4. A person falling from the same height on a cemented or hard floor receives greater injuries than if he falls on sand or soft floor.

When he falls on cemented floor his momentum reduces to zero immediately and he gets large force from the floor and if he falls on sand or soft floor his momentum will reduce to zero in comparatively large time and hence he will receive less injuries.

171. Define concurrent forces and describe equilibrium of concurrent forces.

Ans :

A number of forces acting on the same point are called concurrent forces.

Equilibrium of concurrent forces

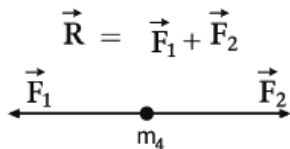
A number of forces are said to be in equilibrium if their resultant is zero.

If $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ be n concurrent forces, and these forces are said to be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = 0$$

- (i) Case of two forces in equilibrium.

If two forces $\vec{F}_1$ and $\vec{F}_2$ are acting as in Fig. 5.44. (a), then



When in equilibrium then

$$\vec{R} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 0 \text{ or } \vec{F}_1 = -\vec{F}_2$$

i.e., their resultant will be zero if these forces are equal and opposite.

- (ii) Case of three forces in equilibrium.

If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are acting at a point O, then the resultant of $\vec{F}_1$ and $\vec{F}_2$ must be equal and opposite to $\vec{F}_3$

i.e. $\vec{OA} + \vec{OB} = \vec{OC}$ [Parallelogram Law]

Add to OC' each side

$$\vec{OA} + \vec{OB} + \vec{OC}' = \vec{OC} + \vec{OC}'$$

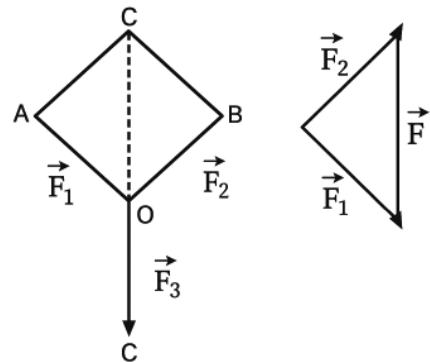
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{OC} + (-\vec{OC})$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

i. e. three concurrent forces will be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$$

i.e., resultant of sum of two forces is equal and opposite to third force $\vec{F}_3$



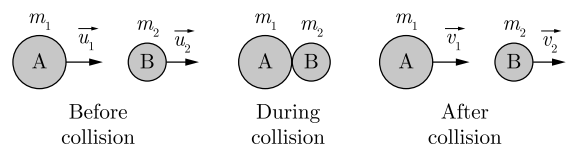
Also, $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are in equilibrium if these can be represented by three sides of a triangle taken in the same order.

In general, if they are in any number, these should be capable of being represented by the sides of a closed polygon taken in the same order. Only then these forces will be in equilibrium.

172. Derive the law of conservation of linear momentum from Newton's third law of motion.

Ans :

Consider an isolated system consisting of two particles A and B of masses m_1 and m_2 moving with velocities $\vec{u}_1$ and $\vec{u}_2$ along the same straight line in the same direction. Let after collision, the velocities of A and B change to $\vec{v}_1$ and $\vec{v}_2$ respectively as shown in Fig. 5.45.



Total momentum of the system before collision

$$= m_1 \vec{u}_1 + m_2 \vec{u}_2$$

Total momentum of the system after collision

$$= m_1 \vec{v}_1 + m_2 \vec{v}_2$$

Let force acting on body A and B in time

$$t = \vec{F}_{AB}$$

Force acting on body B by A in time

$$t = \vec{F}_{BA}$$

Then impulsive acting on A,

$$\vec{F}_{AB} \cdot t = m_1 \vec{v}_1 - m_1 \vec{u}_1$$

Impulsive acting on B,

$$\vec{F}_{BA} \cdot t = m_2 \vec{v}_2 - m_2 \vec{u}_2$$

From Newton's third law of motion,

$$\begin{aligned}\vec{F}_{AB} &= -\vec{F}_{BA} \\ \vec{F}_{AB} \cdot t &= -\vec{F}_{BA} \cdot t \\ m_1 \vec{v}_1 - m_1 \vec{u}_1 &= -(m_2 \vec{v}_2 - m_2 \vec{u}_2) \\ &= m_2 \vec{v}_2 + m_2 \vec{u}_2 \\ m_1 \vec{v}_1 - m_1 \vec{u}_1 &= m_1 \vec{u}_1 + m_2 \vec{u}_2\end{aligned}$$

i.e. total momentum of the system after collision = total momentum of the system before collision. This is law of conservation of linear momentum.

- 173.** (a) What is a frame of reference ? Explain inertial and non-inertial frames of reference.
(b) Is earth an inertial frame of reference ?

Ans :

- (a) A frame of reference is defined as a system relative to which the motion of bodies is observed.

There are two types of frames of reference :

- (i) Inertial frame
- (ii) Non-inertial frame

Inertial frames :

These are frames of reference in which law of inertia ($\vec{F} = m\vec{a}$) is valid i.e., If $\vec{F} = 0$ then $\vec{a} = 0$. In other words, body at rest remains at rest and a body moving with a velocity continues to move with same velocity in the same direction.

All frames of reference moving with respect to one another with uniform relative velocity are also inertial frames of reference.

If a ball is placed in a stationary train, the ball remains at rest if it is not disturbed by external force.

If the ball is placed in train running with uniform velocity, it will be found to be at rest again. i.e. law of inertia is valid in stationary frames or in those frames which are moving with a uniform velocity.

Importance of inertial frames :

1. All the fundamental laws of physics have the same form in all inertial frames.
2. All the fundamental laws of physics are formulated in inertial frames.
3. Inertial frames are isotropic i.e. all the directions of space are equally preferred.

Non-inertial frames :

These are the frames of reference in which the law of inertia ($\vec{F} = m\vec{a}$) is not valid.

i.e. If $\vec{F} = 0$ then $a \neq 0$.

It means that even if no force acts on a body,

a body at rest does not remain at rest and one moving with some velocity does not continue to move with the same velocity, rather it is accelerated.

All accelerated frames are non-inertial frames of reference.

Place a ball in a train which is accelerated or which is gaining velocity. The ball will not be at rest, rather it rolls down in the opposite direction i.e. a body at rest does not remain at rest in an accelerated frame.

Similarly, if we place a ball in the train when it is retarding or slowing down, the ball rolls in forward direction. A rotating frame is also a non-inertial frame as it has acceleration towards its centre.

- (b) Since earth revolves around the sun and also spins about its own axis, so it is a rotating frame, hence in true sense, earth is a non-inertial frame. But the acceleration due to rotation is very small as compared to acceleration due to gravity. Thus for all practical purposes, the earth is considered to be an inertial frame of reference.

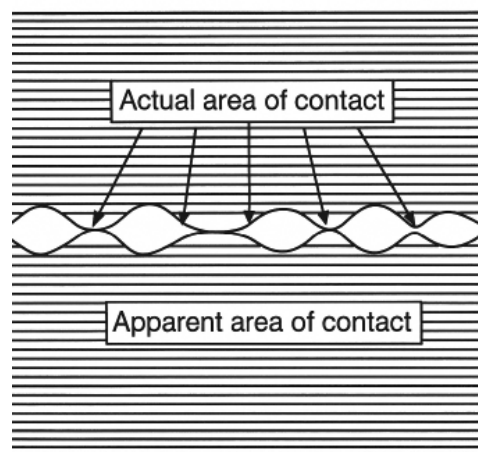
- 174.** Define friction. What is its cause ? Distinguish between static friction, limiting friction and kinetic friction. How do they vary with the applied force ?

Ans :

Friction may be defined as the force which acts when a body slides or rolls or tends to do so on the surface of another body and acts in tangential direction to the surface in contact opposite to the direction of motion of body.

Cause of friction (modern or molecular theory)

The surfaces which appear smooth to the naked eyes are irregular, when viewed under a microscope as shown in the Figure.



$$\vec{F} = 0$$

$$F = 0$$

Hence no external force is required to move a body uniformly along a straight line.

5. Whether force F is constant or changing, Newton's second law of motion is applicable. In case of variable force, the law can be applied for small intervals Δt and then calculating the accelerations.
6. The acceleration produced in the body by force does not depend upon the present velocity of body or velocity it possessed before application of force.
7. In $F = ma$, both quantities m and a can be directly measured.

The product of m and a gives the force which is the external cause.

Newton's second law tells us to look for a cause which is force.

163. State and explain Newton's third law of motion.

Ans :

Newton's third law of motion

It states that "to every action there is always an equal and opposite reaction."

Explanation: When two bodies interact with each other, one of two forces is termed as action and, the other is called reaction. There is no cause and effect relation between action and reaction. They occur simultaneously and any one of two forces may be called action and reaction.

Action and reaction do not act on the same body. If they acted on the same body, the resultant force would be zero and there could never be accelerated motion. It is impossible to have a single isolated force. If we consider two bodies A and B , body A exerts a force $\vec{F}_{AB}$ on body B , simultaneously body B also exerts a force $\vec{F}_{BA}$ on body A , then according to Newton's third law of motion, we have

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Illustration of Newton's 3rd law of motion

1. Walking : In order to walk, the frictional force F due to contact of the foot with the ground acts backward, and the ground exerts reaction to enable us to move in forward direction.
2. Swimming : While swimming, a swimmer pushes the water backward with his hands and as a result, water pushes the swimmer in forward direction.
3. It is difficult to walk on sand or ice : In order to walk, when sand is pressed, it gets displaced

and a very small reaction is available, which makes it difficult to walk on sand. In case of ice, the reaction is very small due to little friction between the feet and ice and hence it is difficult to walk on sand or ice.

4. Gun fire : When a gun is fired, the bullet moves in forward direction and the gun gets reaction in backward direction and it gets recoiled.
5. Driving nail into wooden block by holding the block : On striking the nail with a hammer on a wooden block not resting against a support, both move in forward direction and if the same process is repeated by placing the block against a support, the nail is easily driven into the block due to reaction of support.

164. Determine apparent weight of a person in a lift when

- (i) lift is at rest ;
- (ii) lift is moving upward with uniform velocity
- (iii) lift is moving downward with uniform velocity ;
- (iv) lift is moving up with acceleration a ,
- (v) lift is moving down with acceleration a ;
- (vi) lift is moving down with $a = g$

Ans :

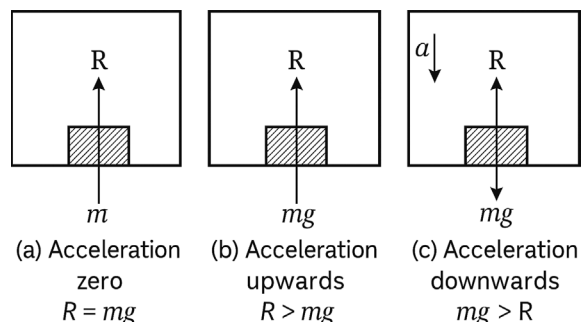
Let a person of mass m is standing on a weighing machine in a lift. Actual weight of person = mg . Machine offers a reaction R given by reading of weighing machine. Reaction gives the apparent weight of the person.

In first three cases :

- (i) when lift is at rest ;
- (ii) when lift is moving up with uniform velocity and
- (iii) when the lift is moving downward with uniform velocity, we have $a = 0$.

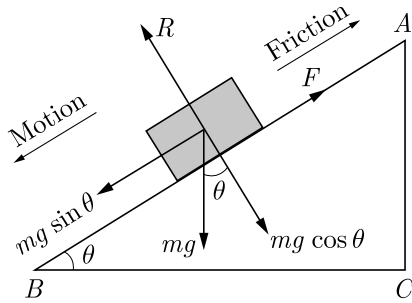
$$\text{Net force on the person} = ma = m \times 0 = 0$$

$$R - mg = 0$$



$$R = mg$$

i.e., apparent weight of the person is equal to the actual weight of the person.



Various forces acting on the body are :

- (i) Weight mg of the body acting vertically downwards.
- (ii) Normal reaction R acting perpendicular to plane.
- (iii) Force of friction F acting opposite to direction of motion.

Force mg can be resolved into two rectangular components $mg \sin \theta$ and $mg \cos \theta$. In equilibrium,

$$mg \sin \theta = F$$

and $mg \cos \theta = R$

$$\frac{mg \sin \theta}{mg \cos \theta} = \frac{F}{R}$$

$$\tan \theta = \mu \quad \dots(1)$$

Coefficient of friction between two surfaces is equal to the tangent of angle of repose between them.

Thus we find the angle of friction α is numerically equal to angle of repose θ between two surfaces.

Since angle of friction is related to coefficient of limiting friction μ by the relation

$$\tan \alpha = \mu \quad \dots(2)$$

From eqs. (1) and (2), we have

$$\tan \theta = \tan \alpha$$

$$\theta = \alpha$$

i.e. angle of repose is equal to angle of friction.

17. (a) Explain rolling friction.
(b) What are the methods of reducing friction ?

Ans :

- (a) **Rolling friction** : When a body rolls over a surface, the force of friction called into play is called rolling friction.

For the same normal pressing force, rolling friction is far less than the sliding friction. Also the rolling friction is inversely proportional to radius of the rolling body and is more for rigid surfaces. The laws of rolling friction are the same as those of sliding friction. However

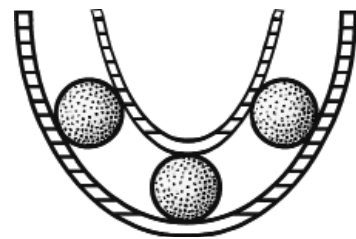
coefficient of rolling friction (μ_r) is very small for the same two surfaces. It is for this reason that all vehicles are provided with wheels.

- (b) Methods of reducing friction :

1. Polishing and Smoothing : According to Rayleigh, the surface layer of a polished solid too behaves like a fluid. Fluid friction being less than sliding friction, polishing reduces friction. Roads are covered with tar to reduce friction.
2. Lubrication : Lubricants do not allow the metal surfaces to come in intimate contact and thus reduce friction. A modern lubricant is a mixture of a mineral oil, a vegetable oil and colloidal graphite. For heavy machinery, thick oil or grease is required, because thin oil would be squeezed out from between the surfaces by the great pressure. For light machinery, e.g., watches, clocks and sewing machines etc. thin oil is always used.

For drawers and other similar wooden surfaces, dry soap acts like a lubricant and prevents 'jamming'.

3. Ball-Bearing : Rolling friction being very small as compared with sliding friction, the axles of the cycles, motor cars and superior fans etc. are provided with ball bearing arrangement instead of sleeve bearing. With it the trouble of repeated oiling too is avoided. If the balls are smeared with lead or iridium the friction is exceedingly lowered.



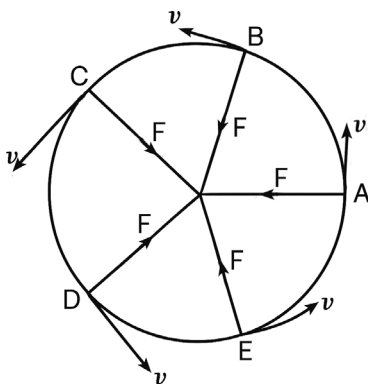
For steel, as steel coefficient of rolling friction is about one-hundredth of the coefficient of sliding friction. Therefore in rotating machinery sliding contact on axles is replaced by rolling contact. To achieve it, the surfaces of the axle and the outer ring are separated by hard steel balls. The arrangement is called ball-bearing (or roller bearing) as shown in Fig. 5.48. In this arrangement, the axle rolls on the balls, while the balls roll in the space called race.

- Proper selection of materials : Friction can be reduced by proper selection of materials.
- Streamlining : Friction can be reduced by streamlining the bodies.

178. State and explain centripetal force.

Ans :

When a body is moving uniformly along a circular path, magnitude of its velocity vector remains constant during motion but the direction changes continuously. According to Newton's laws of motion, the direction of motion of the body can change only if an external force acts on it which continuously deflects it from its straight line path. Since the magnitude of the velocity remains unaffected by external force, its direction should be such that it has no component along the velocity vector. It is possible only if this force acts along the radius of the circle towards its centre so that it is at right angles to the velocity vector. This radially inward force is called centripetal force.



Therefore, the centripetal force is defined as the force which compels a body to keep moving in a circular path with a constant speed and is directed along the radius of the circle towards its centre.

The magnitude of centripetal force is given by

$$F = m \frac{v^2}{r} \quad \dots(1)$$

If ω is the angular velocity of the particle, then

$$v = r\omega$$

$$F = \frac{m}{r} (r\omega)^2 = mr\omega^2 \quad \dots(2)$$

So $F = -r(\omega)^2 = mr\omega^2 \dots(ii)$

Following are a few examples of centripetal force :

- Earth moves around the sun in its orbit. The necessary centripetal force is provided by gravitational pull of sun on earth.
- Moon and other artificial satellites revolve round the earth. Gravitational pull (gravity) of

earth provides the necessary centripetal force.

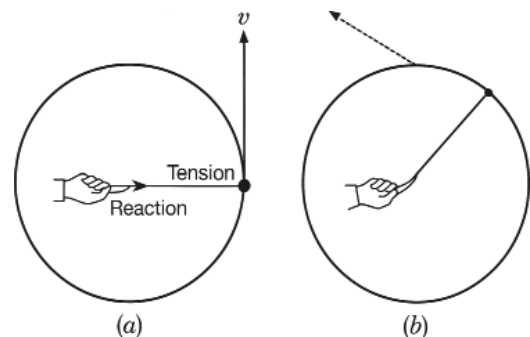
- In an atom, negatively charged electron revolves around the positively charged nucleus. The electrostatic force of attraction between nucleus and the electron is the source of centripetal force.

179. Define centrifugal force. Explain its concept by giving an example.

Ans :

Centrifugal force. The centrifugal force is defined as the force of reaction exerted by a body moving uniformly along a circular path on the external agent which is providing the centripetal force, by virtue of its constant tendency to travel along a straight line path. The centrifugal force acts along the radius but in the outward direction. Magnitude of centrifugal force is equal to that of centripetal force. Hence, the magnitude of centrifugal force

$$\begin{aligned} &= \frac{mv^2}{r} = mr\omega^2 = 4\pi^2 mr v^2 \\ &= \frac{4\pi^2 mr}{T^2} \end{aligned}$$

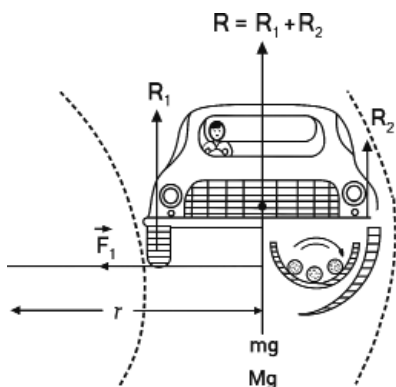


To explain the concept of centrifugal force, let us consider a small rubber cork (or a small piece of stone) tied to one end of a light cord which is whirled about the other end in a horizontal plane. In order to maintain the rubber cork into uniform circular motion our hand is supplying necessary centripetal force through the tension (Inward) in the string. The hand in turn experiences an outward pull along the radius due to the center fugal force exerted by the rubber cork due to its directional inertia. [Figure (a)]. If at some instant the finger holding the cord is slightly allowed to be let loose, we observe the rubber cork flying away tangentially at that point. Because by letting the finger loose our hand stops providing centripetal force and due to its directional inertia rubber cork flies off tangentially and falls after covering some distance. [Figure (b)].

- 180.** Calculate the maximum speed with which a vehicle can travel on a : level circular road without skidding.

Ans :

Consider a vehicle (say a car) of weight mg going around a level circular path (road) of radius r with velocity v as shown in Figure.



Centripetal force acting on the car

$$F_C = \frac{mv^2}{r} \quad \dots(1)$$

Other forces acting on the car are :

- Weight mg acting vertically downwards
- Normal reaction $R (= R_1 + R_2)$ acting upwards. The frictional force F between the tyres and the road provides the necessary centripetal force.

$$F = F_C$$

Using eq. (1), we get

$$F = \frac{mv^2}{r} \quad \dots(2)$$

The weight mg of the car is balanced by the normal reaction R

$$\text{i.e.} \quad R = mg \quad \dots(3)$$

Dividing eq. (2) by (3), we get

$$\frac{F}{R} = \frac{mv^2}{r} \times \frac{1}{mg} = \frac{v^2}{rg} \quad \dots(4)$$

If μ is the coefficient of friction, then

$$\frac{F}{R} = \mu$$

So eq. (4) becomes

$$\mu = \frac{v^2}{rg}$$

$$v = \sqrt{\mu rg}$$

For safe turning,

$$v \leq \sqrt{\mu rg}$$

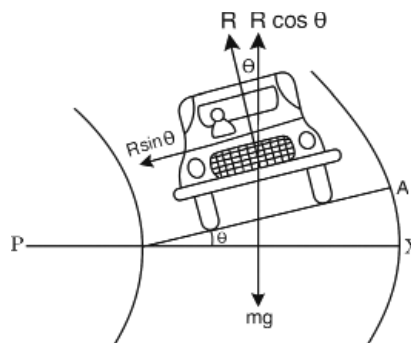
The maximum speed of the vehicle without skidding will be

$$v_{\max} = \sqrt{\mu rg}$$

- 181.** Explain with the help of a neat ; diagram, how banking provides the centripetal force necessary for a car to go in circular track (Ignore friction between tyres and road).

Ans :

To provide the centripetal force for making a vehicle to go round the curved track, the outer edge of the road may be raised above the inner side. This is called banking of road as shown in Figure.



Let θ = angle of banking of road

m = mass of vehicle.

Following forces are acting on the vehicle :

- Weight mg of the vehicle (acting vertically downward)
- The normal reaction R of the ground. Normal reaction R can be resolved into two components.
 - $R \cos \theta$ = (balanced by the weight of the vehicle)

$$\text{i.e.} \quad R \cos \theta = mg \quad \dots(1)$$

- $R \sin \theta$ = (provided centripetal force)

$$\text{i.e.} \quad R \sin \theta = \frac{mv^2}{r} \quad \dots(2)$$

Dividing (2) by (1), we get

$$\frac{R \sin \theta}{R \cos \theta} = \frac{v^2}{rg}$$

$$\tan \theta = \frac{v^2}{rg} \quad \dots(3)$$

$$v = \sqrt{rg \tan \theta} \quad \dots(4)$$

For safe turning

$$v \leq \sqrt{rg \tan \theta}$$

The maximum speed a vehicle can have on the banked road at angle θ without skidding will be

$$v_{\max} = \sqrt{rg \tan \theta}$$

- 194.** A car starts from rest and accelerates uniformly with 2 ms^{-2} . At $t = 10 \text{ s}$, a stone is dropped out of the window (1m high) of the car.

What are the (a) velocity and (b) acceleration of the stone at $t = 10.1 \text{ s}$. (neglect air resistance). Taking $g = 9.8 \text{ ms}^{-2}$

Ans :

- (a) Stone will possess horizontal velocity due to motion of car and vertical downward velocity due to gravity.

$$u_x = u_y = 0$$

$$u = 0, a_x = 2 \text{ ms}^{-2}, t = 10 \text{ s}$$

$$a_y = 9.8 \text{ ms}^{-2}$$

Horizontal velocity of stone.

$$\begin{aligned} \text{Using, } v_x &= u_x + a_x t = 0 + 2 \times 10 \\ &= 20 \text{ ms}^{-1} \end{aligned}$$

Vertical velocity of the stone.

$$\begin{aligned} \text{Using, } v_y &= u_y + a_y t = 0 + 9.8 \times 0.1 \\ &= 0.98 \text{ ms}^{-1} \end{aligned}$$

Resultant velocity of stone

$$\begin{aligned} &= \sqrt{20^2 + (0.98)^2} \\ &= 20.02 \text{ ms}^{-1} \end{aligned}$$

$$\tan \theta = \frac{0.98}{20} = 0.049$$

$$\theta = \tan^{-1}(0.049)$$

$$= 2.8^\circ \text{ with the horizontal direction}$$

Vertical component of acceleration.

$$g = 9.8 \text{ ms}^{-2}$$

CASE BASED QUESTIONS

- 195.** A child is playing with a toy car on a rough floor. She notices that to keep the car moving at a constant slow speed, she has to continuously pull the string attached to it with a gentle force. If she stops pulling, the car quickly comes to rest. Her older sibling mentions that an ancient philosopher also believed that a continuous force was needed to keep bodies in motion.

Questions:

- Which ancient Greek thinker held the view that an external force is required to keep a body in motion?
- What is the flaw in the ancient philosopher's argument, as explained by the concept of friction?

- When the child pulls the toy car at a uniform velocity, what can be said about the net external force acting on it?
- If the floor were perfectly frictionless, and the car was set in motion, would the child need to continuously pull the string to keep it moving at a constant velocity according to Galileo's insights? Explain.

Ans :

- The Greek thinker Aristotle held the view that if a body is moving, something external is required to keep it moving.
- The flaw is that a moving toy car comes to rest because the external force of friction on the car by the floor opposes its motion. To counter this, an external force is needed.
- When the car is in uniform motion (constant velocity), there is no net external force acting on it; the force by the child cancels the force of friction by the floor.
- No, according to Galileo, an object moving on a frictionless horizontal plane must neither have acceleration nor retardation, i.e., it should move with constant velocity without any continuous force.

- 196.** An astronaut working outside a small spaceship in interstellar space, far from any significant gravitational forces, accidentally pushes off the spaceship. After the push, she is no longer holding onto anything, and her jet-pack is off. The spaceship itself is accelerating at a constant rate of 100 m/s^2 due to its engines.

Questions:

- What is Newton's first law of motion?
- What does the first law imply if the net external force on a body is zero?
- Once the astronaut is outside and no longer in contact with the spaceship (and with no jetpack thrust), what is the net external force acting on her, assuming no nearby stars exert gravitational force?
- What will be the acceleration of the astronaut the instant after she is outside the accelerating spaceship?

Ans :

- Newton's first law of motion states: "Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise."
- If the net external force on a body is zero, its acceleration is zero. The body will continue to be in its state of rest or of uniform motion in a straight line.

determined. It is clear that for greater value of v and a sharp curve, the value of h is large.

- 183.** Derive an expression for the angle of bending of a cyclist on a curved track.

Ans :

A cyclist going round a curve bend inward to provide necessary centripetal force to enable him to go along the curved track (Fig. 5.55)

Let

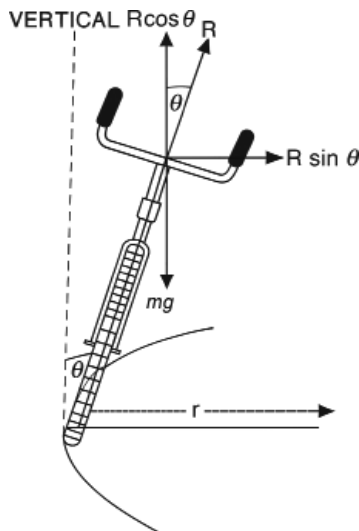
θ = angle of banking of the cyclist with vertical.

m = mass of the cyclist.

v = velocity of the cyclist.

r = radius of the circular path.

Following forces are acting on the cyclist :



- (i) Weight mg (making angle θ with vertical)
- (ii) Reaction R (making angle θ with vertical)
Normal reaction R can be resolved in two components.

(a) $R \cos \theta$ balanced by its own weight.

$$\text{i.e. } R \cos \theta = mg \quad \dots(1)$$

(b) $R \sin \theta$ provides centripetal force

$$\text{i.e. } R \sin \theta = \frac{mv^2}{r} \quad \dots(2)$$

Dividing equation (2) by (1),

$$\begin{aligned} \tan \theta &= \frac{v^2}{rg} \\ v &= \sqrt{rg \tan \theta} \quad \dots(3) \end{aligned}$$

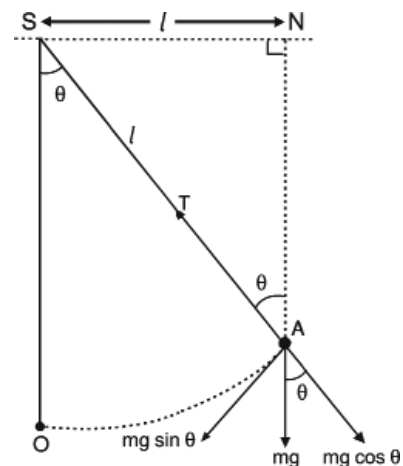
So θ should be large if he moves with greater speed or through a curve of smaller radius.

- 184.** What is a simple pendulum ? Obtain expression for its angular acceleration, when its bob is displaced through an angle.

Ans :

A heavy point mass attached to a weightless, inextensible string suspended from a rigid support about which it is free to oscillate is called an ideal simple pendulum.

In practice it is not possible to have a simple pendulum as defined above because we cannot have a point heavy mass and a weightless and inextensible string, so we take a heavy metallic bob suspended by a thin silken thread from a rigid support.



Let O be the mean position of the bob of the pendulum and let at any instant the bob be at a point A having angular displacement θ as shown in Fig. 5.56

Following forces are acting on the bob at A :

- (i) Tension T of the string towards the point of suspension.
- (ii) Weight mg of the bob acting vertically downwards and can be resolved in two rectangular components
 - (a) $mg \cos \theta$ balanced by tension T .
 - (b) $mg \sin \theta$ acts perpendicular to the length of the string and is responsible to bring the bob back to its position O called restoring force.

The torque τ acting on the bob at A is given by

$$\tau = \text{force} \times \perp \text{ distance from the axis of rotation}$$

$$\text{i.e. } \tau = -mg(SN)$$

$$\tau = -mg l \sin \theta \quad \dots(1)$$

- 182.** Calculate the speed with which a vehicle can safely move on a banked circular road. How can wear and tear of tyres be reduced on such a road ? (taking into consideration the friction between tyres and road)

Ans :

To provide the centripetal force for making a vehicle to go round the curved track, the outer edges of the road may be raised above the inner side. This is called banking of road as shown in Figure.

Let

θ = angle of banking of the road.

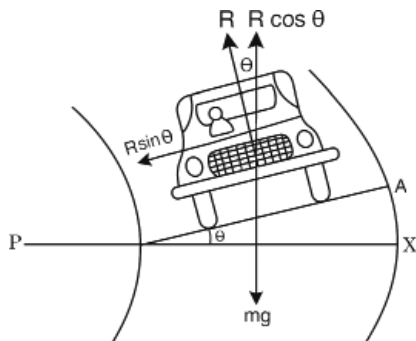
m = mass of vehicle.

μ = coefficient of friction.

Following forces are acting on the vehicle :

- Weight mg of the vehicle acting in vertically downward direction.
- Normal reaction R of the road.
- The force of friction F between the tyres and road.

Normal reaction R can be resolved into two components :



$R \sin \theta$ and $R \cos \theta$

Similarly, frictional force F can be resolved into two components :

$F \sin \theta$ and $F \cos \theta$

From the figure, we find that

- $R \sin \theta$ and $F \cos \theta$ provide the necessary centripetal force $\left(\frac{mv^2}{r}\right)$ to the car.

$$\text{So } R \sin \theta + F \cos \theta = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = R \sin \theta + F \cos \theta \quad \dots(1)$$

- Weight mg and $F \sin \theta$ components are acting in vertically downward direction and $R \cos \theta$ component of R is acting vertically upward direction.

$$R \cos \theta = F \sin \theta + mg$$

$$mg = R \cos \theta - F \sin \theta \quad \dots(2)$$

Dividing (1) by (2) we get

$$\frac{v^2}{rg} = \frac{R \sin \theta + F \cos \theta}{R \sin \theta - F \cos \theta}$$

$$\frac{v^2}{rg} = \frac{\tan \theta + \frac{F}{R}}{1 - \frac{F}{R} \tan \theta}$$

Since $\frac{F}{R} = \mu$ (Coefficient of friction)

$$v^2 = \left[\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right] rg \quad \dots(3)$$

This gives the maximum velocity with which the vehicle can move along the circular road without skidding.

If the friction between the tyres and the road is not taken into consideration i.e. $\mu = 0$, then equation (3) becomes.

$$\text{i.e. } v = \sqrt{rg \tan \theta} \quad \dots(4)$$

For safe turning,

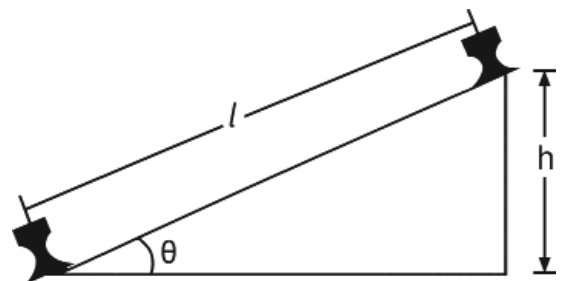
$$v \leq \sqrt{rg \tan \theta}$$

The maximum speed of the vehicle without skidding will be

$$v_{\max} = \sqrt{rg \tan \theta}$$

There will be least wear and tear of the tyres when the vehicle moves with this speed on the circular banked road.

Note. The railway track is also banked in similar manner. If l is the distance between two rails and h is the height through which outer rail is raised above the inner rail [Fig. 5.54], then



$$\sin \theta = \frac{h}{l}$$

In practice, θ is very small so that

$$\sin \theta \simeq \theta \simeq \tan \theta = \frac{h}{l}$$

$$\text{Since } \tan \theta = \frac{v^2}{rg}$$

$$\frac{h}{l} = \frac{v^2}{rg}$$

$$h = \frac{v^2}{rg} l \quad \dots(5)$$

For given values of v , r and l , the value of h i.e. height of outer rail above the inner rail can be

At lowest point L

$$h = 0$$

$$T_L = \frac{m}{r}(v_1^2 + gr)$$

At highest point H

$$h = 2r \quad \dots(4)$$

$$T_H = \frac{m}{r}(v_1^2 - 3g(2r) + gr)$$

$$T_H = \frac{m}{r}(v_1^2 - 5gr) \quad \dots(5)$$

Difference in tension at lowest and highest points.:

$$\begin{aligned} T_L - T_H &= \frac{m}{r}[v_1^2 + gr - v_1^2 + 5gr] \\ &= \frac{m}{r}.6gr \\ T_L - T_H &= 6mg = 6 \times \text{weight of the body} \end{aligned} \quad \dots(6)$$

Hence tension in the string at the lowest point exceeds the tension in the string at the highest point by six times the weight of the body.

Velocity at the lowest point.

The body will be reach the highest point H , if the string does not slacken at H . This is possible only if tension in the string at highest point is positive i.e.

$$T_H \geq 0$$

Let us take $T_H = 0$

Using eq.(5), we get

$$\begin{aligned} \frac{m}{r}(v_1^2 - 5gr) &= 0 \\ v_1^2 &= 5gr \\ v_1 &= \sqrt{5gr} \end{aligned} \quad \dots(7)$$

This is the minimum velocity the body must possess at the lowest point to loop the loop.

Velocity at the highest point

To find velocity of the body at highest point H , replace v by v_H and h by $2r$ in equation (1).

We get $v_H = 5gr - 4gr$

$$v_H = \sqrt{gr} \quad (8)$$

Thus, velocity at highest point is not zero.

NUMERICAL PROBLEMS

- 186.** A bullet of mass 0.04 kg moving with a speed of 90 ms⁻¹ enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet ?

Ans :

Mass of bullet $m = 0.04$ kg

Velocity of bullet $\mu = 90$ ms⁻¹ ; $v = 0$

$$s = 60 \text{ cm} = \frac{60}{100} \text{ m}$$

Using the relation $v^2 - u^2 = 2as$

$$\text{Retardation } a = \frac{0^2 - 90^2}{2 \times \frac{60}{100}}$$

$$\begin{aligned} a &= \frac{-90 \times 90 \times 100}{2 \times 60} \\ &= -6750 \text{ ms}^{-2} \end{aligned}$$

Resistance force,

$$F = ma$$

$$0.04 \times 6750 = 270 \text{ N}$$

- 187.** A rocket of initial mass 6000 kg ejects mass at a constant rate of 16 kgs⁻¹ with constant relative speed of 11 kms⁻¹. What is the acceleration of the rocket a minute after the blast ? (Neglect gravity).

Ans :

$$\begin{aligned} \frac{\Delta m}{\Delta t} &= 16 \text{ kgs}^{-1}, u = 11 \text{ km s}^{-1} \\ &= 11,000 \text{ m s}^{-1} \end{aligned}$$

$$M_0 = 6000 \text{ kg}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta m}{\Delta t} \frac{u}{\left(M_0 - \frac{\Delta m}{\Delta t} t\right)}$$

Acceleration at

$$t = 60$$

$$\begin{aligned} s &= \frac{16 \times 11000}{6000 - 16 \times 60} \\ &= 34.9 \text{ ms}^{-2} \end{aligned}$$

- 188.** A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined at an angle $\theta = 15^\circ$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface ?

Ans :

Since at this angle the body just begins to slide, therefore, this is angle of repose

$$\mu = \tan \theta = \tan 15^\circ$$

$$\mu = 0.27$$

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-ve sign means that the turning effect of the torque is to decrease θ .

Since velocity and momentum are always tangential to the circle, so their level arm will always be l

$$L = mvl = m(\omega)l$$

$$= m l^2 \omega = m l^2 \frac{d\theta}{dt} \quad \dots(2)$$

Since torque τ and angular momentum L are related by relation

$$\tau = \frac{dL}{dt} \quad \dots(3)$$

Using equation (2) and (3), we get

$$\tau = m l^2 \frac{d^2 \theta}{dt^2} \quad \dots(4)$$

From equation (1) and (4), we have

$$-mgl \sin \theta = m l^2 \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = \frac{-g}{l} \sin \theta \text{ or } \alpha$$

$$= \frac{-g}{l} \sin \theta$$

This equation gives the oscillations of a simple pendulum in a vertical plane.

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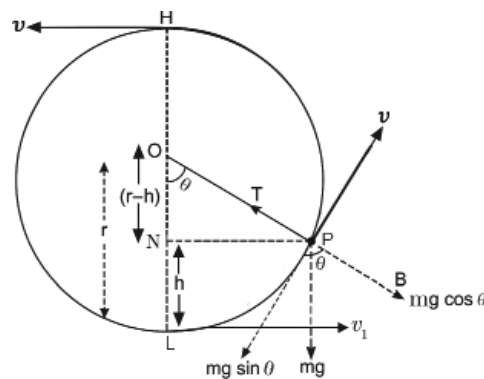
- 185.** (a) A heavy ball of mass m is suspended from a thread of length r and is being rotated in a vertical circle. Show that the tension in the string at the lowest point exceeds the tension in the string at the highest point by six times the weight of the body.
- (b) Find the velocity of mass m at lowest and highest points and the condition for looping the loop

Ans :

Motion in a vertical circle

A ball of mass m suspended from a thread of length and rotated in a vertical circle is shown in Fig. 5.57. As it moves from the lowest point L of the vertical circle to the highest point, the velocity of mass goes on decreasing as gravity opposes its motion. Hence velocity is minimum at the highest point H (say V^2) and maximum at lowest point L (say V_1).

Let v be velocity of the body at any point P at vertical height h (Fig. 5.57).



From law of conservation of energy,

$$[\text{Total energy}]_{\text{at } P} = [\text{Total energy}]_{\text{at } L}$$

$$[KE + PE]_P = [KE + PE]_L$$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_1^2 + 0$$

$$v^2 + 2gh = v_1^2$$

$$v^2 = v_1^2 - 2gh \quad \dots(1)$$

This is the expression for the velocity of the body at point P of the vertical circle.

Tension of the string (T)

At point P , following forces are acting on the body :

- (i) Tension T in the string along PO .
- (ii) Weight mg of the body acting vertically downwards.

It can be resolved in two rectangular components

(a) $mg \cos \theta$ – acting along PB directed away from O .

(b) $mg \sin \theta$ – acting tangential at P .

As the body is moving in a circular path, so $T > mg \cos \theta$ and $T - mg \cos \theta$ provides the required centripetal force.

$$T - mg \cos \theta = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg \cos \theta \quad \dots(2)$$

$$T = \frac{mv^2}{r} + mg \left(\frac{r-h}{r} \right)$$

$$\left[\text{In rt. } \angle d\Delta PNO \right]$$

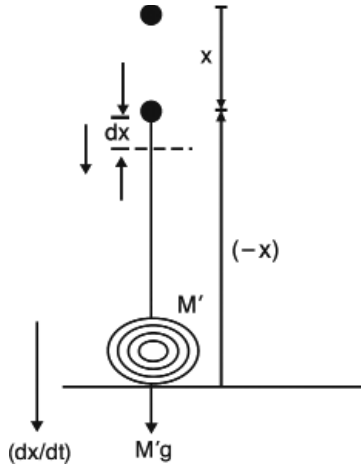
$$\cos \theta = \frac{r-h}{r}$$

Using Eq. (1), we get

$$T = \frac{m}{r} (v_1^2 - 2gh) + mg \left(\frac{r-h}{r} \right)$$

$$T = \frac{m}{r} (v_1^2 - 3gh + gr) \quad \dots(3)$$

This is the General expression for tension in the string.



- 192.** Two particles of equal masses m and m are connected by a light string of length $2l$ as shown in Fig. (5.63). A constant force F is applied continuously at the mid-point of the string, always along the perpendicular bisector of the straight line joining the two particles. Show that when the distance between the two particles is $3x$, the acceleration of the particle is

$$a = \frac{F}{m} \cdot \frac{x}{(l^2 - x^2)^{\frac{1}{2}}}$$

Ans :

The force acting on A towards B is given by $T \cos \theta$
From Fig. 5.63

$$F = 2T \sin \theta$$

$$T = \frac{F}{2 \sin \theta} \quad \dots(1)$$

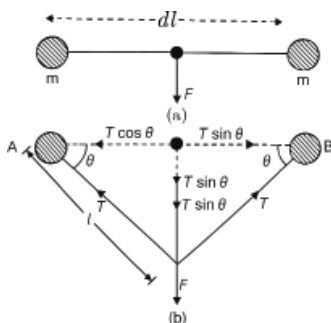
Now, acceleration of A towards B ,

$$a_1 = \frac{T \cos \theta}{m} \quad \dots(2)$$

and acceleration of B towards A ,

$$a^2 = \frac{T \cos \theta}{m} \quad \dots(3)$$

Putting the value of T , from eq. (1) in eq. (2) and (3), we get



$$a_1 = a_2 = \frac{F \cos \theta}{2m \sin \theta} = \frac{F \cot \theta}{2m}$$

Acceleration of approach

$$= 2 \times \frac{F \cot \theta}{2m}$$

$$a = \frac{F}{m} \cdot \frac{x}{(l^2 - x^2)^{\frac{1}{2}}}$$

- 193.** The total mass of an elevator with a 80 kg man in it is 1000 kg. This elevator moving upward with a speed of 8 ms^{-1} is brought to rest over a distance of 16 m. Calculate

- (a) the tension T in the cable supporting the elevator and
(b) the force exerted on man by the elevator floor.

Ans :

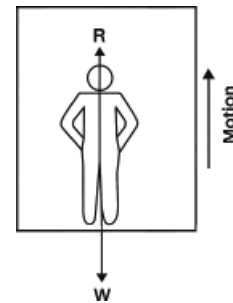
- (a) $u = 8 \text{ ms}^{-1}$
 $s = 16 \text{ m}$

Using the relation,

$$v^2 - u^2 = 2as$$

$$0 = 8^2 + 2 \times a \times 16$$

$$a = -2 \text{ ms}^{-2}$$



$$R - W = 1000 a$$

$$R = W + 1000 a$$

$$= 1000 \times 9.8 + 1000 \times (-2)$$

$$= 1000 \times 7.8 = 7800 \text{ N}$$

Tension in springs is equal to the relation force,

$$\text{i.e. } T = R = 7800 \text{ N}$$

- (b) If ' F ' is the upward force exerted on the man, then weight of the man acting downward

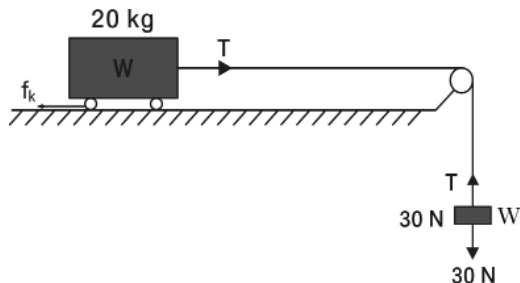
$$mg = 80 \times 9.8 \text{ N}$$

$$F - mg = ma$$

$$= m(g + a) = 80 \times (9.8 - 2)$$

$$= 624.0 \text{ N}$$

189. What is the acceleration of the block and trolley system shown in the figure, if the coefficient of kinetic friction between the trolley and the surface is 0.04 ? What is the tension in the string? (Take $g = 10 \text{ ms}^{-2}$) Neglect the mass of the string



Ans :

Since the length of the string is fixed, the 3 kg block and the 20 kg trolley both have the same acceleration (in magnitude) a .

Apply the second law to the motion of the block

$$30 - T = 3a$$

Apply the second law to the motion of the trolley

$$T - F = 20a$$

$$\text{Now } F = \mu R = \mu mg$$

Therefore,

$$T - 0.04 \times 200 = 20a \text{ or } T - 8 = 20a$$

From these equations,

$$a = \frac{22}{23} \text{ ms}^{-2} = 0.96 \text{ ms}^{-2}$$

$$\text{and } T = 27.1 \text{ N}$$

190. The upper half of an inclined plane with inclination θ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, find the coefficient of friction for the lower half.

Ans :

For upper half motion

$$u = 0, v = ? a = g \sin \theta$$

Let distance covered

$$S = \frac{s}{2}$$

where 'S' is the total length of the inclined plane.

Using equation of motion,

$$\text{As } v^2 - u^2 = 2as$$

$$v^2 - 0^2 = 2(g \sin \theta) \left(\frac{s}{2} \right)$$

$$v = \sqrt{gs \sin \theta}$$

Motion of the body along lower half plane :

Now, initial velocity,

$$u = \sqrt{gs \sin \theta}$$

$$v = 0$$

$$a = -g(\mu \cos \theta - \sin \theta)$$

Stopping distance,

$$s = \frac{S}{2}$$

Again, using equation of motion

$$v^2 - u^2 = 2as$$

$$0^2 - (gs \sin \theta) = 2 [-g(\mu \cos \theta - \sin \theta)] \left(\frac{S}{2} \right)$$

This gives,

$$\mu = 2 \tan \theta$$

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191. A piece of uniform string hangs vertically so that its free end just touches horizontal surface of a table. The upper end of the string is now released. Show that at any instant during the falling of string, the total force on the surface is three times the weight of that part of string lying on the surface.

Ans :

Let m be the mass per unit length of string and x the length of string at any instant on the table. If differential length dx of string further falls in time dt , then velocity v of this element will be given by

$$v^2 = u^2 + 2gx = 0 + 2gx$$

$$v^2 = 2gx \quad \dots(1)$$

Net force on the surface of the table

$$F = \frac{dp}{dt} + \text{weight of string lying on the surface}$$

$$= \frac{mdxv}{dt} + mxg = mv \frac{dx}{dt} + mxg$$

$$= mv^2 + mgx$$

Using(1)

$$F = m(2gx) + mgx = 3(mx)g$$

But $mx = M' = \text{weight of string lying on the table:}$

$$F = 3M'g$$

- Since there are no nearby stars to exert gravitational force on her and the small spaceship exerts negligible gravitational attraction, the net force acting on the astronaut, once she is out of the spaceship, is zero.
- By the first law of motion, the acceleration of the astronaut will be zero.

197. A bullet of mass 0.04 kg is fired with a speed of 90 m/s into a heavy wooden block. The bullet travels a distance of 60 cm into the block before it is stopped. The block exerts an average resistive force on the bullet.

Questions:

- What is Newton's second law of motion relating force, mass, and acceleration?
- To find the average resistive force, which physical quantity related to the bullet's motion must first be calculated assuming it's constant?
- Using the initial speed, final speed (zero), and distance, calculate the magnitude of the bullet's average retardation (acceleration).
- What is the magnitude of the average resistive force exerted by the block on the bullet?

Ans :

- Newton's second law can be written as $F = ma$, where F is the net external force, m is the mass of the body, and a is its acceleration.
- To find the average resistive force, the bullet's average retardation (or negative acceleration) must first be calculated, assuming it's constant.
- The initial velocity $u = 90\text{m/s}$, final velocity $v = 0\text{m/s}$, distance $s = 60\text{cm} = 0.6\text{m}$.
Using $v^2 = u^2 + 2as$:

$$0^2 = (90)^2 + 2a(0.6)$$

$$0 = 8100 + 1.2a$$

$$a = -8100/1.2 = -6750\text{m/s}^2.$$

The retardation is 6750m/s^2 .

- The magnitude of the average resistive force is

$$F = ma$$

$$= (0.04\text{kg}) \times (6750\text{m/s}^2)$$

$$= 270\text{N}.$$

198. A batsman hits a cricket ball of mass 0.15 kg straight back towards the bowler. The ball was initially moving towards the batsman at 12 m/s and, after being hit, it moves away from the batsman also at 12 m/s. The contact between the bat and ball is very brief.

Questions:

- Define impulse. What is its relation to momentum?

- If the initial direction of the ball (towards the batsman) is taken as negative, what is its initial momentum? What is its final momentum after being hit?
- Calculate the change in momentum of the ball.
- Determine the impulse imparted to the ball. Why is the concept of impulse useful in such situations?

Ans :

- Impulse is the product of force and time duration, and it equals the change in momentum of the body.
- Initial momentum

$$\begin{aligned} p_i &= m \times (-v) \\ &= 0.15\text{kg} \times (-12\text{m/s}) \\ &= -1.8\text{kgm/s}. \end{aligned}$$

Final momentum

$$\begin{aligned} p_f &= m \times v \\ &= 0.15\text{kg} \times (12\text{m/s}) \\ &= 1.8\text{kgm/s}. \end{aligned}$$

- Change in momentum

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= 1.8\text{kgm/s} - (-1.8\text{kgm/s}) \\ &= 3.6\text{kgm/s (or Ns)}. \end{aligned}$$

- Impulse imparted to the ball is 3.6 Ns, in the direction from the batsman to the bowler. The concept of impulse is useful here because the force on the ball by the batsman and the time of contact are difficult to know separately, but the impulse (change in momentum) is readily calculated.

199. Two teams, A and B, are engaged in a tug-of-war. Team A pulls on the rope with a force F_{AB} (force on rope by A) and team B pulls with a force F_{BR} (force on rope by B). The rope, in turn, exerts forces on the teams. Consider the forces acting between Team A and the rope.

Questions:

- State Newton's third law of motion.
- According to Newton's third law, if the rope exerts a force F_{RA} on Team A, what is the force F_{AR} exerted by Team A on the rope in relation to F_{RA} ?
- Do the forces F_{RA} (on Team A by rope) and F_{AR} (on rope by Team A) act on the same body or different bodies?
- If Team A is winning, does this mean the force they exert on the rope is greater than the force the rope exerts on them? Explain using Newton's third law.

Ans :

- (a) False, The total momentum and total energy of the system remain conserved and not of each body.
- (b) False, The external forces on a system can increase or decrease (change) the total energy of the system.
- (c) False, Work done in the motion of a body over a closed loop is zero only if the body moves under the action of conservative forces and not when the forces are non-conservative.
- (d) False, True usually but not always, because in an elastic collision, some K.E. usually changes into some other forms of energy.

8. Answer carefully with reasons :

- (a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls i.e., when they are in contact ?
- (b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?
- (c) What are the answers to (a) and (b) for an inelastic collision ?
- (d) If the potential energy of two billiard balls depends only on the separation distance between their centres is the collision elastic or inelastic? (Note : we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Ans :

- (a) No, because some of Kinetic energy is converted into Potential energy.
- (b) Yes, momentum remains conserved.
- (c) Linear momentum is conserved during an elastic collision. Kinetic energy is not conserved even after the collision is over.
- (d) Elastic : Since Potential energy depends upon distance only, therefore, it is zero after a small distance. Thus Kinetic energy remains the same before and after collision.

9. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

- (a) $t^{1/2}$ (b) t
- (c) $t^{3/2}$ (d) t^2

Ans :

- (b) t

As we know that, $a = \frac{dv}{dt} = \frac{F}{m}$

From the first equation of motion, we get,

So

$$v = 0 + at = \frac{F}{m}t$$

$$P = Fv = F \frac{F}{m}t = \frac{F^2}{m^2}(mt)$$

$$= \left(\frac{F}{m}\right)^2 mt$$

$$P = a^2 mt$$

$$P \propto t$$

10. A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to

- (a) $t^{1/2}$ (b) t
- (c) $t^{3/2}$ (d) t^2

Ans :

- (c) $t^{3/2}$

As we know that, $t^{3/2} = Fv = mav \quad \dots(1)$

From the first equation,

$$v = u + at = 0 + at$$

$$u = at$$

From equation (1), we get

$$P = ma(at)$$

$$a = \sqrt{\frac{P}{mt}}$$

Also,

$$x = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}\sqrt{\frac{P}{mt}}t^2$$

$$= \frac{1}{2}\sqrt{\frac{P}{m}}t^{3/2}$$

$$x \propto t^{3/2}$$

11. A body constrained to move along the z-axis of a co-ordinate system is subject to a constant force $\vec{F}$ given by

$$\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})N,$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x -, y - and z - axis of the system respectively. What is the work done by this force in moving the body through a distance of 4 m along the z -axis?

Ans :

Given, $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})N$

$$\vec{d} = (0\hat{i} + 0\hat{j} + 4\hat{k})m$$

Ans :

- The centripetal force is

$$f_c = \frac{mv^2}{R}.$$
- On a level road, the static friction between the tyres and the road provides the necessary centripetal force.
- The maximum static frictional force available is

$$(f_s)_{\max} = \mu_s N = \mu_s mg.$$
- For safe turning, the centripetal force required must be less than or equal to the maximum static friction:

$$\frac{mv^2}{R} \leq \mu_s mg.$$

This simplifies to

$$v^2 \leq \mu_s Rg.$$

Therefore, $v_{\max} = \sqrt{\mu_s Rg}.$

- 203.** A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the mid-point P of the rope. The rope makes an angle θ with the vertical in equilibrium. (Neglect the mass of the rope, take $g = 10 \text{ m/s}^2$).

Questions:

- What does equilibrium of a particle mean in terms of the net external force acting on it?
- Draw a free-body diagram for the 6 kg mass. What forces act on it, and what is the tension (T_2) in the lower part of the rope?
- Draw a free-body diagram for point P. What forces act on point P?
- By resolving forces horizontally and vertically at point P and applying the conditions for equilibrium, find the angle θ the upper part of the rope makes with the vertical.

Ans :

- Equilibrium of a particle means the net external force on the particle is zero.
- Forces on the 6 kg mass are: its weight ($W = 6 \text{ kg} \cdot 10 \text{ m/s}^2 = 60 \text{ N}$) acting downwards, and the tension T_2 in the lower part of the rope acting upwards. In equilibrium, $T_2 = W = 60 \text{ N}$.
- Forces on point P are: the applied horizontal force (50 N), the tension T_2 from the lower rope (acting downwards), and the tension T_1 from the upper rope (acting upwards at an angle θ to the vertical).
- Resolving forces at P
Vertical equilibrium:

$$T_1 \cos \theta = T_2 = 60 \text{ N}.$$

Horizontal equilibrium:

$$T_1 \sin \theta = 50 \text{ N}.$$

Dividing the second equation by the first:

$$\frac{T_1 \sin \theta}{T_1 \cos \theta} = \frac{50}{60}.$$

$$\tan \theta = \frac{5}{6}$$

$$\theta = \tan^{-1}(5/6) \approx 39.8^\circ.$$

- 204.** A person needs to move a heavy box across a rough horizontal floor. They can either push it at an angle downwards or pull it at the same angle upwards. The coefficient of kinetic friction between the box and the floor is μ_k .

Questions:

- What is kinetic friction, and how is its magnitude related to the normal force N and the coefficient of kinetic friction μ_k ?
- How does the normal force exerted by the floor on the box change when you push down on the box at an angle versus pulling up at an angle, compared to just the weight of the box?
- If you push with a force F at an angle α below the horizontal, what is the normal force N ? What is the frictional force f_k ?
- Is it generally easier to move the box by pushing or pulling it at an angle? Explain why in terms of the frictional force involved.

Ans :

- Kinetic friction (f_k) is the frictional force that opposes relative motion between surfaces in contact. Its magnitude is given by

$$f_k = \mu_k N.$$
- When you push down at an angle, the vertical component of your push adds to the weight, increasing the normal force. When you pull up at an angle, the vertical component of your pull reduces the effective force on the ground, decreasing the normal force.
- If you push with force F at an angle α below the horizontal:

Vertical forces:

$$N - mg - F \sin \alpha = 0$$

$$N = mg + F \sin \alpha.$$

Frictional force: $f_k = \mu_k N$

$$= \mu_k (mg + F \sin \alpha).$$

- It is generally easier to pull the box at an angle upwards. When pulling upwards, the normal force $N = mg - F \sin \alpha$, which reduces the frictional force $f_k = \mu_k (mg - F \sin \alpha)$. When pushing downwards, the normal force increases, thus increasing the frictional force. A smaller frictional force means less effort is required to move the box.

$$s = 49 \text{ m}$$

Hence, $W = (\text{force} \times \text{distance})$
 $= -5.04 \times 49$
 $= -247 \text{ J}$

(c) Effective force $= ma$
 $a = \frac{5.04}{2} = 2.52 \text{ ms}^{-2}$

Using this value of a ,

$$s = ut + \frac{1}{2}at^2$$

$$= 0 \times 10 + \frac{1}{2} \times 2.52 \times 10 \times 10$$

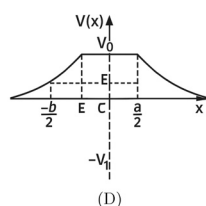
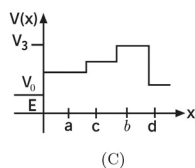
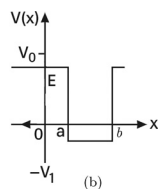
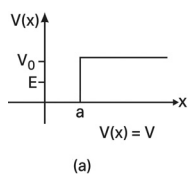
$$= 126 \text{ m}$$

Hence, $W = (\text{effective force} \times \text{distance})$
 $= 5.04 \times 126 = 635 \text{ J}$

(d) $v = u + at$
 $= 0 + 2.52 \times 10 = 25.2 \text{ ms}^{-1}$
 $\text{K.E.} = \frac{1}{2}mv^2$
 $= \frac{1}{2} \times 2 \times 25.2 \times 25.2$
 $= 635 \text{ J}$

Work done by the net force on a body is equal to the change in its kinetic energy.

3. Given in figure are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contents for which these potential energy shapes are relevant.



Ans :

As we know that,

$$E = \text{Potential energy} + \text{Kinetic energy}$$

Here, $E = \text{Total energy}$

$$\text{P.E.} = \text{Potential energy}$$

$$\text{K.E.} = \text{Kinetic energy}$$

$$\text{K.E.} = E - \text{Potential energy}$$

$$\frac{1}{2}mv^2 = E - \text{Potential energy}$$

Since Kinetic energy cannot be negative, so Potential energy cannot be more than total energy.

- (a) In region $x = 0$ and $x = a$, P.E. = 0, so K.E. = +ve. And in region $x > a$, P.E. > E, so K.E. is -ve. Thus the particle can't be present in region $x > a$

The minimum total energy of the particle must be 0.

- (b) In region $x < a$ and $x > b$, P.E. > E, so K.E. will be negative and hence the particle cannot be present in this region. And in region $a < x < b$, P.E. is negative, so K.E. is positive and the particle can be present in this region.

The minimum total energy of the particle must be $-V_1$

- (c) In region $-\infty < x < \infty$ (i.e. in regions $-\infty < x < a$, $a < x < c$, $c < x < b$, $b < x < d$ and $d < x < \infty$, P.E. > E, so it cannot be present in the region $-\infty < x < \infty$.

The minimum total energy of the particle must be positive i.e. V_1

- (d) $-\frac{b}{2} < x < -\frac{a}{2}$, $\frac{a}{2} < x < \frac{b}{2}$, P.E. > E, so K.E. will be negative in these regions and the particle will be present in these regions.

The minimum total energy of the particle must be $-V_1$.

4. The potential energy function of a particle executing linear simple harmonic motion is given by $V(x) = \frac{kx^2}{2}$, where k is the force constant of the oscillator.

For $k_1 = 0.5 \text{ Nm}^{-2}$, the graph of $V(x)$ versus x is shown below. Show that a particle of total energy 1 J moving under this potential must turn back when it reaches $x = \pm 2 \text{ m}$

Ans :

The significance of the above equation is shown by the graph in figure in which energy is plotted vertically and the co-ordinate x horizontally. The curve represents the potential energy $V(x) = (1/2)kx^2$ this curve is parabola. A horizontal line is drawn at a height equal to the total energy E . We see that

7. COLLISION

A collision between two bodies occurs when they either physically come into contact or when the motion of one body is influenced by an interaction with the other, even without direct contact.

7.1 Elastic and Inelastic Collisions

The momentum and kinetic energy are conserved in elastic collisions. In an inelastic collision, the kinetic energy is not conserved.

Laws of collisions are as follows :

- Principle of conservation of linear momentum.
The vector sum of the moments of the impinging bodies before collision is equal to the vector sum of the momenta after collisions.
- When two bodies collide, the only force acting on them at the time of impact is the mutual interaction which acts along the common normal. There is no force acting along the common tangent. Hence, there is change of momentum in that direction.

7.2 Coefficient of Restitution (e)

Coefficient of restitution is given by

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$e = 1$ for elastic collision, $e < 1$ for inelastic collision, $e = 0$ for perfectly elastic collision.

NCERT TEXT-BOOK QUESTIONS

- The sign of work done by a force on body is important to understand. State carefully if the following quantities are positive or negative.
 - Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
 - Work done by gravitational force in the above case.
 - Work done by friction on a body sliding down an inclined plane.
 - Work done by applied force on a body moving on a rough horizontal plane with uniform velocity.
 - Work done by resistive force of air on a vibrating pendulum in bringing it to rest.

Ans :

- Positive, because force and displacement are in the same direction.
- Negative, because force and displacement are in opposite directions.

- Positive, because friction and displacement are in opposite directions.
- Positive, because force and displacement are in the same direction.
- Negative, because force and displacement are in opposite directions.

- A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the

- work done by the applied force in 10 s.
- work done by friction in 10 s.
- work done by the net force on the body in 10 s.
- change in kinetic energy of the body in 10 s and interpret your results.

Ans :

- Given,

Mass of the body = 2 kg.,

Coefficient of kinetic friction,

$$\mu = 0.1$$

Initial velocity, $u = 0 \text{ m/s}$

As we know that,

$$F = ma$$

Hence, $7 = 2 \times a$

$$a = \frac{7}{2} \text{ ms}^{-2}$$

Force of friction $F_1 = \mu R = \mu mg$

$$= 0.1 \times 2 \times 9.8 = 1.96 \text{ N}$$

(Acts in a direction opposite to applied force)

Effective force, $F - F_1 = 7 - 1.96 = 5.04 \text{ N}$

Distance covered in 10 s is given by

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 0 \times 10 + \frac{1}{2} \times \frac{7}{2} \times 10 \times 10 \\ &= 175 \text{ m} \end{aligned}$$

Hence, $W = (F - F_1)S$
 $= 5.04 \times 175 = 882 \text{ J}$

- $F = \mu mg$
 $ma = \mu mg$

$$a = \mu g = 0.1 \times 9.8 = \frac{98}{100} \text{ ms}^{-2}$$

Then, $s = ut + \frac{1}{2}at^2$
 $= 0 \times 10 + \frac{1}{2} \times \frac{98}{100} \times 10 \times 10$

Ans :

1. Newton's third law states: "To every action, there is always an equal and opposite reaction."
2. The force F_{AR} exerted by Team A on the rope is equal in magnitude and opposite in direction to the force F_{RA} exerted by the rope on Team A ($F_{AR} = -F_{RA}$).
3. Action and reaction forces act on different bodies. So, F_{RA} acts on Team A, and F_{AR} acts on the rope.
4. No. According to Newton's third law, the force Team A exerts on the rope (F_{AR}) is always equal in magnitude and opposite in direction to the force the rope exerts on Team A (F_{RA}). Team A accelerates due to the net force acting on Team A. This net force includes the force from the rope and the friction force from the ground on Team A.

- 200.** A rifle of mass 4 kg fires a bullet of mass 20 g (0.02 kg). The bullet leaves the muzzle with a velocity of 300 m/s. The rifle and bullet are initially at rest. Assume this system (rifle + bullet) is isolated during the brief firing process.

Questions:

1. What is the law of conservation of linear momentum?
2. What is the total momentum of the rifle and bullet system before firing?
3. What is the momentum of the bullet after firing?
4. Using the law of conservation of momentum, determine the recoil velocity of the rifle.

Ans :

1. The total momentum of an isolated system of interacting particles is conserved.
2. Since both the rifle and bullet are initially at rest, the total initial momentum of the system is zero.
3. Momentum of the bullet after firing (p_b) = mass of bullet $\times$ velocity of bullet

$$= 0.02\text{kg} \times 300\text{m/s} = 6\text{kgm/s}.$$
4. Let P_b be the momentum of the bullet after firing and P_g be the recoil momentum of the gun. According to momentum conservation, $P_b + P_g = 0$. So, $P_g = -P_b = -6\text{kgm/s}$.
 If v_g is the recoil velocity of the rifle, then

$$M_g v_g = -6\text{kgm/s}.$$

$$4\text{kg} \times v_g = -6\text{kgm/s}$$

$$v_g = -6/4 = -1.5\text{m/s}.$$

The recoil velocity of the rifle is 1.5 m/s in the direction opposite to the bullet.

- 201.** A block of mass 'm' rests on a horizontal plane. The plane is gradually inclined. At an angle $\theta = 15^\circ$ with the horizontal, the mass just begins to slide. The forces acting on the block are its weight (mg), the normal force (N) from the plane, and the static frictional force (f_s).

Questions

1. What does static friction oppose?
2. When the block just begins to slide, what is the magnitude of the static frictional force in relation to its maximum possible value?
3. Resolve the weight 'mg' into components parallel and perpendicular to the inclined plane. At the point of impending motion, how are these components related to f_s and N?
4. Derive an expression for the coefficient of static friction (μ_s) in terms of the angle θ_{max} at which the block just begins to slide. Calculate μ_s for $\theta = 15^\circ$.

Ans :

1. Static friction opposes impending motion.
2. When the block just begins to slide, the static frictional force f_s achieves its maximum value,

$$(f_s)_{max} = \mu_s N.$$

3. Component of weight parallel to the plane

$$= mg \sin \theta.$$

$$\text{Component of weight perpendicular to the plane} = mg \cos \theta.$$

$$\text{At impending motion, } f_s = mg \sin \theta \quad \text{and} \quad N = mg \cos \theta.$$

4. Since $(f_s)_{max} = \mu_s N$, we have

$$mg \sin \theta_{max} = \mu_s (mg \cos \theta_{max}).$$

$$\text{This gives } \tan \theta_{max} = \mu_s.$$

$$\text{For } \theta_{max} = 15^\circ, \mu_s = \tan 15^\circ \approx 0.27.$$

- 202.** A car of mass 'm' is taking a circular turn of radius 'R' on a level (unbanked) road with a speed 'v'. For the car to make the turn safely without skidding, a centripetal force is required.

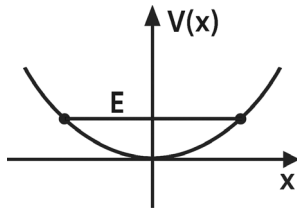
Questions:

1. What is the expression for the centripetal force required for a body of mass 'm' moving in a circle of radius 'R' with speed 'v'?
2. On a level road, which force provides the necessary centripetal force for the car to take a circular turn?
3. If μ_s is the coefficient of static friction between the tyres and the road, what is the maximum static frictional force available? (Assume normal reaction $N = mg$).
4. Derive the expression for the maximum safe speed (v_{max}) with which the car can take the turn without skidding.

the motion is restricted to values of x lying between the points at which the horizontal line intersects the parabola since if x were outside this range, the potential energy would exceed the total energy, and that is impossible.

$$V = \frac{1}{2} kx^2$$

$$V = \frac{1}{2} \times \frac{1}{2} x^2 = \frac{1}{4} x^2$$



When total energy = P.E. = 1 J

$$\frac{1}{4} x^2 = 1$$

$$x^2 = 4$$

$$x = \pm 2 \text{ m}$$

Hence, Maximum height, the particle can attain is $\pm 2 \text{ m}$.

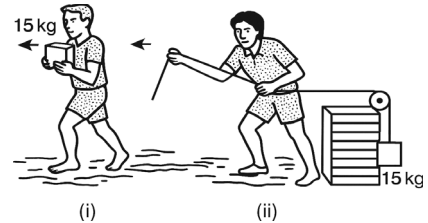
5. Answer the following :

- The casting of a rocket in flight burns up due to friction. At whose expense the heat energy required for burning is obtained ? The rocket or the atmosphere ?
- Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?
- An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?
- In figure (i), the man walks 2 m carrying a mass of 15 kg on his hands. In figure (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley and a mass of 15 kg hangs at its other end. In which case is the work done larger ?

Ans :

- The rocket provides the necessary energy.
- For conservative force, work done over a path is minus of change in potential energy. Over a complete orbit, there is no change in potential energy.

- Kinetic energy increases, but potential energy decreases and the sum decreases due to dissipation against friction.
- In the second case, the work done is larger, because in this case both Kinetic energy as well as Potential energy of the weight increases. Kinetic energy is equal to as in case (i), therefore, Potential energy is extra. Work done in carrying the mass is zero.



6. Underline the correct alternative :

- When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.
- Work done by a body against friction always results in a loss of its kinetic/potential energy.
- The rate of change of total momentum of a many particle system is proportional to the external force/sum of the internal forces of the system.
- In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Ans :

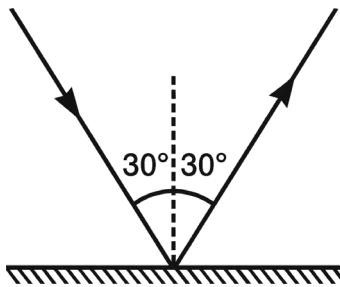
- decreases
- kinetic energy
- external force
- total linear momentum and also total energy of the system of two bodies is conserved.

7. State if each of the following statements is true or false. Give reasons for your answer.

- In an elastic collision of two bodies, the momentum and energy of each body is conserved.
- Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
- Work done in the motion of a body over a closed loop is zero for every force in nature.
- In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

If m = mass of molecule

Momentum before collision
 $= m \times 200 \cos 30^\circ \quad \dots(1)$



Momentum after the collision = Momentum of rebounded molecule + Recoil momentum of wall
 But recoil momentum of wall is zero
 Hence, Momentum of molecule after collision along normal

$$= m \times 200 \cos 30^\circ \quad \dots(2)$$

Momentum of system is conserved along the normal and as the component of velocity along the wall is only that of molecule, momentum along the wall also remains the same. Thus, momentum is conserved.

Kinetic energy before collision

$$\begin{aligned} &= \frac{1}{2}m(200 \cos 30^\circ)^2 + \frac{1}{2}m(200 \sin 30^\circ)^2 \\ &= \frac{1}{2}m \times (200)^2 \end{aligned}$$

i.e. Kinetic energy after collision

$$= \frac{1}{2}m \times (200)^2 + \text{K.E. of wall}$$

(Kinetic energy of wall is practically zero)

$$= \frac{1}{2}m \times (200)^2 + 0$$

Therefore, Kinetic energy is also conserved

Hence, Collision is elastic.

- 15.** A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?

Ans :

Given,

Volume of tank, $V = 30 \text{ m}^3$, $g = 9.8 \text{ m s}^{-2}$,

$d = 40 \text{ m}$

Density of water, $\rho = 1000 \text{ kg/m}^3$

As we know that,

$$\begin{aligned} \text{Mass} &= \text{Volume} \times \text{density} \\ &= 30 \times 1000 \end{aligned}$$

$$m = 3 \times 10^4 \text{ kg}$$

Power, $P = \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$

$$\begin{aligned} &= \frac{3 \times 10^4 \times 9.8 \times 40}{15 \times 60} \\ &= \frac{196}{15} = 13.07 \text{ kW} \end{aligned}$$

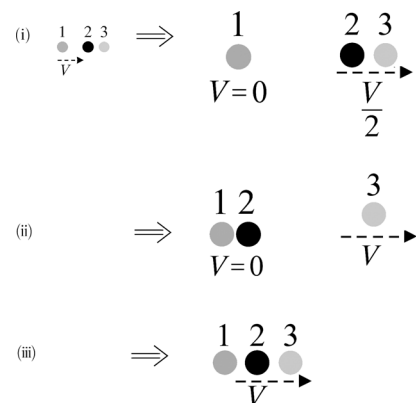
Percentage efficiency,

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100$$

Hence, Input power $= \frac{13.07 \times 100}{30}$

$$= 43.55 \text{ or nearly } 43.6 \text{ kW.}$$

- 16.** Two identical ball-bearings in contact with each other and resting on a frictionless table are hit head-on collision by another ball bearing of same mass moving initially with a speed v . If the collision is elastic, which of the following is a possible result after collision.



Ans :

(ii) is a possible collision.

It can be observed that the total momentum before and after collision in each case is constant.

For an elastic collision, the total kinetic energy of a system remains conserved before and after collision.

For mass of each ball bearing m , we can write :

Total kinetic energy of the system before collision :

$$\begin{aligned} &= \frac{1}{2}mV^2 + \frac{1}{2}(2m)0 \\ &= \frac{1}{2}mV^2 \end{aligned}$$

As we know that,

$$\begin{aligned}\vec{W} &= \vec{F} \cdot \vec{d} \\ &= (-\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k}) \\ &= -1 \times 0 + 2 \times 0 + 3 \times 4 \\ &= 12 \text{ J}\end{aligned}$$

- 12.** An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J).

Ans :

As we know that,

$$\begin{aligned}\text{Kinetic energy of electron} &= \frac{1}{2} m_e v_e^2 \\ 10 \times 10^3 \times 1.60 \times 10^{-19} &= \frac{1}{2} \times 9.11 \times 10^{-31} v_e^2 \\ &\dots(1)\end{aligned}$$

$$\begin{aligned}\text{Kinetic energy of proton} &= \frac{1}{2} m_p v_p^2 \\ 100 \times 10^3 \times 1.60 \times 10^{-19} &= \frac{1}{2} \times 1.67 \times 10^{-27} v_p^2 \\ &\dots(2)\end{aligned}$$

Dividing (1) by (2), we get

$$\begin{aligned}\frac{10 \times 10^3 \times 1.60 \times 10^{-19}}{100 \times 10^3 \times 1.60 \times 10^{-19}} &= \frac{\frac{1}{2} \times 9.11 \times 10^{-31} v_e^2}{\frac{1}{2} \times 1.67 \times 10^{-27} v_p^2} \\ \frac{1}{10} &= \frac{9.11 \times v_e^2}{1.67 \times 10^4 v_p^2} \\ \frac{v_e^2}{v_p^2} &= \frac{1.67 \times 10^4}{9.11 \times 10} = \frac{1670}{9.11} \\ \frac{v_e}{v_p} &= \sqrt{\frac{1670}{9.11}} = 13.5\end{aligned}$$

Thus, the electron moves faster than proton.

- 13.** A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration due to viscous resistance of the air until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is work done by the gravitational force on the drop in the first half and second half of its journey? What is the work done by resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

Ans :

Given,

$$\text{Radius} = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

As we know that,

$$\begin{aligned}\text{Volume of drop} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \times (2 \times 10^{-3})^3 \\ &= 3.35 \times 10^{-8} \text{ m}^3\end{aligned}$$

Distance covered in the first half of its journey i.e.,

$$s = \frac{500}{2} = 250 \text{ m}$$

Density of water, $\rho = 10^3 \text{ kg m}^{-3}$

Work done by the gravitational force on the drop in the first half of its journey.

$$\begin{aligned}&= \text{volume} \times \text{density} \times g \times \text{distance} \\ &= \frac{4}{3} \pi r^3 \rho \cdot g \times 250 \\ &= \frac{4}{3} \times \frac{22}{7} \times (2 \times 10^{-3})^3 \times 10^3 \times 9.8 \times 250 \\ &= 0.082 \text{ J}\end{aligned}$$

Similarly, for the second half work done = 0.082 J.

Let initial velocity of the drop $u = 0$

And final velocity $v = 10 \text{ ms}^{-1}$

Hence, change in Kinetic energy

$$\begin{aligned}\Delta E_k &= \frac{1}{2} m v^2 - \frac{1}{2} m u^2 \\ &= \frac{1}{2} m v^2 - 0 = \frac{1}{2} V \rho v^2 \\ &= \frac{1}{2} \times 3.35 \times 10^{-8} \times 10^3 \times 10^2 \\ &= 0.001675 \text{ J}\end{aligned}$$

Work done by gravitational force,

$$\begin{aligned}W &= mgh \\ &= V \rho gh \\ &= 3.35 \times 10^{-1} \times 10^3 \times 9.8 \times 500 \\ &= 0.16415 \text{ J}\end{aligned}$$

Hence, Work done by resistive force in the entire journey.

$$\begin{aligned}&= \Delta E_k - W \\ &= 0.001675 - 0.16415 \\ &= -0.163 \text{ J}\end{aligned}$$

- 14.** A molecule in a gas container hits the wall with speed 200 ms^{-1} and angle 30° with the normal and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Ans :

Velocity of molecule along the normal to the wall.

$$= 200 \cos 30^\circ$$

CHAPTER 5

Work, Energy and Power

SUMMARY

1. WORK

When a body moves through a certain distance by application of forces, work is said to be done. In general,

$$W = \vec{F} \cdot \vec{S} = F S \cos \theta$$

Work can be positive, negative or zero depending upon whether $\theta < 90^\circ$, $\theta > 90^\circ$ or $\theta = 90^\circ$.

In S.I., the unit of work is joule.

$$1\text{J} = 10^7 \text{ erg.}$$

2. POWER

Power (P) is defined as rate of doing work.

$$P = \frac{W}{t}$$

Its S.I unit is watt.

Dimensional formula

$$P = \frac{W}{t} = \frac{FS}{t} = \frac{[MLT^{-2}][L]}{[T]}$$

$$= [ML^2 T^{-3}]$$

$$1\text{H.P.} = 746 \text{ W}$$

3. ENERGY

It is defined as the capacity to do work. In S.I. the unit of energy is joule.

The energy is classified in the following parts.

3.1 Kinetic Energy

It is defined as the energy possessed by a body by virtue of its motion and is measured as

$$\text{Kinetic energy} = \frac{1}{2}mv^2,$$

where m = mass of body and v = velocity of body. Kinetic energy is always positive and is a scalar quantity.

3.2 Potential Energy

The potential energy of a body is its capacity to do work by virtue of its position or configuration. Potential energy may be gravitational, elastic or electrostatic. It can be positive or negative.

Gravitational Potential Energy

It is the energy possessed by a body by virtue of its position in the gravitational field.

And gravitational Potential energy = mgh .

4. WORK-ENERGY THEOREM

It states that the work done on a body is equal to change in its mechanical energy.

i.e., Work done = Final Kinetic energy

– Initial Kinetic energy

$$W = \text{Increase in Kinetic energy}$$

If the applied force opposes the motion of the body, then

Work done, $W = \text{Decrease in Kinetic energy}$

Hence, In general,

$$W = \text{Change in Kinetic energy}$$

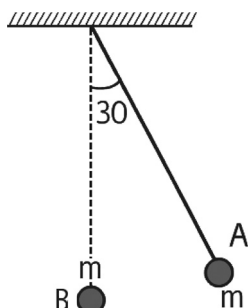
5. CONSERVATIVE FORCE

If the work done by a force in moving a particle between two points is independent of the path taken, the force is called a conservative force. Examples include gravitational force, Coulomb force, and magnetic force.

6. NON-CONSERVATIVE FORCE

If the work done by a force in moving a particle from one point to another depends on the path followed, the force is called a non-conservative force. An example of this is the force of friction.

17. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown. How does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



Ans :

Let velocity of A before collision = v

Velocity of B before collision = 0

Let v_1 and v_2 be the velocities of bob after collisions.

Using law of conservation of momentum and energy,

$$mv = mv_1 + mv_2$$

$$v = v_1 + v_2 \quad \dots(1)$$

$$\text{and } \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$v^2 = v_1^2 + v_2^2 \quad \dots(2)$$

Squaring (1) and subtracting (2),

$$0 = 2v_1v_2$$

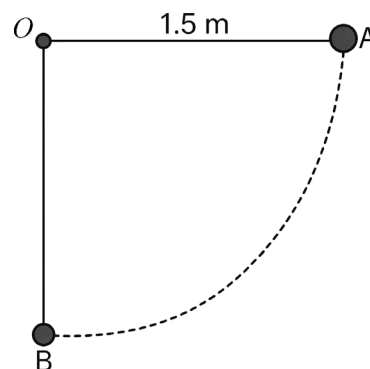
Either $v_1 = 0$ or $v_2 = 0$

But $v_2 \neq 0$,

Hence, $v_1 = 0$

Hence A stops and B moves with a velocity equal to that of A.

Ans :



At point A, total energy of bob =

$$\text{Potential energy} = mgh$$

$$= m \times 9.8 \times 1.5$$

$$= 14.7 \text{ m J}$$

$$\text{Energy lost} = \frac{14.7 \text{ m} \times 5}{100}$$

$$= 0.735 \text{ m J}$$

Hence Energy at point B = $14.7 \text{ m J} - 0.735 \text{ m J}$

$$= 13.96 \text{ m J}$$

At the lowest point, the energy of bob is only kinetic (Potential energy being zero)

$$\text{Hence, } \frac{1}{2}mv^2 = 13.96 \text{ m}$$

$$v_2 = 27.92$$

$$v = \sqrt{2792} = 5.3 \text{ ms}^{-1}$$

19. A trolley of mass 300 kg carrying a sand bag of 25 kg is moving uniformly with a speed of 27 km h^{-1} on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at rate of 0.05 kg s^{-1} . What is the speed of the trolley after the entire sand bag is empty?

Ans :

As there is no friction, the weight of the trolley, carrying a sand bag, acting downwards is exactly balanced by the normal reaction of the floor and thus there is no effect of the horizontal direction. Therefore, the speed of the trolley after the entire sand bag is empty will remain 27 km h^{-1} or there will be no change in speed.

20. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2 \text{ m}$?

Ans :

Given, $m = 0.5 \text{ kg}$

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18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipates 5% of its initial energy against air resistance?

force and viscous drag.

- (A) I, II, IV, III
- (B) II, I, IV, III
- (C) I, IV, II, III
- (D) IV, I, II, III

Ans :

- (A) I, II, IV, III

Initially, as the raindrop falls, its gravitational potential energy is converted into kinetic energy, increasing its speed (I). As the speed increases, the viscous drag force from the air also increases (II). Eventually, the sum of the upward buoyant force and the viscous drag force equals the downward gravitational force (IV), leading to zero net force. At this point, the raindrop stops accelerating and falls with a constant terminal velocity (III).

- 28.** A car of mass 1000 kg is moving at a speed of 20 m/s. The brakes are applied, and the car comes to a stop after traveling 50 m. What is the average resistive force exerted by the brakes?

- (A) 2000 N
- (B) 4000 N
- (C) 8000 N
- (D) 1000 N

Ans :

- (B) 4000 N

First, calculate the initial kinetic energy:

$$K_i = \frac{1}{2}mv^2$$

$$= 200000 \text{ J}$$

The final kinetic energy $K_f = 0$. By the work-energy theorem, work done by brakes

$$W = K_f - K_i = -200000 \text{ J}$$

Work done by resistive force F_r is

$$W = F_r \times d \times \cos 180^\circ = -F_r d$$

So, $-F_r \times 50 = -200000$, which gives $F_r = 4000 \text{ N}$

- 29.** What is kinetic energy?

- (A) Energy possessed by a body due to its position or configuration in a field.
- (B) Energy possessed by a body by virtue of its motion and mass.
- (C) The total energy stored within the chemical bonds of a substance.
- (D) The energy transferred between systems due to a temperature difference.

Ans :

- (B) Energy possessed by a body by virtue of its motion and mass.

Kinetic energy is explicitly defined as the energy an object possesses because it is in motion. This energy depends on the mass of the object and the square of its speed, as quantified by the formula $K = \frac{1}{2}mv^2$.

30. The work done by a variable force $F(x)$ in moving an object from an initial position x_i to a final position x_f is given by:

- (A) $F(x_f) - F(x_i)$
- (B) $(x_f - x_i) \times \text{average Force}$
- (C) $\int_{x_i}^{x_f} F(x) dx$
- (D) $\frac{F(x_f) + F(x_i)}{2} \times (x_f - x_i)$

Ans :

- (C) $\int_{x_i}^{x_f} F(x) dx$

For a variable force, the work done is calculated by integrating the force function $F(x)$ with respect to displacement x over the interval from the initial position x_i to the final position x_f . This represents the area under the force-displacement curve.

- 31.** What is the value of the electron volt (eV) in Joules (J)?

- (A) $1.6 \times 10^{-19} \text{ J}$
- (B) $6.022 \times 10^{23} \text{ J}$
- (C) $3.6 \times 10^6 \text{ J}$
- (D) 4.186J

Ans :

- (A) $1.6 \times 10^{-19} \text{ J}$

An electron volt (eV) is a unit of energy commonly used in atomic and particle physics. It is defined as the amount of kinetic energy gained (or lost) by a single electron accelerating from rest through an electric potential difference of one volt in vacuum.

- 32.** What is potential energy?

- (A) The energy of an object due to its movement through space.
- (B) The 'stored energy' by virtue of the position or configuration of a body.
- (C) The energy associated with the random motion of atoms and molecules.
- (D) The energy transferred from one object to another due to a force.

Ans :

- (B) The 'stored energy' by virtue of the position or configuration of a body.

Potential energy is defined as the energy that an object possesses due to its position relative to other

OBJECTIVE QUESTIONS

- Which of the following best defines work done by a constant force?
 - The product of force and the time over which it acts on an object.
 - The product of the component of force in the direction of displacement and the magnitude of displacement.
 - The product of the mass of an object and its acceleration.
 - The product of force and the velocity of an object in the direction of force.

Ans :

(B) The product of the component of force in the direction of displacement and the magnitude of displacement.

Work is defined as the scalar product of force and displacement. Specifically, it's the component of the force that acts along the direction of the object's displacement, multiplied by the magnitude of that displacement. This accounts for situations where the force is not perfectly aligned with the motion.

- What is the SI unit of work?
 - Watt (W)
 - Newton (N)
 - Joule (J)
 - Pascal (Pa)

Ans :

(C) Joule (J)

The SI unit of work is the joule (J), named after James Prescott Joule. One joule is defined as the work done when a force of one newton displaces an object by one meter in the direction of the force. Watts are units of power, Newtons are units of force, and Pascals are units of pressure.

- Under what condition is no work done by a force, even if the force is large and the object is displaced?
 - When the force is applied at an angle less than 90° to the displacement.
 - When the force and displacement are in the same direction.
 - When the force and displacement are mutually perpendicular.
 - When the force is applied at an angle greater than 90° to the displacement.

Ans :

(C) When the force and displacement are mutually perpendicular.

Work done is given by $W = Fd\cos\theta$, where θ is the angle between the force and displacement. If the force and displacement are mutually perpendicular, then $\theta = 90^\circ$, and $\cos 90^\circ = 0$. Therefore, the work done is zero, regardless of the magnitudes of the force and displacement.

- What form of energy is possessed by an object due to its motion?
 - Potential energy
 - Chemical energy
 - Kinetic energy
 - Thermal energy

Ans :

(C) Kinetic energy

Kinetic energy is the energy an object possesses due to its motion. It is directly proportional to the mass of the object and the square of its velocity. Potential energy is stored energy due to position, chemical energy is stored in bonds, and thermal energy relates to temperature.

- If a force of 10 N displaces an object by 2 m in the direction of the force, how much work is done?
 - 5 J
 - 10 J
 - 20 J
 - 12 J

Ans :

(C) 20 J

Work done (W) is calculated as the product of force (F) and displacement (d) when the force is in the direction of displacement, i.e., $W = F \times d$. In this case, $W = 10 \text{ N} \times 2 \text{ m} = 20 \text{ J}$.

- The work-energy theorem states that the change in kinetic energy of a particle is equal to what?
 - The net force acting on it.
 - The work done on it by the net force.
 - The impulse applied to it.
 - The power delivered to it.

Ans :

(B) The work done on it by the net force.

The work-energy theorem is a fundamental principle in physics that directly relates the work done by the net force on an object to the change in its kinetic energy. It provides a powerful tool for analyzing motion without directly using Newton's laws in all cases.

7. What is the kinetic energy of an object of mass 'm' moving with velocity 'v'?

(A) mv
 (B) mv^2
 (C) $\frac{1}{2}mv^2$
 (D) $2mv^2$

Ans :

(C) $\frac{1}{2}mv^2$

The kinetic energy (K) of an object is given by the formula $K = \frac{1}{2}mv^2$, where 'm' is the mass of the object and 'v' is its velocity. This formula shows that kinetic energy increases with mass and significantly with the square of the velocity.

8. Which of the following statements about work done is incorrect?

(A) Work done is a scalar quantity.
 (B) Work done can be positive or negative.
 (C) Work done is always equal to the force multiplied by distance.
 (D) Work is done when a force causes displacement.

Ans :

(C) Work done is always equal to the force multiplied by distance.

Work done is the product of force and distance only when the force is constant and acts exactly in the direction of displacement. More generally, work is the dot product of force and displacement, $W = \mathbf{F} \cdot \mathbf{d}$, or $W = Fd\cos\theta$, where θ is the angle between force and displacement.

9. Which of the following statements correctly describes the relationship between work and energy?

(A) Work and energy are unrelated concepts in physics.
 (B) Work is a form of energy.
 (C) Energy is the rate at which work is done.
 (D) Work done on an object can change its energy.

Ans :

(D) Work done on an object can change its energy. Work is the transfer of energy. When work is done on an object, its energy changes (e.g., its kinetic energy or potential energy can increase or decrease). While work itself is not a form of energy, it is the process by which energy is transferred or transformed.

10. A weightlifter holds a 100 kg barbell above his head at a constant height for 10 seconds. How much work is done on the barbell by the weightlifter during this time?

(A) 9800 J

(B) 980 J
 (C) 0 J
 (D) 1000 J

Ans :

(C) 0 J

Work is done when a force causes a displacement in the direction of the force. In this case, although the weightlifter is exerting an upward force to counteract gravity, the barbell is not moving (displacement is zero). Therefore, the work done on the barbell by the weightlifter is zero.

11. In which of the following scenarios is the work done by the specified force negative?

(A) Work done by gravity on a ball thrown upwards.
 (B) Work done by a person pushing a car in the direction of its motion.
 (C) Work done by tension in a string when a bob is at the lowest point of its swing.
 (D) Work done by an applied force lifting an object at constant velocity.

Ans :

(A) Work done by gravity on a ball thrown upwards. Work done is negative when the force (or a component of it) acts in the direction opposite to the displacement. When a ball is thrown upwards, gravity acts downwards while the displacement is upwards. Thus, the angle between the gravitational force and displacement is 180° , making the work done by gravity negative.

12. How does kinetic energy differ from potential energy?

(A) Kinetic energy is due to motion, while potential energy is stored energy due to position or configuration.
 (B) Kinetic energy is a vector quantity, while potential energy is a scalar quantity.
 (C) Kinetic energy can be negative, while potential energy is always positive.
 (D) Kinetic energy is measured in Watts, while potential energy is measured in Joules.

Ans :

(A) Kinetic energy is due to motion, while potential energy is stored energy due to position or configuration.

Kinetic energy is the energy associated with an object's movement. Potential energy, on the other hand, is energy stored within an object due to its position (e.g., gravitational potential energy) or its internal configuration (e.g., elastic potential

energy in a spring). Both are scalar quantities and measured in Joules.

- 13.** Assertion (A): The work done by a centripetal force in uniform circular motion is always zero.
Reason (R): In uniform circular motion, the centripetal force is always perpendicular to the instantaneous displacement of the particle.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

In uniform circular motion, the centripetal force acts towards the center of the circle, while the instantaneous displacement (or velocity) of the particle is tangential to the circle. Thus, the force and displacement are always perpendicular ($\theta = 90^\circ$), and work done ($W = Fd\cos 90^\circ$) is zero.

- 14.** Assertion (A): When a conservative force does positive work on a body, the potential energy of the body decreases.
Reason (R): Work done by a conservative force is equal to the negative of the change in potential energy.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The definition of potential energy change (ΔV) associated with a conservative force (F_c) is $\Delta V = -W_c$, where W_c is the work done by the conservative force. If W_c is positive, then ΔV is negative, meaning the potential energy decreases.

- 15.** Assertion (A): The rate of change of total momentum of a many-particle system is proportional to the sum of the internal forces on the system.
Reason (R): Internal forces in a system can change the total momentum of the system.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.

- (C) A is true but R is false.
(D) Both A and R are false.

Ans :

(D) Both A and R are false.

The rate of change of total momentum of a many-particle system is proportional to the external force acting on the system, not the sum of internal forces. Internal forces occur in action-reaction pairs and their vector sum within the system is zero, hence they cannot change the total momentum of the system.

- 16.** Consider the following statements regarding work:
I Work is done when a force produces displacement.
II Work done by friction is always negative.
III Work done by gravity on an object lifted upwards is positive.
IV The SI unit of work is Watt.
Which of the following combinations of statements is correct?
(A) I and II only
(B) I, II and III
(C) I only
(D) I and IV only

Ans :

(A) I and II only

Statement I is correct by definition. Statement II is generally true as friction opposes motion. Statement III is incorrect; work done by gravity when an object is lifted upwards is negative as force and displacement are opposite. Statement IV is incorrect; Watt is the unit of power, while Joule is the unit of work.

- 17.** Which of the following statements about kinetic energy are true?
I Kinetic energy is always a positive scalar quantity.
II Kinetic energy of an object depends on its direction of motion.
III If the speed of an object is doubled, its kinetic energy becomes four times.
IV The work-energy theorem relates work done to the change in potential energy.
(A) I and III only
(B) II and IV only
(C) I, III, and IV
(D) I, II, and III

Ans :

(A) I and III only

The speed increases by a factor of the square root of 2.

39. Which of the following is NOT an example of a conservative force?

(A) Gravitational force
(B) Electrostatic force
(C) Spring force
(D) Frictional force

Ans :

(D) Frictional force

A conservative force is one for which the work done in moving a particle between two points is independent of the path taken. Gravitational, electrostatic, and ideal spring forces are conservative. Frictional force is non-conservative because the work done by friction depends on the path length and always opposes motion, dissipating energy as heat.

40. What is the primary difference between an elastic collision and a completely inelastic collision?

(A) Momentum is conserved in elastic collisions but not in completely inelastic collisions.
(B) Kinetic energy is conserved in elastic collisions but not in completely inelastic collisions where objects stick together.
(C) Objects bounce off each other in completely inelastic collisions but stick together in elastic collisions.
(D) External forces are present in elastic collisions but absent in completely inelastic collisions.

Ans :

(B) Kinetic energy is conserved in elastic collisions but not in completely inelastic collisions where objects stick together.

The defining characteristic distinguishing elastic and completely inelastic collisions is the conservation of kinetic energy. In elastic collisions, kinetic energy is conserved. In completely inelastic collisions, the objects stick together after impact, and the maximum possible kinetic energy is lost (converted to other forms). Momentum is conserved in both types if the system is isolated.

41. A pump lifts 100 kg of water to a height of 20 m in 10 seconds. What is the power developed by the pump? (Take $g = 9.8 \text{ m/s}^2$)

(A) 196 W
(B) 1960 W
(C) 2000 W
(D) 200 W

Ans :

(B) 1960 W

Work done to lift the water is

$$\begin{aligned} W &= mgh \\ &= 100\text{kg} \times 9.8\text{m/s}^2 \times 20\text{m} \\ &= 19600\text{J} \end{aligned}$$

Power (P) is work done per unit time, so

$$\begin{aligned} P &= W/t \\ &= 19600\text{J}/10\text{s} \\ &= 1960\text{W} \end{aligned}$$

42. What is meant by the term 'power' in physics?

(A) The total amount of energy a system possesses.
(B) The force applied per unit area on an object.
(C) The time rate at which work is done or energy is transferred.
(D) The capacity of a body to perform work due to its motion.

Ans :

(C) The time rate at which work is done or energy is transferred.

Power is fundamentally defined as the rate at which work is performed or the rate at which energy is converted from one form to another or transferred from one system to another. Its SI unit is the Watt (Joules per second).

43. In a completely inelastic collision between two objects, what happens to them after the collision?

(A) They bounce off each other with their original speeds.
(B) They exchange their velocities.
(C) They stick together and move with a common velocity.
(D) They both come to rest immediately after the collision.

Ans :

(C) They stick together and move with a common velocity.

A defining characteristic of a completely inelastic collision is that the colliding objects adhere to each other after impact and subsequently move as a single combined mass with a common final velocity. Kinetic energy is not conserved in such collisions.

44. The law of conservation of linear momentum states that if no external forces act on a system, its total linear momentum:

(A) Increases at a constant rate.
(B) Decreases at a constant rate.

energy (s). (c) A stretched rubber band releasing an object converts elastic potential energy to kinetic energy (p). (d) Burning of coal converts chemical energy to thermal energy (q).

22. Which of the following pairs correctly matches a physical concept with its characteristic?

(A) Inelastic Collision : Kinetic energy is conserved.
 (B) Conservative Force : Work done in a closed loop is non-zero.
 (C) Power : Unit is Joule.
 (D) Work : Can be negative.

Ans :

(D) Work : Can be negative.

Work done can be positive, negative, or zero depending on the angle between force and displacement. In an inelastic collision, kinetic energy is not conserved. For a conservative force, work done in a closed loop is zero. The unit of power is Watt.

23. Identify the correct pair relating a type of energy and its definition.

(A) Gravitational Potential Energy : Energy due to motion under gravity.
 (B) Elastic Potential Energy : Energy stored in a body due to its deformation.
 (C) Kinetic Energy : Energy stored in chemical bonds.
 (D) Thermal Energy : Energy associated with an object's height above ground.

Ans :

(B) Elastic Potential Energy : Energy stored in a body due to its deformation.

Elastic potential energy is the energy stored in an elastic material (like a spring or rubber band) when it is stretched or compressed. Gravitational potential energy is due to height. Kinetic energy is due to motion. Thermal energy relates to temperature/heat.

24. Which of the following pairs incorrectly matches a term with its description?

(A) Scalar Product : Results in a scalar quantity.
 (B) Work done by friction : Generally opposes motion.
 (C) Watt : Unit of energy.
 (D) Conservative force : Work done is path independent.

Ans :

(C) Watt : Unit of energy.

Watt is the SI unit of power, which is the rate at which energy is transferred or work is done. The SI

unit of energy (and work) is the Joule. The other pairs are correctly matched descriptions.

25. Identify the incorrectly matched pair from the following:

(A) Collision : Transfer of momentum and energy.
 (B) Perfectly Inelastic Collision : Objects stick together after collision.
 (C) Elastic Collision : Only momentum is conserved.
 (D) Potential Energy of a spring : $\frac{1}{2}kx^2$

Ans :

(C) Elastic Collision : Only momentum is conserved. In an elastic collision, both momentum and kinetic energy are conserved. In all types of collisions, momentum is conserved if no external forces act. The other pairs accurately describe their respective concepts.

26. Arrange the following steps in the correct sequence to calculate the work done by a variable force $F(x)$ in moving an object from an initial position x_i to a final position x_f :

I Divide the total displacement into small infinitesimal displacements dx .
 II Integrate $F(x) dx$ from x_i to x_f .
 III Consider the force $F(x)$ to be approximately constant over each small displacement dx .
 IV Calculate the small work done $dW = F(x) dx$ for each small displacement.

(A) I, III, IV, II
 (B) III, I, IV, II
 (C) I, IV, III, II
 (D) IV, III, I, II

Ans :

(A) I, III, IV, II

To find the work done by a variable force, we first divide the path into infinitesimal displacements (I). Over each such small displacement, the force is assumed to be constant (III). Then, the small work done for each segment is calculated (IV). Finally, these small work contributions are summed up (integrated) over the entire path (II).

27. Consider the process of a raindrop falling from a height and eventually reaching terminal velocity. Arrange the following energy transformations and force balances in the likely sequence of occurrence:

I Gravitational potential energy converts to kinetic energy.
 II Viscous drag force increases with speed.
 III Net force becomes zero, and the drop falls with constant terminal velocity.
 IV Gravitational force balances the sum of buoyant

(I) Kinetic energy ($\frac{1}{2}mv^2$) is always positive as mass and v^2 are positive, and it's a scalar. (II) Kinetic energy depends on speed (magnitude of velocity), not direction. (III) Since $K \propto v^2$, if v doubles, K becomes $2^2 = 4$ times. (IV) The work-energy theorem relates work done to the change in kinetic energy.

18. Consider the following statements about potential energy:

I Potential energy is the energy possessed by a body by virtue of its position or configuration.
 II Gravitational potential energy increases when an object is lifted to a greater height.
 III The elastic potential energy of a spring is proportional to the square of its extension or compression.
 IV Potential energy is a vector quantity.
 Identify the correct statements.

- (A) I, II, and III only
 (B) II and IV only
 (C) I and III only
 (D) All statements are correct.

Ans :

(A) I, II, and III only
 Statement I correctly defines potential energy. Statement II is true for gravitational potential energy (mgh). Statement III correctly describes the elastic potential energy of a spring ($\frac{1}{2}kx^2$). Statement IV is incorrect; potential energy is a scalar quantity, representing stored energy.

19. Match the physical quantities in Column I with their corresponding SI units in Column II.

	Column I		Column II
a.	Work	p.	Newton
b.	Kinetic Energy	q.	Watt
c.	Power	r.	Joule
d.	Force	s.	Pascal

- (A) a-r, b-r, c-q, d-p
 (B) a-p, b-q, c-r, d-s
 (C) a-r, b-s, c-p, d-q
 (D) a-q, b-r, c-s, d-p

Ans :

(A) a-r, b-r, c-q, d-p
 Work and Kinetic Energy are both forms of energy and are measured in Joules (r). Power is the rate of doing work and is measured in Watts (q). Force is measured in Newtons (p). Pascal is the unit of pressure.

20. Match the concepts in Column I with their appropriate descriptions in Column II.

	Column I		Column II
a.	Work-Energy Theorem	p.	Energy stored due to position
b.	Conservative Force	q.	Work done is independent of path
c.	Potential Energy	r.	Rate of doing work
d.	Power	s.	Change in kinetic energy equals net work done

- (A) a-s, b-q, c-p, d-r
 (B) a-p, b-r, c-q, d-s
 (C) a-s, b-p, c-q, d-r
 (D) a-r, b-q, c-s, d-p

Ans :

(A) a-s, b-q, c-p, d-r
 The Work-Energy Theorem states that the change in kinetic energy equals the net work done (s). A Conservative Force is one for which the work done is independent of the path taken (q). Potential Energy is energy stored due to an object's position or configuration (p). Power is the rate at which work is done (r).

21. Match the type of energy transformation in Column I with the example in Column II.

	Column I		Column II
a.	Kinetic to Potential	p.	A stretched rubber band releasing an object
b.	Potential to Kinetic	q.	Burning of coal to produce heat
c.	Elastic Potential to Kinetic	r.	A ball thrown upwards reaching its highest point
d.	Chemical to Thermal	s.	Water falling from a dam turning a turbine

- (A) a-r, b-s, c-p, d-q
 (B) a-s, b-p, c-r, d-q
 (C) a-r, b-p, c-s, d-q
 (D) a-p, b-s, c-q, d-r

Ans :

(A) a-r, b-s, c-p, d-q
 A ball thrown upwards converts kinetic energy to potential energy as it rises (r). (b) Water falling from a dam converts potential energy to kinetic

objects, stresses within itself, its electric charge, or other factors. It represents the potential to do work.

- 33.** A force $F = (3\hat{i} + 4\hat{j})$ N acts on a particle which undergoes a displacement $d = (2\hat{i} - \hat{j})$ m. Calculate the work done by the force.

(A) 2 J
(B) 10 J
(C) 6 J
(D) 14 J

Ans :

(A) 2 J

Work done (W) is the dot product of force ($\mathbf{F}$) and displacement ($\mathbf{d}$):

$$W = \mathbf{F} \cdot \mathbf{d} = (F_x d_x) + (F_y d_y)$$

$$\begin{aligned} \text{So, } W &= (3 \times 2) + (4 \times -1) \\ &= 6 - 4 \\ &= 2 \text{ J} \end{aligned}$$

- 34.** The principle of conservation of mechanical energy states that the total mechanical energy of a system is conserved if:

(A) Only non-conservative forces are doing work on it.
(B) The net external force acting on the system is zero.
(C) The forces, doing work on it, are conservative.
(D) The system is in thermal equilibrium with its surroundings.

Ans :

(C) The forces, doing work on it, are conservative. The principle of conservation of mechanical energy holds true when all forces doing work within the system are conservative forces. This means that no energy is lost to dissipative forces like friction or air resistance.

- 35.** What is the potential energy stored in a spring with spring constant 'k' when it is compressed or extended by a distance 'x' from its equilibrium position?

(A) kx
(B) kx^2
(C) $\frac{1}{2}kx^2$
(D) $\frac{1}{2}kx$

Ans :

(C) $\frac{1}{2}kx^2$

The elastic potential energy stored in a spring is given by the formula $V(x) = \frac{1}{2}kx^2$, where 'k' is the spring constant (a measure of the spring's stiffness)

and 'x' is the displacement from its equilibrium (unstretched or uncompressed) position.

- 36.** Which of the following statements regarding power is incorrect?

(A) Power is the rate at which work is done.
(B) The SI unit of power is the Watt.
(C) Power is a vector quantity.
(D) Power can be calculated as the dot product of force and velocity.

Ans :

(C) Power is a vector quantity.

Power is defined as the rate of doing work or transferring energy, $P = dW/dt = \mathbf{F} \cdot \mathbf{v}$. Since work and time are scalars, power is also a scalar quantity, not a vector. It has magnitude but no direction.

- 37.** Which statement accurately describes an elastic collision?

(A) Both kinetic energy and momentum are conserved.
(B) Only momentum is conserved, kinetic energy is lost.
(C) Only kinetic energy is conserved, momentum is lost.
(D) The objects stick together after the collision.

Ans :

(A) Both kinetic energy and momentum are conserved.

An elastic collision is defined as a collision in which the total kinetic energy of the system is conserved (i.e., no kinetic energy is converted into other forms like heat or sound). Linear momentum is also conserved in all types of collisions, provided no external forces act.

- 38.** A ball is dropped from a height 'h'. Just before hitting the ground, its speed is 'v'. If the same ball is dropped from a height '2h', what will be its speed just before hitting the ground (neglecting air resistance)?

(A) v
(B) $2v$
(C) $\sqrt{2}v$
(D) $4v$

Ans :

(C) $\sqrt{2}v$

Using conservation of mechanical energy, $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$.

If the height is doubled to 2h, the new speed

$$\begin{aligned} v' &= \sqrt{2g(2h)} \\ &= \sqrt{2}v \end{aligned}$$

$$v = ax^{3/2} \text{ and } a = 5 \text{ m}^{-1/2} \text{ s}^{-1},$$

$$x = 2 - 0 = 2 \text{ m}$$

Hence, work done,

$$\begin{aligned} W &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2}m[v^2 - 0] \\ &= \frac{1}{2}m[ax^{3/2}]^2 \\ &= \frac{1}{2} \times 0.5 \times [5 \times 2^{3/2}]^2 \\ &= \frac{0.5}{2} \times 25 \times 8 = 50 \text{ J} \end{aligned}$$

21. The blades of a windmill sweep out a circle of area A.

- If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ?
- What is the kinetic energy of the air ?
- Assume that the windmill converts 25% of the wind energy into electrical energy and that $A = 30 \text{ m}^2$, $v = 36 \text{ km/h}$ and the density of air is 1.2 kg m^{-3} . What is the electrical power produced ?

Ans :

$$\begin{aligned} \text{(a) Volume of air, } V &= \frac{\text{mass}(m)}{\text{density}(\rho)} \\ m &= V\rho = \text{Area} \times \text{distance} \times \rho \\ &= A \times (\text{velocity} \times \text{time}) \times \rho \\ m &= Avt\rho = \rho Avt. \end{aligned}$$

- (b) Kinetic energy of air

$$= \frac{mv^2}{2} = \frac{\rho Avt.v^2}{2} = \frac{\rho Av^3 t}{2}$$

- (c) Given, $A = 30 \text{ m}^2$,

$$v = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1},$$

$$\rho = 1.2 \text{ kg m}^{-3}$$

$$\begin{aligned} \text{Wind power, } P_1 &= \frac{W}{t} = \frac{K.E.}{t} = \frac{\rho Av^3 t}{2t} \\ &= \frac{1.2 \times 30 \times 10^3}{2} \text{ W} \\ &= 18000 \text{ W} \end{aligned}$$

Wind power converted into electrical power

$$\begin{aligned} &= 18000 \times \frac{25}{100} = 4500 \text{ W} \\ &= 4.5 \text{ kW}. \end{aligned}$$

22. A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated.

- How much work does she do against the gravitational force ?
- Fat supplies $3.8 \times 10^7 \text{ J}$ of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?

Ans :

- (a) Total work done by the dieter,

$$\begin{aligned} W &= (mgh)1000 \\ &= 10 \times 9.8 \times \frac{5}{10} \times 1000 \\ &= 49000 \text{ J} \end{aligned}$$

- (b) Mass of fats used up by the dieter having efficiency 20%.

$$\begin{aligned} &= \frac{49000}{3.8 \times 10^7} \times \frac{100}{20} \\ &= \frac{24.5 \times 10^{-3}}{3.8} = 6.45 \times 10^{-3} \text{ kg}. \end{aligned}$$

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23. A family uses 8 kW of power

- Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW?
- Compare this area to that of the roof of a typical house.

Ans :

- (a) Given, power needed,

$$P = 8 \text{ kW} = 8000 \text{ W}$$

$$\text{Energy incident per square metre} = 200 \text{ Wm}^{-2}$$

$$\text{Useful energy} = 200 \times \frac{20}{100} = 40 \text{ Wm}^{-2}$$

$$\text{Since } 40 \text{ W energy is provided by area} = 1 \text{ m}^2$$

So 8000 W energy is provided by area

$$= \frac{1}{40} \times 8000 = 200 \text{ m}^2$$

- (b) This area is comparable to the roof of a large house of dimension $14 \text{ m} \times 14 \text{ m}$.

- (C) Remains constant.
 (D) Becomes zero after some time.

Ans :

(C) Remains constant.

The law of conservation of linear momentum is a fundamental principle stating that in an isolated system (a system not acted upon by external forces), the total linear momentum of the system remains unchanged over time, regardless of interactions between particles within the system.

45. What is the relationship between the force F , velocity v , and instantaneous power P ?
- (A) $P = F/v$
 (B) $P = Fv$
 (C) $P = Fv^2$
 (D) $P = F^2v$

Ans :

(B) $P = Fv$ (when force and velocity are in the same direction)

Instantaneous power can be expressed as the dot product of the force vector and the velocity vector, $P = \mathbf{F} \cdot \mathbf{v}$. If the force and velocity are in the same direction, this simplifies to $P = Fv$. This formula is derived from $P = dW/dt$ and $dW = \mathbf{F} \cdot d\mathbf{r}$.

46. If the kinetic energy of a body is 100 J and its mass is 2 kg, what is its speed?
- (A) 5 m/s
 (B) 10 m/s
 (C) 20 m/s
 (D) 50 m/s

Ans :

(B) 10 m/s

Kinetic energy $K = \frac{1}{2}mv^2$.

Given $K = 100$ J and $m = 2$ kg.

So, $100 = \frac{1}{2} \times 2 \times v^2$

which simplifies to $100 = v^2$.

Therefore,

$$v = \sqrt{100} = 10 \text{ m/s}$$

47. When is the work done by a force considered positive?
- (A) When the force acts perpendicular to the displacement.
 (B) When the force has a component in the direction opposite to the displacement.
 (C) When the force has a component in the same direction as the displacement.

- (D) When there is no displacement, but the force is large.

Ans :

(C) When the force has a component in the same direction as the displacement.

Work done is positive when the angle θ between the force vector and the displacement vector is between 0° (inclusive) and 90° (exclusive), meaning $\cos\theta$ is positive. This occurs when the force, or at least a component of it, acts in the direction of motion.

48. What condition must be met for the total mechanical energy of a system to be conserved?
- (A) The system must be at rest.
 (B) All forces acting on the system must be external.
 (C) All forces doing work within the system must be conservative.
 (D) The kinetic energy of the system must be zero.

Ans :

(C) All forces doing work within the system must be conservative.

The conservation of total mechanical energy (sum of kinetic and potential energies) applies when the work done by non-conservative forces (like friction or air resistance) is zero. This means all forces performing work within the system are conservative forces.

49. Which statement concerning collisions is incorrect?
- (A) In an elastic collision, kinetic energy is conserved.
 (B) In all collisions (elastic or inelastic), linear momentum is conserved if no net external force acts.
 (C) In a perfectly inelastic collision, the objects stick together after impact.
 (D) In an inelastic collision, momentum is lost but kinetic energy is conserved.

Ans :

(D) In an inelastic collision, momentum is lost but kinetic energy is conserved.

In an inelastic collision, linear momentum is conserved (if the system is isolated), but kinetic energy is not conserved; some kinetic energy is transformed into other forms of energy like heat or sound. The statement incorrectly claims momentum is lost and kinetic energy is conserved.

50. Which of the following correctly describes a situation where gravitational potential energy is converted into kinetic energy?
- (A) A spring being compressed.

82. Can a body have energy without momentum ?

Ans :

Yes, the internal energy of a body is due to random motion of the particles of the body, while the vector sum of the momenta of the moving particles may be zero.

83. Is linear momentum of a system always conserved ?

Ans :

No, the linear momentum is conserved only when the system is isolated.

84. A cake of mud is thrown on a wall where it sticks. What happens to its initial kinetic energy ?

Ans :

A part of the kinetic energy is used in deforming the cake and the remaining part is converted into heat and sound energy.

85. A body is moving at constant speed over a frictionless surface. What is the work done by the weight ?

Ans :

Zero, because the angle between force (weight) and displacement is 90° .

So, $W = FS \cos 90^\circ = 0$

86. Name two situations where variable force acts.

Ans :

- (i) Gravitational force between two bodies as distance of two varies.
- (ii) Restoring force in a stretched string.

87. Where from the kinetic energy of falling rain drops come ?

Ans :

From the gravitational potential energy of water molecules in clouds.

88. Name the physical quantity which is expressed as product of force and velocity. Is it a scalar or vector quantity ?

Ans :

$\vec{F}, \vec{v}$ is called power. It is a scalar quantity.

89. What is represented by area under the force displacement curve ?

Ans :

It represents the work done by the force over the given displacement.

90. What is spring constant ? What are its SI units ?

Ans :

Spring constant is the restoring force developed in the spring per unit elongation/compression. SI units of spring constant are Nm^{-1} .

91. When an air bubble rises in water, what happens to its potential energy ?

Ans :

Its potential energy decreases as work is done by upthrust on the bubble.

92. Can you associate potential energy with a non-conservative force ?

Ans :

No, potential energy can be associated only with conservative force.

93. Is it possible to have a collision in which the whole of kinetic energy is lost ?

Ans :

Yes, in perfectly elastic collision of two bodies moving towards each other with equal linear momenta.

94. Can a body have momentum without energy ?

Ans :

No, if a body has momentum, it must be in motion and consequently possess kinetic energy.

95. Define work.

Ans :

Work is said to be done if a force acting upon a body displaces it through a certain distance in the direction of force.

Work is measured by the product of force and distance through which the body moves in the direction of force.

96. Can acceleration be produced without doing any work?

Ans :

Yes, When a body is moving in circular path, it moves under uniform acceleration, yet it is not doing any work

97. Two protons are brought together, how will Potential energy of the system alter ?.

Ans :

Potential energy of system will increase since work

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56. What type of quantity is power?

- (A) Scalar quantity
- (B) Vector quantity
- (C) Tensor quantity
- (D) Dimensionless quantity

Ans :

(A) Scalar quantity

Power, defined as the rate of doing work ($P = W/t$) or the rate of energy transfer, is a scalar quantity. Although force and velocity (used in $P = \mathbf{F} \cdot \mathbf{v}$) are vectors, their dot product results in a scalar. Power has magnitude but no direction.

57. In a collision, if the two colliding bodies stick together after the impact, the collision is said to be:

- (A) Perfectly elastic
- (B) Partially elastic
- (C) Perfectly inelastic
- (D) Partially inelastic

Ans :

(C) Perfectly inelastic

A perfectly inelastic collision is one in which the colliding objects stick together after the collision and move with a common final velocity. In such collisions, the maximum possible amount of kinetic energy is converted into other forms of energy.

58. What is the SI unit of power?

- (A) Joule (J)
- (B) Newton (N)
- (C) Watt (W)
- (D) Pascal (Pa)

Ans :

(C) Watt (W)

The SI unit of power is the watt (W), named after James Watt. One watt is defined as the rate of energy transfer of one joule per second. Joules are units of energy/work, Newtons are units of force, and Pascals are units of pressure.

59. A body of mass 5 kg is lifted to a height of 10 m. What is the potential energy gained by the body? (Take $g = 9.8 \text{ m/s}^2$)

- (A) 49 J
- (B) 50 J
- (C) 490 J
- (D) 500 J

Ans :

(C) 490 J

Gravitational potential energy (PE) is given by $PE = mgh$, where m is mass, g is acceleration due to gravity, and h is height.

So,

$$PE = 5\text{kg} \times 9.8\text{m/s}^2 \times 10\text{m} \\ = 490\text{J}$$

60. For an elastic collision, which of the following quantities is/are conserved (assuming an isolated system)?

- (A) Only kinetic energy
- (B) Only linear momentum
- (C) Both kinetic energy and linear momentum
- (D) Neither kinetic energy nor linear momentum

Ans :

(C) Both kinetic energy and linear momentum

In an elastic collision occurring within an isolated system (no net external forces), both the total linear momentum and the total kinetic energy of the system are conserved. This distinguishes elastic collisions from inelastic collisions, where kinetic energy is not conserved.

61. The work done by a force is given by the dot product of force $\vec{F}$ and displacement $\vec{S}$. If the work done is zero, then :

- (a) $\vec{F}$ and $\vec{S}$ act in the same direction.
- (b) $\vec{F}$ and $\vec{S}$ act in the opposite direction.
- (c) $\vec{F}$ and $\vec{S}$ are parallel to each other.
- (d) $\vec{F}$ and $\vec{S}$ are at right angles.

Ans :

(d) $\vec{F}$ and $\vec{S}$ are at right angles.

$$W = FS \cos \theta$$

Here, $W = 0$, so $\cos \theta = 0$ or $\theta = 90^\circ$

i.e. $\vec{F}$ and $\vec{S}$ are at right angles.

62. A particle moves in the $y-z$ plane under the influence of a force. The rectangular components of the momentum of the particle at time t are as under :

$$p_x = 1.2345 \sin t \text{ and } p_y = 1.2345 \cos t$$

the angle between the applied force and the momentum at time t is :

- (a) 0°
- (b) 30°
- (c) 60°
- (d) 90°

Ans :

(d) 90°

The magnitude of total momentum is independent of time, hence the force vector perpendicular to the velocity vector i.e. $\theta = 90^\circ$

63. The kinetic energy of a body becomes 4 times its initial value. The new momentum will be :
- same as initial value.
 - four times the initial value.
 - twice the initial value.
 - eight times the initial value.

Ans :

(c) twice the initial value.

$p = \sqrt{2mE}$, when E is made 4E, p becomes 2p.

64. A bullet of mass M hits a block of mass M'. The transfer of energy is maximum, when :
- M' = M
 - M' = 2M
 - M < M'
 - M' > M

Ans :

(a) M' = M

The transfer of energy is maximum when M' = M, because then velocity of bullet becomes zero and block acquires the velocity equal to velocity of bullet.

65. One coolie takes two minutes to raise a box from platform to train. Another coolie takes one minute, work done
- by the first is more
 - by the first is less
 - is same by both
 - becomes $\frac{1}{4}$ in the first.

Ans :

(c) is same by both

Work done is independent of time, hence is same by both.

66. The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of constant force is :
- directly proportional to $\sqrt{m}$
 - directly proportional to m
 - independent of m.
 - directly proportional to $1/\sqrt{m}$

Ans :

(c) independent of m.

Kinetic energy is the energy stored by the work done by a force ($W = K.E. = F.S.$), so it is independent of mass.

67. In a gravitational field, the work done in transporting mass from one point to another depends on :
- end positions
 - distance between them
 - actual path of motion
 - velocity of transport

Ans :

(a) end positions

As gravitational forces are conservative forces, so the work done is independent of the path and depends only on initial and final positions (i.e. end positions).

68. A lorry and a car moving with the same Kinetic energy are brought to rest by the application of brakes which provide equal retarding forces. Which of them will come to rest in a shorter distance?
- Lorry
 - Car
 - Both will come to rest after covering equal distances
 - It depends on their velocities.

Ans :

(c) Both will come to rest after covering equal distances

Applying work-energy theorem ($W = FS$) is same in both case. As F is same, so S will also be same i.e., both will come to rest after covering equal distances.

69. When Kinetic energy of a body is increased by 300%, the momentum of the body is increased by
- 20%
 - 50%
 - 100%
 - 200%

Ans :

(c) 100%

$$E' = E + \Delta E = E + 3E = 4E$$

$$\frac{p'}{p} = \sqrt{\frac{2mE'}{2mE}} = \sqrt{\frac{4E}{E}} = 2 \text{ or } p' = 2p$$

$$\text{Hence, } \frac{\Delta p}{p} \% = \frac{p' - p}{p} \times 100 = \frac{2p - p}{p} \times 100 = 100\%$$

70. Two solid rubber balls A and B having masses 0.2 and 0.4 are moving in opposite directions with velocity of A equal to 0.3 ms^{-1} . After collision, the two balls come to rest when the velocity of B is :
- 0.15 ms^{-1}
 - 1.5 ms^{-1}
 - -0.15 ms^{-1}
 - zero

Ans :

(c) -0.15 ms^{-1}

From law of conservations of linear momentum

$$m_1 v_1 + m_2 v_2 = mv$$

$$0.2 \times 0.3 + 0.4 + v = 0.6 \times 0$$

$$v = -0.15 \text{ ms}^{-1}$$

71. If a stone is thrown up vertically and return to ground, its potential energy is maximum.

- (a) during the upward journey
(b) at the maximum height
(c) during the return journey
(d) at the bottom

Ans :

- (b) at the maximum height

Because Potential energy = mgh and is maximum at maximum height.

72. A ball is dropped from a height of 20 m and rebounds to a height of 10 m. The loss of energy is :

- (a) 5% (b) 25%
(c) 50% (d) 75%

Ans :

- (c) 50%

$$\text{Loss of energy} = \frac{mg(h - h')}{mgh} \times 100$$

$$= \left(1 - \frac{h'}{h}\right) 100 = \left(1 - \frac{10}{20}\right) 100$$

$$= 50\%$$

73. Two masses of 1g and 9g are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momentum is :

- (a) 1 : 9 (b) 9 : 1
(c) 3 : 1 (d) 1 : 3

Ans :

- (d) 1 : 3

Since,

$$p = \sqrt{2mE}$$

Hence,

$$\frac{p_1}{p_2} = \sqrt{\frac{1}{9}} = \frac{1}{3} = 1:3$$

ONE MARK QUESTIONS

74. What is the work done in holding a 15 kg suitcase while waiting for a bus for 15 minutes?

Ans :

Zero, because $W = FS \cos \theta = F \cdot 0 \cdot \cos \theta = 0$

75. What do you mean by work done by a force?

Ans :

Work is said to be done when a force applied on a body displaces the body through certain distance in the direction of force.

76. What is the work done by tension in the string of a simple pendulum?

Ans :

Zero, because, $W = F S \cos 90^\circ = 0$.

77. What is SI unit of work?

Ans :

joule (J)

78. What is the work done by centripetal force ?

Ans :

Zero, because centripetal force is always perpendicular to the displacement.

79. What is the amount of work done by sun's gravitational force if earth is rotating in almost circular orbit?

Ans :

Zero, because here the gravitational force is centripetal force and hence perpendicular to the path of earth.

80. A body is moving at constant speed over a frictionless surface. What is the work done by the weight?

Ans :

Since the force and displacement are at right angle to each other, the work done by the weight is zero.

81. What sort of energy is associated with a bird flying in air ?

Ans :

Since the bird is flying with certain velocity above the ground so it possesses both potential energy and kinetic energy.

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114. With what type of forces, potential energy is associated?

Ans :

Conservative forces.

TWO MARKS QUESTIONS

115. Can Kinetic energy of a system be increased or decreased without the application of external force?

Ans :

Yes, in some cases the work is done by the internal forces and hence the Kinetic energy of the system increases e.g. in case of explosion of a bomb, the Kinetic energy of the system increases.

116. Can overall energy of a body be negative ?

Ans :

Yes, since total energy $E = KE + PE$, if Potential energy is negative having magnitude more than that of Kinetic energy, E will be negative. For example, the energy possessed by an electron of an atom is negative.

117. Since work is a scalar quantity, can it have negative value ?

Ans :

Yes, it can be negative. Work is said to be negative if the direction of motion of body is opposite to the direction of applied force.

118. What work is done on a body in maintaining its circular path ?

Ans :

Zero, since angle between force of attraction (acting towards the centre) and direction of motion (always tangential to the circle) is 90°

Hence, $W = F S \cos 90^\circ = 0$.

119. An arrow is shot from a bow, from where does it get kinetic energy ?

Ans :

In order to stretch a bow, work has to be done, which is stored in the form of elastic potential energy and on releasing the arrow, this stored potential energy is converted into the kinetic energy and it goes forward to hit the target.

120. A person is trying to push a box, the whole day he makes use of his stored energy but cannot move it. In physics we say he has not done any work. Explain, why ?

Ans :

In physics he does work only when he can bring about motion in the body because $\text{Work} = \text{Force} \times \text{distance}$. In case no motion is produced, whatever the force applied, no work is done.

121. An electron is moving towards a proton. Is the work done positive or negative ?

Ans :

In this case positive work is done by the force of attraction. Similarly, when a body is allowed to move down under gravity, positive work is done by gravity on the body.

122. Can a body have energy without momentum or vice versa ? Explain.

Ans :

Yes, a body can have energy without momentum. And the energy possessed by the body is potential energy because it is due to position or configuration of the body e.g. potential energy possessed by a body lying at rest in a gravitational field. But a body cannot have momentum without energy because if a body has momentum, it always possesses kinetic energy.

123. What are the conditions so that transfer of kinetic energy is maximum during collision ?

Ans :

- (i) Collision should be head-on.
- (ii) The target should be at rest.
- (iii) Collision should be perfectly elastic.
- (iv) The mass of incident particle should be equal to the target.

124. A light and a heavy body have same linear momentum. Which one has greater kinetic energy ?

Ans :

Given,

$$p_1 = p_2$$

$$p_1^2 = p_2^2$$

$$2m_1 E_1 = 2m_2 E_2 \quad \left[\text{since, } E = \frac{p^2}{2m} \right]$$

$$\frac{E_2}{E_1} = \frac{m_1}{m_2}$$

has to be done to bring the two protons closer against the force of repulsion.

- 98.** An electron is brought near the proton flow will its Potential energy change ?

Ans :

It will decrease. Work will be done by the force of attraction between them.

- 99.** What is kinetic energy ?

Ans :

Energy possessed by a body due to its motion is called kinetic energy.

- 100.** What is potential energy ?

Ans :

Energy possessed by a body due to its position or configuration is called potential energy.

- 101.** What is the work done by gravitational force in taking a mass m to a height h with uniform motion?

Ans :

$$W = - mgh,$$

because the direction of gravitational force and displacement are opposite.

- 102.** A man is rowing upstream and is at rest w.r.t. shore. Is he doing work w.r.t. shore ?

Ans :

No. Because he is at rest w.r.t. shore, so work done

$$W = mg(0) = 0$$

- 103.** What is a collision ?

Ans :

Collision between two particles is said to have occurred, when they actually strike against each other or path of motion of the particle or body is affected by the other.

- 104.** What is conservative force ?

Ans :

A force is said to be conservative if the work done by or against the force moving a body depends upon only the initial and final positions of the body and not on the path followed.

Gravitational, electrical and magnetic forces are conservative forces.

- 105.** What is non-conservative force ?

Ans :

A force is said to be non-conservative if the work done by or against the force in moving a body from one position to another depends upon the path followed.

Friction is a non-conservative force.

- 106.** Can a body have momentum without energy ?

Ans :

No, because for momentum, it must have some velocity and hence energy.

- 107.** With what type of forces, potential energy is associated?

Ans :

Conservative forces.

- 108.** Give some examples of bodies possessing Potential energy other than gravitational Potential energy.

Ans :

- (i) Compressed or wound spring
- (ii) Stretched bow
- (iv) Gas compressed in a cylinder.

- 109.** Give one example of dot product of vectors. Also give its unit.

Ans :

Work done $W = \vec{F} \cdot \vec{S}$. Work is measured in joule.

- 110.** Name the physical quantity which is expressed as force time velocity. Is it a scalar or vector quantity ?

Ans :

$\vec{F} \cdot \vec{v}$ is power. It is a scalar quantity.

- 111.** What is the relation between joule and kg m?

Ans :

$$1 \text{ kg m} = 9.8 \text{ J.}$$

- 112.** Is work scalar or vector quantity ?

Ans :

Scalar quantity.

- 113.** Does the work done in raising a box on a platform depend upon how fast it is raised up ? If not, why ?

Ans :

No. Because work is independent of path followed.

- 135.** Explain the condition for positive and negative work done by a force.

Ans :

Work done by a force is positive if the angle θ between the force and displacement is between 0° and 90° (e.g., lifting an object). Work done is negative if θ is between 90° and 180° (e.g., frictional force opposing motion).

- 136.** If the kinetic energy of a body increases, what can be inferred about the net work done on it?

Ans :

According to the work-energy theorem ($K_f - K_i = W$), if the kinetic energy of a body increases ($K_f > K_i$), the change in kinetic energy (ΔK) is positive. This implies that the net work done (W) on the body is positive.

- 137.** What is gravitational potential energy near the Earth's surface? On what factors does it depend?

Ans :

Gravitational potential energy $V(h)$ of an object near the Earth's surface is given by mgh , where m is the mass, g is acceleration due to gravity, and h is the height from a reference level. It depends on the object's mass and its height.

- 138.** What is a conservative force? Give two examples.

Ans :

A conservative force is a force where the work done by it in moving a particle between two points is independent of the path taken. Examples include gravitational force and the spring force. The work done by such a force in a closed path is zero.

- 139.** How does the potential energy of a spring change when it is compressed or stretched?

Ans :

The potential energy $V(x)$ of a spring, when compressed or stretched by a displacement x from its equilibrium position, is given by $V(x) = \frac{1}{2}kx^2$, where k is the spring constant. The energy increases quadratically with displacement.

- 140.** What is meant by a completely inelastic collision? What happens to the colliding bodies?

Ans :

A completely inelastic collision is one in which the two particles move together after the collision, meaning they have the same final velocity. While linear momentum is conserved, kinetic energy is not, with some typically lost as heat or sound.

- 141.** Explain the concept of 'projection of a vector' in the context of scalar product.

Ans :

In the scalar product $A \cdot B = AB \cos \theta$, the term $B \cos \theta$ represents the projection (or component) of vector B along the direction of vector A . Similarly, $A \cos \theta$ is the projection of A along B .

- 142.** A weightlifter holds a 150 kg mass steadily on his shoulder for 30 s. How much work is done on the load?

Ans :

No work is done on the load by the weightlifter during the time it is held steady. This is because, for work to be done, there must be a displacement in the direction of the applied force, and here the displacement of the load is zero.

- 143.** How is the work done by friction generally characterized in terms of energy change?

Ans :

Work done by friction is generally negative as the frictional force opposes the displacement. This negative work results in a decrease in the kinetic energy of the moving body, often converting it into heat.

- 144.** If an object's speed is constant, can its kinetic energy change? Explain.

Ans :

No, if an object's speed (v) is constant and its mass (m) is constant, its kinetic energy ($K = \frac{1}{2}mv^2$) will also remain constant. A change in kinetic energy requires a change in speed, assuming mass is unchanged.

- 145.** Explain how the area under a force versus displacement graph represents work done.

Ans :

For a variable force, the work done over a small displacement Δx is approximately $F(x) \Delta x$, which is the area of a small rectangle on the graph. Summing these areas for infinitesimal Δx gives the total work, which corresponds to the definite integral $\int F(x) dx$, or the total area under the $F(x)$ vs x curve.

- 146.** What is the significance of the negative sign in the relation $F(x) = -dV/dx$?

Ans :

The negative sign in $F(x) = -dV/dx$ indicates that the conservative force $F(x)$ acts in the direction

i.e. Kinetic energy is inversely proportional to mass
i.e. Kinetic energy of lighter body is more than heavier body.

- 125.** Explain the scalar product of two vectors A and B. How is its magnitude determined?

Ans :

The scalar product (or dot product) of two vectors A and B, denoted $A \cdot B$, is a scalar quantity. Its magnitude is defined by $A \cdot B = AB \cos \theta$, where A and B are the magnitudes of the vectors and θ is the angle between them.

- 126.** What is the work-energy theorem? Explain its significance in relating work and kinetic energy.

Ans :

The work-energy theorem states that the change in kinetic energy of a particle is equal to the work done on it by the net force. This means if positive work is done, kinetic energy increases, and if negative work is done, kinetic energy decreases, directly linking energy transfer and motion.

- 127.** Define kinetic energy. On what factors does it depend?

Ans :

Kinetic energy (K) is the energy possessed by an object by virtue of its motion. It is a scalar quantity and depends on the mass (m) of the object and the square of its speed (v), given by the formula $K = \frac{1}{2}mv^2$.

- 128.** How is work done by a variable force determined? Explain with reference to a graph.

Ans :

Work done by a variable force $F(x)$ over a displacement from x_i to x_f is determined by integrating the force over the displacement, $W = \int_{x_i}^{x_f} F(x) dx$. Graphically, it is represented by the area under the force-displacement ($F-x$) curve between the initial and final positions.

- 129.** What is potential energy? Give an example of a system possessing potential energy.

Ans :

Potential energy is the 'stored energy' by virtue of the position or configuration of a body. For example, a stretched bow-string possesses potential energy, which is converted into kinetic energy of the arrow when released.

- 130.** Explain the principle of conservation of mechanical energy. What are its limitations?

Ans :

The principle of conservation of mechanical energy states that the total mechanical energy (sum of kinetic and potential energy) of a system remains constant if the forces doing work on it are conservative. Its limitation is that it does not hold if non-conservative forces, like friction, are present and do work.

- 131.** Describe Hooke's Law for an ideal spring. What does the spring constant signify?

Ans :

Hooke's Law states that for an ideal spring, the spring force (F_s) is directly proportional to the displacement (x) from its equilibrium position and acts in the opposite direction: $F_s = -kx$. The spring constant (k) signifies the stiffness of the spring; a larger k means a stiffer spring.

- 132.** Differentiate between average power and instantaneous power. Provide their SI units.

Ans :

Average power (P_{av}) is the ratio of the total work (W) done to the total time (t) taken, $P_{av} = W/t$. Instantaneous power (P) is the limiting value of the average power as the time interval approaches zero, $P = dW/dt = F \cdot v$. The SI unit for both is the watt (W).

- 133.** Distinguish between elastic and inelastic collisions. What quantity is conserved in both?

Ans :

In an elastic collision, both total linear momentum and total kinetic energy of the system are conserved. In an inelastic collision, total linear momentum is conserved, but the total kinetic energy of the system is not conserved, often being transformed into other forms of energy like heat or sound.

- 134.** How does the scalar product $A \cdot B$ relate to the magnitudes of A and B and the angle θ between them?

Ans :

The scalar product $A \cdot B$ is defined as $AB \cos \theta$, where A and B are the magnitudes of the vectors A and B respectively, and θ is the angle between them. This product is a scalar quantity.

- (B) A car accelerating on a flat road.
- (C) Water falling from a waterfall.
- (D) A battery powering a flashlight.

Ans :

(C) Water falling from a waterfall.

As water falls from a higher elevation to a lower one, its stored gravitational potential energy (due to its height) is converted into kinetic energy (energy of motion), causing its speed to increase (neglecting air resistance and other losses).

51. Two objects of mass m_1 and m_2 ($m_1 > m_2$) have the same kinetic energy. Which object has greater momentum?

- (A) The object with mass m_1 .
- (B) The object with mass m_2 .
- (C) Both have the same momentum.
- (D) Cannot be determined without knowing their velocities.

Ans :

(A) The object with mass m_1 .

Kinetic energy $K = p^2 / (2m)$, where p is momentum. If K is the same for both, then $p^2 = 2mK$, so $p = \sqrt{2mK}$. Since K is the same, $p \propto \sqrt{m}$. Therefore, the object with the larger mass (m_1) will have greater momentum.

52. All of the following are units of energy EXCEPT:

- (A) Joule (J)
- (B) Kilowatt-hour (kWh)
- (C) Electron volt (eV)
- (D) Watt (W)

Ans :

(D) Watt (W)

Joule, kilowatt-hour, and electron volt are all units of energy. A Watt, however, is the SI unit of power, which is the rate at which energy is used or transferred (energy per unit time, e.g., Joules per second).

53. How does the scalar product (dot product) of two vectors differ from their vector product (cross product)?

- (A) Scalar product results in a vector, while vector product results in a scalar.
- (B) Scalar product results in a scalar, while vector product results in a vector perpendicular to both original vectors.
- (C) Both scalar and vector products result in scalar quantities.

- (D) Both scalar and vector products result in vector quantities.

Ans :

(B) Scalar product results in a scalar, while vector product results in a vector perpendicular to both original vectors.

The scalar (dot) product of two vectors (e.g., $\mathbf{A} \cdot \mathbf{B}$) yields a scalar quantity representing the projection of one vector onto another, multiplied by the magnitude of the other. The vector (cross) product (e.g., $\mathbf{A} \times \mathbf{B}$) yields a new vector that is perpendicular to the plane containing the original two vectors.

54. A spring with a spring constant of 200N/m is compressed by 0.1m. What is the potential energy stored in the spring?

- (A) 1 J
- (B) 2 J
- (C) 10 J
- (D) 20 J

Ans :

(A) 1 J

The elastic potential energy (v) stored in a spring is given by $V = \frac{1}{2}kx^2$, where K is the spring constant and x is the compression or extension. Here, $k = 200 \text{ N/m}$ and $x = 0.1 \text{ m}$.

So,

$$\begin{aligned} V &= \frac{1}{2} \times 200 \times (0.1)^2 \\ &= \frac{1}{2} \times 200 \times 0.01 \\ &= 1 \text{ J} \end{aligned}$$

55. What is the work-energy theorem?

- (A) Work done is equal to the force multiplied by displacement always.
- (B) The net work done on an object equals the change in its kinetic energy.
- (C) The total energy of an isolated system remains constant over time.
- (D) Power is the product of force and velocity for any type of force.

Ans :

(B) The net work done on an object equals the change in its kinetic energy.

The work-energy theorem specifically states that the total work done by all forces acting on a particle (the net work) is equal to the change in the kinetic energy of that particle. This is a fundamental relationship between work and kinetic energy.

in which the potential energy $V(x)$ decreases most rapidly. For example, gravitational force is downward, and potential energy decreases downward.

- 147.** If only conservative forces act on a system, what can be said about its total mechanical energy?

Ans :

If only conservative forces act on a system and do work, its total mechanical energy (the sum of kinetic and potential energy) is conserved, meaning it remains constant. This is the principle of conservation of mechanical energy.

- 148.** Describe the energy transformations in a block attached to a spring oscillating on a smooth horizontal surface.

Ans :

In an oscillating spring-mass system, energy transforms between kinetic and potential energy. At extreme positions, potential energy ($kx^2/2$) is maximum and kinetic energy is zero. At the equilibrium position, kinetic energy ($mv^2/2$) is maximum and potential energy is zero.

THREE MARKS QUESTIONS

- 149.** What is coefficient of restitution ? What is its value for perfectly elastic and perfectly elastic collisions ?

Ans :

Coefficient of restitution

$$= \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

During perfectly elastic collision, relative velocity of separation is equal to relative velocity of approach, hence coefficient of restitution is unity.

During perfectly inelastic collision, the relative velocity of separation is zero i.e., bodies stick together, therefore, coefficient of restitution is zero.

- 150.** State the conditions under which a force does no work.

Ans :

Work done by a force is given by

$$W = \vec{F} \cdot \vec{S} = FS \cos \theta$$

Now work will be zero, if either

$$S = 0 \text{ or } \theta = 90^\circ$$

Thus, conditions for zero work are :

- Distance moved by the body due to applied force is zero.
- Force is acting perpendicular to the direction of motion of the body.

- 151.** Why is electrical power required at all when the elevator is descending ? Why should there be a limit on the number of passengers in this case ?

Ans :

- The electric power is required so that the downward acceleration remains less than the acceleration due to gravity.
- Because as the number of passengers is increased, the downward force increases and we have to prevent the elevator from falling freely under gravity, hence there is a limit on the number of passengers on the elevator.

- 152.** What is work ? What is it equal to ?

Ans :

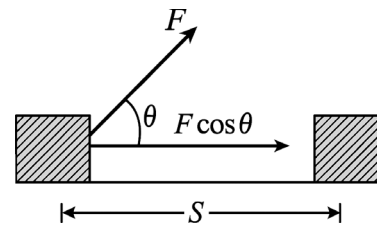
Work is said to be done when a body moves actually through some distance in the direction of force.

If force F is applied as shown in the figure and the body is moved through distance S , then

$$W = \vec{F} \cdot \vec{S} = FS \cos \theta$$

If $\theta = 0^\circ$;

$$W = FS \cos 0^\circ = FS$$



- 153.** A lighter body and a heavier body have same kinetic energy. Which one has greater momentum ?

Ans :

Given,

$$E_1 = E_2$$

$$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2} \quad \left[E = \frac{p^2}{2m} \right]$$

$$\left(\frac{p_2}{p_1} \right)^2 = \frac{m_2}{m_1}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}}$$

If $m_2 > m_1$, then $p_2 > p_1$

Heavier body has greater linear momentum as compared to lighter body.

154. For a force to do maximum work, what should be the angle between force and displacement vectors ?

Ans :

Since work done by a force is given by

$$W = \vec{F} \cdot \vec{S}$$

$$W = FS \cos \theta$$

The work done, (W), will be maximum

if $\cos \theta = \max.$

$$\cos \theta = +1$$

$$\theta = 0^\circ$$

Hence a force will do maximum work, if it acts in the direction of motion of the body i.e. angle between $\vec{F}$ and $\vec{S}$ is zero.

155. Define potential energy. Give examples.

Ans :

Potential energy is the energy stored in a body by virtue of its position in a field of force or by its configuration.

Examples.

- (1) A stone lying on the table possesses potential energy by virtue of its position.
 - (2) A compressed spring has potential energy due to change in configuration.
 - (3) A wound spring of a watch has potential energy which is stored in the spring.
 - (4) Due to potential energy of stretched bow, the arrow goes forward with a large velocity on releasing the bow.
 - (5) Potential energy of water in a dam is used to run the turbines of the electric generator.
156. Two springs of spring constants k_1 and k_2 are stretched by the same force. First spring undergoes more extension than the second one. In which of the two springs, energy stored will be maximum ?

Ans :

Since applied force is directly proportional to the extension (Hooke's law),

i.e. $F \propto x$

$$F = kx$$

$$k = \frac{F}{x}$$

Hence, Energy stored in a spring,

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \frac{F}{x} x^2 = \frac{1}{2} Fx$$

$$U \propto x$$

Hence more the extension in the spring, more will be the energy stored (for a given force). Thus, in first spring, the energy stored is more than the second spring.

157. In what part of its orbit is the earth's potential energy greatest with respect to the sun ? In what part of its orbit is its kinetic energy greatest ? Explain.

Ans :

When the earth is farthest from the sun. The work required to pull the earth away from the sun to a given distance increases with the distance. Hence earth's potential energy is greatest when it is farthest from the sun.

When it is closest to the sun. The gravitational pull of the sun on the earth is greatest when it is closest to the sun. Therefore, the speed is also greatest there in order that the gravitational and the centripetal forces should be in balance. In this position, kinetic energy is greatest.

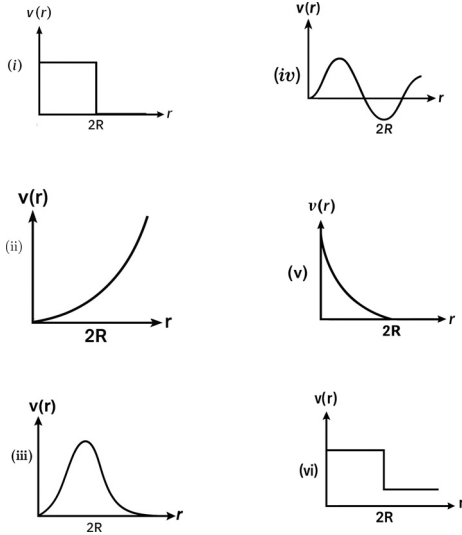
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158. Given that
- (i) linear momentum of a system of particles is zero.
 - (ii) kinetic energy of a system of particle is zero.
- Show that (i) does not imply (ii) but implies (i).

Ans :

When linear momentum of a system of particles is zero, velocities of individual particles may not be zero. Thus, kinetic energy of the system of particle may be non-zero. Now, when kinetic energy of a system of particles is zero, then both kinetic energy and velocity of each particle is zero. Therefore, the linear momentum of the system of particles is also zero.

159. Which of the following potential energy curves cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls, R is the radius of each ball.



Ans :

Potential energy, $U = -\frac{Gm_1m_2}{r}$

If $r < 2R$, the Potential energy is there due to attraction but as soon as $r = 2R$, it becomes zero and remains zero.

- | | |
|------------------|------------------|
| (i) Impossible | (ii) Impossible |
| (iii) Impossible | (iv) Impossible |
| (v) Possible | (vi) Impossible. |

160. Explain the scalar product of two vectors. What are its important properties?

Ans :

The scalar product, or dot product, of two vectors A and B is a fundamental mathematical operation in physics.

- It is defined as $A \cdot B = AB \cos \theta$, where A and B are the magnitudes of the vectors and θ is the angle between them.
- The result of a scalar product is a scalar quantity.
- It is commutative, meaning $A \cdot B = B \cdot A$.
- It is distributive over addition:

$$A \cdot (B + C) = A \cdot B + A \cdot C.$$

- If two vectors are perpendicular, their scalar product is zero, as $\cos(90^\circ) = 0$.

161. Derive the work-energy theorem for a constant force. What does it signify?

Ans :

The work-energy theorem establishes a direct relationship between the work done on an object and its change in kinetic energy. For a constant force F causing a displacement d along the x-axis:

From kinematics, for constant acceleration a, $v^2 - u^2 = 2as$, where u is initial and v is final speed, and s is displacement.

Multiplying by $m/2$, we get $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$.

Since $F = ma$, this becomes $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$.

Thus, $K_f - K_i = W$. It signifies that the net work done on an object changes its kinetic energy.

162. Discuss the concept of work done by a constant force, including positive, negative, and zero work.

Ans :

Work done (W) by a constant force (F) causing a displacement (d) is defined as $W = (F \cos \theta) d = F \cdot d$, where θ is the angle between F and d.

- **Positive work:** Done when $0^\circ \leq \theta < 90^\circ$; force component is in the direction of displacement, increasing kinetic energy.
- **Negative work:** Done when $90^\circ < \theta \leq 180^\circ$; force component opposes displacement, decreasing kinetic energy (e.g., work by friction).
- **Zero work:** Done if displacement is zero, force is zero, or force is perpendicular to displacement ($\theta = 90^\circ$).

163. Define kinetic energy and explain its dependence on mass and velocity. Is it a scalar or vector?

Ans :

Kinetic energy (K) is the energy possessed by an object due to its motion.

- It is defined by the formula $K = \frac{1}{2}mv^2$, where 'm' is the mass of the object and 'v' is its speed.
- Kinetic energy depends quadratically on the speed; doubling the speed quadruples the kinetic energy.
- It depends linearly on the mass; doubling the mass doubles the kinetic energy for the same speed.
- Kinetic energy is a scalar quantity, as both mass and speed squared are scalars.

164. Explain how work done by a variable force in one dimension is calculated using integration.

Ans :

When a force $F(x)$ varies with position x, the work done cannot be found by simply multiplying force and displacement.

- We consider a small displacement Δx over which the force $F(x)$ is approximately constant. The work done is $\Delta W = F(x) \Delta x$.
- To find the total work done in moving from an initial position x_i to a final position x_f , we sum these infinitesimal works.

- This summation, in the limit as $\Delta x \rightarrow 0$, becomes a definite integral: $W = \int_{x_i}^{x_f} F(x) dx$.
- Graphically, this represents the area under the force-displacement curve.

165. What are conservative forces? Explain their properties with respect to work done and potential energy.

Ans :

A conservative force is a force with the property that the total work done in moving a particle between two points is independent of the path taken.

- The work done by a conservative force depends only on the initial and final positions of the particle.
- Consequently, the work done by a conservative force in moving a particle around any closed path is zero.
- For every conservative force, a potential energy function $V(x)$ can be defined such that $F(x) = -dV/dx$. Examples include gravitational force and spring force.

166. State and explain the principle of conservation of mechanical energy. Provide an example.

Ans :

The principle of conservation of mechanical energy states that the total mechanical energy (sum of kinetic and potential energy) of a system remains constant if the forces doing work on it are conservative.

- Mathematically, if K is kinetic energy and V is potential energy, then $K + V = \text{constant}$, or $\Delta(K + V) = 0$.
- This means energy can transform between kinetic and potential forms, but their total sum is unchanged.
- An example is a freely falling body (neglecting air resistance), where gravitational potential energy converts to kinetic energy, but the total mechanical energy remains constant.

167. Describe the potential energy stored in a spring. How is it related to the spring constant and displacement?

Ans :

The potential energy stored in a spring is a consequence of its deformation (stretching or compression) and the conservative nature of the spring force.

- According to Hooke's Law, the spring force is $F_s = -kx$, where k is the spring constant and x is the displacement from equilibrium.

- The potential energy $V(x)$ of the spring when displaced by x from its equilibrium position is given by $V(x) = \frac{1}{2}kx^2$.
- This energy is maximum at maximum displacement and zero at the equilibrium position.

168. Define instantaneous power. How is it expressed in terms of force and velocity?

Ans :

Instantaneous power is the rate at which work is done or energy is transferred at a particular instant in time.

- It is defined as the limiting value of the average power as the time interval Δt approaches zero, $P = dW/dt$.
- If a force F acts on an object moving with an instantaneous velocity v , the instantaneous power can also be expressed as the scalar product of force and velocity: $P = F \cdot v$.
- This means $P = Fv \cos \theta$, where θ is the angle between F and v .

169. Distinguish clearly between elastic and completely inelastic collisions in one dimension.

Ans :

Collisions are interactions between bodies where momentum conservation is a key principle.

- **Elastic Collision:** In an elastic collision, both the total linear momentum and the total kinetic energy of the system are conserved. The bodies separate after impact without loss of kinetic energy.
- **Completely Inelastic Collision:** In a completely inelastic collision, the total linear momentum is conserved, but the total kinetic energy is not. The colliding bodies stick together after the collision and move with a common final velocity. This type of collision involves the maximum possible loss of kinetic energy (consistent with momentum conservation).

170. Explain the conditions under which the work done by a force can be negative, with an example.

Ans :

Work done by a force is negative when the component of the force in the direction of displacement opposes the displacement.

This occurs when the angle θ between the force vector and the displacement vector is greater than 90° and less than or equal to 180° .

Mathematically, $W = Fd \cos \theta$; if $90^\circ < \theta \leq 180^\circ$, then $\cos \theta$ is negative, making W negative.

An example is the work done by the force of kinetic friction on a sliding block, as friction always opposes motion.

171. How is the work-energy theorem applied when non-conservative forces are present in a system?

Ans :

When non-conservative forces (like friction) are present, the principle of conservation of mechanical energy is modified. The work-energy theorem in its general form still holds: the total work done by all forces (conservative and non-conservative) equals the change in kinetic energy.

- If W_c is work by conservative forces and W_{nc} by non-conservative forces, then $W_c + W_{nc} = \Delta K$.
- Since $W_c = -\Delta V$ (change in potential energy), the equation becomes $-\Delta V + W_{nc} = \Delta K$.
- Rearranging, $W_{nc} = \Delta K + \Delta V = \Delta E$, meaning the work done by non-conservative forces equals the change in total mechanical energy.

FIVE MARKS QUESTIONS

172. What do you mean by conservative and non-conservative forces? Explain the various properties of conservative force.

Ans :

Conservative force : If the amount of work done by a force in moving a body from initial to final location is independent of the path followed, the force is said to be conservative force.

e.g., gravitational force, electrostatic force, magnetic force etc.

Non-conservative force : If the amount of work done by a force in moving a body from initial to final location depends on the path followed by the body, the force is said to be non-conservative force.

e.g. frictional force, force of viscosity etc.

Properties of conservative force :

- (1) The amount of work done against a conservative force is independent of the path followed by the body.
- (2) The amount of work done against a conservative force depends on the initial and final positions of the body moved.
- (3) In a closed path, the amount of work done by a body against a conservative force is zero.
- (4) All central forces are conservative forces.

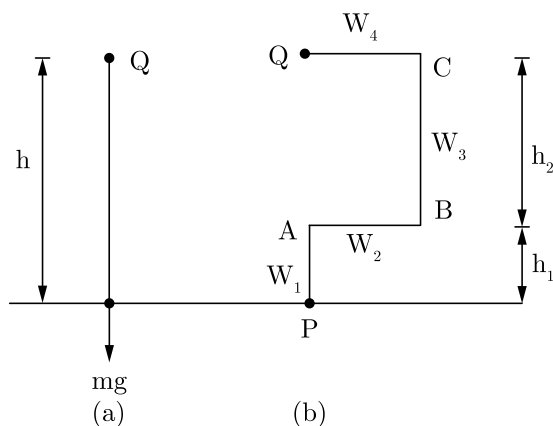
173. Show that gravitational force is a conservative force.

Ans :

Consider a body of mass m lying at point P . Let us move it from point P to point Q through a height h as shown in the figure.

Hence, Work done, $W = mgh$

Now let us move the body from P to Q through path $PABCQ$ as shown in figure.



$$\text{Total work done} = W_1 + W_2 + W_3 + W_4$$

$$= mgh_1 + mg0 + mgh_2 + mg0$$

$$= mgh_1 + mgh_2$$

$$= mg(h_1 + h_2) = mgh$$

$$[\text{since, } h_1 + h_2 = h]$$

$$\text{Total work done} = W$$

Hence, we find that work done against gravitational force is independent of the path followed and hence gravitational force is conservative.

174. Show that sum of potential energy and kinetic energy of a freely falling body is conserved.

Ans :

Conservation of kinetic energy and potential energy of a freely falling body.

Consider a body of mass m at a height h from the ground at point A .

$$\text{Potential energy} = mgh$$

$$\text{Kinetic energy} = \frac{1}{2}m(0)^2 = 0$$

Since, velocity at A is zero.

Hence, sum of kinetic energy and potential energy at

$$A = mgh + 0 = mgh \quad \dots(1)$$

$$\Delta W = F(x) \Delta x = F \Delta x$$

The total work done is equal to sum of all small amounts of work during each interval which is expressed as

$$W = \sum_{x_1}^{x_2} F \Delta x = \text{Shaded area}$$

where symbol Σ (sigma) stands for summation. When $\Delta x \rightarrow 0$, the work done between x_1 and x_2 is

$$W = \lim_{\Delta x \rightarrow 0} \sum_{x_1}^{x_2} F \Delta x = \int_{x_1}^{x_2} F dx = \text{Area } x_1' x_1 x_2 x_2'$$

- 178.** State and explain unit of work. Derive relation between joule and erg. Also derive dimensional formula of work.

Ans :

- (i) **Absolute unit of work :**

Since work done,

$$W = FS (\theta = 0^\circ)$$

In SI, if $F = 1\text{ N}$, $S = 1\text{ m}$, then $W = 1\text{ joule}$

Hence, work is said to be 1 joule, if a force of 1 N displaces a body through 1 m in its own direction.

$$1\text{ J} = 1\text{ N} \cdot 1\text{ m}$$

$$1\text{ J} = 1\text{ N m}$$

In cgs system, if

$$F = 1\text{ dyne}, S = 1\text{ cm}, \text{ then}$$

$$W = 1\text{ erg}$$

Hence work is said to be 1 erg, if a force of 1 dyne displaces a body through 1 cm in its own direction.

$$\text{So, } 1\text{ erg} = 1\text{ dyne } 1\text{ cm}$$

$$1\text{ erg} = 1\text{ dyne cm}$$

Relation between joule and erg

$$\text{Since, } 1\text{ J} = 1\text{ N} \cdot 1\text{ m}$$

$$= 10^5\text{ dyne } 10^2\text{ cm}$$

$$= 10^7\text{ dyne cm} = 10^7\text{ erg}$$

$$\text{So, } 1\text{ J} = 10^7\text{ erg}$$

- (ii) **Gravitational unit of work :**

In SI, the gravitational unit of work is kg m.

Work is said to be 1 kg m, if a force of 1 kg f displaces a body through 1 m in the direction of force.

$$\text{Hence, } 1\text{ kg m} = 1\text{ kgf} \times 1\text{ m} = 9.5\text{ J}$$

In cgs system, the gravitational unit of work is g cm.

Work is said to be 1 g cm, if a force of 1 g f displaces a body through 1 cm in the direction of force.

$$\text{Hence, } 1\text{ g cm} = 1\text{ gf} \times 1\text{ cm} = 980\text{ erg}$$

Dimensional formula of work

$$\text{Since, } \text{Work} = \text{force} \times \text{distance}$$

$$\text{i.e., } = FS = maS$$

$$= [M] [LT^{-2}] [L] = [ML^2T^{-2}]$$

- 179.** State and prove work energy theorem.

Ans :

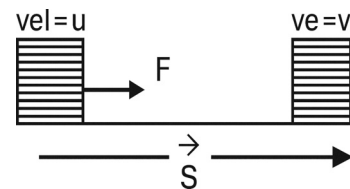
Work energy theorem states that work done by a force on a body is equal to change in kinetic energy of the body.

Consider a body of mass m moving with velocity u and a force $\vec{F}$ displaces it through a displacement $\vec{S}$ and changes its velocity to v as shown in figure. Work done by the force,

$$W = \vec{F} \cdot \vec{S} = FS \cos 0^\circ = FS \quad \dots(1)$$

If a is the acceleration produced in the body, then

$$F = ma$$



So, equation (1) becomes

$$W = maS \quad \dots(2)$$

If the velocity of the body at point B is v , then using the relation,

$$v^2 - u^2 = 2aS \quad \dots(3)$$

$$aS = \frac{v^2 - u^2}{2}$$

Putting equation (3) in (2), we get

$$W = m \left(\frac{v^2 - u^2}{2} \right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \quad \dots(4)$$

Equation (4) is work-energy theorem.

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Thus, work is equal to the dot product of force and displacement. Work is a scalar quantity. SI unit of work is joule (J).

- 176.** What do you understand by positive work, negative work and zero work? Give at least two examples of each type.

Ans :

(i) Positive work

Since work done, $W = \vec{F} \cdot \vec{S} = FS \cos \theta$,

So if $\theta < 90^\circ$, $\cos \theta$ is +ve, hence W is +ve,

Examples :

1. In case a body falling freely towards the earth, the work done by gravity is +ve, because in this case $\theta = 0^\circ$. Hence $W = FS \cos 0^\circ = FS$.
2. When a spring is stretched, work done by stretching force is +ve, because the force and displacement are in the same direction i.e. $\theta = 0^\circ$, so $W = FS \cos 0^\circ = FS$.

(ii) Negative work

Since work done, $W = \vec{F} \cdot \vec{S} = FS \cos \theta$

So if $\theta > 90^\circ$, $\cos \theta$ is -ve, hence W is -ve.

Examples :

1. Work done by friction is always -ve because it always acts opposite to the direction of motion of the body

i.e. $\theta = 180^\circ$ so $W = \vec{F} \cdot \vec{S}$
 $= FS \cos 180^\circ = -FS$

2. When brakes are applied on a moving vehicle, the work done by braking force is always -ve, because braking force and motion of vehicle are in opposite directions i.e. $\theta = 180^\circ$, so

$$W = \vec{F} \cdot \vec{S} = FS \cos 180^\circ = -FS$$

3. When we lift a body, work done by gravity is -ve, because the displacement of the body is opposite to the direction of force due to gravity.

(iii) Zero work

If either $\theta = 90^\circ$ or $S = 0$ i.e. either the force is acting perpendicular to the displacement or unable to change the position of a body, work done by force is zero.

Since, $W = FS \cos 90^\circ$ or $W = 0$
 $= F(0) \cos \theta = 0$

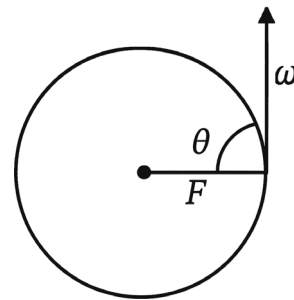
Examples :

1. When we try to push a stationary locomotive, work done by the person is zero, because

$$W = \vec{F} \cdot \vec{S} = F(0) \cos \theta = 0$$

2. If a body moves along a frictionless horizontal surface, its weight and reaction of the surface, both of which are normal to surface to direction of motion i.e. $\theta = 90^\circ$
3. When a stone tied to the end of a string is whirled in a circle with uniform speed, the force is directed towards the centre and is always normal to direction of motion.

Hence, the force does no work.



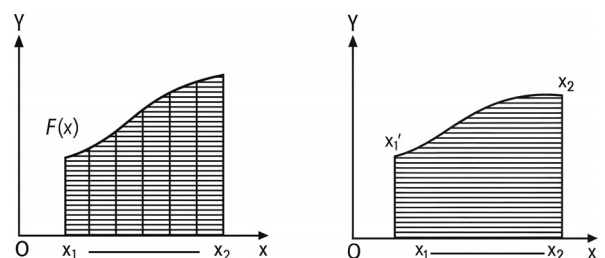
NODIA 7. Explain work done by a variable force.

Ans :

A force whose magnitude may keep on varying but direction is always the same is called a variable force.

Consider a force which always acts in one direction, which is assumed to be along x -axis. The magnitude of force is a function of x . It means for each value of x , force $F(x)$ is known.

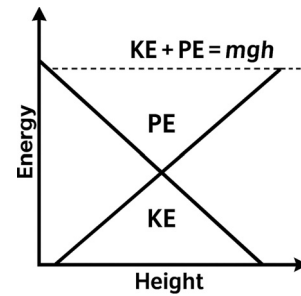
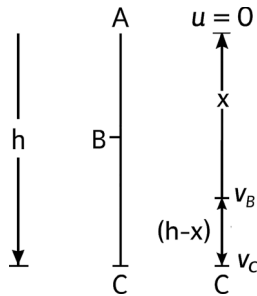
Force is plotted as a function of x and is shown in figure.



Divide the total displacement into a number of equal intervals Δx .

We assume that during each small interval Δx the force is constant.

Therefore, the work done in moving the body by Δx at the given point x is



At point B.

To find velocity at B, consider motion AB.

Here, $u = 0, s = x, a = g, v = v_B$

$$v_B^2 - (0)^2 = 2gx \quad [\text{Using } v^2 - u^2 = 2aS]$$

$$\begin{aligned} \text{Kinetic energy at B} &= \frac{1}{2} m v_B^2 \\ &= \frac{1}{2} m 2gx = mgx \end{aligned}$$

$$\begin{aligned} \text{Potential energy at B} &= mg(h - x) \\ &= mgh - mgx \end{aligned}$$

$$\begin{aligned} \text{Sum of Kinetic energy and Potential energy at B} \\ &= mgx + (mgh - mgx) \\ &= mgh \quad \dots(2) \end{aligned}$$

At point C (Just before touching the earth).

Consider motion AC

Here, $u = 0, s = h, a = g, v = v_C$

$$v_C^2 - (0) = 2gh$$

$$\text{Kinetic energy at C} = \frac{1}{2} m v_C^2 = \frac{1}{2} m 2gh = mgh$$

$$\text{Potential energy at C} = mg(0) = 0$$

$$\begin{aligned} \text{Sum of kinetic energy and potential energy at C} \\ &= mgh + 0 = mgh \quad \dots(3) \end{aligned}$$

We find from (1), (2) and (3) that the sum of kinetic energy and potential energy (i.e. total mechanical energy) of a freely falling body is conserved.

This shows that when a body falls freely under gravity, the potential energy of the body goes on converting into kinetic energy and energy is fully kinetic when it reaches the ground.

It is also clear that total energy (sum of kinetic energy and potential energy) remains constant.

The variation of potential energy into kinetic energy is as shown in Figure.

175. Explain the term work. Obtain an expression for work done by a constant force.

Ans :

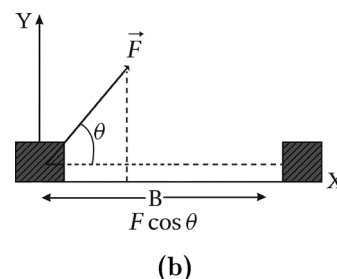
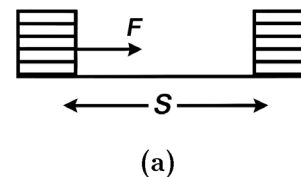
Work is said to be done if a force acting upon a body displaces it through a certain distance in the direction of force.

Work is measured by the product of force and distance through which the body moves in the direction of force.

Various cases are as follows :

1. If a constant force F displaces a body through a distance S in the direction of force, [as shown in the figure (a)],

$$\text{Work done} = FS$$



2. When constant force $\vec{F}$ makes an angle θ with direction of displacement, the component of force in the direction of displacement is $F \cos \theta$. In this case, the work done is measured by the product of component and distance S . As shown in figure (b).

$$\begin{aligned} W &= F \cos \theta \times S = FS \cos \theta \\ &= \vec{F} \cdot \vec{S} \quad (\text{since, } \vec{A} \cdot \vec{B} = AB \cos \theta) \end{aligned}$$

- 180.** State law of conservation of energy. Illustrate with examples. Explain the transformation of energy in a simple pendulum.

Ans :

Law of conservation of energy states that energy can neither be created nor destroyed but can be transformed from one form to other equivalent form. The total energy in a closed system remains constant.

Illustrations.

- (1) In hydroelectric power generation, the potential energy of water stored is converted into kinetic energy and it gains speed. The water is then made to run turbines of the electric generators that produce electrical energy. It is this hydroelectrical energy that is used in a number of ways in our daily life.
- (2) In a cell the chemical energy is converted into electric energy.
- (3) The work done in winding the watch is stored as potential energy. This potential energy gets converted into kinetic energy of rotation of hands of watch.
- (4) Radiant energy from the sun heats up water in the oceans and converts part of it into water vapours. As the water molecules rise up, their gravitational potential energy increases. When they condense into clouds, they are transformed into rain and pour down. The gravitational potential energy is converted into kinetic energy.

Transformation of energy in a simple pendulum :

In a simple pendulum executing vibrations, there is a repeated transformation of potential energy into kinetic energy and vice versa.

In a simple pendulum, when the bob is displaced from mean position O to the extreme position say A , the bob gets raised through a vertical height h [as shown in figure] and acquires potential energy given by

$$\text{Potential energy} = mgh.$$

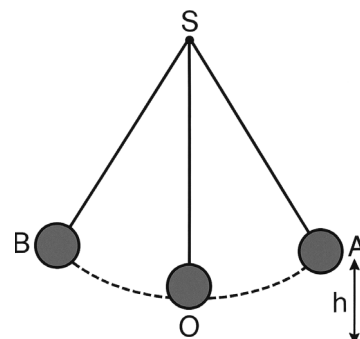
where m is the mass of the bob.

Since at the extreme position, the bob is at rest, its energy is wholly potential at A .

As soon as the bob is released, it falls downward and moves towards the mean position O . In this process, the potential energy of the bob goes on decreasing and kinetic energy continuously increases due to increase in velocity. On reaching point O , the bob has energy which is totally kinetic in nature. Between points A and O , the energy of the bob

is partly potential and partly kinetic. If v is the velocity of the bob on reaching point O , then

$$\frac{1}{2}mv^2 = mgh$$



On reaching point O , the bob does not come to rest but overshoots the mean position O due to inertia. As the bob rises up in moving towards point B (the other extreme position), the velocity of the bob goes on decreasing till it becomes zero at point B . Thus, at point B , the energy of the bob is again totally potential in nature.

After coming to rest at point B momentarily, the bob again starts moving towards the extreme position A and so on. Therefore, as the pendulum executes oscillations, there is a repeated transformation of potential energy into kinetic energy and vice versa.

However, in practice, during each oscillation, a part of the energy of the bob is converted into heat energy, while overcoming the air resistance. Due to this, amplitude of the bob goes on decreasing and ultimately, it comes to rest.

- 181.** Discuss potential energy of a coiled spring.

Ans :

Consider a massless spring with its one end attached firmly to rigid support, in a wall. The other end of the spring is attached to a block of mass m . The block can move on a smooth frictionless surface.

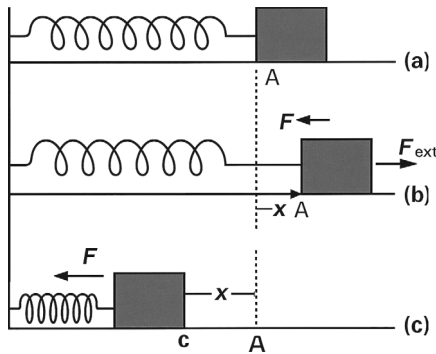
For simplicity, let us assume that stretching of the spring is one dimensional i.e. along x -axis.

Let $x = 0$ be the original position of the block. Suppose the spring is stretched from its initial position to a distance x . A restoring force will be developed in the spring bringing the spring back to equilibrium position.

For ideal elastic spring, using Hooke's law,

$$F \propto x$$

$$F = -kx,$$



NODIA

where k is called spring constant.

Hence, Force to be applied on spring to stretch it through distance $x = kx$ (in the opposite direction)

The graph between various values of forces and their corresponding displacements is shown in figure.

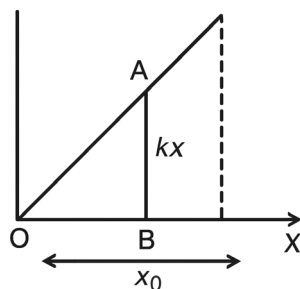
Consider a point A on the line of graph. Corresponding force for displacement x is $F = kx$

Since work done = area of triangle

$$= \frac{1}{2} \times OB \times AB$$

$$= \frac{1}{2} \times x \times kx$$

$$W = \frac{1}{2} kx^2$$



This work done is equal to potential energy of the spring. Potential energy of spring,

Hence, potential energy of spring

$$U = \frac{1}{2} kx^2$$

Suppose the spring is stretched to a distance x_0 and let it go. The speed of the block increases from zero to a maximum value at $x = 0$.

If the potential energy is totally converted into kinetic energy, assuming that there is no heat energy loss, the block reaches the mean position $x = 0$, the potential energy goes on converting into kinetic energy. The block then overshoots the mean position $x = 0$, and moves towards the other side with decrease in velocity. The block comes to rest at $x = -x_0$.

In ideal conditions, the whole of kinetic energy gets converted into potential energy, which is maximum.

At $x = -x_0$, the restoring force again brings the block towards the mean position. Thus, the block keeps on oscillating and the total energy remains conserved.

At extreme position i.e. at $x = x_0$, $v = 0$

Hence, Kinetic energy $= \frac{1}{2} mv^2 = 0$ and

$$\text{Potential energy} = \frac{1}{2} kx^2 = \frac{1}{2} kx_0^2$$

At any position between mean and extreme

i.e. $x = x_1$ (say)

Potential energy of block $= \frac{1}{2} kx_1^2$ and

$$\text{Kinetic energy of block} = \frac{1}{2} k(x_0^2 - x_1^2)$$

$$\text{Total energy} = \text{P.E.} + \text{K.E.}$$

$$= \frac{1}{2} kx_1^2 + \frac{1}{2} k(x_0^2 - x_1^2)$$

$$= \frac{1}{2} kx_0^2$$

At mean position, $x = 0$

$$\text{Potential energy of block} = \frac{1}{2} k(x_0^2 - x_1^2)$$

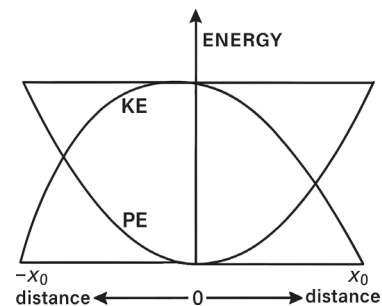
If $v = v_{\max}$ at $x = 0$

$$\text{Kinetic energy of block} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} kx_0^2$$

$$\text{Total energy of block} = \text{P.E.} + \text{K.E.}$$

$$= 0 + \frac{1}{2} kx_0^2 = \frac{1}{2} kx_0^2$$

At other extreme position



i.e. $x = -x_0$

$$\text{Potential energy of block} = \frac{1}{2} kx^2 = \frac{1}{2} kx_0^2$$

$$\text{Kinetic energy of block} = \frac{1}{2} k \times 0 = 0$$

$$\text{Total energy of block} = \frac{1}{2} kx_0^2$$

The variation of kinetic energy with distance from the mean position is shown in figure.

- 182.** (a) Define the term collision. Discuss its types with examples.
 (b) Discuss the elastic collisions in one dimension and calculate the velocities of bodies after the collision. Also discuss special cases.

Ans :

- (a) A collision between two bodies is said to take place if they physically collide with each other or even the path of a body is affected due to some interaction between them.

Types of Collisions :

It is of following types :

- (i) **Perfectly elastic collisions :** Those collisions in which the momentum and kinetic energy of system are conserved, are called perfectly elastic collisions.

Examples : The collisions between atoms, molecules, atomic particles are perfectly elastic collisions.

- (ii) **Inelastic collisions :** Those collisions in which the momentum of the system is conserved and kinetic energy of the system is not conserved are called inelastic collisions.

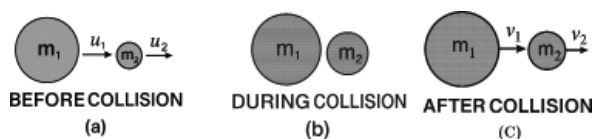
In such collisions, some kinetic energy is always lost and is converted into heat and sound energy. For example, in nature most of the collisions are inelastic collisions.

- (iii) **Perfectly inelastic collisions :** In these types of collisions, momentum of the system is conserved but kinetic energy of the system is not conserved and two bodies stick together after collision.

For example, a collision between bullet and a target when the bullet remains embedded in the target.

- (b) Elastic Collision in One Dimension :

When Two bodies moving along the same straight line collide with each other and after collision move along same line it is called collision in one dimension or head-on collision. Consider two bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively, such that $u_1 > u_2$ [as shown in figure (a)]. The two bodies will collide with each other and then move in the same direction with velocities v_1 and v_2 [as shown in figure (c)].



Since the collision is perfectly elastic, therefore the kinetic energy of the system is conserved i.e.

Total kinetic energy before collision = Total kinetic energy after collision

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2$$

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \quad \dots(1)$$

According to law of conservation of momentum,
 Total momentum before collision

= Total momentum after collision

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad \dots(2)$$

Dividing (1) by (2), we get

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \quad (3)$$

Relative velocity of approach before collision

= Relative velocity of separation after collision

Calculation for velocities after collision

$$\text{From (3),} \quad v_2 = u_1 - u_2 + v_1 \quad (4)$$

Putting this value of v_2 in (2), we get

$$m_1 (u_1 - v_1) = m_2 (u_1 - u_2 + v_1 - u_2)$$

To calculate v_2 from (3), we get

$$v_1 = v_2 + u_2 - u_1$$

Putting this value of v_1 in (2), we get

$$m_1 (u_1 - v_2 - u_2 + u_1) = m_2 (v_2 - u_2)$$

$$2m_1 u_1 - m_1 u_2 = m_2 v_2 - m_2 u_2$$

$$2m_1 u_1 - (m_1 - m_2) u_2 = (m_1 + m_2) v_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_2 \quad \dots(5)$$

To calculate v_2 from (3), we get

$$v_1 = v_2 + u_2 - u_1$$

Putting this value of v_1 in (2), we get

$$m_1 (u_1 - v_2 - u_2 + u_1) = m_2 (v_2 - u_2)$$

$$2m_1 u_1 - m_1 v_1 - m_1 u_2 = (m_1 + m_2) v_2$$

$$2m_1 u_1 - (m_1 - m_2) u_2 = (m_1 + m_2) v_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_2 \quad \dots(6)$$

Special cases

- (i) When both the bodies are of the same Mass

i.e. $m_1 = m_2$, then from (5), we get $v_1 = v_2$ and from (6), we get $v_2 = u_1$

Thus, the velocities of the two bodies of the same mass are exchanged after perfectly elastic collision.

final particles. Thus disintegration energy is given by the relation.

$$E_{\beta} = {}_z M_n^A - [{}_{z+1} M_n^A + m_e]$$

where M_n represents nuclear mass (A, z) and ($A, z+1$), the mass and atomic number of the parent and the daughter nuclei respectively and the mass of the β -particle. Thus the condition for β -decay is that

$${}_z M^A > {}_{z+1} M^A$$

This shows that the β -decay disintegration energy is given by the difference between the initial and final atomic masses and as such it must be a definite and discrete amount of energy.

NUMERICAL QUESTIONS

- 186.** The potential energy of a certain particle is given by $U = 30x^2 - 20y^2$. Find the force acting on the particle.

Ans :

We know that $F_x = -\frac{dU}{dx}$ and $F_y = -\frac{dU}{dy}$

Finding these values from the equation

$$U = 30x^2 - 20y^2$$

Hence $F_x = -\frac{dU}{dx} = -60x$

and $F_y = -\frac{dU}{dy} = 40y$

In vector notation, $\vec{F} = -60xi + 40yj$

- 187.** A bomb of mass 9 kg explodes into pieces of masses 3 kg and 6 kg. The velocity of 3 kg mass is 16 m/s. Find the kinetic energy associated with the 6 kg mass.

Ans :

Since momentum before explosion is equal to momentum after explosion, velocity of 6 kg mass is given by

$$3 \times 16 = 6 \times v$$

$$v = 8 \text{ m/s}$$

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2} \times 6 \times 64 = 192 \text{ J}$$

- 188.** 1 g of water cools down from its boiling point to its freezing point, it gives off about 420 J of heat energy. By how much does its mass change because of this loss in energy ?

Ans :

According to Einstein's equation,

$$\Delta E = (\Delta m)(c)^2$$

Hence, $420 = (\Delta m) \times (9 \times 10^{16})$

so that, $\Delta m = 4.6 \times 10^{-15} \text{ Kg}$

- 189.** A railway carriage of mass 10,000 kg moving with a speed of 54 km h⁻¹ strikes a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? Is the collision elastic ?

Ans :

Since carriage gets coupled with other carriage, therefore, collision is inelastic. Momentum of system is conserved.

$$\text{Speed} = 54 \text{ kmh}^{-1}$$

$$= 54 \times \frac{5}{18} = 15 \text{ ms}^{-1}$$

$$m_1 v_1 + m_2 \times v_2 = MV$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V$$

$$10,000 \times 15 + 10,000 \times 0 = (10,000 + 10,000) v$$

$$v = \frac{15 \times 10,000}{20,000}$$

$$v = 7.5 \text{ ms}^{-1}$$

- 190.** A shot travelling at the rate of 100 ms⁻¹ is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 mm thick ?

Ans :

Given, $v_1 = 100 \text{ ms}^{-1}$, $d_1 = 4 \text{ cm} = 0.04 \text{ m}$,

$$v_2 = ?, d_2 = 0.09 \text{ m}$$

Since kinetic energy of shot is spent in doing work against plank resistance F

$$\frac{1}{2}mv^2 = Fd$$

Case (i), $\frac{1}{2}mv_1^2 = Fd_1 \quad \dots(1)$

Case (ii), $\frac{1}{2}mv_2^2 = Fd_2 \quad \dots(2)$

Dividing (2) by (1), we get

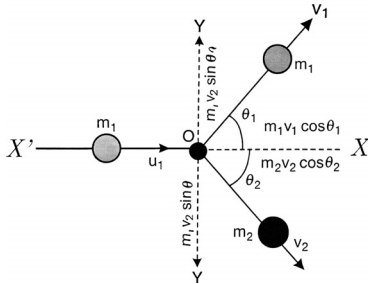
$$\frac{v_2^2}{v_1^2} = \frac{d_2}{d_1}$$

$$v_2^2 = v_1^2 \times \frac{d_2}{d_1}$$

$$= (100)^2 \times \frac{.09}{.04}$$

two dimensions.

Consider a body of mass m_1 moving with velocity u_1 colliding with another body of mass m_2 at rest at a point O.



Let v_1 and v_2 be the velocities of the two bodies after elastic collision in the directions making angles θ_1 and θ_2 with the original direction as shown in figure.

Resolving velocity in two rectangular components, we get

$v_1 \cos \theta_1$ acts along OX, i.e. along the original direction and $v_1 \sin \theta_1$ acts along OY i.e. perpendicular to the original direction.

Similarly, the rectangular components of velocity v_2 are $v_2 \cos \theta_2$ along OX i.e. along the original direction and $v_2 \sin \theta_2$ along OY' i.e. perpendicular to the original direction.

From law of conservation of linear momentum,
Along OX,

$$m_1 u_1 = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2 \quad \dots(1)$$

Along OY,

$$0 = m_1 v_1 \sin \theta_1 - m_2 v_2 \sin \theta_2 \quad \dots(2)$$

Since the collision is elastic, kinetic energy is also conserved. From the principle of conservation of energy, we get

$$\frac{1}{2} m_1 u_1^2 + 0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2 \quad \dots(3)$$

From equations (1), (2) and (3), we note that there are four unknown quantities v_1 , v_2 , θ_1 and θ_2 . These four values cannot be determined from these three equations. Thus, in order to find the value of v_1 and v_2 , we must find out the values of θ_1 and θ_2 experimentally or otherwise.

Case of one-dimensional collision

Let $\theta_1 = \theta_2 = 0$ i.e. after collision bodies move in the same straight line.

Eqn. (1) becomes

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \quad \dots(4)$$

$$m_1 (u_1 - v_1) = m_2 v_2 \quad \dots(5)$$

$$\text{From Eqn. (3), } m_1 (u_1^2 - v_1^2) = m_2 v_2^2 \quad \dots(6)$$

Dividing (6) and (5),

$$\frac{m_1 (u_1^2 - v_1^2)}{m_1 (u_1 - v_1)} = \frac{m_2 v_2^2}{m_2 v_2} \quad \dots(7)$$

Substituting the value of v_1 from (7) in Eqn. (4),

$$m_1 u_1 = m_1 v_1 + m_2 (u_1 + v_1)$$

$$m_1 u_1 = m_1 v_1 + m_2 u_1 + m_2 v_1$$

$$u_1 (m_1 - m_2) = (m_1 + m_2) v_1$$

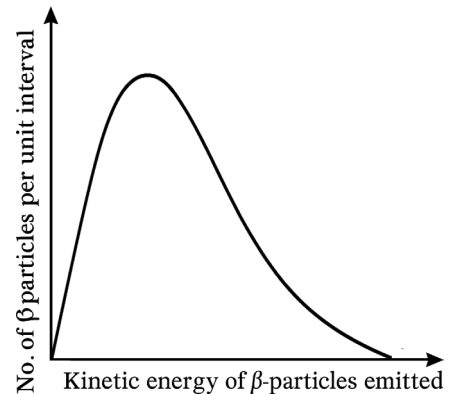
$$v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2}$$

Substituting the value of v_1 in Eq. (7),

$$\begin{aligned} v_1 &= u_1 + \frac{(m_1 - m_2) u_1}{m_1 + m_2} \\ &= \frac{m_1 u_1 + m_2 u_1 + m_1 u_1 - m_2 u_1}{m_1 + m_2} \\ &= \frac{2m_1 u_1}{m_1 + m_2} \end{aligned}$$

- 185.** Consider the decay of the free neutron at rest :
 $n = p + e^-$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and therefore, cannot account for the observed continuous energy distribution in the β decay of a neutron or a nucleus.



Ans :

The emission of β -particles is due to the transformation of a neutron into a proton, an electron (β -particle) is ejected whereas the proton remains in the nucleus. The emission of a particle, therefore, increases the atomic number by 1 and keeps the mass number unaffected.

The energy available for disintegration is provided by the difference in mass between the initial nucleus and the sum of the masses of the

1. What is the formula for kinetic energy of an object of mass m moving with speed v ?
2. Is kinetic energy a scalar or a vector quantity?
3. Calculate the initial kinetic energy of the bullet.
4. What is the emergent speed of the bullet as it leaves the plywood?

Ans :

1. The kinetic energy K is given by $K = \frac{1}{2}mv^2$.
2. Kinetic energy is a scalar quantity.
3. Initial kinetic energy

$$\begin{aligned} K_i &= \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(50.0 \times 10^{-3}\text{kg})(200\text{m/s})^2 \\ &= \frac{1}{2} \times 0.05 \times 40000 = 1000\text{J}. \end{aligned}$$

4. Final kinetic energy

$$\begin{aligned} K_f &= 10\% \text{ of } K_i \\ &= 0.10 \times 1000\text{J} = 100\text{J}. \end{aligned}$$

$$K_f = \frac{1}{2}mv_f^2$$

$$100\text{J} = \frac{1}{2}(50.0 \times 10^{-3}\text{kg})v_f^2.$$

$$\begin{aligned} v_f^2 &= \frac{2 \times 100}{0.05} = \frac{200}{0.05} \\ &= 4000. \end{aligned}$$

$$v_f = \sqrt{4000} \approx 63.2\text{m/s}.$$

- 207.** A bob of mass ' m ' is suspended by a light string of length L . It is imparted a horizontal velocity v_0 at its lowest point A, such that it completes a semi-circular trajectory in the vertical plane. The string becomes slack (tension becomes zero) only on reaching the topmost point, C, of its trajectory. The potential energy is taken to be zero at the lowest point A.

Questions:

1. What is the principle of conservation of mechanical energy?
2. Which forces act on the bob? Which of these forces does no work on the bob during its motion?
3. If v_C is the speed at the topmost point C, what is the mechanical energy at C in terms of v_C , m , g , and L ? What is the condition for the string to become slack at C?
4. Using the conservation of mechanical energy and the condition for slackening at C, find the initial velocity v_0 required at point A.

Ans :

1. The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.
2. The two external forces on the bob are gravity and the tension (T) in the string. The tension (T) does no work since the displacement of the bob is always normal (perpendicular) to the string.
3. At C, the height is $2L$. The mechanical energy at C is

$$E_C = \frac{1}{2}mv_C^2 + mg(2L).$$

For the string to become slack at C, the tension $T_C = 0$. At this point, the centripetal force is provided by gravity:

$$mg = \frac{mv_C^2}{L}.$$

4. Mechanical energy at A is $E_A = \frac{1}{2}mv_0^2$. From the condition at C, $v_C^2 = gL$. Substituting into energy at C:

$$E_C = \frac{1}{2}m(gL) + 2mgL = \frac{5}{2}mgL.$$

By conservation of mechanical energy,

$$E_A = E_C:$$

$$\frac{1}{2}mv_0^2 = \frac{5}{2}mgL$$

$$v_0^2 = 5gL$$

$$v_0 = \sqrt{5gL}.$$

- 208.** An elevator can carry a maximum load of 1800 kg (elevator + passengers). It is moving up with a constant speed of 2 m/s. The frictional force opposing the motion is 4000 N. (Take $g = 10\text{m/s}^2$). Questions:

1. How is instantaneous power P delivered by a force F related to the velocity v of the object?
2. If the elevator is moving up at a constant speed, what is the net force acting on it?
3. Calculate the total downward force on the elevator due to its weight and the frictional force.
4. Determine the minimum power that must be delivered by the motor to the elevator to maintain this constant upward speed. Express this in watts and horsepower ($1\text{hp} = 746\text{W}$).

Ans :

1. Instantaneous power $P = \mathbf{F} \cdot \mathbf{v}$.
2. If the elevator is moving at a constant speed, its acceleration is zero. Therefore, by Newton's first law, the net force acting on it is zero.

$$= \left(\frac{100 \times 0.3}{0.2} \right)^2$$

Hence, $v_2 = \frac{100 \times 0.3}{0.2} = 150 \text{ ms}^{-1}$

- 191.** A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley's direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Ans :

Given, $u_1 = 36 \text{ km/h} = 10 \text{ ms}^{-1}$

The child gives an impulse to the trolley at the beginning and then runs on the trolley with a uniform speed of 4 ms^{-1} with respect to the trolley's new velocity. If v be the change in speed of the trolley due to this impulse, then according to the principle of conservation of momentum for an observer outside, we get,

$$m_1 u_1 + m_2 u_2 = m_1 u_1 + m_2 v_2$$

$$200(10 + v) + 20(10 + v - 4) = 200 \times 10 + 20 \times 10$$

$$2000 + 200v + 120 + 20v = 2200$$

$$220v = 80$$

$$v = 0.36 \text{ ms}^{-1}$$

$$\text{Trolley's new speed} = 10 + 0.36 = 10.36 \text{ ms}^{-1}$$

Time taken by the child to cross the trolley

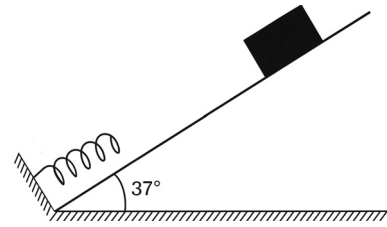
$$t = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{10}{4} = 2.5 \text{ s}$$

Distance moved by the trolley from the time the child begins to run = new speed $\times$ time = 10.36×2.5

$$= 25.9 \text{ m}$$

- 192.** Figure shows a spring fixed at the bottom end of all incline of inclination 37° . A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and block and (b) the spring constant of the spring. Take $g = 10 \text{ m s}^{-2}$



Ans :

Given, $S_1 = 4.8 \text{ m}$

The compression $x = 20 \text{ cm} = 0.2 \text{ m}$

Gain in kinetic energy = Work done in compressing the spring + Work done against friction

$$mg \sin 37^\circ (x + 4.8)$$

$$= \frac{1}{2} kx^2 + \mu NS mg \times \frac{3}{5} (0.2 + 4.8)$$

$$= \frac{1}{2} k(0.2)^2 + \mu \times mg \cos 37^\circ \times (0.2 + 4.8)$$

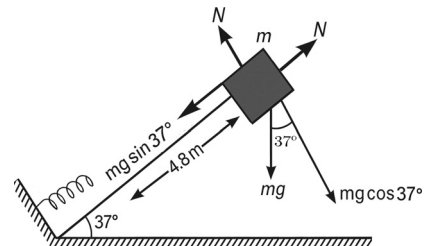
$$2 \times 10 \times \frac{3}{5} \times 5 = \frac{1}{2} k \times 0.4 + \mu \times 2 \times 10 \times \frac{4}{5} \times 5$$

$$= 0.02k + 80\mu$$

$$80\mu + 0.02k = 60 \quad \dots(1)$$

When the spring goes up by a distance of 1 m then

$$\frac{1}{2} k(0.2)^2 = mg \sin 37^\circ \times 1 - \mu \times 2 \times 10 \times \frac{4}{5} \times 5$$



$$60 = 0.02k + 80\mu$$

$$80\mu + 0.02k = 60$$

When the spring goes up by a distance of 1 m, then

$$\frac{1}{2} k(0.2)^2 = mg \sin 37^\circ \times 1 - \mu mg \cos 37^\circ \times 1$$

$$\frac{1}{2} k(.04) = 2 \times 10 \times \frac{3}{5} - \mu \times 2 \times 10 \times \frac{4}{5}$$

$$0.2k = 12 - 16\mu$$

$$16\mu - 0.02k = -12 \quad \dots(2)$$

(1) and (2) gives

$$96\mu = 48 \quad \dots(3)$$

$$\mu = \frac{48}{96} = \frac{3}{5} = \frac{1}{2}$$

(ii) When the body B is at rest *i.e.*, $u_2 = 0$

From equations (5) and (6),

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 \quad \dots(7)$$

and
$$v_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1 \quad (8)$$

Sub case. (a) when,

$$m_1 = m_2$$

From (7),
$$v_1 = 0$$

From (8),
$$v_2 = \frac{2m_1 u_1}{2m_1} = u_1$$

Thus, if a body suffers an elastic collision with another body of the same mass at rest, the velocities of two bodies are interchanged after collision. There is a complete transfer of energy.

(b) When body in motion has negligible mass

i.e.
$$m_1 \ll m_2$$

From (7),
$$v_1 = -u_1$$

From (8),
$$v_2 = 0$$

if a light body suffers an elastic collision with a heavy body at rest, the light body rebounds with almost an equal and opposite velocity and the heavy body, remains at rest. There is no transfer of energy.

(c) When body at rest has negligible mass *i.e.*

$$m_2 \ll m_1$$

From (7),
$$v_1 = u_1$$

From (8),
$$v_2 = 2u_1$$

Thus, velocity of heavy body remains unchanged but lighter body moves with velocity twice the its original velocity.

183. Define elastic and inelastic collisions. A lighter body collides with a much more massive body at rest. Prove that the direction of the lighter body is reversed and massive body remains at rest.

Ans :

Elastic collision is one in which there is absolutely no loss of kinetic energy and linear momentums *i.e.*, linear momentum and kinetic energy are conserved in an elastic collision.

Inelastic collision is one in which there is some loss of kinetic energy but linear momentum of the system remains the same. Hence, kinetic energy is not conserved in an inelastic collision.

Let m and M be the masses of light and massive bodies respectively. If u_1 is the velocity of lighter body before it strikes the massive body at rest

(*i.e.* at $u_2 = 0$) and v_1, v_2 be their velocities after collisions, then

$$mu_1 + M \times 0 = mv_1 + Mv_2$$

$$mu_1 = mv_1 + Mv_2$$

$$m(u_1 + v_1) = Mv_2 \quad \dots(1)$$

Again,
$$\frac{1}{2}mu_1^2 + \frac{1}{2}M(0)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$\frac{1}{2}m(u_1^2 - v_1^2) = \frac{1}{2}Mv_2^2$$

$$m(u_1^2 - v_1^2) = \frac{1}{2}Mv_2^2 \quad \dots(2)$$

Dividing (2) by (1), we have

$$u_1 + v_1 = v_2 \text{ So } v_1 = v_2 - u_1$$

Substituting in (1), we have

$$m(u_1 - v_1) = M(u_1 + v_1)$$

$$mu_1 - mv_1 = Mu_1 + Mv_1$$

$$(m - M)u_1 = v_1(M + m)$$

$$v_1 = \left(\frac{m - M}{M + m} \right) u_1 \quad \dots(3)$$

Substituting the value of $v_1 = u_1$ in equation (1), we have

$$m[u_1 - (v_2 - u_1)] = Mv_2$$

$$mu_1 - mv_2 + mu_1 = Mv_2$$

$$2mu_1 = (M + m)v_2$$

$$v_2 = \left(\frac{2m}{M + m} \right) u_1$$

If m is very light as compared to massive body of mass M , then $m \rightarrow 0$ when compared to M . Hence equations (3) and (4) reduce to

$$v_1 = \frac{0 - M}{M + 0} u_1$$

$$v_1 = -u_1$$

Light body is reflected back with reversed velocity

Also,
$$v_2 = \left(\frac{2 \times 0}{M + 0} \right) u_1$$

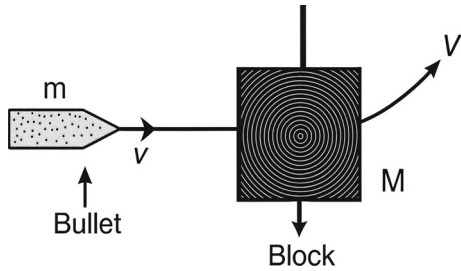
$$v_2 = 0$$

Massive body remains at rest.

184. What is two dimensional elastic collision ? Discuss collision of two bodies two dimensions. Deduce from it the velocities of bodies in one dimension.

Ans :

When two bodies moving along the same straight line collide with each other and after collision move in two different directions, it is called a collision in



Mass of block of wood,

$$M = 0.4 \text{ kg}$$

Let V be the velocity of system consisting of block and the bullet having a mass $M + m$.

According to the law of conservation of momentum,

$$mv = (m + M) V$$

$$0.012 \times 70 = (0.012 + 0.4) V$$

$$V = \frac{0.012 \times 70}{0.412} = 2.04 \text{ ms}^{-1}$$

$$V^2 - u^2 = 2gh$$

$$(2.04)^2 = 2 \times 9.8 \times h$$

$$h = \frac{(2.04)^2}{2 \times 9.8} = 0.212 \text{ m}$$

$$h = 21.2 \text{ cm}$$

Heat produced in the block

$$= \text{loss in Kinetic energy}$$

$$= \text{Kinetic energy of bullet}$$

$$- \text{Kinetic energy of (bullet+block)}$$

Heat produced in the block

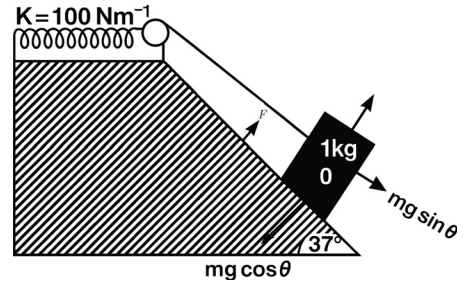
$$= \frac{1}{2}mv^2 - \frac{1}{2}(M + m)V^2$$

$$= \frac{1}{2} \times 12 \times 70 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

$$= 29.4 - 0.9 = 28.5 \text{ J.}$$

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- 196.** A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in the figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assuming that the spring has negligible mass and the pulley is frictionless.



Ans :

Given, $m = 1 \text{ kg}, k = 100 \text{ N m}^{-1}$

$$x = 0.1$$

From the figure, we have

$$R = mg \cos \theta$$

$$F = \mu R = \mu mg \cos \theta$$

Net force in the block along the plane

$$= mg \sin \theta - F$$

$$= mg \sin \theta - \mu mg \cos \theta$$

Hence, work done,

$$W = mg (\sin \theta - \mu \cos \theta) x$$

In equilibrium,

$$W = \text{Potential energy of stretched spring}$$

$$mg (\sin \theta - \mu \cos \theta) x = \frac{1}{2} kx^2$$

$$1 \times 10 (\sin 37^\circ - \mu \cos 37^\circ) (0.1)$$

$$= \frac{1}{2} \times 100 \times (0.1)^2$$

$$= 0.601 - \mu (0.789) = 0.5$$

$$\mu (0.789) = 0.601 - 0.5$$

$$\mu = 0.125$$

- 197.** A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1} . It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

Ans :

Given, Mass of the bolt = 0.3 kg

Length of the elevator = 3 m

Uniform speed of the elevator = 7 ms^{-1}

As the bolt will hit the floor of the elevator, the potential energy will be changed into the heat given by

$$H = mgh = 0.3 \times 9.8 \times 3$$

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(a) $\mu = 0.5$

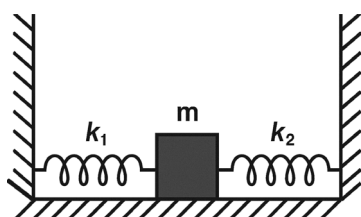
(b) $16 \times \frac{1}{2} - 0.02k = 12$

$$-0.02k = -12 - 8$$

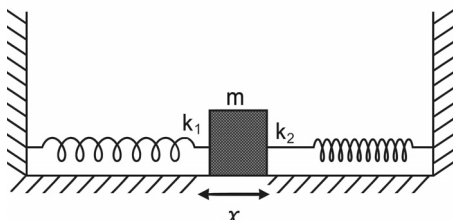
$$k = \frac{-20}{-0.02}$$

$$k = 1000 \text{ Nm}^{-1}$$

- 193.** A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.



Ans :



When the body of mass ' m ' is displaced by distance ' x ' spring on right is compressed by ' x ' and spring on left is extended by ' x '. So it is this potential energy.

$\frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$ which is converted into kinetic energy $\frac{1}{2}mv^2$. So,

$$\frac{1}{2}mv^2 = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$$

$$v^2 = \frac{k_1}{m}x^2 + \frac{k_2}{m}x^2$$

$$v = \sqrt{\frac{k_1 + k_2}{m}}x$$

- 194.** A 20 g bullet pierces through a plate of mass $M_1 = 1 \text{ kg}$ and then comes to rest inside a second plate of mass $M_2 = 2.98 \text{ kg}$ as shown in Figure. It is found that the two plates, initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is

between M_1 and M_2 . Neglect any loss of material of the plates, due to action of bullet.

Ans :

Let v_1 = Initial velocity of the bullet

v_2 = Velocity with which each plate moves

Given,

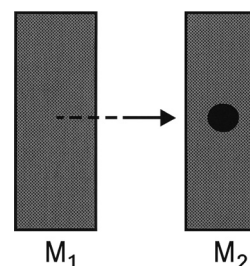
Momentum of bullet = momentum of plate M_1 + momentum of the plate M_2 and the bullet. Hence Initial momentum

= Final momentum

$$mv_1 = M_1v_2 + (M_2 + m)v_2$$

$$0.02v_1 = 1 \times v_2 + (2.98 + 0.02)v_2$$

$$v_1 = 200v_2 \quad \dots(1)$$



Let the velocity of the bullet when it comes out of the plate M_1 be v_3 . The momentum of the bullet between plates M_1 and M_2 is equal to the momentum of plates M_2 plus the bullet

$$0.02v_3 = (0.02 + 2.98)v_2$$

$$v_3 = 150v_2 \quad \dots(2)$$

Now, $\% \text{ loss} = \frac{v_1 - v_2}{v_1} \times 100$

$$= \frac{200v_2 - 150v_2}{200v_2} \times 100 = 25\%$$

- 195.** A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Ans :

Given,

Mass of bullet, $m = 0.012 \text{ kg}$

Speed of bullet, $v = 70 \text{ ms}^{-1}$

CASE BASED QUESTIONS

- 204.** A woman pushes a trunk on a railway platform which has a rough surface. Initially, she applies a constant force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired, and her applied force reduces linearly with distance from 100 N at 10 m to 50 N at 20 m. The frictional force opposing the motion is constant at 50 N throughout the 20 m displacement.

Questions:

- How is work done by a constant force defined?
- When a force varies with displacement, how can the work done be calculated from a force-displacement graph?
- Calculate the work done by the woman in the first 10 m of displacement.
- Calculate the total work done by the woman over the entire 20 m. What is the work done by the frictional force over the total 20 m displacement?

Ans :

- Work done by a constant force is defined as the product of the component of the force in the direction of the displacement and the magnitude of this displacement ($W = (F \cos \theta) d = \mathbf{F} \cdot \mathbf{d}$).
- When a force varies with displacement, the work done is equal to the area under the force-displacement curve. It can be calculated by summing the areas of small rectangles or, more precisely, by integration ($W = \int F(x) dx$).
- In the first 10 m, the force is constant at 100 N. So, work done

$$W_1 = F \times d$$

$$= 100\text{N} \times 10\text{m} = 1000\text{J}.$$

- Work done in the first 10 m = 1000 J. For the next 10 m (from 10 m to 20 m), the force reduces linearly from 100 N to 50 N. The work done in this part is the area of a trapezium:

$$W_2 = \frac{1}{2}(100\text{N} + 50\text{N}) \times (20\text{m} - 10\text{m})$$

$$= \frac{1}{2} \times 150\text{N} \times 10\text{m} = 750\text{J}$$

Total work done by the woman

$$= W_1 + W_2$$

$$= 1000\text{J} + 750\text{J} = 1750\text{J}.$$

Work done by the frictional force ($f = -50\text{N}$ since it opposes motion) over 20 m:

$$W_f = f \times d$$

$$= (-50\text{N}) \times 20\text{m} = -1000\text{J}.$$

- 205.** A raindrop of mass 1.00 g falls from a height of 1.00 km. It experiences a downward gravitational force and an opposing resistive (drag) force from the air. It hits the ground with a speed of 50.0 m/s. Assume the drop is initially at rest. (Take $g = 10\text{m/s}^2$).

Questions:

- State the work-energy theorem.
- What form of energy does the raindrop possess initially due to its height, and what form does it possess due to its motion?
- Calculate the work done by the gravitational force on the raindrop.
- What is the work done by the unknown resistive force on the raindrop during its fall? Is this work positive or negative?

Ans :

- The work-energy theorem states that the change in kinetic energy of a particle is equal to the work done on it by the net force.
- Initially, due to its height, the raindrop possesses gravitational potential energy. Due to its motion, it possesses kinetic energy. The problem phrasing guides to calculate change in kinetic energy and work by gravity, then use WE theorem for work by resistive force.
- Work done by gravitational force

$$W_g = mgh$$

$$= (1.00 \times 10^{-3}\text{kg}) \times (10\text{m/s}^2) \times (1.00 \times 10^3\text{m})$$

$$= 10.0\text{J}.$$

- Initial kinetic energy $K_i = 0$. Final kinetic energy

$$K_f = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(1.00 \times 10^{-3}\text{kg})(50.0\text{m/s})^2$$

$$= \frac{1}{2} \times 10^{-3} \times 2500 = 1.25\text{J}.$$

Change in kinetic energy

$$\Delta K = K_f - K_i = 1.25\text{J}.$$

According to the work-energy theorem, $\Delta K = W_g + W_r$, where W_r is the work done by the resistive force.

So,

$$W_r = \Delta K - W_g$$

$$= 1.25\text{J} - 10.0\text{J} = -8.75\text{J}.$$

The work is negative.

- 206.** A police officer fires a bullet of mass 50.0 g with an initial speed of 200 m/s into soft plywood of thickness 2.00 cm. The bullet emerges from the plywood with only 10% of its initial kinetic energy.

Questions:

$$= \frac{9 \times 98}{100}$$

$$H = 8.82 \text{ J}$$

It is immaterial in this case whether the elevator is moving or not. So, if the elevator were stationary even then the heat produced will be equal to 8.82 J.

- 198.** A trolley of mass 200 kg moves with a uniform speed of 36 km h^{-1} on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Ans :

The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms^{-1} with respect to the trolley's new velocity. Applying the law of conservation of momentum,

$$(200 + 20) \times 10 = 20(10 + v - 4) + 200(10 + v') \quad \dots(1)$$

where $10 + v' =$ new velocity of the trolley

and v' is the change in speed of the trolley

and $v' = 36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}$

Substituting in (1),

$$\text{Hence, } v' = \frac{4}{11} \text{ ms}^{-1}$$

The trolley's new speed

$$= 10 + \frac{4}{11} = 10.36 \text{ ms}^{-1}$$

The child takes 2.5 s to run

Distance covered in 2.5 s

$$= 10.36 \times 2.5 = 25.9 \text{ m}$$

- 199.** How does kinetic energy of a body change, if its momentum is doubled?

Ans :

$$\text{Since, kinetic energy, } E = \frac{p^2}{2m}$$

If momentum is doubled i.e.

$$\text{when, } p_1 = 2p$$

$$\begin{aligned} \text{Kinetic energy, } E_1 &= \frac{p_1^2}{2m} \\ &= \frac{(2p)^2}{2m} = 4 \left(\frac{p^2}{2m} \right) \\ &= 4E \end{aligned}$$

Hence, kinetic energy will become 4 times.

- 200.** A body is being raised to a height h from the surface of earth. What is the sign of work done by
(a) applied force
(b) gravitational force.

Ans :

Since, Work done, $W = F s \cos \theta$

- (a) The work done by applied force is positive as

$$\theta = 0^\circ, \text{ so}$$

$$W = F s \cos 0^\circ = F s = +ve$$

- 201.** Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

Ans :

Here,

$$m = 400 \text{ kg, } s = 2 \text{ m}$$

$$g = 9.8 \text{ ms}^{-2} \text{ and } \theta = 90^\circ$$

$$W = F s \cos 90^\circ = 0$$

$$[\text{since, } \cos 90^\circ = 0]$$

- 202.** What is the relation between kinetic energy and momentum?

Ans :

Kinetic energy of a body of mass m moving with velocity v is given by

$$\begin{aligned} E &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} \frac{m^2 v^2}{m} = \frac{m^2 v^2}{2m} \\ &= \frac{p^2}{2m} \quad [\because p = mv] \end{aligned}$$

- 203.** An electron and proton have equal momentum. Which has more kinetic energy and what is the ratio between the kinetic energy of electron and proton?

Ans :

$$\text{Kinetic energy of electron, } K_e = \frac{p^2}{2m_e}$$

$$\text{Kinetic energy of proton, } K_p = \frac{p^2}{2m_p}$$

$$\begin{aligned} \text{Hence, } \frac{K_e}{K_p} &= \frac{\frac{p^2}{2m_e}}{\frac{p^2}{2m_p}} \times \frac{2m_p}{p^2} \\ \frac{K_e}{K_p} &= \frac{m_p}{m_e} = \frac{1837m_e}{m_e} = \frac{1837}{1} \end{aligned}$$

3. Weight $W = mg$

$$= 1800\text{kg} \times 10\text{m/s}^2 = 18000\text{N}.$$

Total downward force = Weight + Frictional force = $18000\text{N} + 4000\text{N} = 22000\text{N}$.

4. To move up at a constant speed, the motor must exert an upward force F_{motor} equal to the total downward force. So, $F_{\text{motor}} = 22000\text{N}$.

Power

$$P = F_{\text{motor}} \times v$$

$$= 22000\text{N} \times 2\text{m/s} = 44000\text{W}.$$

In horsepower,

$$P = \frac{44000}{746}\text{hp} \approx 58.98\text{hp}$$

(The rounds to 59 hp).

- 209.** Two masses, m_1 and m_2 , undergo a one-dimensional elastic collision. Mass m_1 has an initial velocity v_{1i} , and mass m_2 is initially at rest ($v_{2i} = 0$). After the collision, their final velocities are v_{1f} and v_{2f} .

Questions:

1. In an elastic collision, which two physical quantities are conserved?
2. Write down the equations for the conservation of linear momentum and kinetic energy for this collision.
3. If $m_1 = m_2$, what are the final velocities v_{1f} and v_{2f} in terms of v_{1i} ?
4. If m_2 is very large compared to m_1 (e.g., $m_2 \gg m_1$), what happens to v_{1f} and v_{2f} ?

Ans :

1. In an elastic collision, both total linear momentum and total kinetic energy are conserved.
2. Conservation of linear momentum:

$$m_1 v_{1i} + m_2(0) = m_1 v_{1f} + m_2 v_{2f}.$$

Conservation of kinetic energy:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2(0)^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2.$$

3. If $m_1 = m_2$, then $v_{1f} = 0$ and $v_{2f} = v_{1i}$. The first mass comes to rest, and the second mass moves off with the initial speed of the first.
4. If $m_2 \gg m_1$, then $v_{1f} \approx -v_{1i}$ (the lighter mass reverses its velocity) and $v_{2f} \approx 0$ (the heavier mass remains almost undisturbed).

- 210.** A force $\mathbf{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})$ units acts on an object, causing a displacement $\mathbf{d} = (5\hat{i} + 4\hat{j} + 3\hat{k})$ units. The work done by this force is given by the scalar product $\mathbf{F} \cdot \mathbf{d}$.

Questions:

1. How is the scalar product (or dot product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ defined in terms of their magnitudes and the angle θ between them?

2. Is the scalar product of two vectors a scalar or a vector? Does it follow the commutative law?
3. Calculate the work done $W = \mathbf{F} \cdot \mathbf{d}$ using the component form of the scalar product.
4. What does it mean if the scalar product of two non-zero vectors is zero?

Ans :

1. The scalar product is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta.$$

2. The scalar product of two vectors is a scalar quantity. Yes, it follows the commutative law:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}.$$

3. $W = \mathbf{F} \cdot \mathbf{d}$

$$= F_x d_x + F_y d_y + F_z d_z$$

$$= (3)(5) + (4)(4) + (-5)(3)$$

$$= 15 + 16 - 15 = 16 \text{ units.}$$

4. If the scalar product of two non-zero vectors is zero ($\mathbf{A} \cdot \mathbf{B} = 0$), it means that the two vectors are perpendicular to each other (since $A \neq 0$, $B \neq 0$, so $\cos \theta$ must be 0, which implies $\theta = 90^\circ$).

- 211.** A man lifts a bucket of mass 10 kg out of a well by means of a rope tied to the bucket. He lifts it steadily to a height of 5 m. (Take $g = 9.8\text{m/s}^2$).

Questions:

1. What is the direction of the force applied by the man relative to the displacement of the bucket? What is the direction of the gravitational force relative to the displacement?
2. Calculate the work done by the man in lifting the bucket. Is this work positive or negative?
3. Calculate the work done by the gravitational force in this case. Is this work positive or negative?
4. If the man holds the 10 kg bucket steady at the top for 30 seconds, how much work does he do on the bucket during this time?

Ans :

1. The force applied by the man is upwards, and the displacement is upwards. So, the angle is 0° . The gravitational force is downwards, while the displacement is upwards. So, the angle is 180° .
2. The force applied by the man

$$F_m = mg$$

$$= 10\text{kg} \times 9.8\text{m/s}^2 = 98\text{N}$$

(to lift it steadily).

Work done by man

$$W_m = F_m d \cos(0^\circ)$$

2. In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27 \text{ \AA}$ ($1 \text{ \AA} = 10^{-10} \text{ m}$). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Ans :

Take H atom as the origin.

The coordinate r_{cm} of the centre of mass is

$$r_{\text{cm}} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

Let Mass of Hydrogen = m

Mass of Chlorine = 35.5

$$\begin{aligned} r_{\text{cm}} &= \frac{m \times 0 + 35.5 \text{ m} \times 27}{m + 35.5 \text{ m}} \\ &= \frac{35.5 + 27}{36.5} = 1.24 \text{ \AA} \end{aligned}$$

Therefore, location is on the line joining H and Cl nuclei at a distance of $1.24 \text{ \AA}$ from the H end.

3. A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner. What is the speed of the center of mass of the (trolley + child) system ?

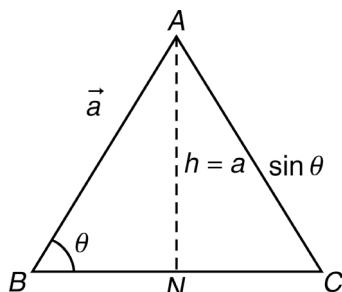
Ans :

The speed of the center of mass of the (trolley + child) system remains unchanged (equal to v) because no external force acts on the system. The forces involved in running on the trolley are internal to this system.

4. Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.

Ans :

Let the two sides AB and BC of triangle ABC is represented by two vectors $\vec{a}$ and $\vec{b}$ as shown in the figure.



$$\begin{aligned} \text{The area of triangle} &= \frac{1}{2} \times \text{base} \times \perp \text{ distance} \\ &= \frac{1}{2} BC \times AN \\ &= \frac{1}{2} b \times h = \frac{1}{2} b a \sin \theta \\ &= \frac{1}{2} |\vec{a} \times \vec{b}| \end{aligned}$$

$$\text{Area of } \Delta = \frac{1}{2} \times \text{magnitude of } (\vec{a} \times \vec{b})$$

5. Show that $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Ans :

Let a parallelepiped OLMFEHGN be formed by three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ as shown in the figure. Here the area of the face OLMN being a parallelogram bounded by $\vec{b}$ and $\vec{c}$ is given by

$$\vec{s} = \vec{b} \times \vec{c}$$

and is normal to the surface OLMN.

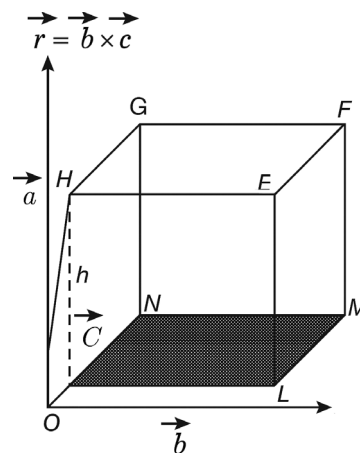
The height h of the normal from point H, on the face OLMN is given by

$$h = |\vec{a}| \cos \theta$$

where θ is the angle between $\vec{s}$ and $\vec{a}$. Volume of the parallelepiped,

$V = \text{Area of face OLMN} \times \text{Normal from H on OLMN}$

$$\begin{aligned} V &= |\vec{s}| h = |\vec{s}| |\vec{a}| \cos \theta \\ V &= |\vec{a}| |\vec{s}| \cos \theta = |\vec{a}| |\vec{b} \times \vec{c}| \cos \theta \\ V &= \vec{a} \cdot (\vec{b} \times \vec{c}) \end{aligned}$$



So the triple scalar product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to the volume of the parallelepiped formed by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

CHAPTER 6

System of Particles and Rotational Motion

SUMMARY

1. CENTRE OF MASS

The centre of mass of a system is the point at which the entire mass of the system can be considered to be concentrated, and the system moves as if all external forces are applied at that point.

1.1 Significance of Centre of Mass

Using the concept of the centre of mass simplifies the analysis of motion, momentum, and energy, as it allows us to study the system as a whole instead of dealing with each individual particle separately.

1.2 Centre of Mass of Two Particle System

In a simple case of system of two particles of masses m_1 and m_2 at distances r_1 and r_2 respectively, from same origin O , the centre of mass of the system is defined as the distance r_{cm} from the origin O .

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

This point has the property that the product of the total mass of the system ($M = m_1 + m_2$) times the distance of this point from the origin is equal to the sum of the products of the mass of each particle by its distance from the origin.

$$(m_1 + m_2)r_{cm} = Mr_{cm} = m_1 r_1 + m_2 r_2 \quad \dots(1)$$

If two particles are of same mass i.e. $m_1 = m_2 = m$ (say) then equation (i) becomes

$$(m + m)r_{cm} = mr_1 + mr_2$$

Thus we find that in a system consisting of two particles of same masses, the centre of mass lies at the centre of two masses.

1.3 Centre of Mass of n -particles System

If a system consists of n -particles of masses $m_1, m_2, \dots, m_n$ having position vectors $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$ respectively then the position vector of centre of

mass of the system $\vec{r}_{cm}$ is given by

$$\begin{aligned} \vec{r}_{cm} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{m_1 + m_2 + \dots + m_n} \\ &= \sum_{i=1}^n \frac{m_i \vec{r}_i}{m_i} \end{aligned}$$

Examples of Motion of Centre of Mass

Following are a few examples of motion of centre of mass of a systems :

- Motion of Indian club.
- Motion of the centre of mass of fire crackers exploding in air.
- Earth moon system
- Binary star system.

2. RIGID BODY

A rigid body is an object that maintains a fixed shape and size, which do not change during its motion. It can be considered as being composed of many small parts, each treated as a point mass.

The rigidity means that the internal forces among the point mass particles are such that the mutual distances between the points never change.

	Shape of body	Position of centre of mass
1.	Uniform rod	Mid point of the rod
2.	Circular ring	Centre of ring
3.	Circular disc	Centre of disc
4.	A plane lamina such as rectangle, parallelogram, square.	Point of intersection of diagonals.
5.	Sphere	Centre
6.	Cylinder	Mid point of axis
7.	Triangular plane lamina	Point of intersection of median
8.	Cubical block	Point of intersection of diagonals.

3. ROTATIONAL MOTION

A body exhibits rotational motion when all its particles move in circular paths within parallel planes. The centers of these circles lie along a fixed straight line, known as the axis of rotation, which is perpendicular to these planes.

4. TORQUE (τ) (OR MOMENT OF FORCE)

The turning effect of a force about an axis of rotation is called torque. It is a vector quantity, and its magnitude is equal to the product of the force's magnitude and the perpendicular distance from the axis of rotation to the line of action of the force.

So, $\tau = \text{magnitude of force} \times \text{perpendicular distance from the axis of rotation}$

In vector form,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

In a pure rotational motion, every particle moves in a circle about the same straight line, which is called axis of rotation. A force can be suitably applied to a rigid body to make it rotate. The turning effect of the force depends on :

- the magnitude of the force,
- the direction of the force and
- the distance from the axis of rotation.

Thus the torque is another word for movement and is a vector quantity having unit in SI as newton metre (Nm) and dimensional formula $[ML^2T^{-2}]$, same as that of work.

4.1 Principle of Moments of Rotational Equilibrium

When a body is in rotational equilibrium, the sum of clockwise moments about any point is equal to the sum of anticlockwise moments about that point i.e., clockwise moments = anticlockwise moments

$$\vec{F}_1 \times \vec{d}_1 = \vec{F}_2 \times \vec{d}_2$$

$$\text{Load} \times \text{load arm} = \text{Effort} \times \text{effort arm}$$

5. COUPLE

A pair of equal and opposite parallel forces acting on a rigid body along different lines of action constitute a couple.

The turning effect of a couple is called moment of the couple or torque exerted by the couple.

$$\text{Torque} = \text{force} \times \text{arm of the couple.}$$

Moment of couple is said to be positive, if the couple rotates the body in anticlockwise direction and negative, if it rotates the body in the clockwise direction.

6. ANGULAR MOMENTUM ($\vec{L}$)

The turning moment of the particle about the axis of rotation is called angular momentum and is measured as the product of the linear momentum of the particle and perpendicular distance of the particle from the axis of rotation.

So $L = \text{linear momentum} \times \text{perpendicular distance from the axis of rotation}$

In vector form

$$\vec{L} = \vec{r} \times \vec{p}$$

Relation between torque and angular momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

6.1 Law of Conservation of Angular Momentum

In an isolated system, the total angular momentum of the system remains conserved.

So $L = I\omega = \text{constant}$

$$I_1\omega_1 = I_2\omega_2$$

7. ANGULAR VELOCITY (ω)

The time rate of change of angular displacement is called angular velocity

$$\text{i.e. } \omega = \frac{\theta}{t} = \frac{d\theta}{dt}$$

$$\text{and } \vec{\omega} = \vec{r} \times \vec{\omega}$$

8. MOMENT OF INERTIA (I)

The moment of inertia of a rigid body about a given axis of rotation is the sum of the products of masses of the various particles and squares of their perpendicular distance from the axis of rotation

$$\text{i.e., } I = \sum_{i=1}^n m_i r_i^2$$

Dimensional formula of moment of inertia

$$[M][L^2] = [ML^2]$$

Moment of inertia of a body depends upon :

- Distribution of constituent particles of the body about the axis of rotation.
- Distance of particles from the axis of rotation.
- Mass of the body.

9. RADIUS OF GYRATION (K)

The radius of gyration is the distance from the axis of rotation at which, if whole mass of a body was supposed to be concentrated, the moment of inertia would be the same as with the actual distribution of the mass of the body into small particles.

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$

9.1 Moment of Inertia of Different Bodies

1. Circular ring

- (a) About an axis $\perp$ through its centre

$$I = MR^2$$

- (b) About its diameter

$$I_d = \frac{1}{2} MR^2$$

2. Circular disc

- (a) About an axis $\perp$ through its centre

$$I = \frac{1}{2} MR^2$$

- (b) About its diameter

$$I_d = \frac{1}{4} MR^2$$

3. A circular cylinder

- (a) About axis of symmetry

$$I = \frac{1}{2} MR^2$$

- (b) About an axis passing through its centre and $\perp$ to axis of symmetry

$$I = M \left[\frac{R^2}{4} + \frac{l^2}{12} \right]$$

4. Hollow cylinder

- (a) About its axis of symmetry

$$I = MR^2$$

- (b) About an axis through the centre and $\perp$ to axis of symmetry

$$I = M \left[\frac{R^2}{2} + \frac{l^2}{12} \right]$$

5. A sphere

- (a) About an axis through its centre

$$I = \frac{2}{5} MR^2$$

- (b) About tangent axis

$$I = \frac{7}{5} MR^2$$

6. Hollow sphere

- (a) About an axis through its centre

$$I = \frac{2}{3} MR^2$$

- (b) About tangent axis

$$I = \frac{5}{3} MR^2$$

10. BODY ROLLING DOWN AN INCLINED PLANE

When a body is rolling down an inclined plane without slipping, we have

$$\mu \geq \frac{I_c}{MR^2 + I_c} \tan \theta$$

and Kinetic energy = $\frac{1}{2} [MR^2 + I_c] \omega^2$

$$= \frac{1}{2} Mv^2 \left[1 + \frac{k^2}{R^2} \right]$$

Motion of a point mass tied to the string wound on a cylinder. The acceleration of mass m tied to a string which passes over a cylinder of radius R is given by

$$a = \frac{g}{1 + \frac{I}{mR^2}} \text{ or } a < g$$

And tension in the string,

$$T = \frac{mg}{1 + \frac{mR^2}{I}} \text{ or } T < mg$$

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NCERT TEXT-BOOK QUESTIONS

- Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

Ans :

- The geometrical centre.
- The geometrical centre.
- The geometrical centre.
- The geometrical centre.

The centre of mass of a body does not necessarily lie inside the body. The centre of mass may lie outside the body, as in case of a ring, a hollow sphere, a hollow cylinder, a hollow cube etc.

$$2.827d = 1.5016$$

$$d = 0.72 \text{ m} = 72 \text{ cm}$$

9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Ans :

Let ω_1 = forces acting on each front wheel

ω_2 = forces acting on each back wheel

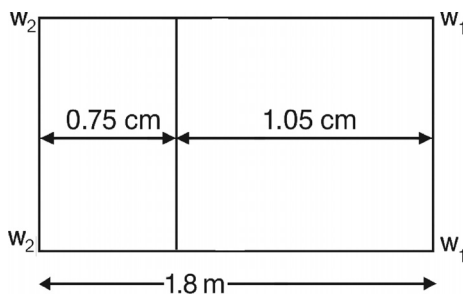
If ω = weight of the car, then

$$\omega = 2\omega_1 + 2\omega_2$$

$$\text{So } \omega_1 + \omega_2 = \frac{\omega}{2} = \frac{1800 \times 9.8}{2} = 8820 \text{ N} \quad \dots(1)$$

If the torque is acting about the centre of gravity, then

$$2\omega_1 \times 1.05 = 2\omega_2 \times 0.75$$



$$\begin{aligned} \omega_1 &= \frac{0.5\omega_2}{1.05} \\ \omega_1 &= 0.714\omega_2 \quad \dots(2) \end{aligned}$$

Putting equation (2) in (1), we get

$$\begin{aligned} 0.714\omega_2 + \omega_2 &= 8820 \\ \omega_2 &= \frac{8820}{1.714} = 5145 \text{ kg} \\ \omega_1 &= 8820 - 5145 = 3675 \text{ Nkg} \\ \omega_1 &= 8820 - 5145 = 3675 \text{ Nkg} \end{aligned}$$

10. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time ?

Ans :

Let ω_1 and ω_2 be the angular velocities of hollow cylinder and solid sphere respectively

m = mass of solid sphere and hollow sphere as well

$$\text{Moment of inertia of hollow cylinder} = \frac{1}{2}MR^2$$

$$\text{Moment of inertia of solid sphere} = \frac{2}{5}MR^2$$

Since torque is same, i.e.

$$\tau_1 = \tau_2$$

$$\frac{dL_1}{dt} = \frac{dL_2}{dt}$$

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{2}{5}MR^2\omega_2 = \frac{1}{2}MR^2\omega_1$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{5} \times \frac{2}{1} = \frac{4}{5}$$

$$\omega_1 = \frac{4}{5}\omega_2$$

Hence, $\omega_1 < \omega_2$

Therefore, angular speed of solid sphere is more after a given interval of time.

11. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?

Ans :

Kinetic energy associated with the rotation of the cylinder

$$\begin{aligned} E &= \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2}MR^2 \times \omega^2 \\ &= \frac{1}{4} \times 20 \times 25 \times 25 \times 100 \times 100 \\ &= 3125 \text{ J} \end{aligned}$$

Angular momentum = $L\omega$

$$\begin{aligned} &= \frac{1}{2} \times 20 \times .25 \times .25 \times 100 \\ &= 62.5 \text{ Js} \end{aligned}$$

12. (a) A child stands at the centre of a turn-table with his two arms outstretched. The turn-table is set rotating with an angular speed of 40 rev/min. How much is angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value ? Assume that the turn-table rotates without friction.
- (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy ?

6. Find the components along the x , y , z axes of the angular momentum $\vec{L}$ of a particle, whose position vector is $\vec{r}$ with components x , y , z and momentum is $\vec{p}$ with components p_x , p_y and p_z . Show that if the particle moves only in the x - y plane the angular momentum has only a z -component.

Ans :

Angular momentum $\vec{L}$, $\vec{r}$ and $\vec{p}$ are related by the relation

$$\vec{L} = \vec{r} \times \vec{p}$$

In three dimensions $\vec{L}$, $\vec{r}$ and $\vec{p}$ are written as

$$L_x \hat{i} + L_y \hat{j} + L_z \hat{k} = (x \hat{i} + y \hat{j} + z \hat{k}) \times (p_x \hat{i} + p_y \hat{j} + p_z \hat{k})$$

$$L_x \hat{i} + L_y \hat{j} + L_z \hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$$

Comparing the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on both sides, we get

$$L_x = yp_z - zp_y, L_y = zp_x - xp_z, L_z = xp_y - yp_x \quad \dots(1)$$

If the particle moves only in x - y plane, then $z = 0$ and $p_z = 0$

$$\text{Hence, } L = k(xp_y - yp_x)$$

As $\hat{k}$ vector corresponds to the z -direction, so we find that the angular momentum $\vec{L}$ has component in the z -direction only.

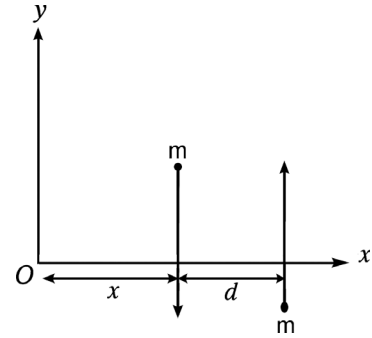
7. Two particles each of mass m and velocity v travel in opposite directions along parallel lines separated by a distance d . Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

Ans :

Let us consider a special case in which the two particles travel parallel to y -axis as shown in the figure.

The total angular momentum of the particles m and m is given by

$$\begin{aligned} \vec{L} &= \vec{L}_1 + \vec{L}_2 \\ &= (\vec{r}_1 \times \vec{p}_1) + (\vec{r}_2 \times \vec{p}_2) \\ &= -mvx(\hat{i} \times \hat{j}) + (mvx + mvd)(\hat{i} \times \hat{j}) \end{aligned}$$



$$\vec{L} = -mvx\hat{k} + mvx\hat{k} + mvd\hat{k}$$

$$\vec{L} = mvd\hat{k}$$

Thus we find that the total angular momentum is independent of the location of the origin.

8. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in the figure. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.

Ans :

If T_1 and T_2 be tensions in the strings, then For translational equilibrium

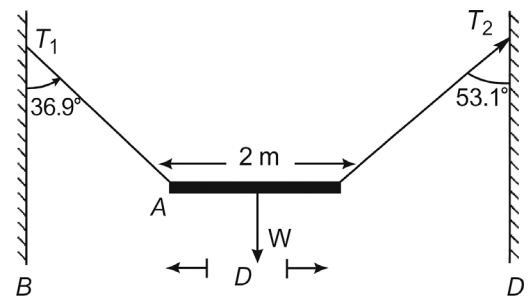
Horizontal component of the forces,

$$T_1 \sin 36.9^\circ = T_2 \sin 53.1^\circ \quad \dots(1)$$

and vertical component of the forces

$$T_1 \cos 36.9^\circ + T_2 \cos 53.1^\circ = W \quad \dots(2)$$

Given, length of the rod, $l = 2$ m



If the torque is acting about the centre of mass, then

For rotational equilibrium,

Clockwise torque = anticlockwise torque

$$[T_1 \cos 36.9^\circ]d = [T_2 \cos 53.1^\circ](l - d) \quad \dots(3)$$

Dividing equation (3) by equation (1), we get

$$d \cot 36.9^\circ = (2 - d) \cot 53.1^\circ$$

$$1.3319d = 0.7508(2 - d)$$

If we take torque about the centre of the original disc which is also the centre of gravity of the original body, then

$$\left(\frac{3}{4}\pi R^2\right)r = \left(\frac{\pi R^2}{4}\right)\frac{R}{2}$$

$$3r = \frac{R}{2}$$

$$r = \frac{R}{6}$$

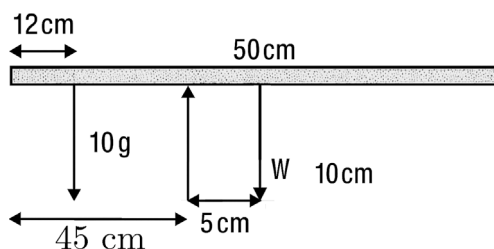
16. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick ?

Ans :

Let mass of the metre stick = m

Weight of the stick, $W = mg$

This weight is acting at the centre of the stick at 50cm from the edge as shown in the figure.



If we take the torque about 45.0 cm mark so for equilibrium, we have

Clockwise torque = anticlockwise torque

$$(50 - 45) mg = (45 - 12) g$$

$$5 mg = 330 g$$

$$m = 66 \text{ grams} = 66.0 \text{ grams}$$

17. The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m² about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 ms^{-1} and that its kinetic energy of rotation is two-thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

Ans :

Rotational kinetic energy = $\frac{2}{3}$ Translational kinetic energy

$$\text{i.e.} \quad \frac{1}{2} I \omega^2 = \frac{2}{3} \frac{1}{2} m v^2$$

$$\text{So} \quad \omega = v \sqrt{\frac{2}{3} \frac{m}{I}}$$

$$= 500 \sqrt{\frac{2}{3} \times \frac{5.30 \times 10^{-26}}{1.94 \times 10^{-46}}}$$

$$= 6.75 \times 10^{12} \text{ rad s}^{-1}$$

OBJECTIVE QUESTIONS

- What is a rigid body ideally defined as?
 - A body that easily changes shape under force.
 - A body with a changing and indefinite shape.
 - A body with a perfectly definite and unchanging shape.
 - A body where distances between particles vary greatly.

Ans :

(C) A body with a perfectly definite and unchanging shape.

A rigid body is an idealized concept where the body maintains a perfectly definite and unchanging shape, meaning the distances between all pairs of particles within the body remain constant, regardless of external forces. This is a foundational concept in rotational motion.

- In pure translational motion, what can be said about the velocity of all particles of the body at any instant?
 - All particles have different velocities at any instant.
 - All particles have the same velocity at any instant.
 - Only the center particles have the same velocity.
 - Particle velocities vary randomly across the entire body.

Ans :

(B) All particles have the same velocity at any instant.

Pure translational motion is characterized by every particle of the body moving with the same velocity at any given moment in time. There is no rotation or change in orientation of the body during this type of motion.

- What is the motion of a rigid body called when it is fixed along a straight line?
 - Pure translational motion only.
 - A combination of translation and rotation.
 - Only rotational motion.
 - Complex oscillatory motion.

Ans :

- (a) Let I_1 and I_2 be initial and final moment of inertia of child and ω_1, ω_2 be angular velocities

$$\omega_1 = \frac{40}{60} \text{ rps}$$

By angular momentum conservation,

$$I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{I_1}{I_2} \omega_1 = \frac{5}{2} \times 40$$

$$\omega_2 = 100 \text{ rpm}$$

- (b) Initial kinetic energy,

$$E_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_1 \left(\frac{40}{60} \right)^2$$

Final kinetic energy,

$$E_2 = \frac{1}{2} I_1 \omega_2^2 = \frac{1}{2} \frac{2}{5} I_1 \omega_2^2$$

$$= \frac{1}{5} I_1 \times \left(\frac{100}{60} \right)^2 \quad \left[\text{since, } I_2 = \frac{2}{5} I_1 \right]$$

$$\frac{E_2}{E_1} = \frac{I_1 \times 100 \times 100 \times 2 \times 60 \times 60}{5 \times I_1 \times 40 \times 40 \times 60 \times 60}$$

$$\frac{E_2}{E_1} = 2.5$$

$$E_2 = 2.5 E_1$$

The new kinetic energy is 2.5 times the initial kinetic energy of rotation. The child uses this internal energy to increase his rotational kinetic energy.

13. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

Ans :

$$M = 3 \text{ kg}, R = 40 \times 10^{-2} \text{ m}$$

- (i) Moment of inertia of cylinder about its own axis,

$$\begin{aligned} I &= MR^2 \\ &= 3 \times (40 \times 10^{-2})^2 \\ &= 48 \times 10^{-2} \text{ kgm}^2 \end{aligned}$$

Torque,

$$\tau = F \times R$$

$$\tau = 30 \times 40 \times 10^{-2}$$

$$\tau = 12 \text{ Nm}$$

$$I\alpha = 12$$

$$\begin{aligned} \alpha &= \frac{12}{I} = \frac{12}{48 \times 10^{-2}} = \frac{100}{4} \\ &= 25 \text{ rad s}^{-1} \end{aligned}$$

(ii)

$$F = ma$$

$$a = \frac{F}{m} = \frac{30}{3} = 10 \text{ ms}^{-2}$$

14. To maintain a rotor at a uniform angular speed of 200 rad s^{-1} , an engine needs to transmit a torque of 180 N m . What is the power required by the engine? (Note : uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

Ans :

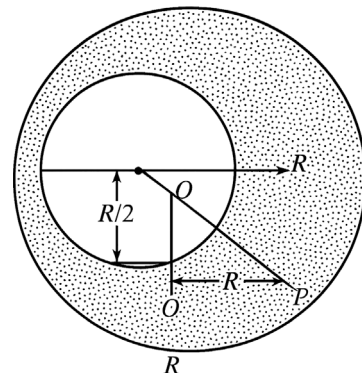
$$\begin{aligned} \text{Power} &= \text{Torque} \times \text{angular velocity} \\ &= 180 \times 200 \\ &= 36000 \text{ W} = 36 \text{ kW} \end{aligned}$$

15. From a uniform disc of radius R , a circular section of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

Ans :

When a circular section of radius $R/2$ is cut out, let the centre of gravity of the resulting body be located at point P at a distance r from the centre O of the original disc.

As the disc is uniform, so the mass of any portion If the disc will be proportional to the corresponding area.



So mass of the disc of radius $R \propto \pi R^2$

And mass of the circular section of radius

$$\frac{R}{2} \propto \frac{\pi R^2}{4}$$

Mass of resulting body

$$\propto \pi R^2 - \frac{\pi R^2}{4}$$

$$\propto \frac{3}{4} \pi R^2$$

$$= (98\text{N})(5\text{m})(1) = 490\text{J}.$$

This work is positive.

3. Work done by gravity

$$\begin{aligned} W_g &= (mg) d \cos(180^\circ) \\ &= (98\text{N})(5\text{m})(-1) = -490\text{J}. \end{aligned}$$

This work is negative.

4. If the man holds the bucket steady, the displacement is zero. Therefore, the work done by the man on the bucket during this time is zero, even if he exerts a force.

- 212.** In a nuclear reactor, a neutron of high speed (e.g., 10^7 m/s) must be slowed down to about 10^3 m/s to effectively cause fission in ^{235}U . This slowing is achieved by elastic collisions with light nuclei (like deuterium or carbon) in a material called a moderator. Consider an elastic head-on collision of a neutron (m_1) with a stationary deuterium nucleus ($m_2 \approx 2m_1$).

Questions:

- For an elastic head-on collision where the target (m_2) is initially at rest, what is the formula for the final velocity of the incident particle (v_{1f})?
- What is the fractional kinetic energy lost by the neutron, $f_i = K_{1f}/K_{1i}$?
- Using $m_2 = 2m_1$, calculate the value of f_i . What percentage of its initial kinetic energy does the neutron retain?
- What is the fractional kinetic energy gained by the moderating deuterium nucleus ($f_f = K_{2f}/K_{1i}$)? Why are light nuclei preferred for moderators?

Ans :

1. For an elastic head-on collision with m_2 at rest,

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i}.$$

2. The fractional kinetic energy retained by the neutron is

$$f_i = \frac{K_{1f}}{K_{1i}} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2.$$

The fractional kinetic energy lost would be $1 - f_i$.

3. Given $m_2 = 2m_1$,

$$\begin{aligned} f_i &= \left(\frac{m_1 - 2m_1}{m_1 + 2m_1} \right)^2 \\ &= \left(\frac{-m_1}{3m_1} \right)^2 = \left(-\frac{1}{3} \right)^2 = \frac{1}{9}. \end{aligned}$$

The neutron retains $1/9$ (or approx 11.1%) of its initial kinetic energy. Therefore, it loses $8/9$ or about 88.9%.

4. The fractional kinetic energy gained by the deuterium nucleus

$$f_f = 1 - f_i$$

$$= 1 - \frac{1}{9} = \frac{8}{9}$$

.Light nuclei are preferred because a larger fraction of the neutron's kinetic energy is transferred to them in each collision, making the slowing down process more efficient.

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Ans :

(C) Only rotational motion.

When a rigid body is constrained by being fixed along a straight line (its axis of rotation), it cannot undergo translational motion. The only possible motion for such a body is rotation around that fixed axis.

4. In the rotation of a rigid body about a fixed axis, how does every particle of the body move?

- (A) In a straight line parallel to the fixed axis always.
 (B) In an elliptical path around the central axis.
 (C) In a circle, which lies in a plane perpendicular to the axis.
 (D) In a random path within the rotating body.

Ans :

(C) In a circle, which lies in a plane perpendicular to the axis.

During the rotation of a rigid body around a fixed axis, every particle within that body traces a circular path. Each of these circles lies in a plane that is perpendicular to the axis of rotation, and the center of each circle is located on the axis.

5. What is the term for the movement of the axis of a spinning top around the vertical through its point of contact with the ground?

- (A) Oscillation
 (B) Precession
 (C) Revolution
 (D) Gyration

Ans :

(B) Precession

Precession is the term used to describe the slow, conical movement of the axis of a spinning body, such as a top, around another axis due to an external torque. In the case of a top, this is often seen as the axis sweeping out a cone shape.

6. What is the general motion of a rigid body that is not pivoted or fixed in some way?

- (A) Only pure rotation always.
 (B) Either pure translation or a combination of translation and rotation.
 (C) Only pure translation always.
 (D) Only oscillatory motion.

Ans :

(B) Either pure translation or a combination of translation and rotation.

A rigid body that is not fixed or pivoted can exhibit either pure translational motion (where all particles

move with the same velocity) or a combination of translational and rotational motion simultaneously.

7. How is the center of mass of a system of two particles with masses m_1 and m_2 at distances x_1 and x_2 from an origin O defined?

- (A) $X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$
 (B) $X = \frac{m_1 x_1 + m_2 x_2}{m_1 - m_2}$
 (C) $X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$
 (D) $X = \frac{m_1 + m_2}{m_1 x_1 + m_2 x_2}$

Ans :

(C) $X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

The center of mass (X) of a two-particle system along the x-axis is calculated as the sum of the products of each mass and its respective position, divided by the total mass of the system. This represents the mass-weighted average position.

8. Which of the following statements about the center of mass of a rigid body is incorrect?

- (A) The center of mass of a homogeneous thin rod coincides with its geometric center due to reflection symmetry.
 (B) For homogeneous rings, discs, and spheres, the center of mass coincides with their geometric center.
 (C) The center of mass of a body must always lie physically within the material of the body itself.
 (D) If the origin of the coordinate system is chosen at the center of mass, then $\sum m_i r_i = 0$.

Ans :

(C) The center of mass of a body must always lie physically within the material of the body itself.

The center of mass does not necessarily have to be within the physical material of the body. For example, the center of mass of a ring or a hollow sphere is at its geometric center, which is a point in space not occupied by material.

9. Which of the following statements correctly describes the motion of the center of mass of a system of particles?

- (A) The center of mass moves as if only the internal forces were applied at that specific point.
 (B) The center of mass moves as if all the mass of the system was concentrated at it and all external forces were applied there.
 (C) The motion of the center of mass is solely dependent on the internal forces within the

particle system.

- (D) The center of mass always remains stationary regardless of the external forces acting on the entire system.

Ans :

(B) The center of mass moves as if all the mass of the system was concentrated at it and all external forces were applied there.

The motion of the center of mass of a system of particles can be described as if the entire mass of the system is concentrated at that point, and all external forces acting on the system are applied directly to this center of mass. Internal forces do not affect the motion of the center of mass.

10. If the total external force acting on a system of particles is zero, what can be concluded about the velocity of its center of mass?

- (A) The velocity of the center of mass must be zero always.
 (B) The velocity of the center of mass will continuously increase.
 (C) The velocity of the center of mass remains constant or zero.
 (D) The velocity of the center of mass will continuously decrease.

Ans :

(C) The velocity of the center of mass remains constant or zero.

According to Newton's second law for a system of particles ($MA = F_{ext}$), if the total external force (F_{ext}) is zero, then the acceleration (A) of the center of mass is zero. Zero acceleration implies that the velocity of the center of mass remains constant (which includes the case of being zero if it was initially at rest).

11. Which of the following properties is NOT always true for the vector product (cross product) of two vectors a and b ?

- (A) The magnitude of $a \times b$ is $ab \sin \theta$.
 (B) $a \times b$ is perpendicular to the plane containing a and b .
 (C) $a \times b = b \times a$ (commutative property).
 (D) $a \times (b + c) = a \times b + a \times c$ (distributive property).

Ans :

(C) $a \times b = b \times a$ (commutative property).

The vector product is not commutative; in fact, it is anti-commutative, meaning $a \times b = -(b \times a)$. The order of the vectors matters in a cross product, affecting the direction of the resultant vector. The

other properties listed are correct for the vector product.

12. How does the role of moment of inertia in rotational motion compare to the role of mass in linear motion?

- (A) Moment of inertia is analogous to force, while mass is analogous to torque always.
 (B) Moment of inertia is the rotational analogue of mass; both resist changes in motion.
 (C) Moment of inertia determines linear acceleration, while mass determines angular acceleration.
 (D) Moment of inertia is inversely proportional to mass for all rigid bodies.

Ans :

(B) Moment of inertia is the rotational analogue of mass; both resist changes in motion.

Moment of inertia (I) in rotational motion plays a role analogous to mass (m) in linear motion. Mass is a measure of an object's resistance to changes in its linear motion (linear inertia), while moment of inertia is a measure of an object's resistance to changes in its rotational motion (rotational inertia) about a given axis.

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13. Assertion (A): The center of mass of a system of particles moves as if all the mass of the system was concentrated at the center of mass and all the external forces were applied at that point.

Reason (R): Internal forces within a system of particles cancel out in pairs according to Newton's third law and do not affect the motion of the center of mass.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The assertion accurately describes how the center of mass behaves. The reason correctly explains why this is the case: internal forces, due to Newton's third law, occur in equal and opposite pairs within the system, so their net effect on the overall motion of the center of mass is zero. Thus, only external forces dictate the motion of the center of mass.

- 14.** Assertion (A): The total angular momentum of a system of particles is conserved if the total external torque on the system is zero.

Reason (R): The rate of change of the total angular momentum of a system of particles about a point is equal to the sum of the external torques acting on the system taken about the same point.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

- (A) Both A and R are true and R is the correct explanation of A.

The reason states the fundamental relationship $dL/dt = \tau_{ext}$. If the total external torque τ_{ext} is zero, then $dL/dt = 0$, which implies that L (total angular momentum) is constant (conserved). Therefore, the reason correctly explains the assertion.

- 15.** Assertion (A): When a body is rotating about a fixed axis, its angular velocity vector lies along the axis of rotation.

Reason (R): For rotation about a fixed axis, the direction of angular velocity changes continuously with time.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

- (C) A is true but R is false.

The assertion is correct; for rotation about a fixed axis, the angular velocity vector is directed along that axis. However, the reason is false. For rotation about a fixed axis, the direction of the angular velocity vector remains constant (along the axis); its magnitude may change, leading to angular acceleration.

- 16.** Consider the following statements regarding the motion of a rigid body:

- I. In pure translational motion, all particles of the body have the same velocity at any instant.
- II. In pure rotational motion about a fixed axis, every particle of the body moves in a circle.
- III. The rolling motion of a cylinder down an inclined plane is an example of pure translational motion.

- IV. A rigid body fixed at one point can only have translational motion.

Which of the above statements are correct?

- (A) I and II only
 (B) I, II, and III only
 (C) II and IV only
 (D) I, III, and IV only

Ans :

- (A) I and II only

Statement I is correct by definition of pure translation. Statement II is correct as particles in a body rotating about a fixed axis trace circular paths. Statement III is incorrect; rolling motion is a combination of translation and rotation. Statement IV is incorrect; a body fixed at one point can have rotational motion, and if that point is on an axis, it can rotate about that axis.

- 17.** Consider the following statements about the center of mass:

- I. The center of mass of a system of particles always coincides with one of the particles in the system.
- II. The center of mass of a homogeneous sphere is at its geometric center.
- III. The velocity of the center of mass of a system of particles changes only if there is a net external force acting on the system.
- IV. The center of mass of a system can be located outside the physical boundary of the system.

Which of the above statements are correct?

- (A) I and III only
 (B) II, III, and IV only
 (C) I, II, and IV only
 (D) II and IV only

Ans :

- (B) II, III, and IV only

Statement I is incorrect; the center of mass is a weighted average position and may not coincide with any particle. Statement II is correct due to symmetry. Statement III is correct, based on $M\mathbf{A} = \mathbf{F}_{ext}$. Statement IV is correct (e.g., a ring).

- 18.** Which of the following statements are true regarding torque and angular momentum?

- I. Torque is the rotational analogue of force in linear motion.
- II. Angular momentum is conserved if the net external force on the system is zero.
- III. The SI unit of torque is Newton-meter (Nm).
- IV. The time rate of change of the angular momentum of a particle is equal to the torque acting on it.

- (A) I, II, and III only
 (B) I, III, and IV only
 (C) II, III, and IV only
 (D) I, II, III, and IV

Ans :

(B) I, III, and IV only

Statement I is a fundamental analogy. Statement II is incorrect; angular momentum is conserved if the net external torque is zero. Statement III is correct. Statement IV is the rotational analogue of Newton's second law ($dL/dt = \tau$).

19. Match the type of motion in Column I with its description in Column II.

	Column I		Column II
1.	Pure Translational Motion	P.	Every particle moves in a circle
2.	Pure Rotational Motion about a fixed axis	Q.	Motion of the center of mass as if all mass is concentrated there
3.	Motion of Center of Mass	R.	All particles have the same velocity at any instant
4.	Rolling Motion	S.	Combination of translation and rotation

- (A) 1-R, 2-P, 3-Q, 4-S
 (B) 1-Q, 2-R, 3-S, 4-P
 (C) 1-R, 2-S, 3-Q, 4-P
 (D) 1-S, 2-P, 3-R, 4-Q

Ans :

(A) 1-R, 2-P, 3-Q, 4-S

Pure translational motion (1) means all particles have the same velocity (R). Pure rotational motion about a fixed axis (2) means every particle moves in a circle (P). The motion of the center of mass (3) is described as if all mass is concentrated there and external forces act on it (Q). Rolling motion (4) is a combination of translation and rotation (S).

20. Match the physical quantity in Column I with its corresponding vector nature or definition in Column II.

	Column I		Column II
1.	Center of Mass Position Vector	P.	$r \times p$
2.	Linear Momentum of a System	Q.	MV

3.	Angular Momentum of a Particle	R.	$\frac{\sum m_i r_i}{M}$
4.	Torque on a Particle	S.	$r \times F$

- (A) 1-R, 2-Q, 3-P, 4-S
 (B) 1-Q, 2-R, 3-S, 4-P
 (C) 1-R, 2-P, 3-Q, 4-S
 (D) 1-S, 2-Q, 3-P, 4-R

Ans :

(A) 1-R, 2-Q, 3-P, 4-S

The position vector of the center of mass (1) is $\frac{\sum m_i r_i}{M}$ (R).

The linear momentum of a system (2) is MV (Q).

The angular momentum of a particle (3) is $r \times p$ (P).

The torque on a particle (4) is $r \times F$ (S).

21. Match the concept in Column I with its mathematical representation or defining characteristic in Column II.

	Column I		Column II
1.	Law of Conservation of Linear Momentum	P.	$L = I\omega$ (for symmetric body, fixed axis)
2.	Newton's Second Law for Rotational Motion (fixed axis)	Q.	If $\tau_{ext} = 0$, then $L = \text{constant}$
3.	Angular Momentum (fixed axis, symmetric body)	R.	If $F_{ext} = 0$, then $P = \text{constant}$
4.	Law of Conservation of Angular Momentum	S.	$\tau = I\alpha$

- (A) 1-S, 2-R, 3-P, 4-Q
 (B) 1-R, 2-S, 3-P, 4-Q
 (C) 1-R, 2-P, 3-S, 4-Q
 (D) 1-Q, 2-S, 3-P, 4-R

Ans :

(B) 1-R, 2-S, 3-P, 4-Q

The Law of Conservation of Linear Momentum (1) states that if $F_{ext} = 0$, then $P = \text{constant}$ (R). Newton's Second Law for Rotational Motion (fixed axis) (2) is $\tau = I\alpha$ (S). Angular Momentum for a symmetric body about a fixed axis (3) can be expressed as $L = I\omega$ (P). The Law of Conservation of Angular Momentum (4) states that if $\tau_{ext} = 0$, then $L = \text{constant}$ (Q).

22. Which of the following pairs correctly matches a type of motion with its key characteristic?

- (A) Pure translation : Different particles have different velocities always.
- (B) Fixed axis rotation : All particles move in elliptical paths.
- (C) Precession : Axis of rotation remains completely fixed.
- (D) Rolling motion : Combination of translation and rotation.

Ans :

(D) Rolling motion : Combination of translation and rotation.

Rolling motion, such as a wheel moving along a surface, involves both the translational movement of its center of mass and rotational movement about its axis. The other pairs are incorrect: pure translation involves same velocity for all particles, fixed axis rotation involves circular paths, and precession involves a moving axis of rotation.

23. Identify the correct pair relating a physical quantity to its definition or property.

- (A) Linear momentum of a particle : $p = m/v$ always
- (B) Vector product $a \times a$: Results in a non-zero vector
- (C) Center of mass : Always located at the geometric center of any body
- (D) Torque : Rotational analogue of force causing angular acceleration

Ans :

(D) Torque : Rotational analogue of force causing angular acceleration

Torque is the rotational equivalent of force; it causes or tends to cause angular acceleration. Linear momentum is $p = mv$. The vector product $a \times a$ is always a null vector. The center of mass only coincides with the geometric center for homogeneous bodies with certain symmetries.

24. Which of the following pairs incorrectly matches a concept with its description?

- (A) Rigid body : Distances between all pairs of particles do not change.
- (B) Axis of rotation : The line about which a body undergoes rotational motion.
- (C) Center of gravity : Coincides with the center of mass only if gravity is non-uniform.
- (D) Moment of inertia : Measure of rotational inertia, depends on mass distribution.

Ans :

(C) Center of gravity : Coincides with the center of mass only if gravity is non-uniform.

The center of gravity coincides with the center of mass if the gravitational field is uniform over the body. If gravity is non-uniform, they may not coincide. The other pairs correctly define the concepts.

25. Identify the incorrect pair from the following options regarding rotational dynamics.

- (A) Angular velocity ω : Rate of change of angular displacement ($d\theta/dt$).
- (B) Angular acceleration (α) : Time rate of change of angular velocity ($d\omega/dt$).
- (C) Kinetic energy of rotation : $\frac{1}{2}I\omega^2$ for rotation about a fixed axis.
- (D) $\tau = I\alpha$: Valid only if torque and angular acceleration are perpendicular always.

Ans :

(D) $\tau = I\alpha$: Valid only if torque and angular acceleration are perpendicular always.

The equation $\tau = I\alpha$ relates the net external torque (τ) to the angular acceleration (α) and moment of inertia (I) about a fixed axis. Torque and angular acceleration are vectors along the axis of rotation (or parallel to it), not necessarily perpendicular. The other pairs are correct definitions or formulas.

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26. Arrange the following steps in the logical order for defining the vector product $c = a \times b$:

- I. The direction of c is given by the right-handed screw rule.
 - II. The magnitude of c is $ab\sin\theta$.
 - III. c is perpendicular to the plane containing vectors a and b .
- (A) I, II, III
 - (B) II, III, I
 - (C) III, II, I
 - (D) II, I, III

Ans :

(B) II, III, I

The definition of a vector product typically starts with its magnitude (II: $ab\sin\theta$), then establishes its orientation relative to the operand vectors (III: c is perpendicular to the plane of a and b), and finally specifies its precise direction using a convention like the right-handed screw rule (I).

- 27.** Consider the steps involved in deriving the equation for the motion of the center of mass, $MA = F_{ext}$. What is the correct sequence of differentiation starting from the definition of the center of mass position vector $R = \frac{\sum m_i r_i}{M}$?
- Differentiate with respect to time to get $MV = \sum m_i v_i$.
 - Differentiate with respect to time to get $MA = \sum m_i a_i$.
 - Relate $\sum m_i a_i$ to the sum of all forces, and then to F_{ext} .
- (A) I, II, III
(B) II, I, III
(C) I, III, II
(D) III, I, II

Ans :

(A) I, II, III

The derivation begins by differentiating the position vector of the center of mass (R) with respect to time to obtain the velocity of the center of mass (V) (Step I). Differentiating V with respect to time gives the acceleration of the center of mass (A) (Step II). Finally, $\sum m_i a_i$ is identified as the sum of all forces, and recognizing that internal forces cancel, this sum equals the net external force F_{ext} (Step III).

- 28.** A figure skater is spinning with her arms outstretched. When she brings her arms in close to her body, her angular speed increases. This phenomenon is a direct consequence of:
- (A) Conservation of linear momentum only.
(B) Conservation of angular momentum.
(C) Conservation of kinetic energy only.
(D) Newton's first law of motion only.

Ans :

(B) Conservation of angular momentum.

When the skater brings her arms in, her moment of inertia (I) decreases. Since no significant external torque acts on her about the vertical axis of rotation, her angular momentum ($L = I\omega$) is conserved. Therefore, as I decreases, her angular speed (ω) must increase to keep L constant.

- 29.** What is the 'axis of rotation'?
- (A) A point around which a body always translates.
(B) The line about which a rigid body is rotating.
(C) A plane within which all particles of a body move.
(D) The direction of the net force on a body.

Ans :

(B) The line about which a rigid body is rotating.

The axis of rotation is the specific straight line, which can be fixed or moving, around which all points of a rotating rigid body move in circles or arcs of circles. This line itself remains stationary relative to the rotating parts if the axis is fixed.

- 30.** If a rigid body undergoes pure translational motion, what is true about the orientation of any line segment connecting two particles within the body?
- (A) The orientation changes continuously during motion.
(B) The orientation remains the same during motion.
(C) The orientation becomes perpendicular to velocity.
(D) The orientation aligns with the direction of motion.

Ans :

(B) The orientation remains the same during motion.

In pure translational motion, every particle in the body moves with the same velocity. This means the body as a whole moves without any rotation, so the orientation (direction) of any line segment connecting two particles within the body remains unchanged.

- 31.** For a system of n particles, where is the center of mass located if the origin of the frame of reference is chosen such that $\sum m_i r_i = 0$?
- (A) At an infinite distance from all the particles.
(B) At the position of the most massive particle.
(C) At the origin of the frame of reference.
(D) At the position of the least massive particle.

Ans :

(C) At the origin of the frame of reference.

The position vector of the center of mass R is given by $R = \frac{\sum m_i r_i}{M}$. If $\sum m_i r_i = 0$, then R must be zero, which means the center of mass is located at the origin of the chosen frame of reference.

- 32.** What is the relationship between the total linear momentum (P) of a system of particles, its total mass (M), and the velocity of its center of mass (V)?
- (A) $P = M/V$
(B) $P = MV^2$
(C) $P = MV$
(D) $P = V/M$

Ans :

(C) $P = MV$

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass. This is a fundamental relationship in the study of systems of particles.

33. What is the value of the vector product $\hat{i} \times \hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along the x and y axes respectively?
- (A) 0 (null vector)
 (B) $\hat{k}$
 (C) $-\hat{k}$
 (D) 1 (scalar)

Ans :

(B) $\hat{k}$

According to the right-hand rule for vector products of orthogonal unit vectors, $\hat{i} \times \hat{j}$ results in $\hat{k}$, which is the unit vector along the z-axis. The magnitude is $1 \times 1 \times \sin(90^\circ) = 1$.

34. The time rate of change of the angular momentum of a particle is equal to:
- (A) The net force acting on it.
 (B) The linear momentum of the particle.
 (C) The torque acting on it.
 (D) The angular velocity of the particle.

Ans :

(C) The torque acting on it.

This is the rotational analogue of Newton's second law for translational motion. The equation is $d\mathbf{l}/dt = \boldsymbol{\tau}$, where $\mathbf{l}$ is the angular momentum and $\boldsymbol{\tau}$ is the net torque acting on the particle.

35. What are the conditions for a rigid body to be in mechanical equilibrium?
- (A) The total force is zero, but total torque can be non-zero.
 (B) The total torque is zero, but total force can be non-zero.
 (C) Both the total external force and the total external torque on it are zero.
 (D) Only the angular momentum is constant, linear momentum can change.

Ans :

(C) Both the total external force and the total external torque on it are zero.

For a rigid body to be in mechanical equilibrium, two conditions must be met simultaneously: translational equilibrium (the vector sum of all external forces is zero) and rotational equilibrium (the vector sum of all external torques is zero).

36. Which of the following statements about the moment of inertia is incorrect?
- (A) Moment of inertia depends on the distribution of mass about the axis of rotation.
 (B) Moment of inertia is always a fixed quantity for a given rigid body regardless of the axis.
 (C) Moment of inertia is the rotational analogue of mass in linear motion.
 (D) The SI unit of moment of inertia is kg-m^2 .

Ans :

(B) Moment of inertia is always a fixed quantity for a given rigid body regardless of the axis.

The moment of inertia of a rigid body is not a single fixed quantity; it depends on the choice of the axis of rotation. Different axes of rotation for the same body will generally yield different moments of inertia.

37. Which statement accurately describes the radius of gyration (k) of a rigid body about an axis?
- (A) It is the average distance of all particles from the axis of rotation always.
 (B) It is defined such that $I = Mk^2$, where I is moment of inertia and M is total mass.
 (C) It is always equal to the physical radius of the rigid body for any shape.
 (D) It has units of kg-m^2 , similar to the moment of inertia itself.

Ans :

(B) It is defined such that $I = Mk^2$, where I is moment of inertia and M is total mass.

The radius of gyration (k) is a length parameter defined such that if the entire mass (M) of the body were concentrated at a distance k from the axis, its moment of inertia (I) would be Mk^2 . It represents how mass is distributed relative to the axis.

38. Why does a flywheel, a disc with a large moment of inertia, help in maintaining smooth rotational motion in engines?
- (A) It significantly reduces the overall mass of the engine system.
 (B) It generates additional torque to speed up the engine quickly.
 (C) It resists sudden changes in angular speed due to its large rotational inertia.
 (D) It converts rotational energy into translational energy very efficiently.

Ans :

(C) It resists sudden changes in angular speed due to its large rotational inertia.

A flywheel has a large moment of inertia (I). According to $\tau = I\alpha$, for a given torque, a larger I means a smaller angular acceleration (α). This resistance to changes in angular speed helps to smooth out fluctuations and prevent jerky motions in engines.

39. Which of these quantities is NOT necessarily conserved for an isolated system of interacting particles (i.e., no external forces or torques)?
- Total linear momentum of the system.
 - Total angular momentum of the system.
 - Total kinetic energy of the system.
 - Velocity of the center of mass.

Ans :

(C) Total kinetic energy of the system.

For an isolated system, total linear momentum and total angular momentum are conserved. The velocity of the center of mass also remains constant (implying conservation of MV_{CM}). However, kinetic energy is not necessarily conserved, as internal forces can do work and convert kinetic energy into other forms of energy (e.g., heat in an inelastic collision).

40. What is a key difference between the center of mass and the center of gravity of an extended body?
- Center of mass depends on gravity, while center of gravity depends only on mass distribution.
 - Center of gravity depends on gravity, while center of mass depends only on mass distribution.
 - Center of mass is always inside the body, while center of gravity can be outside.
 - Center of gravity is always inside the body, while center of mass can be outside.

Ans :

(B) Center of gravity depends on gravity, while center of mass depends only on mass distribution. The center of mass is defined based purely on the distribution of mass within a body. The center of gravity is the point where the net gravitational torque on the body is zero. These two points coincide if the gravitational field is uniform over the body, but they are conceptually distinct and can differ if the gravitational field varies.

41. Assertion (A): The motion of a rigid body can always be described as a pure rotation or a pure translation.
Reason (R): A rigid body is defined as a body with a perfectly definite and unchanging shape.
- Both A and R are true and R is the correct explanation of A.
 - Both A and R are true but R is NOT the correct

explanation of A.

- A is true but R is false.
- A is false but R is true.

Ans :

(D) A is false but R is true.

The assertion (A) is false because the motion of a rigid body, in general, can be a combination of translation and rotation (e.g., a rolling wheel). The reason (R) is a correct definition of an ideal rigid body.

42. Consider a system of particles. Which of the following statements are correct?
- The position of the center of mass depends on the choice of the origin of the coordinate system.
 - The velocity of the center of mass is independent of the frame of reference.
 - If the net external force on the system is zero, the center of mass must be at rest.
 - Internal forces do not affect the acceleration of the center of mass.
- I and IV only
 - II and III only
 - I, III and IV only
 - IV only

Ans :

(D) IV only

I is incorrect; while coordinates change with origin, the physical point of CM is fixed relative to the system. II is incorrect; velocity is frame-dependent. III is incorrect; if $F_{ext} = 0$, V_{CM} is constant, not necessarily zero. IV is correct because internal forces occur in equal and opposite pairs.

43. Match the kinematic equations for linear motion with their rotational motion analogues.

	Column I		Column II
1.	$v = v_0 + at$	P.	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
2.	$x = x_0 + v_0 t + \frac{1}{2}at^2$	Q.	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
3.	$v^2 = v_0^2 + 2ax$ (if $x_0 = 0$)	R.	$\omega = \omega_0 + \alpha t$

- 1-R, 2-P, 3-Q
- 1-P, 2-R, 3-Q
- 1-Q, 2-P, 3-R
- 1-R, 2-Q, 3-P

Ans :

(A) 1-R, 2-P, 3-Q

The rotational analogue of linear velocity v is angular velocity ω , linear acceleration a is angular acceleration α , and linear displacement x is angular displacement θ . Thus, $v = v_0 + at$ corresponds to $\omega = \omega_0 + \alpha t$ (1-R).

$$x = x_0 + v_0 t + \frac{1}{2} at^2 \text{ corresponds to}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \text{ (2-P).]}$$

$$v^2 = v_0^2 + 2ax \text{ corresponds to}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \text{ (3-Q).}$$

44. Which one of the following pairs represents quantities that are analogous in translational and rotational motion, respectively?

- (A) Force : Angular displacement
(B) Mass : Torque
(C) Linear momentum : Angular velocity
(D) Velocity : Angular velocity

Ans :

(D) Velocity : Angular velocity

Linear velocity (v) in translational motion is analogous to angular velocity (ω) in rotational motion. Force is analogous to torque, mass to moment of inertia, and linear momentum to angular momentum.

45. Identify the incorrectly matched pair regarding concepts in rotational motion.

- (A) Couple : Produces rotation without translation.
(B) Principle of moments : Load arm $\times$ effort = Effort arm $\times$ load.
(C) Radius of gyration : A distance related to mass distribution and moment of inertia.
(D) Kinematics of rotational motion : Deals with the causes of rotational motion.

Ans :

(B) Principle of moments : Load arm $\times$ effort = Effort arm $\times$ load.

The principle of moments for a lever in equilibrium is Load arm $\times$ Load = Effort arm $\times$ Effort. Statement (B) has the terms “effort” and “load” swapped in the equation. Kinematics deals with the description of motion without considering its causes (forces/torques); dynamics deals with the causes.

46. What is the correct sequence to determine the temperature (T_0) of the junction of two bars (iron and brass) of equal length and area, soldered end to end, with free ends at temperatures T_1 and T_2 ($T_1 > T_2$), and thermal conductivities K_1 and K_2 respectively, under steady state?

I. Use $T_0 = \frac{(K_1 T_1 + K_2 T_2)}{(K_1 + K_2)}$.

- II. Equate the heat current through the iron bar $H_1 = \frac{K_1 A (T_1 - T_0)}{L}$ to the heat current through the brass bar $H_2 = \frac{K_2 A (T_0 - T_2)}{L}$.

- III. State that under steady state condition, $H_1 = H_2$.

- (A) I, II, III
(B) III, II, I
(C) II, III, I
(D) III, I, II

Ans :

(B) III, II, I

The first step is to recognize that in steady state, the rate of heat flow (heat current) through both bars must be equal (III). Then, express these heat currents mathematically for each bar (II). Finally, solving the equality from step II for the junction temperature T_0 yields the formula in (I).

47. A dancer performing a pirouette (spinning on one foot) pulls her arms in towards her body. What is the primary reason her angular speed increases?

- (A) Her mass decreases significantly when she pulls her arms in.
(B) The torque due to gravity increases when her arms are in.
(C) Her moment of inertia decreases, and angular momentum is conserved.
(D) Frictional forces from the floor provide an accelerating torque.

Ans :

(C) Her moment of inertia decreases, and angular momentum is conserved.

By pulling her arms closer to her axis of rotation, the dancer reduces her moment of inertia (I). In the absence of significant external torques (friction is usually minimized), her angular momentum ($L = I\omega$) is conserved. Thus, a decrease in I leads to an increase in angular speed (ω).

48. Angular momentum of a particle with respect to an origin O is defined as:

- (A) $l = r \cdot p$
(B) $l = m(r \times v)$
(C) $l = r \times F$
(D) $l = I\omega$

Ans :

(B) $l = m(r \times v)$

The angular momentum (l) of a particle about an origin is defined as the vector product of its position vector (R) relative to the origin and its

linear momentum (p). Since $p = mv$, this can also be written as $l = r \times mv$ or $l = m(r \times v)$. L is for a rigid body.

49. If the total external torque on a system of particles is zero, which of the following quantities is conserved?
- The total linear momentum of the entire system.
 - The total kinetic energy of the entire system.
 - The total angular momentum of the entire system.
 - The total potential energy of the entire system.

Ans :

(C) The total angular momentum of the entire system.

The relationship $dL/dt = \tau_{ext}$ states that the rate of change of total angular momentum (L) of a system is equal to the total external torque (τ_{ext}) acting on it. If $\tau_{ext} = 0$, then $dL/dt = 0$, which implies L is constant (conserved).

50. The moment of inertia of a thin circular ring of radius R and mass M , about an axis perpendicular to its plane and passing through its center is:
- $\frac{1}{2}MR^2$
 - MR^2
 - $\frac{2}{5}MR^2$
 - $ML^2/12$

Ans :

(B) MR^2

For a thin circular ring, all its mass (M) is located at the same distance (R) from the axis of rotation passing through its center and perpendicular to its plane. Therefore, its moment of inertia is $I = \sum m_i r_i^2 = MR^2$.

51. What does the symbol $L_z = I\omega\hat{k}$ represent in the context of a symmetric rigid body rotating about a fixed z-axis?
- The component of linear momentum along the z-axis for the body.
 - The total angular momentum vector of the rotating body.
 - Only the kinetic energy component along the z-axis.
 - The perpendicular component of angular momentum only.

Ans :

(B) The total angular momentum vector of the rotating body.

For a rigid body that is symmetric about the axis of rotation (chosen as the z-axis), the component of angular momentum perpendicular to the axis ($L_{\perp}$)

is zero. Therefore, the total angular momentum L is equal to its component along the z-axis (L_z), which is given by $L_z = I\omega\hat{k}$.

52. What is the rotational analogue of the linear motion equation $P = Fv$ (Power = Force $\times$ velocity)?

- $P = \tau/\omega$
- $P = I\omega$
- $P = \tau\omega$
- $P = L/\omega$

Ans :

(C) $P = \tau\omega$

Power (P) in rotational motion is the product of torque (τ) and angular velocity (ω). This is analogous to power in linear motion being the product of force (F) and linear velocity (v).

53. The dynamics of rotational motion about a fixed axis is described by an equation analogous to Newton's second law for linear motion ($F = ma$). What is this rotational analogue?

- $L = I\alpha$
- $\tau = I\omega$
- $\tau = I\alpha$
- $K = \frac{1}{2}I\omega^2$

Ans :

(C) $\tau = I\alpha$

The equation $\tau = I\alpha$ is Newton's second law for rotational motion about a fixed axis. It states that the net external torque (τ) is equal to the product of the moment of inertia (I) and the angular acceleration (α).

54. What property of a rigid body does its moment of inertia (I) represent?

- Its resistance to changes in linear motion only.
- Its total mass irrespective of distribution.
- Its resistance to changes in rotational motion.
- The total angular velocity it can achieve.

Ans :

(C) Its resistance to changes in rotational motion.

The moment of inertia (I) is a measure of an object's rotational inertia, which is its resistance to changes in its state of rotational motion about a particular axis. A larger moment of inertia means it is harder to change the object's angular velocity.

55. Which of the following statements regarding the vector product (cross product) $c = a \times b$ is incorrect?

- If a and b are parallel, then $c = 0$.

- (B) The magnitude of c is maximum when a and b are perpendicular.
- (C) The direction of c changes if the direction of a is reversed but b is unchanged.
- (D) The vector product $a \times b$ always results in a scalar quantity.

Ans :

(D) The vector product $a \times b$ always results in a scalar quantity.

The vector product (or cross product) of two vectors results in another vector, not a scalar quantity. This resultant vector is perpendicular to the plane containing the original two vectors. All other statements are correct properties of the vector product.

- 56.** Which of the following correctly describes a property of the center of mass of an isolated system (no net external force)?
- (A) The center of mass must always accelerate if internal forces are present within.
- (B) The center of mass moves with a constant velocity (which can be zero).
- (C) The total linear momentum associated with the center of mass is always zero.
- (D) The center of mass can only be located at the geometric center of the system.

Ans :

(B) The center of mass moves with a constant velocity (which can be zero).

If the net external force on a system is zero ($F_{ext} = 0$), then the acceleration of the center of mass (A) is zero ($MA = F_{ext}$). Zero acceleration implies that the velocity of the center of mass (V) remains constant. This constant velocity can be zero if the system was initially at rest.

- 57.** Consider two particles of masses m_1 and m_2 located at positions x_1 and x_2 respectively along the x-axis. If $m_1 > m_2$, where will their center of mass be located relative to the midpoint between them?
- (A) Exactly at the midpoint between x_1 and x_2 .
- (B) Closer to the particle with mass m_2 .
- (C) Closer to the particle with mass m_1 .
- (D) At the position of the particle with mass m_2 .

Ans :

(C) Closer to the particle with mass m_1 .

The center of mass is a weighted average of the positions, weighted by the masses. Therefore, the center of mass will be closer to the more massive particle. Since $m_1 > m_2$, the center of mass will be closer to m_1 .

- 58.** Which of the following is NOT a characteristic of pure translational motion of a rigid body?

- (A) All particles of the body have the same velocity at any instant of time.
- (B) The orientation of the body in space changes during the motion.
- (C) Any line segment connecting two points in the body remains parallel to its initial orientation.
- (D) The body moves without any rotation or spinning motion.

Ans :

(B) The orientation of the body in space changes during the motion.

In pure translational motion, the body moves without any rotation. This means its orientation in space remains constant; it does not change. All particles move with the same velocity, maintaining their relative positions and the body's overall orientation.

- 59.** How does the definition of torque ($\tau = r \times F$) differ fundamentally from the definition of work done by a constant force ($W = F \cdot d$)?

- (A) Torque is a scalar quantity, while work is a vector quantity always.
- (B) Torque involves a vector product, while work involves a scalar product.
- (C) Torque depends only on force magnitude, while work depends on displacement.
- (D) Torque and work have completely identical mathematical definitions.

Ans :

(B) Torque involves a vector product, while work involves a scalar product.

Torque ($\tau = r \times F$) is defined as the vector (cross) product of the position vector (r) and the force vector (F), resulting in a vector quantity. Work ($W = F \cdot d$) is defined as the scalar (dot) product of the force vector (F) and the displacement vector (d), resulting in a scalar quantity.

- 60.** A child is sitting on a rotating merry-go-round. If the child moves from the outer edge towards the center of the merry-go-round, what happens to the moment of inertia of the (child + merry-go-round) system, assuming no external torques?

- (A) The moment of inertia increases significantly.
- (B) The moment of inertia remains exactly the same.
- (C) The moment of inertia decreases.
- (D) The moment of inertia becomes zero.

rotational energy, the change in angular velocity is negligible. Thus maintains uniform motion.

- 141.** If the ice on the polar caps of the earth melts, how will it affect the duration of the day ? Explain.

Ans :

If ice on the polar caps of the earth melts, the moment of inertia of earth about the axis of rotation increases as ice spreads out.

Angular velocity decreases.

Hence $\frac{2\pi}{\omega}$ duration of day will increase.

- 142.** Why does a cyclist lean to one side while going along a curve ? In which direction does he lean ?

Ans :

While going along a curve, the cyclist lean to one side, so as to provide himself the necessary centripetal force. He leans to the inward direction i.e. towards the centre of circular path.

- 143.** Moment of inertia of the sphere about its diameter is $\frac{2}{5}MR^2$. What is its moment of inertia about an axis $\perp$ to point of intersection of two diameters ?

Ans :

It will be again $\frac{2}{5}MR^2$ because the axis in question is also diameter of the sphere.

- 144.** Two solid spheres of same mass are made of metals of different densities. Which one of them has a larger moment of inertia about a diameter ?

Ans :

The sphere with smaller density will be of more radius than the other. Moment of inertia of sphere about diameter $= \frac{2}{5}MR^2$. Hence moment of inertia of sphere with smaller density will be more.

- 145.** Two spheres are of same mass and same external radius, one is solid and the other is hollow. Which one of them has larger moment of inertia about its diameter ?

Ans :

The entire mass of hollow sphere is concentrated away from its centre and hence its moment of inertia about diameter will be greater than that of solid sphere.

- 146.** Do the internal forces affect the location of the centre of mass ?

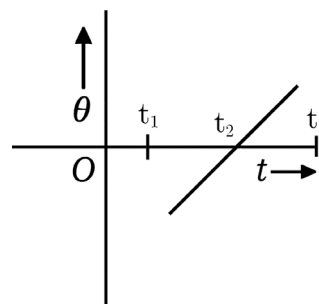
Ans :

No, the internal forces do not affect the motion of the system. The centre of mass of the system is affected only by external force.

- 147.** The variation of angular position θ of a point on a rotating rigid body, with time t is shown in figure. Is the body rotating clockwise or anticlockwise ?

Ans :

Since the slope of θ - t graph is positive, it indicates that the body is rotating anticlockwise which is taken as positive.



- 148.** Why does cream collect in the middle when milk is churned up ?

Ans :

Mass of cream particles is less than the mass of milk particles, so when milk is rotated, the centripetal force experienced by cream particles is less than that experienced by milk particles. Hence the cream gets collected in the middle.

- 149.** It is easier to balance a bicycle in motion. Why ?

Ans :

A moving bicycle possesses angular momentum and in the absence of external torque total angular momentum remains conserved. Thus the direction of angular momentum also remains conserved (angular momentum is a vector quantity) and hence the axis of rotation will not tilt and the bicycle remains balanced.

- 150.** How does the motion of particles differ in pure translation versus general rigid body motion?

Ans :

In pure translational motion, all particles of the body have the same velocity at any instant of time. In general rigid body motion, such as rolling, different particles may have different velocities at any instant.

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Ans :

(d) None of these

Since, $\frac{dL}{dt} = \tau = 0$

So $L = \text{constant}$

69. A rigid body is rotating about an axis. To stop the rotation, we have to apply:

- (a) pressure
- (b) force
- (c) momentum
- (d) torque

Ans :

(d) torque

To stop rotating rigid body, an equal and opposite torque is required.

70. A couple produces a:

- (a) pure linear motion
- (b) pure rotational motion
- (c) both linear and rotational motion
- (d) no motion

Ans :

(b) pure rotational motion

Pure rotational motion can be produced by a couple only.

71. Angular momentum is the vector product of :

- (a) linear momentum and radius vector
- (b) moment of inertia and angular acceleration
- (b) linear momentum and angular velocity
- (c) linear velocity and radius vector

Ans :

(a) linear momentum and radius vector

Because, $\vec{L} = \vec{r} \times \vec{p}$

72. A planet is revolving round the sun in an elliptical orbit. The maximum and minimum distances of the planet from the sun are $3 \times 10^2 \text{ m}$ and $2 \times 10^{10} \text{ m}$ respectively. The speed of the planet when it is nearest to sun is $2 \times 10^7 \text{ ms}^{-1}$. What is the speed when it is farthest to the sun ?

- (a) $1.33 \times 10^9 \text{ ms}^{-1}$
- (b) $1.51 \times 10^7 \text{ ms}^{-1}$
- (c) $1.33 \times 10^5 \text{ ms}^{-1}$
- (d) $2 \times 10^7 \text{ ms}^{-1}$

Ans :

(c) $1.33 \times 10^5 \text{ ms}^{-1}$

$$v_1 r_1 = v_2 r_2$$

$$v_2 = \frac{2 \times 10^7 \times 2 \times 10^{10}}{3 \times 10^{12}} = 1.33 \times 10^5 \text{ ms}^{-1}$$

73. A car of mass 1000 kg moves on a circular road with a speed of 20 ms^{-1} . Its direction changes by 90° after travelling 628 m on the road. The centripetal force acting on the car is

- (a) 1000 N
- (b) 1500 N
- (c) 2000 N
- (d) 3000 N

Ans :

(a) 1000 N

As we know that,

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{1000 \times (20)^2}{400} \\ &= 1000 \text{ N} \end{aligned}$$

74. A particle revolves round a circular path with a constant speed. The acceleration of the particle is

- (a) along circumference of the circle
- (b) along the tangent
- (c) along the radius
- (d) zero centripetal

Ans :

(c) along the radius

The acceleration is always along the radius towards the centre.

ONE MARK QUESTIONS

75. Where does the centre of mass of a rectangle lie ?

Ans :

At point of intersection of its diagonals.

76. What is an isolated system ?

Ans :

It is a system where no external force is acting.

77. Is centre of mass a reality ?

Ans :

No, centre of mass of a system is a hypothetical point which acts as a single mass particle of a system for an external system.

78. What is the position vector of centre of mass of two particles of equal masses ?

Ans :

It is the average of the position vectors of two particles.

- 163.** Define the radius of gyration (k) of a body about an axis.

Ans :

The radius of gyration (k) of a body about an axis is the distance from the axis to a mass point (whose mass equals the whole body's mass) whose moment of inertia equals that of the body about the axis ($I = Mk^2$).

- 164.** Write the three kinematic equations for rotational motion with uniform angular acceleration.

Ans :

The kinematic equations are:

- $\omega = \omega_0 + \alpha t$
- $\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Where symbols have their usual meanings.

- 165.** State the rotational analogue of Newton's second law for motion about a fixed axis.

Ans :

The rotational analogue of Newton's second law is $\tau = I\alpha$. This means that torque (τ) produces angular acceleration (α) in a body, inversely proportional to its moment of inertia (I).

- 166.** Explain the term angular momentum of a particle (l) concerning its components along and perpendicular to the axis of rotation.

Ans :

For a particle in a rotating rigid body, $l = r \times p$. The component l_z along the fixed axis is $mr^2\omega\hat{k}$. There is also a component $OC \times mv$ perpendicular to the axis, so l is not generally along the axis.

- 167.** Under what condition is the component of angular momentum perpendicular to the fixed axis ($L_\perp$) constant?

Ans :

For rotation about a fixed axis, the component of total angular momentum perpendicular to the fixed axis ($L_\perp$) is constant. This follows from $dL_\perp/dt = 0$ when only torques along the axis are considered.

- 168.** How does stretching arms affect a rotating person on a swivel chair? Explain using conservation of angular momentum.

Ans :

Stretching arms horizontally increases the person's moment of inertia (I) about the axis of rotation. Since no external torque acts (neglecting friction),

angular momentum ($L_z = I\omega$) is conserved. Thus, as I increases, angular speed ω decreases.

- 169.** What are the key characteristics of a rigid body as defined in mechanics?

Ans :

Ideally, a rigid body is one for which the distances between different particles of the body do not change, even when forces act on them. It has a perfectly definite and unchanging shape.

- 170.** Describe the motion of particles in a rigid body during pure translational motion.

Ans :

In pure translational motion, every particle of the body moves with the same velocity at any instant of time. This means all particles trace out identical trajectories, though shifted in position.

- 171.** Explain the concept of centre of mass for a system of particles.

Ans :

The centre of mass is a point whose position vector R is the mass-weighted average of the position vectors of the individual particles ($R = \sum m_i r_i / M$). It moves as if the entire mass is concentrated there.

- 172.** Discuss the relationship between total linear momentum of a system and the velocity of its centre of mass.

Ans :

The total linear momentum P of a system of particles is equal to the product of the total mass M of the system and the velocity V of its centre of mass ($P = MV$).

- 173.** What is the condition for the conservation of linear momentum of a system of particles?

Ans :

The total linear momentum of a system of particles is conserved (remains constant) if the total external force acting on the system is zero ($F_{ext} = 0$). This means the velocity of the centre of mass remains constant.

- 174.** Describe the distributive property of the vector product with respect to vector addition.

Ans :

The vector product is distributive with respect to vector addition. This means for three vectors a , b , and c , the relation $a \times (b + c) = (a \times b) + (a \times c)$ holds true.

- 97.** Does moment of inertia change with change of the axis of rotation ?

Ans :

Yes

- 98.** What is physical significance of moment of inertia ?

Ans :

Moment of inertia of a body plays the same role in rotational motion as its mass does in linear motion.

- 99.** Why are spokes fitted in a cycle wheel ?

Ans :

To ensure uniform motion of the cycle wheel, it should possess large moment of inertia. To increase moment of inertia of the wheel without large increase in its mass, it is provided with spokes.

- 100.** About which axis of a body is its moment of inertia the least ?

Ans :

About an axis passing through the centre of mass of the body.

- 101.** How does moment of inertia change with speed of rotation ?

Ans :

Moment of inertia is not affected by speed of rotation of body.

- 102.** What is the moment of inertia of a (i) ring (ii) disc about their diameters ?

Ans :

(i) $I = \frac{1}{2}MR^2$

(ii) $I = \frac{1}{4}MR^2$

- 103.** An electric drill comes to rest quicker than an electric grinder after the power is switched off. Why

Ans :

$$I_{\text{drill}} > I_{\text{grinder}}$$

- 104.** What is the position of centre of mass of two particles of equal masses ?

Ans :

It is the average of the position vectors of two particles.

- 105.** What is an isolated system ?

Ans :

It is a system where no external force is acting.

- 106.** What is dimensional formula of angular momentum? What are its units ? Is it scalar quantity ?

Ans :

Dimensions and unit of angular momentum are $[ML^2T^{-1}]$ and $\text{kg m}^2\text{s}^{-1}$ respectively. It is a vector quantity.

- 107.** A wrench with large arm is preferred. Why ?

Ans :

The turning effect of the force $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = rF\sin\theta$ larger, the r , larger the torque or the turning effect.

- 108.** On what factors does the moment of inertia of a body depend ?

Ans :

It depends upon the mass and the distribution of mass about the axis of rotation.

- 109.** Under what conditions angular momentum of a moving body is zero ?

Ans :

Since angular momentum,

$$\vec{L} = \vec{r} \times \vec{p}$$

So $L = 0$, if either $r = 0$ or $\vec{r} \parallel \vec{p}$

- 110.** Give mathematical expression for centre of mass of N-particle system.

Ans :

The expression of centre of mass of N-particle system is given by

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots\dots\dots m_N\vec{r}_N}{m_1 + m_2 + \dots\dots\dots m_N}$$

- 111.** Can centre of mass a body coincide with geometric centre of the body ?

Ans :

If the body has uniform density, the centre of mass coincides with geometric centre of the body.

- 112.** Is there any difference between moment of inertia and rotational inertia ?

Ans :

No

Ans :

(C) The moment of inertia decreases.

The moment of inertia depends on the mass and its distribution relative to the axis of rotation ($I = \sum m_i r_i^2$). When the child moves closer to the center (axis of rotation), their contribution to the total moment of inertia decreases because their distance 'r' from the axis decreases. Thus, the total moment of inertia of the system decreases.

61. If two bodies are moving towards each other without any external forces and join, then centre of mass:

- (a) moves
- (b) remain stationary
- (c) moves away from bodies
- (d) none of the above

Ans :

(b) remain stationary

Since no external force is acting on the system, its centre of mass remains stationary.

62. Two bodies of different masses of 2 kg and 4 kg are moving with velocities 2 ms^{-1} and 10 ms^{-1} towards each other due to mutual gravitational attraction. What is velocity of their centre of mass?

- (a) 5 ms^{-1}
- (b) 6 ms^{-1}
- (c) 8 ms^{-1}
- (d) zero

Ans :

(d) zero

Since gravitational forces are mutual forces, so velocity of centre of mass is zero.

63. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ?

- (a) Moment of inertia
- (b) Angular momentum
- (c) Angular velocity
- (d) Rotational kinetic energy

Ans :

(b) Angular momentum

In free space, assuming that $g = 0$ and no torque is acting on the sphere, and its radius is increased due to internal forces, so law of conservation of angular momentum holds good.

64. Which of these statements is not correct ?

- (a) Moment of inertia is dependent on shape and size of the body.
- (b) Moment of inertia depends on choice of axes.

(c) Moment of inertia does not depend on mass of body.

(d) None of these.

Ans :

(d) None of these

Moment of inertia depends on the mass of the body.

65. The mass of a flywheel is concentrated at its rim so as to have:

- (a) a large moment of inertia
- (b) a small moment of inertia
- (c) a stable moment of inertia
- (d) an unstable equilibrium

Ans :

(a) a large moment of inertia

Larger the distance of distribution of mass from the axis of rotation more will be the moment of inertia.

66. If a body is moving in a circle of radius r centimetre at a constant speed of v cm s⁻¹, then its angular velocity is:

- (a) $\frac{v^2}{r}$
- (b) vr
- (c) $\frac{v}{r}$
- (d) $\frac{r}{v}$

Ans :

(c) $\frac{v}{r}$

Since, $v = r\omega$, so $\omega = \frac{v}{r}$

67. A closed tube, partly filled with a liquid and set horizontal, is rotated about a vertical axis passing through its centre. In the process, the moment of inertia of the system about its axis would:

- (a) always increase
- (b) always decrease
- (c) remain constant
- (d) increase if tube is less than half filled but otherwise decrease.

Ans :

(a) always increase

The centrifugal force pushes the liquid outwards, hence moment of inertia would always increase.

68. When a torque acting upon a system is zero, which of the following will be constant?

- (a) Force
- (b) Linear momentum
- (c) Linear impulse
- (d) None of these

- 129.** Is it necessary that centre of mass should always lie inside the body ? Give one example.

Ans :

No, it is not necessary that the centre of mass of a body lie inside the body e.g., in a ring, the centre of mass lies in the centre i.e., outside the body.

- 130.** What is torque ? Give its S.I. unit.

Ans :

The turning effect (or moment) of a force about an axis of rotation is called torque and is a measure of the rotational effect of force.

Unit : In S.I., the unit of torque is newton metre.

- 131.** The cap of the pen can be easily opened with the help of two fingers than with one finger. Explain why.

Ans :

With the help of two fingers, we are able to apply a couple. This couple enables us to open the cap of the pen easily. With one finger, we can apply a single force.

- 132.** Why do all helicopters have two propellers ?

Ans :

If a helicopter has a single propeller, to conserve angular momentum, the helicopter itself will start rotating in a direction opposite to that of the rotation of the propeller.

- 133.** On what factors the radius of gyration depend.

Ans :

Radius of gyration depends upon

- Position of axis of rotation
- distribution of mass about the axis of rotation

- 134.** On what factors does the moment of inertia depend?

Ans :

It depends upon

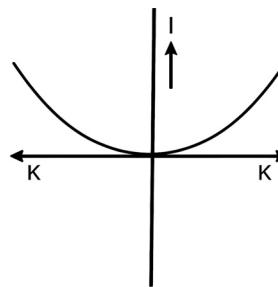
- mass of the body,
- position and direction of axis of rotation,
- shape of the body.

- 135.** Draw a graph between moment of inertia and radius of gyration.

Ans :

Since, $I = mk^2$
or I and k^2

So the graph between I and K is a parabola as shown in the given figure.



- 136.** Two satellites of equal masses are revolving around at different heights. Will their moment of inertia same or different ?

Ans :

The satellites can be taken as point masses. Moment of inertia of a point mass m , revolving at a distance r , from the axis of rotation is $I = mr^2$. Now for two satellites, r is different, so their moment of inertia will be different.

- 137.** A disc is recast into hollow and thin cylinder of same radius, which will have larger moment of inertia.

Ans :

Hollow cylinder will have larger moment of inertia as most of its mass is located at comparatively larger distances from the axis of rotation.

- 138.** Define radius of gyration.

Ans :

Radius of gyration of a body about an axis is the distance between axis of rotation and that point where if whole of the mass of the body is concentrated, it will have the same moment of inertia as with the actual distribution of mass.

- 139.** Torque and work are both defined as force times distance. How do they differ?

Ans :

	Torque	Work
1.	Torque is a vector quantity	Work is a scalar quantity
2.	$\tau = \vec{r} \times \vec{p}$	$W = \vec{F} \times \vec{S}$

- 140.** In a flywheel most of mass is concentrated at the rim. Explain why ?

Ans :

In flywheel most of mass is concentrated at rim so that its moment of inertia is large. If it loses or gains

- 113.** If no external torque acts on a body, will its angular velocity remain conserved ?

Ans :

$$\tau = \frac{dL}{dt}, \text{ if } \tau = 0, L = \text{constant}$$

i.e. L is constant ω will be constant as long as I , the moment of inertia remains constant.

- 114.** If a body is rotating, is it necessary being acted upon by an external torque ?

Ans :

Not necessary, Torque is required if angular acceleration is to be produced.

- 115.** Moment of inertia of a sphere about its diameter is $\frac{2}{5}MR^2$. What is its radius of gyration about that axis ?

Ans :

$$I = MK^2 = \frac{2}{5}MR^2$$

$$K = \sqrt{\frac{2}{5}}R$$

- 116.** A bucket containing water is rotated in a vertical circle. Explain why does not water fall down.

Ans :

Weight of the water is spent in supplying the centripetal force.

- 117.** During motion in the vertical circle, what is the difference in tension at the top and bottom of the circle ?

Ans :

It is 6 mg. i.e. six times the weight of the body.

- 118.** What is rotational analogue of mass ?

Ans :

Rotational inertia or moment of inertia.

- 119.** What happens when a spinning ballet-dancer draws her arms close to her chest ?

Ans :

She will spin faster due to conservation of angular momentum.

- 120.** How a swimmer jumping from a height is able to increase the number of loops made in the air ?

Ans :

By decreasing his moment of inertia by folding his legs and arms.

- 121.** Can centripetal force produce rotation ?

Ans :

Yes

- 122.** What is SI unit of torque ?

Ans :

Nm

- 123.** Is radius of gyration of a body a constant quantity ?

Ans :

No

- 124.** Does moment of inertia change with change of axis of rotation ?

Ans :

Yes

- 125.** Will two spheres of equal mass, one solid and other hollow have equal moment of inertia ?

Ans :

No

- 126.** What are the dimensions and unit of moment of inertia ?

Ans :

These are $[M^1L^2T^0]$ and kg m^2 respectively.

TWO MARKS QUESTIONS

- 127.** Can centre of mass of a body be outside it ? If so, give example.

Ans :

Yes, the centre of mass of a body can be outside it e.g. in a ring, the centre of mass is at the centre where there is no mass.

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- 128.** Define centre of mass of a system of particles.

Ans :

Centre of mass of a system of particles is the point where the entire mass of the system can be supposed to be concentrated and its motion would be the same as if external forces acting on the system are acting at that point.

- 151.** Explain rotation about a fixed axis with an example.

Ans :

In rotation about a fixed axis, every particle of the rigid body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis. A ceiling fan rotating about its central rod is an example.

- 152.** Describe the motion of particles in a rigid body rotating about a fixed z-axis.

Ans :

Each particle, like P_1 at a distance r_1 from the axis, describes a circle of radius r_1 in a plane perpendicular to the z-axis, with its centre on the axis. Particles on the axis remain stationary.

- 153.** Distinguish between rotation with a fixed axis and rotation with a fixed point.

Ans :

In fixed-axis rotation, a line in the rigid body is fixed, like a ceiling fan's axle. In fixed-point rotation, one point of the body is fixed, but the axis passing through this point may change orientation, like a spinning top.

- 154.** What are the characteristics of pure translational motion of a rigid body?

Ans :

In pure translational motion, all particles of the body have the same velocity at any instant. The orientation of any line on the body, like OP, with respect to a fixed direction remains unchanged throughout the motion.

- 155.** How does the motion of a rigid body in combined translation and rotation differ from pure translation?

Ans :

In pure translation, all particles have identical velocities. In combined translation and rotation, different particles like O (centre of mass) and P (any other point) have different velocities, and the body's orientation changes.

- 156.** What is a couple? Describe its effect on a rigid body.

Ans :

A couple is a pair of forces of equal magnitude but acting in opposite directions with different lines of action. A couple produces rotation without translation in a rigid body.

- 157.** State the principle of moments for a lever in equilibrium.

Ans :

The principle of moments for a lever states that for rotational equilibrium, the load arm multiplied by the load equals the effort arm multiplied by the effort ($d_1 F_1 = d_2 F_2$). This means anticlockwise moments balance clockwise moments about the fulcrum.

- 158.** Define the centre of gravity (CG) of an extended body.

Ans :

The centre of gravity (CG) of an extended body is defined as that point where the total gravitational torque on the body is zero. If g is uniform, $\sum m_i r_i \times g = (\sum m_i r_i) \times g = 0$, implying $\sum m_i r_i = 0$.

- 159.** When does the centre of gravity of a body coincide with its centre of mass?

Ans :

The centre of gravity coincides with the centre of mass if the gravitational field g is uniform over the entire body, meaning g does not vary from one part of the body to another. For large bodies, they may not coincide.

- 160.** Define the moment of inertia (I) of a rigid body about an axis.

Ans :

The moment of inertia (I) of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$. Here, m_i is the mass of the i^{th} particle and r_i is its perpendicular distance from the axis of rotation.

- 161.** How is the kinetic energy (K) of a rotating rigid body expressed in terms of its moment of inertia (I) and angular velocity (ω)?

Ans :

The total kinetic energy K of a rigid body rotating about a fixed axis is given by $K = (1/2) I \omega^2$. This is analogous to the translational kinetic energy expression $K = (1/2) m v^2$.

- 162.** On what factors does the moment of inertia of a rigid body depend?

Ans :

The moment of inertia depends on the mass of the body, its shape and size, the distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

- 79.** Is it correct to say that centre of mass of a system of n -particles is always given by the average position vectors of the constituent particles ? If not, when is this statement true ?

Ans :

No. This is true when all particles of the system are of same mass.

- 80.** Can centre of mass of a body coincide with geometrical centre of the body ?

Ans :

Yes, when the body has a uniform mass density.

- 81.** Where does centre of mass of a rectangle lie ?

Ans :

It lies at the point of intersection of the diagonals.

- 82.** If one of the particle is heavier than the other, to which side will their centre of mass shift ?

Ans :

The centre of mass will shift to the heavier particle.

- 83.** Is torque a scalar or a vector quantity ?

Ans :

Torque is a vector quantity.

- 84.** What is S.I. unit of torque ?

Ans :

The S.I. unit of torque is Nm.

- 85.** Which physical quantities are expressed by the following ?

- (a) Rate of change of angular momentum.
(b) Moment of linear momentum.

Ans :

- (a) Torque τ
(b) Angular momentum (L).

- 86.** State law of conservation of angular momentum.

Ans :

It states that if no external torque is acting on an object then, its total angular momentum remains conserved.

- 87.** A body is rotating, it is necessary being acted upon by an external torque ?

Ans :

No, torque is required only for angular acceleration.

- 88.** What is the difference between torque and work ?

Ans :

Torque and work has same dimensional formula. Work is a scalar and torque is a vector.

- 89.** What is physical meaning of angular momentum ?

Ans :

Physically, angular momentum measures the turning movement of a body due to its linear momentum.

- 90.** What is lever arm ?

Ans :

Lever arm is the perpendicular distance between the axis of rotation of a body and line of action of applied force or linear momentum.

- 91.** What is the relation between torque and power ?

Ans :

$$\text{Power} = \text{Torque} \times \text{angular velocity}$$

i.e.,

$$P = \tau \omega$$

- 92.** How torque is related to angular momentum ?

Ans :

$$\text{Torque} \quad \tau = \frac{dL}{dt}$$

- 93.** How angular momentum related to areal velocity ?

Ans :

$$\text{Angular momentum} = 2 \times \text{mass} \times \text{areal velocity}$$

- 94.** Write an expression for torque. Which rule is used for finding its direction ?

Ans :

$\vec{\tau} = \vec{r} \times \vec{F}$. The direction of torque $\vec{\tau}$ is given by right hand screw rule.

- 95.** Define moment of inertia.

Ans :

Moment of inertia of a body about a given axis of rotation is the sum of products of masses of the various particles and square of their perpendicular distances from the axis of rotation.

- 96.** Is there any difference between moment of inertia and rotational inertia ?

Ans :

No, the moment of inertia is another name for rotational inertia.

- 175.** Explain how the angular velocity vector is defined for rotation about a fixed axis.

Ans :

For rotation about a fixed axis, the angular velocity vector ω lies along the axis of rotation. Its magnitude is the rate of change of angular displacement ($d\theta/dt$), and its direction is given by the right-hand rule.

THREE MARKS QUESTIONS

- 176.** The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard ?

For which of the following the two coincides ? A building, a pond, a lake a mountain ?

Ans :

A body is said to be small, if its vertical height is very small as compared to the radius of earth otherwise it is called an extended object.

A building and a pond are called small objects and a lake and a mountain are called extend objects.

- 177.** Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius about an axis passing through their axes of symmetry?

Ans :

The moment of inertia of a body about an axis of rotation is the sum of product of mass and square of distance of constituent particles from the axis of rotation.

i.e.,
$$I = \sum m_i r_i^2$$

In case of a solid sphere, the most of the mass lies at a smaller distance than its radius R and in case of hollow cylinder, all the mass lies at a distance R from the axis of symmetry.

- 178.** What is centre of mass ? Locate its position in two particle system.

Ans :

Centre of mass of a system is the point at which the entire mass of the system can be supposed to be concentrated. The nature of motion of the system shall remain unaffected if all the forces acting on the system were applied directly on the centre of mass of the system.

Let m_1 and m_2 be situated at position vectors $\vec{r}_1$ and $\vec{r}_2$ with respect to origin O . Then the centre of mass is a point which will have a position vector $\vec{r}_{cm}$ defined as

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

- 179.** Define a rigid body. Discuss centre of mass of rigid bodies.

Ans :

Rigid body is a body with a fixed geometrical shape and size, both of which do not change during the motion. We can think of such a body made up of a large number of small parts, each regarded as a point mass.

The rigidity means that the internal forces among the point mass particle are such that the mutual distances between the points never change.

	Shape of body	Position of centre of mass
1.	Uniform rod	Mid point of the rod
2.	Circular ring	Centre of ring
3.	Circular disc	Centre of disc
4.	A plane lamina such as rectangle, Ilgm, square	Point of intersection of diagonals
5.	Sphere	Centre
6.	Cylinder	Mid point of axis
7.	Triangular plane lamina	Point of intersection of medians
8.	Cubical block	Point of intersection of diagonals

- 180.** Show that for an isolated system the centre of mass moves with a uniform velocity along a straight line path.

Ans :

Let M be the total mass of the system concentrated at the centre of mass whose position vector is $\vec{r}_{cm}$, then

$$M \frac{d^2 \vec{r}_{cm}}{dt^2} = \vec{F}$$

$$M \frac{d}{dt} \left(\frac{d\vec{r}_{cm}}{dt} \right) = \vec{F}$$

$$M \frac{d}{dt} (\vec{v}_{cm}) = \vec{F}$$

For isolated system,

$$\vec{F} = 0$$

- 192.** Define moment of inertia. What factors influence the moment of inertia of a rigid body?

Ans :

The moment of inertia (I) of a rigid body about an axis is a measure of its rotational inertia, analogous to mass in linear motion. It is defined as $I = \sum m_i r_i^2$, where m_i is the mass of the i^{th} particle and r_i is its perpendicular distance from the axis of rotation. Factors influencing I are:

- The mass of the body.
- Its shape and size.
- The distribution of mass about the axis of rotation.
- The position and orientation of the axis of rotation.

- 193.** How is the kinetic energy of a rotating body related to its moment of inertia and angular velocity?

Ans :

The kinetic energy (K) of a rigid body rotating about a fixed axis is given by the expression $K = (1/2) I \omega^2$.

- I is the moment of inertia of the body about the axis of rotation.
- ω is the angular velocity of the body.

This formula is analogous to the translational kinetic energy formula $K = (1/2) m v^2$, with I replacing m and ω replacing v.

- 194.** Derive the kinematic equation $\omega = \omega_0 + \alpha t$ for rotational motion from first principles.

Ans :

This equation relates final angular velocity (ω) to initial angular velocity (ω_0), uniform angular acceleration (α), and time (t).

By definition, uniform angular acceleration is $\alpha = d\omega/dt$.

Rearranging gives $d\omega = \alpha dt$.

Integrating both sides, $\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha dt$.

Since α is constant, $\omega - \omega_0 = \alpha(t - 0)$.

Thus, $\omega = \omega_0 + \alpha t$.

- 195.** Explain how work done by a torque is calculated for a body rotating about a fixed axis.

Ans :

The work done by a torque on a body rotating about a fixed axis is analogous to work done by a force in linear motion. If a torque τ acts on the body, causing an infinitesimal angular displacement $d\theta$, the work done dW is:

$$dW = \tau d\theta$$

For a finite angular displacement from θ_1 to θ_2 , the total work done is $W = \int_{\theta_1}^{\theta_2} \tau d\theta$. If τ is constant, $W = \tau(\theta_2 - \theta_1)$.

- 196.** State and explain Newton's second law for rotational motion about a fixed axis.

Ans :

Newton's second law for rotational motion about a fixed axis states that the net external torque (τ) acting on a rigid body is equal to the product of its moment of inertia (I) about that axis and its angular acceleration (α).

Mathematically, $\tau = I\alpha$.

This law is analogous to $F = ma$ for linear motion, where torque causes angular acceleration, and moment of inertia represents resistance to this change in rotational motion.

- 197.** Discuss the angular momentum of a system of particles when rotating about a fixed axis.

Ans :

For a system of particles rotating about a fixed axis (say, the z-axis), the component of the total angular momentum along this axis (L_z) is given by $L_z = I\omega$, where I is the moment of inertia about the z-axis and ω is the angular velocity. The total angular momentum L also has a component $L_{\perp}$ perpendicular to the axis, so $L = L_z + L_{\perp}$. For bodies symmetric about the axis of rotation, $L_{\perp}$ is often zero, making $L = L_z = I\omega \hat{k}$.

- 198.** Explain the principle of conservation of angular momentum in the context of rotation about a fixed axis.

Ans :

The principle of conservation of angular momentum states that if the total external torque acting on a system about a fixed axis is zero, then the component of the total angular momentum of the system along that axis (L_z) remains constant.

Mathematically, if $\tau_{ext,z} = 0$, then $L_z = I\omega = \text{constant}$. This implies that if the moment of inertia I changes, the angular velocity ω must change inversely to keep $I\omega$ constant.

- 199.** A block slides down an inclined plane. Describe its motion if it's purely translational versus rolling.

Ans :

The motion of a block down an inclined plane can be described as:

- **Pure Translational (Sliding):** If the block slides without any rotation, all its particles move together with the same velocity at any given

$$\text{So} \quad \frac{E_1}{E_2} = \frac{I_2}{I_1}$$

$$\text{As} \quad I_2 < I_1$$

$$\text{Hence,} \quad E_1 < E_2$$

Kinetic energy is not conserved.

- 185.** State and prove law of conservation of angular momentum.

Ans :

Law of conservation of angular momentum states that, if no external torque is acting on a system (i.e. in isolated system), the total angular momentum of the system remains constant.

If $\vec{L}$ is the angular momentum and $\vec{\tau}$ is the torque acting on a system, then

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau} = 0$, then

$$\frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{constant}$$

$$I\vec{\omega} = \text{constant}$$

- 186.** Define torque. What is the dimensional formula and SI unit of torque?

Ans :

Torque, or moment of force, is the rotational analogue of linear force and measures the turning effect of a force. If a force F acts on a particle at a position r relative to an origin O , the torque τ about O is defined as $\tau = r \times F$.

- Its magnitude is $\tau = rF \sin \theta$.
- The dimensional formula for torque is $[ML^2 T^{-2}]$
- The SI unit of torque is newton-metre (Nm).

- 187.** Define angular momentum of a particle and state its relation with torque.

Ans :

The angular momentum l of a particle with linear momentum p , at a position r relative to an origin O , is defined as the vector product $l = r \times p$. It is the rotational analogue of linear momentum.

The relation between torque τ and angular momentum l for a particle is that the time rate of change of the angular momentum of the particle is equal to the torque acting on it: $dl/dt = \tau$.

- 188.** Explain the conditions for the mechanical equilibrium of a rigid body.

Ans :

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time. This implies two conditions:

- **Translational Equilibrium:** The vector sum of all external forces acting on the body must be zero ($\sum F_i = 0$). This ensures the body has no linear acceleration.
- **Rotational Equilibrium:** The vector sum of all external torques acting on the body (about any origin) must be zero ($\sum \tau_i = 0$). This ensures the body has no angular acceleration.

- 189.** What is meant by a couple? Show that the moment of a couple is independent of the point about which moments are taken.

Ans :

A couple is defined as a pair of forces of equal magnitude acting in opposite directions with different lines of action. It produces rotation without translation.

Consider forces F and $-F$ acting at points B and A with position vectors r_2 and r_1 from origin O . The total moment is $r_2 \times F + r_1 \times (-F) = (r_2 - r_1) \times F$. Since $r_2 - r_1 = AB$ (the vector from A to B), the moment of the couple is $AB \times F$, which only depends on the vector AB and F , not on the origin O .

- 190.** State and explain the principle of moments as applied to a lever.

Ans :

The principle of moments for a lever in equilibrium states that the sum of clockwise moments about the fulcrum equals the sum of anticlockwise moments. For a simple lever with load F_1 at load arm d_1 and effort F_2 at effort arm d_2 , this is expressed as:

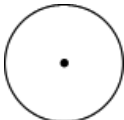
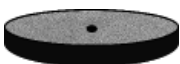
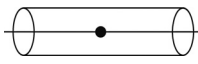
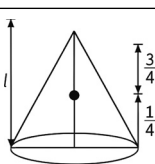
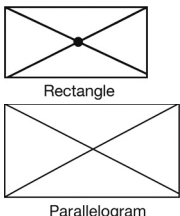
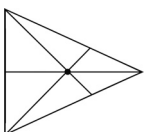
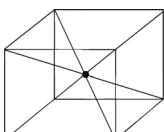
$$d_1 F_1 = d_2 F_2$$

This implies that "load arm $\times$ load = effort arm $\times$ effort". The ratio F_1/F_2 is the mechanical advantage.

- 191.** Explain the concept of centre of gravity and its relation to the centre of mass.

Ans :

The centre of gravity (CG) of an extended body is the point where the total gravitational torque on the body is zero. If r_i is the position vector of the i^{th} particle with respect to CG, then $\sum r_i \times m_i g = 0$. The CG coincides with the centre of mass (CM) if the gravitational field g is uniform over the body. The CM depends only on mass distribution, while CG also involves gravity.

S. NO	Shape of the body	Position of centre of mass	Diagram
1.	Uniform Rod	Mid point of rod	
2.	Circular Ring	Centre of the ring	
3.	Solid or Hollow Sphere	Centre of the sphere	
4.	Circular Disc	Centre of the disc	
5.	Solid or Hollow Cylinder	Mid point of the axis	
6.	Cone or Pyramid	At the height of one third from the apex	
7.	Plane Rectangle, Parallelogram or Square	At the mid point of the intersection of two diagonals	
8.	Triangular Lamina	Point of intersection of medians	
9.	Rectangular or Cubical Block	Point of intersection of diagonals	

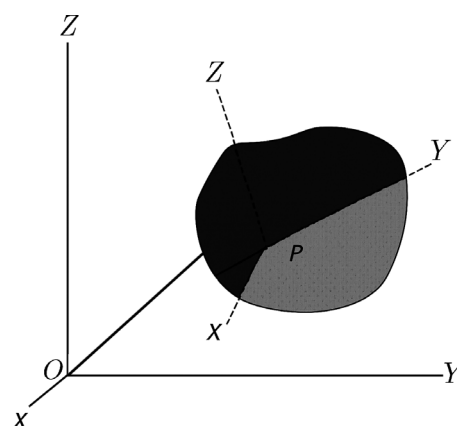
The rigidity means that the internal force among the point mass particle are such that the mutual distances between the points never change.

Centre of mass of a rigid body

Centre of mass of a rigid body is a fixed point where whole mass of the body is supposed to be concentrated and does not change when external forces are applied on it.

The centre of mass of a rigid body depends upon the shape and distribution of mass in a body. The centre of mass may or may not lie within the body. The centre of mass of various bodies is as shown on previous page :

- (b) The general motion of a rigid body. Suppose an inertial frame with origin O and mutually perpendicular co-ordinate axes OX, OY, OZ are given. We first pick once and for all some convenient point which is fixed in the body. Then, if we give the coordinates of P_0 with respect to O and the given axis, and after that the orientation of the body about P_0 the position of the entire body will be completely fixed. For the latter purpose, we could imagine a set of mutually perpendicular axes X', Y', Z' fixed in the body at P_0 . Their directions with respect to the axes X, Y, Z of the inertial frame gives us the orientation of the body.



In a general motion of the body, both the position vector of P_0 and the orientation about P_0 will change with time. We call these the translational and rotational parts of the motion respectively.

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- 216.** Define torque. Discuss the rotational motion of a single particle in a plane.

Ans :

The turning effect of force is called torque and is measured as the product of magnitude of force and perpendicular distance from the axis of rotation.

If a force $\vec{F}$ acts on a single particle at a point P whose position with respect to the origin O of the inertial frame is given by displacement vector $\vec{r}$ the torque $\vec{\tau}$ acting on the particle with respect to origin O is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

instant. The motion is governed by net force along the incline.

- **Rolling:** If the block rolls, it undergoes both translational motion of its centre of mass down the incline and rotational motion about an axis through its centre of mass. Different particles have different velocities.

200. What is precession? Explain with the example of a spinning top.

Ans :

Precession is the phenomenon where the axis of rotation of a spinning object itself rotates around another axis. A prominent example is a spinning top.

- The top spins rapidly about its own axis.
- Due to torque (often from gravity acting on its tilted axis), this axis of spin moves around the vertical line passing through its point of contact with the ground, sweeping out a cone. This movement of the spin axis is precession.

201. How does the concept of centre of mass simplify the analysis of projectile motion after an explosion?

Ans :

The concept of centre of mass simplifies the analysis because the motion of the centre of mass (CM) of the projectile's fragments is unaffected by internal forces of the explosion.

- The CM continues to follow the same parabolic trajectory it would have if no explosion occurred.
- Only external forces like gravity influence the CM's path. This allows prediction of the CM's trajectory regardless of the complex motion of individual fragments.

202. Explain the non-commutative nature of the vector product using unit vectors.

Ans :

The vector product is non-commutative, meaning $a \times b \neq b \times a$; specifically, $a \times b = -b \times a$.

For orthogonal unit vectors:

- $\hat{i} \times \hat{j} = \hat{k}$
- However, $\hat{j} \times \hat{i} = -\hat{k}$

This clearly shows that reversing the order of multiplication reverses the direction of the resultant vector, demonstrating non-commutativity.

203. Describe the conditions under which the linear velocity of a particle in a rotating body is zero.

Ans :

The linear velocity (v) of a particle in a rotating rigid body is given by $v = \omega r_{\perp}$, where ω is the angular

velocity and $r_{\perp}$ is the perpendicular distance of the particle from the axis of rotation.

The linear velocity is zero if:

- The particle lies on the axis of rotation, meaning $r_{\perp} = 0$.
- The body is not rotating, meaning $\omega = 0$.

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204. Why is torque considered the rotational analogue of force? Justify with an example.

Ans :

Torque is considered the rotational analogue of force because it causes a change in the state of rotational motion (i.e., angular acceleration), just as force causes a change in linear motion (linear acceleration).

For example, to rotate a door (a rigid body) about its hinges (axis), a force must be applied. The effectiveness of this force in causing rotation (torque) depends on its magnitude and where it's applied relative to the hinges.

FIVE MARKS QUESTIONS

- 205.** (a) Define moment of inertia.
(b) Give quantitative definition of moment of inertia. Find an expression of moment of inertia of a body about XY-axis.

Ans :

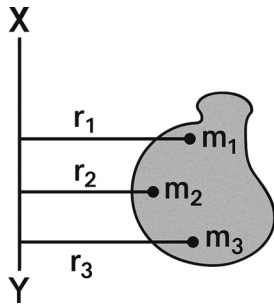
- (a) Moment of inertia of a body is defined as its property by virtue of which it opposes any change in its state of rest or rotational motion.
(b) Quantitatively, the moment of inertia of a body about an axis is defined as the sum of product of mass and square of distance of constituent particles from the axis of rotation.

Let $m_1, m_2, m_3, \dots$ be the masses of constituent particles, and $r_1, r_2, r_3, \dots$ be the distance of particles from the axis of rotation XY (as shown in the figure).

Moment of inertia of body about an axis XY.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

$$= \sum_{i=1}^n m_i r_i^2$$



Unit of moment of inertia is kg m^2 . It is a scalar quantity.

Dimensional formula of moment of inertia

$$= [M][L^2] = [ML^2]$$

Moment of inertia of a body depends upon

1. Distribution of constituent particles of the body about the axis of rotation.
2. Distance of particles from the axis of rotation.
3. Mass of body.

206. Derive equation of motion of rigid body. What does it infer ?

Ans :

Suppose a rigid body moment of inertia I rotates about an axis (say z -axis) with angular velocity ω .

Angular momentum of body about z -axis $= L_z$

Rate of change of angular momentum

$$= \frac{d}{dt}(L_z) \frac{d}{dt}(I\omega) = I \frac{d\omega}{dt}$$

$$= I\alpha = \tau_z$$

$$= \text{Total external torque about } z\text{-axis}$$

Inference :

- (1) If torque is known, then we can predict the behaviour of $\omega(t)$ and $\alpha(t)$ i.e. we predict the kind of motion, and
- (2) If kind of motion is known then equation guides us in arranging the external force to produce the necessary total torque about OZ .

207. What is radius of gyration ? Find expression for it ?

Ans :

Radius of gyration of a body is the perpendicular distance from the axis of rotation of a point, at which whole mass of body is supposed to be concentrated, such that its moment of inertia about the axis comes out to be the same as with the actual mass distribution.

Expression : Let M = Mass of body supposed to be concentrated at a distance K (radius of gyration).
Moment of inertia of a body, $I = MK^2$

If the body consists of n particles each of mass m , then

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \dots + mr_n^2$$

$$MK^2 = mr_1^2 + mr_2^2 + mr_3^2 + \dots + mr_n^2$$

$$= m(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2) \quad (M = mn)$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots}{n}}$$

i.e., radius of gyration is the square root of mean of squares of $\perp$ distances of particles from the axis of rotation. Radius of gyration changes with location of axis of rotation.

Hence radius of gyration depends upon :

- (i) distribution of mass from the axis of rotation.
- (ii) the position and direction of the axis of rotation.

Unit. In S.I. the units of K is m.

The dimensional formula of radius of gyration is $[L] = [M^0 L T^0]$.

208. State law of conservation of angular momentum. Give its applications.

Ans :

Law of conservation of angular momentum states that if no torque acts on a system, the angular momentum of the system remains constant.

1. The angular velocity of a planet having elliptical orbit about sun increases as it comes near the sun and decreases as it goes away from the sun due to conservation of angular momentum $[L = I\omega]$.
2. A rotating ice skater can increase his angular velocity by folding his stretched arms and bringing his stretched leg close to the other leg.
3. The speed of inner layer of the whirlwind about its axis in a tornado is very high because the moment of inertia of inner layer is very small, so ω is very large ($I_\omega = \text{constant}$).
4. A diver jumping from spring board sometimes exhibits somersaults in air before touching the water surface by using law of conservation of angular momentum.
5. A person having some weights in his hands and sitting on a rotating turn table can increase his speed of rotation drawing his weights close to his chest.

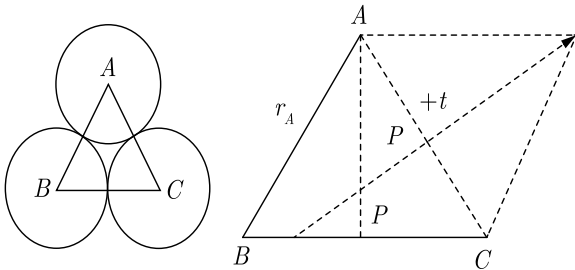
209. Three identical spheres each of radius R are placed touching each other on a horizontal table. Where is the centre of mass of the system located ?

Ans :

The centre of mass of each sphere (assumed to be of uniform mass density) is at its centre. The center

of mass of the system is then the center of mass of three equal mass points located at the vertices of the equilateral triangle ABC of side $2R$, where A , B and C are the centres of the three spheres. To locate the center of mass, choose one of the points, say B , as the origin. The position vector of the center of mass with respect to this origin is given by

$$R = \frac{1}{3M}(Mr_A + Mr_C) = \frac{1}{3}(r_A + r_C)$$



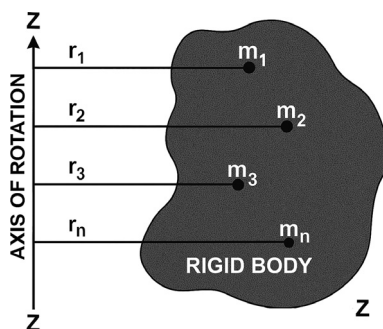
where, M is the mass of each sphere r_A and r_C are the position vectors A and C respectively relative to the origin at B . Using the parallelogram rule of addition of vectors and the fact that ABC is an equilateral triangle we see clearly from the figure that the center of mass of the system P is located at the point of intersection of the medians (or angle or side bisectors) of the equilateral triangle ABC . The result is what you would expect anyway from the symmetry of the problem.

- 210.** Establish the relation $L = I\omega$ and hence define moment of inertia.

Ans :

Consider a rigid body rotating about a given axis XY with constant angular velocity ω .

Let $m_1, m_2, m_3, \dots, m_n$ be the masses of the various particles of the body and $r_1, r_2, \dots, r_n$ be their respective perpendicular distances from the axis of rotation. Let $v_1, v_2, \dots, v_n$ be tangential velocities of the particles.



$$v_1 = r_1\omega, v_2 = r_2\omega \text{ and so on}$$

Moment of momentum of particles of mass m_i about XY axis

$$(m_i v_i) r_i = (m_i r_i \omega) r_i = m_i r_i^2 \omega$$

Similarly moment of momentum of particles of mass m_2 about XY axis $= m_2 r_2^2 \omega$ and so on.

Angular momentum of the body about XY = Sum of moments of momenta of various particles

$$\begin{aligned} \text{i.e. } L &= m_1 r_1^2 \omega + m_2 r_2^2 \omega + \dots + m_n r_n^2 \omega \\ L &= \omega [m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2] \\ &= \omega \left(\sum_{i=1}^n m_i r_i^2 \right) = \omega I \quad \left[\because \sum_{i=1}^n m_i r_i^2 = I \right] \end{aligned} \quad \dots(1)$$

$$L = I\omega$$

From Equation (1),

$$I = \frac{L}{\omega}$$

If $\omega = 1$, then $I = L$

So moment of inertia of a rigid body about an axis is numerically equal to angular momentum of the body when rotating with unit angular velocity.

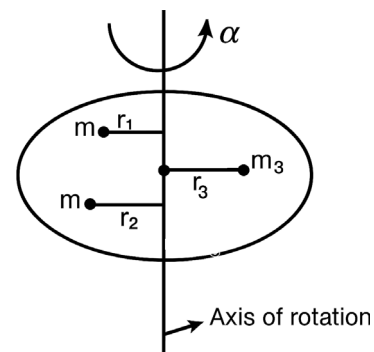
- 211.** Deduce the relation between torque and moment of inertia.

Ans :

Let a body consists of n particles of masses $m_1, m_2, \dots, m_n$ at perpendicular distances $r_1, r_2, \dots, r_n$ respectively from axis of rotation. Since the body is rigid, therefore, angular acceleration of all the particles is the same i.e. it is α in each case.

If $a_1, a_2, a_3, \dots, a_n$ are the respective linear accelerations,

$$\text{then, } a_1 = r_1 \alpha; a_2 = r_2 \alpha; \dots, a_n = r_n \alpha$$



Torque acting on the body

$$\tau = \tau_1 + \tau_2 + \dots + \tau_n,$$

where $\tau_1, \tau_2, \dots$ are the torques on individual particles.

$$\tau = r_1 F_1 + r_2 F_2 + \dots r_n F_n$$

$$\tau = r_1 m_1 a_1 + r_2 m_2 a_2 + \dots r_n m_n a_n$$

$$\tau = r_1 m_1 (r_1 \alpha) + r_2 m_2 (r_2 \alpha) + \dots + r_n m_n (r_n \alpha)$$

$$\tau = (\sum m r_i^2) \alpha = I \alpha$$

$$\tau = I \alpha$$

If $\alpha = 1$, $I = \tau$

Hence, moment of inertia of a rigid body about an axis is the torque experienced by body about that axis when rotating with a unit angular acceleration.

- 212.** What do you mean by centre of mass of a system ? What is its significance ? Obtain expression for the position vector of centre of mass of two particle

Ans :

Centre of mass (Concept of centre of mass)

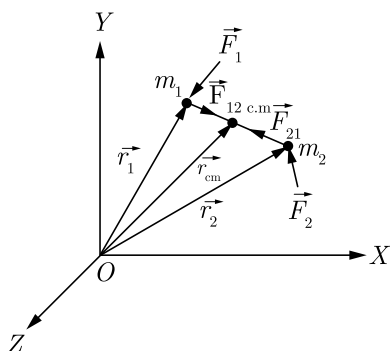
Centre of mass of a system is defined as that point at which whole mass of the system is supposed to be concentrated and its motion would be the same as if external forces acting on the system are acting at that point.

Significance of centre of mass

It becomes very easy to deal with the motion, momentum and energy using the concept of centre of mass rather than the individual particles.

Centre of mass of two particle system

Consider a system consisting of two particles of masses m_1 and m_2 acted upon by external forces $\vec{F}_1$ and $\vec{F}_2$ respectively. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors from the origin O as shown in figure.



If $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the particles of masses m_1 and m_2 respectively, then

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt} \text{ and } \vec{v}_2 = \frac{d\vec{r}_2}{dt}$$

Let $\vec{F}_{12}$ = internal force exerted on m_1 by m_2

and $\vec{F}_{21}$ = internal force exerted on m_2 by m_1

Force Newton's 3rd law of motion, we have

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0 \quad \dots(1)$$

Total force acting on particle $m_1 = \vec{F}_1 + \vec{F}_{12}$

For particle of mass m_1

From Newton's second law of motion, we have

$$\frac{d}{dt}(m_1 \vec{v}_1) = \vec{F}_1 + \vec{F}_{12} \quad \dots(2)$$

Similarly, for particle of mass m_2

$$\frac{d}{dt}(m_2 \vec{v}_2) = \vec{F}_2 + \vec{F}_{21} \quad \dots(3)$$

Adding equations (2) and (3), we get

$$\frac{d}{dt}(m_1 \vec{v}_1) + \frac{d}{dt}(m_2 \vec{v}_2) = \vec{F}_1 + \vec{F}_{12} + \vec{F}_2 + \vec{F}_{21}$$

$$\frac{d}{dt}(m_1 \vec{v}_1 + m_2 \vec{v}_2) = (\vec{F}_1 + \vec{F}_2) + (\vec{F}_{12} + \vec{F}_{21})$$

Using equation (1), we get

$$\frac{d}{dt}(m_1 \vec{v}_1 + m_2 \vec{v}_2) = (\vec{F}_1 + \vec{F}_2)$$

$$\frac{d}{dt}\left(m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt}\right) = \vec{F}$$

(where $\vec{F} = \vec{F}_1 + \vec{F}_2$ is the total external force acting on the system)

$$\frac{d^2}{dt^2}(m_1 \vec{r}_1 + m_2 \vec{r}_2) = \vec{F}$$

$$\vec{F} = \frac{d^2}{dt^2}(m_1 \vec{r}_1 + m_2 \vec{r}_2)$$

$$= (m_1 + m_2) \frac{d^2}{dt^2} \frac{(m_1 \vec{r}_1 + m_2 \vec{r}_2)}{m_1 + m_2}$$

$$\vec{F} = (m_1 + m_2) \frac{d^2}{dt^2} \vec{r}_{cm}$$

$$\text{where } \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

is the position vector of centre of mass of two particle system.

- 213.** Determine the co-ordinates of centre of mass of a system of n particles.

Ans :

Let a system consists of n particles of masses $m_1, m_2, \dots, m_n$ whose positions in the co-ordinate system are given by $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$. Let $\vec{r}_1$ be the position vector of centre of mass of n particle system. Let $\vec{f}_1, \vec{f}_2, \dots, \vec{f}_n$ be the external forces acting on n particles of the system. Each particle experiences internal forces due to rest of $(n-1)$ particles, Internal force of i th particle is

$$\vec{F}_i = \sum_{\substack{j=1 \\ j \neq i}}^n \vec{F}_{ij}$$

If $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are the instantaneous velocities of the particles, then

$$\frac{d}{dt}(m_i \vec{v}_i) = \vec{F}_i + \vec{f}_i$$

In general, for n particles, above equation modifies to

$$\sum_{i=1}^n \frac{d}{dt}(m_i \vec{v}_i) = \sum_{i=1}^n \vec{F}_i + \sum_{i=1}^n \vec{f}_i$$

Internal forces cancel out in pairs, therefore,

$$\sum_{i=1}^n \vec{F}_i = 0$$

$$\sum_{i=1}^n \frac{d}{dt}(m_i \vec{v}_i) = \vec{f}, \text{ where } \vec{f} = \sum_{i=1}^n \vec{f}_i$$

i.e. The vector sum of all external forces

$$\sum_{i=1}^n \frac{d}{dt}(m_i \frac{d\vec{r}_i}{dt}) = \vec{f}$$

$$\sum_{i=1}^n \frac{d^2}{dt^2}(m_i \vec{r}_i) = \vec{f}$$

But $M \frac{d^2 \vec{r}_{cm}}{dt^2} = \vec{f}$

$$\sum_{i=1}^n \frac{d^2}{dt^2}(m_i \vec{r}_i) = M \frac{d^2}{dt^2} \vec{r}_{cm}$$

$$\vec{r}_{cm} = \sum_{i=1}^n \frac{m_i \vec{r}_i}{M}$$

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{m_1 + m_2 + \dots + m_n}$$

214. Illustrate with examples, the concept of centre of mass.

Ans :

- Exploding fire cracker :** Consider a fire cracker which goes up into the sky, and then at some point explodes into many fiery pieces, which creates a spectacular star. Before exploding, the cracker would be going as a parabola : that is the path of any object moving with constant acceleration due to the earth's gravity. After the explosion, each fragment will travel on its own parabolic path, but since the explosion is caused due to internal forces, the centre of mass continues in parabolic path shown by dotted lines in the figure (a). Thus after explosion the centre of mass is just a mathematical point.

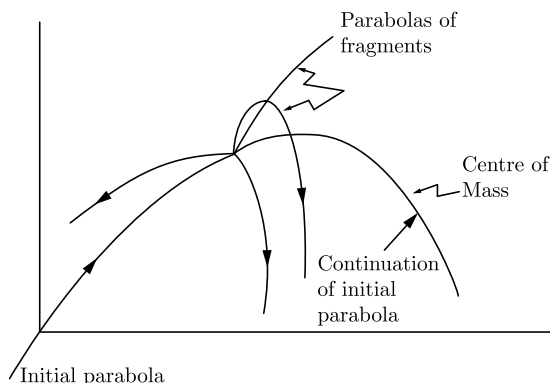


Figure: (a)

- Earth moon system :** Another example is the Earth-Moon system. The Earth and the Moon both travel in circles about their centre of mass, always being on opposite sides of it. The centre of mass describes an ellipse round the Sun. This is shown in figure (b).

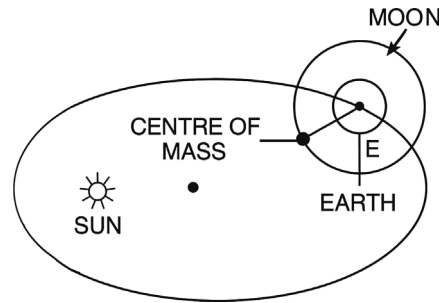


Figure: (b)

Here the force of attraction between the Earth and Moon is internal to the Earth-Moon System, while the Sun's attraction of both Earth and Moon are external forces.

- Decay of nucleus :** A third example is the spontaneous decay of radioactive nucleus, say, into two fragments. If the parent nucleus is initially at rest, then since the decay is caused by internal processes alone, the centre of mass of the decay products continues at rest. Momentum is conserved in the decay so it remains zero. The heavier and the lighter fragments then fly away in opposite directions with former having a proportionably smaller speed than the latter.

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- 215.** (a) Define a rigid body. Discuss centre of mass of rigid bodies.
 (b) Discuss general motion of a rigid body.

Ans :

- (a) **Rigid body :** Rigid body is a body with a fixed geometrical shape and size, both of which do not change during the motion. We can think of such a body made up of a large number of small parts, each regarded as a point mass.

Thus, $M \frac{d}{dt}(\vec{v}_{cm}) = 0$

$$\frac{d}{dt}(\vec{v}_{cm}) = 0$$

Hence, $\vec{v}_{cm} = \text{constant}$

Hence centre of mass of an isolated system moves with a uniform velocity along a straight line path.

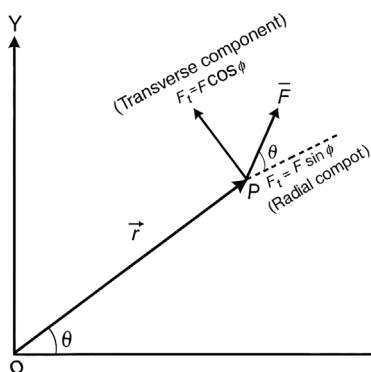
- 181.** Show that torque is due to transverse component only and does not depend on the radial component.

Ans :

Consider a particle P having position vector $\vec{r}$ making an angle θ with x -axis and let a force $\vec{F}$ on it at an angle ϕ with $\vec{r}$ as shown in Figure.

The torque τ acting on particle P is given by

$$\tau = rF \sin \phi \quad \dots(1)$$



Resolve the force $\vec{F}$ into two rectangular components F_r called radial component of $\vec{F}$ and F_θ is called transverse component of $\vec{F}$, then

$$F_r = F \cos \phi$$

and $F_\theta = F \sin \phi \quad \dots(2)$

Putting equations (2) in (1), we get

$$\tau = rF_\theta$$

Thus the torque does not depend on the radial component F_r of the force at all. This component has no turning effect on the particle, only a pushing or pulling from the origin. It is only the component F_θ that has turning effect.

- 182.** State law of conservation of angular momentum and illustrate with example.

Ans :

Equation of motion of angular momentum of a particle is :

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

If there is no external torque i.e. $\vec{\tau} = 0$

$$\frac{d\vec{L}}{dt} = 0$$

$\vec{L}$ is constant of motion i.e. L_x, L_y, L_z , is constant of time.

The law of conservation of angular momentum of particles states that if no external torque acts on the particle the angular momentum vector is conserved.

As angular momentum is a vector quantity, so in the absence of torque, both the magnitude and direction of angular momentum remains constant e.g. when a planet revolves around the sun, no external torque is acting on it, hence magnitude of angular momentum is constant and thus areal velocity of the planet is constant and fixed due to fixed direction of angular momentum shows that the plane of orbital motion of the planet around the sun also must remain fixed.

- 183.** State triangle law of vector addition. Show that the vector $\vec{A} = -6\hat{i} + 9\hat{j} - 12\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ are parallel to each other.

Ans :

Triangle law of vector addition states that if two vectors are represented both in magnitude and direction by the two sides of a triangle taken in same order, then the resultant is represented both in magnitude and direction of the third side of the triangle in opposite order.

Two vectors are said to be parallel, if their cross product is zero

$$\begin{aligned} \vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 9 & -12 \\ 2 & -3 & 4 \end{vmatrix} \\ &= \hat{i}(36 - 36) + \hat{j}(-24 + 24) + \hat{k}(18 - 18) \end{aligned}$$

The vectors $\vec{A}$ and $\vec{B}$ are parallel.

- 184.** If the angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved ? Explain.

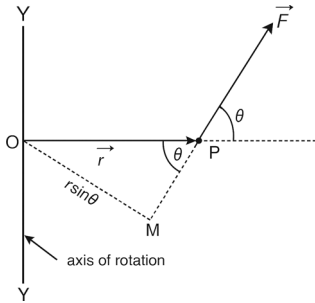
Ans :

Since, $I\omega = \text{constant}$

$$I_1\omega_1 = I_2\omega_2 \quad \dots(1)$$

Squaring (1), and multiplying by $\frac{1}{2}$ on both sides, we get

$$\begin{aligned} \frac{1}{2} I_1^2 \omega_1^2 &= \frac{1}{2} I_2^2 \omega_2^2 \\ \frac{1}{2} I_1 \omega_1^2 \times I_1 &= \frac{1}{2} I_2 \omega_2^2 \times I_2 \\ I_1 E_1 &= I_2 E_2 \end{aligned}$$



Magnitude of torque is given by $\tau = rF \sin \theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$. Direction torque is given by right hand rule and is perpendicular to the plane of $\vec{r}$ and $\vec{F}$.

Units of torque

Unit of torque is Nm and dimensions of torque are $[M^1 L^2 T^{-2}]$.

- When $\theta = 0^\circ$ or 180° , $\tau = 0$ i.e. a force has no torque. If its line of action passes through origin or intersects the axis of rotation.
- When $\theta = 90^\circ$, $\tau = rF$ i.e., the force has the maximum torque.

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- 217.** Derive expression for torque in Cartesian co-ordinate system.

or

Show that $\tau = xF_y - yF_x$

Ans :

Consider a particle P having position vector $\vec{r}$ w.r.t. origin O . This particle can be rotated about an axis passing through O and perpendicular to the plane containing $\vec{r}$ and $\vec{F}$ i.e. along z -axis. Let a force $\vec{F}$ applied on the particle at P displaces it to point Q in small time interval Δt s.t. $\angle POQ = \Delta \theta$.

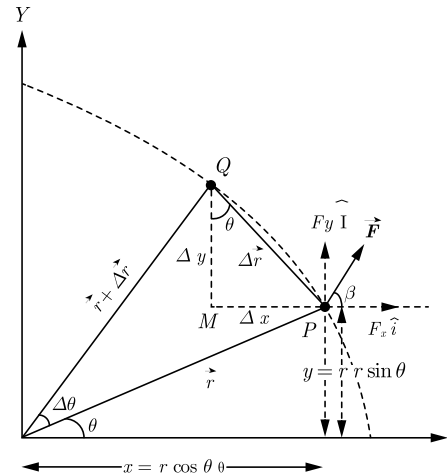
Then $\vec{PQ} = \Delta \vec{r}$ the displacement of the particle as shown in figure.

Small amount of work done,

$$\Delta W = \vec{F} \cdot \Delta \vec{r} \quad \dots(1)$$

If F_x and F_y are rectangular components of $\vec{F}$ along x and y -axis respectively, then

$$\vec{F} = F_x \hat{i} + F_y \hat{j} \quad \dots(2)$$



If Δx and Δy are rectangular components of $\Delta \vec{r}$ along X and Y -axis respectively, then

$$\begin{aligned} \Delta \vec{r} &= \Delta x(-\hat{i}) + \Delta y(\hat{j}) \\ &= -\Delta x \hat{i} + \Delta y \hat{j} \end{aligned} \quad \dots(3)$$

Putting equations (2) and (3) in (1), we get

$$\begin{aligned} \Delta W &= (F_x \hat{i} + F_y \hat{j}) \cdot (-\Delta x \hat{i} + \Delta y \hat{j}) \\ \Delta W &= -\Delta x F_x + \Delta y F_y \end{aligned} \quad \dots(4)$$

In. rt. $\angle \Delta PMQ$

$$\begin{aligned} \frac{PM}{PQ} &= \sin \theta \\ PM &= PQ \sin \theta \\ \Delta x &= \Delta r \cdot \frac{y}{r} \quad \left[\text{Since, In rt. } \angle \Delta ONP \right. \\ &\quad \left. \sin \theta = \frac{y}{r} \right] \\ \Delta x &= y \frac{\Delta r}{r} \\ \Delta x &= y \Delta \theta \end{aligned} \quad \dots(5)$$

$$\left[\text{since, angle} = \frac{\text{arc}}{\text{radius}} \text{ i.e. } \Delta \theta = \frac{\Delta r}{r} \right]$$

Similarly $\Delta y = x \Delta \theta$

Putting equations (5) and (4), we get

$$\begin{aligned} \Delta W &= -y \Delta \theta F_x + \Delta \theta F_y \\ \Delta W &= (xF_y - yF_x) \Delta \theta \\ \Delta W &= \tau \Delta \theta \end{aligned} \quad \dots(6)$$

$$\text{where } \tau = xF_y - yF_x \quad \dots(7)$$

Hence, torque of force F about an axis acts $\perp$ to the plane.

- 218.** Derive expression for torque in polar coordinates.

or

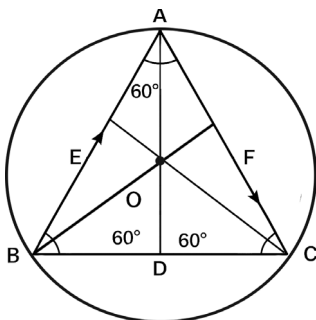
Show that $\tau = F \cdot d$

where τ is torque, F is force applied and d is the $\perp$ distance of the particle from the axis of rotation called lever arm.

(b) Comparison of relations in linear and rotational motions :

	Linear motion	Rotational motion
1.	Position x	Angle θ
2.	Velocity	Angular velocity
3.	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
4.	Acceleration	Angular acceleration
5.	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$
6.	Mass m	Moment of inertia I
7.	Momentum	Angular momentum
8.	$p = mv$	$L = I\omega$
9.	Force F	Torque τ
10.	Newton's law	Consequence of Newton's law
11.	$F = \frac{dp}{dt}$	$\tau = \frac{dL}{dt}$
12.	Kinetic energy	Kinetic energy
13.	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
14.	Power $= Fv$	Power $= \tau\omega$

- 227.** Three particles, each of mass ' m ' are situated at the vertices of an equilateral triangle of side ' a '. The only force acting on the particles are their mutual gravitational forces. It is desired that each particle move on a circle, while maintaining the original mutual separation ' a '. Find the initial velocity that should be given to each particle and also the time period of the circular motion.



Ans :

Gravitational force between any two particles is;

$$F = \frac{Gmm}{a^2} = \frac{Gm^2}{a^2}$$

Resultant force on each particle due to the other two particles is,

$$R = \sqrt{F^2 + F^2 + 2FF\cos 60^\circ}$$

$$= \sqrt{F^2 + F^2 + F^2} = \sqrt{3} F$$

Hence, $R = \frac{\sqrt{3} Gm^2}{a^2}$

If the particles were at rest, each particle would move under the action of resultant force R (on each) and meet at the centroid ' O ' of the triangle.

If each particle is given a tangential velocity v so that R acts as the centripetal force and they would move in a circle of radius

$$r = OA = OB = OC$$

$$r = \frac{2}{3} a \sin 60^\circ = \frac{2}{3} a \frac{\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

The original mutual separation will be maintained.

As $R = \frac{mv^2}{r}$

We have, $v = \sqrt{\frac{Rr}{m}} = \sqrt{\frac{\sqrt{3} Gm^2}{a^2} \times \frac{a}{\sqrt{3} m}}$

$$v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = 2\pi \frac{a}{\sqrt{3}} \times \sqrt{\frac{a}{Gm}} = 2\pi \sqrt{\frac{a^3}{3Gm}}$$

NUMERICAL PROBLEMS

- 228.** Prove that the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

Ans :

We know that the angular momentum,

$$L = I\omega$$

$$\frac{dL}{dt} = I \frac{d\omega}{dt} = I\alpha, \quad \dots(1)$$

where,

$$\frac{d\omega}{dt} = \alpha, \text{ the angular acceleration.}$$

Also, torque

$$\tau = I\alpha \quad \dots(2)$$

From (1) and (2), we have

$$\tau = \frac{dL}{dt}$$

- 229.** Deduce an expression for rectangular components of torque in the three dimensions.

Ans :

We know, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\begin{aligned} \hat{i}\tau_x + \hat{j}\tau_y + \hat{k}\tau_z &= (\hat{i}x + \hat{j}y + \hat{k}z) \times (\hat{i}F_x + \hat{j}F_y + \hat{k}F_z) \end{aligned}$$

acceleration and in rotational motion torque produces angular acceleration. In rotational motion mass is replaced by moment of inertia (I) of the body about the axis of rotation.

In linear motion, $\vec{F} = m\vec{a}$

In rotational motion, $\vec{\tau} = I\vec{\alpha}$

- 220.** Define angular momentum. Give its units and dimensional formula. Establish the relation between angular momentum and torque.

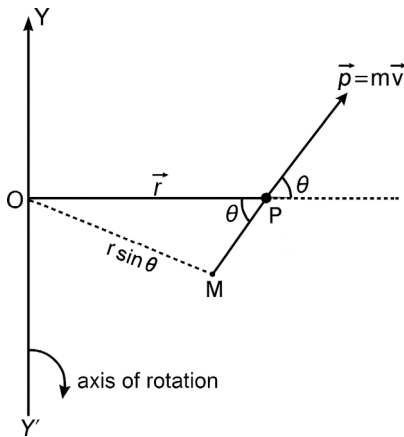
Ans :

The turning movement of a particle about the axis of rotation is called angular momentum and is measured as the product of linear momentum and perpendicular distance of the particle from the axis of rotation.

If $\vec{p}$ is the linear momentum of a single particle at a point P whose position w.r.t. origin O is the $\vec{r}$. Angular momentum of the particle w.r.t. origin O is defined as

$$\vec{L} = \vec{r} \times \vec{p} \quad \dots(1)$$

In magnitude,



$$L = rp \sin \theta$$

$$L = p(r \sin \theta) = p(OM)$$

= linear momentum $\times$ $\perp$ distance of the particle from the axis of rotation.

Unit of $\vec{L}$

In SI, the unit of angular momentum is $\text{kg m}^2 \text{s}^{-1}$

Dimensional formula = $[M][L^2 T^{-1}][L]$

Relation between angular momentum and torque.

From equation (1), angular momentum of a particle is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

Differentiating both sides w.r.t. t , we get

$$\begin{aligned} \frac{d\vec{L}}{dt} &= \frac{d}{dt}(\vec{r} \times \vec{p}) \\ \frac{d\vec{L}}{dt} &= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \\ &= \vec{v} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \quad \left[\frac{d\vec{r}}{dt} = \vec{v} \right] \\ &= \vec{v} \times m\vec{v} + \vec{r} \times \frac{d\vec{p}}{dt} \quad [\vec{p} = m\vec{v}] \\ \frac{d\vec{L}}{dt} &= m(\vec{v} \times \vec{v}) + \vec{r} \times \frac{d\vec{p}}{dt} \\ &= 0 + \vec{r} \times \frac{d\vec{p}}{dt} \quad [\vec{v} \times \vec{v} = 0] \\ \frac{d\vec{L}}{dt} &= \vec{r} \times \vec{F} \quad \left[\frac{d\vec{p}}{dt} = \vec{F} \right] \\ \frac{d\vec{L}}{dt} &= \vec{\tau} \quad [\vec{r} \times \vec{F} = \vec{\tau}] \\ \vec{\tau} &= \frac{d\vec{L}}{dt} \quad \dots(2) \end{aligned}$$

This is the required relation between $\vec{\tau}$ and $\vec{L}$.

- 221.** Derive the expression for angular momentum of a particle in two dimensions (or in a plane).

or

Show that $L = xp_y - yp_x$, where letters have their usual meanings.

Ans :

From Newton's 2nd law of motion, we have

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \dots(1)$$

In cartesian coordinates, the force $\vec{F}$ and linear momentum $\vec{p}$ can be written as

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

and

$$\vec{p} = p_x \hat{i} + p_y \hat{j}$$

Hence equation (1) becomes

$$F_x \hat{i} + F_y \hat{j} = \frac{d}{dt}(p_x \hat{i} + p_y \hat{j})$$

$$F_x \hat{i} + F_y \hat{j} = \frac{dp_x}{dt} \hat{i} + \frac{dp_y}{dt} \hat{j}$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$F_x = \frac{dp_x}{dt} \quad \dots(2)$$

and

$$F_y = \frac{dp_y}{dt} \quad \dots(3)$$

Since torque τ in the plane rotation is given by

$$\tau = xF_y - yF_x \quad \dots(4)$$

Putting the values of F_x and F_y from equations (2) and (3) in (4), we get

$$\begin{aligned} \tau &= x \frac{dp_y}{dt} - y \frac{dp_x}{dt} \\ \tau &= m \left[x \frac{dv_y}{dt} - y \frac{dv_x}{dt} \right] \quad \dots(5) \end{aligned}$$

$$\begin{aligned}
 &= \frac{5/2}{3} = \frac{5}{6} \text{ m.} \\
 Y &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \\
 &= \frac{1\text{kg}(1/2\text{m}) + 1\text{kg}(1/2\text{m}) + 1\text{kg}(3/2\text{m})}{3\text{kg}} \\
 &= \frac{5/2}{3} = \frac{5}{6} \text{ m.}
 \end{aligned}$$

- 238.** A cord of negligible mass is wound around the rim of a flywheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord. The flywheel is mounted on a horizontal axle with frictionless bearings and starts from rest.

Questions:

1. What is the rotational analogue of Newton's second law for linear motion ($F = ma$) for rotation about a fixed axis?
2. How is the torque due to the applied force on the cord related to the force and the radius of the flywheel?
3. Calculate the moment of inertia of the flywheel, assuming it's a solid disc ($I = MR^2/2$).
4. Compute the angular acceleration (α) of the wheel.

Ans :

1. The rotational analogue is $\tau = I\alpha$, where τ is the torque, I is the moment of inertia, and α is the angular acceleration.
2. The torque $\tau = F \times R$, where F is the pull and R is the radius of the flywheel.
3. $M = 20\text{kg}$, $R = 20\text{cm} = 0.20\text{m}$.

Moment of inertia

$$\begin{aligned}
 I &= \frac{MR^2}{2} \\
 &= \frac{20.0\text{kg} \times (0.20\text{m})^2}{2} \\
 &= \frac{20.0 \times 0.04}{2} = 0.4\text{kgm}^2.
 \end{aligned}$$

4. Torque

$$\begin{aligned}
 \tau &= F \times R \\
 &= 25\text{N} \times 0.20\text{m} = 5.0\text{Nm}.
 \end{aligned}$$

Angular acceleration

$$\begin{aligned}
 \alpha &= \frac{\tau}{I} \\
 &= \frac{5.0\text{Nm}}{0.4\text{kgm}^2} = 12.5\text{rad/s}^2.
 \end{aligned}$$

- 239.** An ideal lever, like a see-saw, is pivoted at a fulcrum. Two children sit on either side. Child 1 (load F_1) is at a distance d_1 from the fulcrum, and Child 2 (effort F_2) is at a distance d_2 . The lever is in mechanical equilibrium.

Questions:

1. What are the two conditions for a rigid body to be in mechanical equilibrium?
2. What is the "principle of moments" for a lever in equilibrium?
3. If Child 1 weighs 200 N and sits 1.5 m from the fulcrum, and Child 2 weighs 150 N, how far from the fulcrum (d_2) must Child 2 sit for the seesaw to be balanced?
4. What is the mechanical advantage (M.A.) in this situation, defined as F_1/F_2 ? What does it imply if M.A. > 1 ?

Ans :

1. For mechanical equilibrium:
 - (1) The total external force on the rigid body is zero ($\Sigma F_i = 0$).
 - (2) The total external torque on the rigid body is zero ($\Sigma \tau_i = 0$).
2. The principle of moments for a lever in equilibrium states that load arm $\times$ load = effort arm $\times$ effort, or $d_1 F_1 = d_2 F_2$ (when forces are perpendicular to the lever).
3. Given $F_1 = 200\text{N}$, $d_1 = 1.5\text{m}$, $F_2 = 150\text{N}$.
Using $d_1 F_1 = d_2 F_2$:

$$(1.5\text{m})(200\text{N}) = d_2(150\text{N})$$

$$300 = 150d_2$$

$$d_2 = \frac{300}{150} = 2.0\text{m}.$$

Child 2 must sit 2.0 m from the fulcrum.

4. Mechanical Advantage (M.A.)

$$\begin{aligned}
 &= \frac{F_1}{F_2} \\
 &= \frac{200\text{N}}{150\text{N}} = \frac{4}{3} \approx 1.33.
 \end{aligned}$$

If M.A. > 1 , it means that a smaller effort can be used to lift a larger load.

- 240.** A solid cylinder rolls down an inclined plane without slipping. Its motion is observed. Particles at different points on the cylinder, like the topmost point, the centre, and the point of contact with the plane, are analyzed at a particular instant.

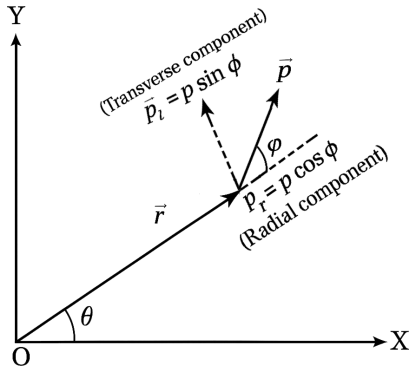
Questions:

1. What kind of motion does a rigid body have if it is not pivoted or fixed in some way?
2. Is the motion of the rolling cylinder pure translational motion? Why or why not?
3. The rolling motion of a cylinder is a combination of what two types of motion?
4. What is the velocity of the point of contact of the cylinder with the inclined plane if it rolls without slipping?

- 224.** Show that angular momentum is due to transverse component and does not depend on the radial component i.e. $L = rp_\theta$

Ans :

Consider a particle P having position vector $\vec{r}$ from origin O of cartesian coordinate XY making an angle θ with X -axis. Let linear momentum $\vec{p}$ of the particle at P makes an angle ϕ with the position vector as shown in figure.



The angular momentum L of the particle at P is given by

$$L = rp \sin \phi \quad \dots(1)$$

Resolve the momentum $\vec{p}$ into two rectangular components p_r called radial component of $\vec{p}$ and p_θ called transverse component of $\vec{p}$, then

$$p_r = p \cos \phi$$

$$p_\theta = p \sin \phi$$

So equation (1) becomes,

$$L = rp_\theta$$

Thus angular momentum does not depend on the radial component p_r of momentum. This component has no turning effect on the particle. It is only the component p_θ that has turning effect.

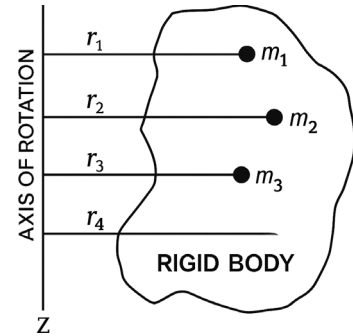
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- 225.** Derive an expression for kinetic energy of rotation.

Ans :

Consider a rigid body of mass M , having a large number of particles of masses $m_1, m_2, m_3, \dots, m_n$.

Let $r_1, r_2, r_3, \dots, r_n$ be their perpendicular distances from the axis of rotation ZZ' .



Let the angular velocity of the body be ω .

$$v_1 = r_1 \omega, v_2 = r_2 \omega, v_3 = r_3 \omega \text{ and so on}$$

Kinetic energy of particle of mass m_1

$$= \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 r_1^2 \omega^2$$

Kinetic energy of particle of mass m_2

$$= \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 r_2^2 \omega^2 \text{ and so on}$$

Kinetic energy of rotation of the body about ZZ'
= sum of kinetic energy of particles forming the body

Kinetic energy of rotation of body about AB

$$\begin{aligned} E &= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2 \\ &= \frac{1}{2} \omega^2 (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \\ &= \frac{1}{2} \omega^2 \sum m r^2 = \frac{1}{2} I \omega^2, \text{ where } \sum m r^2 = I \end{aligned}$$

Definition of I

$$E = \frac{1}{2} I \omega^2$$

If $\omega = 1$

$$I = 2E$$

Moment of inertia of a body about the axis of rotation is defined as twice the kinetic energy of body rotating with unit angular velocity.

- 226.** (a) Explain the physical meaning of moment of inertia of body.
(b) Compare relations in linear motion and rotational motion.

Ans :

- (a) Whatever role mass plays in linear motion, same role is played by moment of inertia in rotational motion i.e. just as mass m is inertia against motion, the moment of inertia I is an inertia against turning. It is also clear from the following table in (b).

For a given torque produced by external force, the angular acceleration will be more if moment of inertia is less and angular acceleration will be less if moment of inertia is more.

We know that,

$$\begin{aligned}\frac{d}{dt}[xv_y - yv_x] &= x\frac{dv_y}{dt} + v_y\frac{dx}{dt} - y\frac{dv_x}{dt} - v_x\frac{dy}{dt} \\ &= x\frac{dv_y}{dt} - v_yv_x - y\frac{dv_x}{dt} - v_xv_y \\ &= x\frac{dv_y}{dt} - y\frac{dv_x}{dt} \quad \dots(6)\end{aligned}$$

Using equation (6),

$$\begin{aligned}\tau &= m\frac{d}{dt}(xv_y - yv_x) \\ &= \frac{d}{dt}(xmv_y - ymv_x) \\ \tau &= \frac{d}{dt}(xp_y - yp_x)\end{aligned}$$

Since

$$\begin{aligned}\tau &= \frac{dL}{dt} \\ \frac{d}{dt}L &= \frac{d}{dt}(xp_y - yp_x) \\ L &= xp_y - yp_x \quad \dots(7)\end{aligned}$$

Equation (7) represents an expression for angular momentum in cartesian coordinates.

- 222.** Derive the expression for angular momentum in three dimensions.

Ans :

The angular momentum of a particle $\vec{L}$ having linear momentum $\vec{p}$ at any point whose position w.r.t. origin O is given by position vector $\vec{r}$ is given by

$$\vec{L} = \vec{r} \times \vec{p} \quad \dots(1)$$

In three dimensions, $\vec{L}$, $\vec{r}$ and $\vec{P}$ can be written as

$$\vec{L} = L_x\hat{i} + L_y\hat{j} + L_z\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{and } \vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

So equation (1) can be written as

$$\begin{aligned}L_x\hat{i} + L_y\hat{j} + L_z\hat{k} \\ = (x\hat{i} + y\hat{j} + z\hat{k}) \times (p_x\hat{i} + p_y\hat{j} + p_z\hat{k})\end{aligned}$$

$$L_x\hat{i} + L_y\hat{j} + L_z\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$\begin{aligned}L_x\hat{i} + L_y\hat{j} + L_z\hat{k} \\ = \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)\end{aligned}$$

Comparing the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on both sides, we get

$$L_x = yp_z - zp_y$$

$$L_y = zp_x - xp_z$$

$$L_z = xp_y - yp_x$$

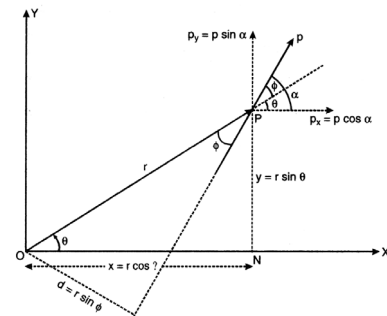
- 223.** Find the relation between angular momentum and lever arm.

or

Show that the angular momentum is the product of linear momentum and per distance from the axis of rotation (lever arm) i.e. $L = p \cdot d$.

Ans :

Consider a particle at P having position vector from the origin O of X - Y coordinate making an angle θ with x -axis. Let $\vec{P}$ be linear momentum of the particle making angle ϕ with position vector as shown in figure.



From the figure, we find that

$$\alpha = \theta + \phi$$

$$\phi = \alpha - \theta \quad (1)$$

Since angular momentum in X - Y plane is given by

$$L = xp_y - yp_x \quad \dots(2)$$

And from figure, we find that

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$p_x = p \cos \alpha, \quad p_y = p \sin \alpha$$

So equation (2) becomes

$$\begin{aligned}L &= r \cos \theta p \sin \alpha - r \sin \theta p \cos \alpha \\ &= rp(\sin \alpha \cos \theta - \cos \alpha \sin \theta) \\ &= rp \sin(\alpha - \theta) \quad \dots(3)\end{aligned}$$

$$[\sin A \cos B - \cos A \sin B = \sin(A - B)]$$

Using equation (1), we get

$$L = rp \sin \phi = p(r \sin \phi)$$

$$L = pd$$

where $d = r \sin \phi = OM$ is the $\perp$ distance from the axis of rotation and is called lever arm.

Angular momentum = linear momentum $\times$ lever arm.

$$\begin{aligned}
 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \\
 &= \hat{i}(yF_z - zF_y) + \hat{j}(zF_x - xF_z) + \hat{k}(xF_y - yF_x) \\
 \tau_x &= yF_z - zF_y; \quad \tau_y = zF_x - xF_z; \\
 \tau_z &= xF_y - yF_x
 \end{aligned}$$

- 230.** A 2 kg body and a 3 kg body are moving along x -axis. At a particular instant, the 2 kg body is 1 m from the origin and has a velocity of 3 ms^{-1} and the 3 kg body is 2 m away from the origin has a velocity of -1 ms^{-1} . Find the position and velocity of the centre of mass and also find the total momentum.

Ans :

Position of centre of mass is given by

$$\begin{aligned}
 r_{\text{cm}} &= \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \\
 &= \frac{2 \times 1 + 3 \times 2}{2 + 3} = \frac{8}{5} = 1.6 \text{ m}
 \end{aligned}$$

Velocity of centre of mass is given by

$$\begin{aligned}
 v_{\text{cm}} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \\
 &= \frac{2 \times 3 + 3 \times (-1)}{2 + 3} = \frac{3}{5} \\
 &= 0.6 \text{ ms}^{-1}
 \end{aligned}$$

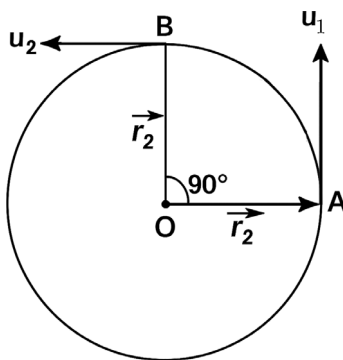
Total momentum is given by

$$\begin{aligned}
 p &= (m_1 + m_2) v_{\text{cm}} = (2 + 3) \times 0.6 \\
 &= 3 \text{ kg ms}^{-1}
 \end{aligned}$$

- 231.** A particle moves in a circle of radius r with constant angular velocity ω . What is the change in velocity when the particle describe an angle of 90° ?

Ans :

Let $\vec{v}_1$ and $\vec{v}_2$ be velocities of the particle at points A and B as shown in the figure.



so $\vec{v}_1 = \vec{r}_1 \omega$

and $\vec{v}_2 = \vec{r}_2 \omega$

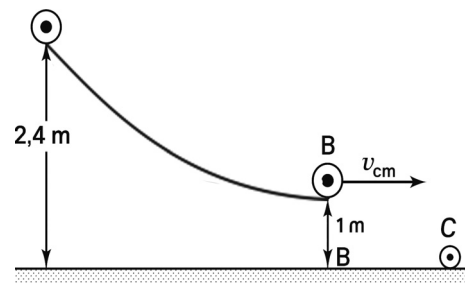
But $|\vec{r}_1| = |\vec{r}_2| = r$

$|\vec{v}_1| = |\vec{v}_2| = r\omega$

Change in velocity $(\vec{v}_2 - \vec{v}_1)$

i.e. $|\vec{v}_2 - \vec{v}_1| = \sqrt{v_2^2 + v_1^2 - 2v_1 v_2 \cos 90^\circ}$
 $= \sqrt{r^2 \omega^2 + r^2 \omega^2 - 0} = \sqrt{2} r\omega$

- 232.** A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part is 1.0 m above the ground level and the top of the track is 2.4 m above the ground. Find the distance on the ground with respect to the point B (which is vertically) below the end of the track as shown in the figure where the sphere lands. During its flight as a projectile does the sphere continue to rotate about its centre of mass? Explain.



Ans :

Let m be the mass and r be the radius of the sphere rolling down without slipping from the top of the track AB.

If v_{cm} is the velocity of the sphere on reaching B when it falls through a height h (where $h = 2.4 \text{ m} - 1.0 \text{ m} = 1.4 \text{ m}$), then From the law of conservation of energy

$$(1/2)I\omega^2 + (1/2)mv_{\text{cm}}^2 = mgh \quad \dots(1)$$

Here ω is the angular velocity of the sphere on reaching A.

As $I = (2/5)mr^2$, from (1)

$$(1/2)(2/5)mr^2\omega^2 + (1/2)mv_{\text{cm}}^2 = mgh$$

$$(7/10)mv_{\text{cm}}^2 = mgh$$

$$\begin{aligned}
 v_{\text{cm}} &= \sqrt{\frac{10}{7}gh} \\
 &= \sqrt{\frac{10}{7} \times g \times 1.4} \\
 &= \sqrt{2g} \quad \dots(2)
 \end{aligned}$$

If t is the time taken by the sphere to fall through a height h from B to C, then

$$t = \sqrt{2h/g}$$

$$h = 1 \text{ m}$$

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$$t = \sqrt{\frac{2 \times 1}{g}} = \sqrt{\frac{2}{g}} \quad \dots(3)$$

Further, if the sphere lands at C , then,

$$B'C = v_{cm} t \quad \dots(4)$$

From (2), (3) and (4),

$$B'C = \sqrt{2g} \times \sqrt{2/g} = 2 \text{ m}$$

According to the law of conservation of angular momentum, the sphere continues to rotate about its centre of mass in the absence of external torque.

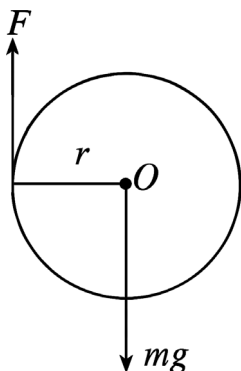
- 233.** A circular cylinder has an inextensible string wrapped around it as shown in the figure. What is the linear acceleration of the cylinder when released?

Ans :

Let F be the tension in the string. If a is the acceleration of the centre of mass of the cylinder,

$$mg - F = ma \quad \dots(1)$$

If τ is the torque produced by the force about the axis of the cylinder through O , then



$$\tau = rF$$

$$\tau = Ia, \quad rF = Ia$$

$$F = \frac{Ia}{r} = \left(\frac{1}{2}mr^2\right)\frac{a}{r} \quad (I = \frac{1}{2}mr^2)$$

$$F = \frac{1}{2}ma = \frac{1}{2}mr\alpha \quad (a = r\alpha)$$

$$\text{from (1), } mg - \frac{1}{2}ma = ma$$

$$mg = \frac{3}{2}ma$$

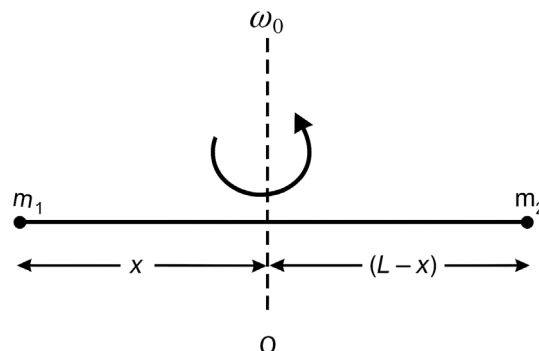
$$a = (2/3)g$$

- 234.** Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L , and negligible mass. The rod is to be set rotating about an axis perpendicular to it. Find the position on this rod through which the axis should pass in order that the work required to set the rod rotating with angular velocity ω_0 should be minimum.

Ans :

Let the axis of rotation be at a distance x from m_1 , as shown in the figure.

Kinetic energy of translation = 0



$$\text{Kinetic energy of rotation} = \frac{1}{2}(I_1 + I_2)\omega_0^2$$

According to work energy principle,

Work done = Increase in energy

$$W = \frac{1}{2}(I_1 + I_2)\omega_0^2$$

$$= \frac{1}{2}[m_1 x^2 + m_2 (L-x)^2]\omega_0^2 \quad \dots(1)$$

For W to be minimum, $\frac{dW}{dx} = 0$

$$\text{From (1), } \frac{dW}{dx} = \frac{1}{2}m_1 2x + \frac{1}{2}m_2 \times 2(L-x)(-1)$$

$$= 0$$

$$m_1 x - m_2 (L-x) = 0$$

$$(m_1 + m_2)x = m_2 L$$

$$x = \frac{m_2 L}{m_1 + m_2}$$

This is the position of centre of mass of the rod from m_1 . Hence the required axis should pass through centre of mass of the rod and perpendicular to the length of the rod as shown in the figure.

CASE BASED QUESTIONS

- 235.** A projectile is fired into the air and follows a parabolic trajectory. Midway through its flight, it explodes into several fragments due to internal forces. The forces leading to the explosion are purely internal to the projectile system. The only external force acting on the system throughout its flight (both before and after the explosion) is gravity.

Questions:

- How does the centre of mass of a system of particles move?

2. Do internal forces, like those causing the explosion, affect the motion of the centre of mass of the system?
3. What path will the centre of mass of the fragments of the projectile continue to follow after the explosion?
4. If the projectile had not exploded, it would have landed at a certain range R. If the explosion happens such that some fragments fall short, what must happen to other fragments for the centre of mass to maintain its original trajectory?

Ans :

1. The centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.
2. No, internal forces contribute nothing to the motion of the centre of mass. The equation $MA = F_{ext}$ shows that only external forces determine the acceleration of the centre of mass.
3. The centre of mass under the influence of the external force (gravity) continues along the same parabolic trajectory it would have followed if there were no explosion.
4. For the centre of mass to maintain its original trajectory, if some fragments fall short of range R, other fragments must travel beyond range R, such that the mass-weighted average position of all fragments (the centre of mass) lands at range R.

236. An ice skater is spinning on a frictionless ice surface with her arms outstretched. She then pulls her arms in close to her body. As she does this, her angular speed noticeably increases. Assume no external torque acts on the skater about her axis of rotation. Questions:

1. What is the law of conservation of angular momentum for rotation about a fixed axis if the external torque is zero?
2. How does pulling her arms in affect the skater's moment of inertia (I) about her axis of rotation?
3. If her initial moment of inertia is I_1 and angular speed is ω_1 , and her final moment of inertia is I_2 and angular speed is ω_2 , what is the relationship between these quantities?
4. Explain why her angular speed increases when she pulls her arms in, using the principle of conservation of angular momentum.

Ans :

1. If the external torque is zero, the component of angular momentum along the fixed axis

of rotation, $L_z = I\omega$, remains constant. For symmetric bodies, L can be used instead of L_z .

2. Pulling her arms in close to her body decreases her moment of inertia about the axis of rotation because more of her mass is distributed closer to the axis.
3. Since angular momentum is conserved ($L = I\omega = \text{constant}$), we have $I_1\omega_1 = I_2\omega_2$.
4. When the skater pulls her arms in, her moment of inertia (I) decreases. To conserve angular momentum ($L = I\omega$), if I decreases, her angular speed (ω) must increase.

237. An L-shaped lamina (a thin flat plate) is formed by three identical squares, each of side 1m and mass 1kg. The lamina is uniform. We need to find the coordinates of its centre of mass, choosing the origin at the bottom-left corner, with the x and y axes along the outer edges of the L-shape.

Questions:

1. For a system of n particles, how is the position vector $\mathbf{R}$ of the centre of mass defined?
2. Where is the centre of mass of a uniform square located?
3. Determine the coordinates of the centre of mass for each of the three 1kg squares forming the L-shape.
4. Calculate the x and y coordinates of the centre of mass of the entire L-shaped lamina.

Ans :

1. The position vector $\mathbf{R}$ of the centre of mass of a system of n particles is given by

$$\mathbf{R} = \frac{\sum m_i \mathbf{r}_i}{M},$$

Where $M = \sum m_i$ is the total mass.

2. By symmetry, the centre of mass of a uniform square is at its geometric centre.
3. Let the squares be C_1 (bottom-left), C_2 (bottom-right), and C_3 (top-left).
Coordinates of CM of C_1 : (1/2, 1/2).
Coordinates of CM of C_2 : (3/2, 1/2)
(if the L-shape extends 2m along x at base, and 1m width).
Coordinates of CM of C_3 : (1/2, 3/2) (if the L-shape extends 2m along y in height, and 1m width).
(Using the squares described in Example 6.3):
 $C_1(1/2, 1/2)$, $C_2(3/2, 1/2)$, $C_3(1/2, 3/2)$ for an L-shape with two squares along x and one above the first.

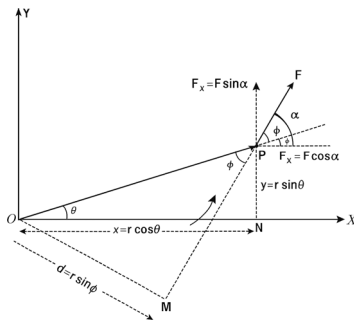
$$\begin{aligned} 4. \quad X &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ &= \frac{1\text{kg}(1/2\text{m}) + 1\text{kg}(3/2\text{m}) + 1\text{kg}(1/2\text{m})}{1\text{kg} + 1\text{kg} + 1\text{kg}} \end{aligned}$$

Ans :

Consider a particle P having position vector $\vec{r}$ making an angle θ with x -axis. A force $\vec{F}$ acts on the particle at angle ϕ with position vector $\vec{r}$ and angle α with x -axis as shown in figure.

From the fig, we find that

$$\begin{aligned}\alpha &= \theta + \phi \\ \phi &= \alpha - \theta\end{aligned}\quad \dots(1)$$



Since torque τ in Cartesian coordinates is given by

$$\tau = xF_y - yF_x \quad \dots(2)$$

In rt. $\triangle ONP$,

$$x = r \cos \theta$$

and $y = r \sin \theta$

In the figure, the components of $\vec{F}$ along X and Y -axis are

$$F_x = F \cos \alpha$$

and $F_y = F \sin \alpha$

So equation (2) can be written as

$$\begin{aligned}\tau &= r \cos \theta \cdot F \sin \alpha - r \sin \theta \cdot F \cos \alpha \\ &= rF [\cos \theta \sin \alpha - \sin \theta \cos \alpha] \\ \tau &= rF \sin(\alpha - \theta)\end{aligned}$$

[since, $\sin(A - B) = \sin A \cos B - \cos A \sin B$]

Using equation (1), we get

$$\begin{aligned}\tau &= rF \sin \phi \\ \tau &= F(r \sin \phi) \\ \tau &= F \cdot d,\end{aligned}\quad \dots(3)$$

where $d = r \sin \phi$ is the $\perp$ distance from the axis of rotation called lever arm.

Hence torque = Force $\times$ lever arm.

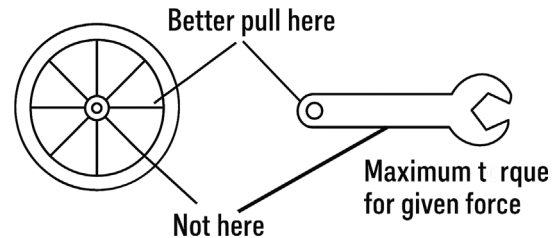
Dependence of torque on lever arm

$$\tau = Fd = Fr \sin(\alpha - \theta)$$

For a given magnitude F of the force, we can have a greater or a lesser torque (about O) depending on

the point where F is applied.

Suppose we want to make a ceiling fan or a cycle wheel go around or we have to unscrew a nut which is very tightly fitted on a bolt. With our limited strength (i.e. magnitude of force), we know it is wiser to apply our hands so far away as possible from the centre O , and apply push or pull perpendicular to radial direction.



219. (a) Derive expression for torque in three dimensions.
(b) What is the significance of torque ?

Ans :

- (a) If a force $\vec{F}$ acts on a single particle at a point P whose position w.r.t. the origin O of the inertial frame is given by displacement vector $\vec{r}$, the torque $\vec{\tau}$ acting on the particle w.r.t. origin O is defined as

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \dots(1)$$

In three dimensions, $\vec{\tau}$, $\vec{r}$ and $\vec{F}$ can be written as

$$\vec{\tau} = \tau_x \hat{i} + \tau_y \hat{j} + \tau_z \hat{k}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

So equation (1) can be written as

$$\begin{aligned}\tau_x \hat{i} + \tau_y \hat{j} + \tau_z \hat{k} &= (x \hat{i} + y \hat{j} + z \hat{k}) \times (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \\ &= \hat{i}(F_z - zF_y) - \hat{j}(zF_x - xF_z) + \hat{k}(xF_y - yF_x)\end{aligned}$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on both sides, we get

$$\tau_x = yF_z - zF_y$$

$$\tau_y = zF_x - xF_z$$

$$\tau_z = xF_y - yF_x$$

- (b) Torque plays the same role in rotational motion as force does in translational motion. In translational motion, force produces

Ans :

1. The motion of a rigid body which is not pivoted or fixed is either a pure translation or a combination of translation and rotation.
2. No, the motion of the rolling cylinder is not pure translational motion because all its particles are not moving with the same velocity at any instant.
3. The rolling motion of a cylinder down an inclined plane is a combination of rotation about a fixed axis and translation. (More accurately, rotation about an axis that is itself translating).
4. The velocity of the point of contact is zero at any instant if the cylinder rolls without slipping.

- 241.** A thin circular ring of radius R and mass M is rotating in its own plane around its centre with an angular velocity ω . Each mass element of the ring is at a distance R from the axis of rotation.

Questions:

1. What is the general formula for the kinetic energy K of a body rotating about a fixed axis with angular velocity ω and moment of inertia I ?
2. How is the moment of inertia (I) of a rigid body about an axis defined?
3. If each mass element dm of the ring moves with a linear speed $v = R\omega$, what is the kinetic energy of this element?
4. By summing the kinetic energies of all mass elements (or by comparing with the general formula), determine the moment of inertia I of the thin circular ring about an axis perpendicular to its plane and passing through its centre.

Ans :

1. The kinetic energy of a rotating body is
2. The moment of inertia I of a rigid body about an axis is defined by $I = \sum m_i r_i^2$, where r_i is the perpendicular distance of the i^{th} particle of mass m_i from the axis.
3. The kinetic energy of an element dm is

$$\begin{aligned} dk &= \frac{1}{2}(dm)v^2 \\ &= \frac{1}{2}(dm)(R\omega)^2 \\ &= \frac{1}{2}(dm)R^2\omega^2. \end{aligned}$$

4. The total kinetic energy

$$\begin{aligned} K &= \sum \frac{1}{2}(dm)R^2\omega^2 \\ &= \frac{1}{2}(\sum dm)R^2\omega^2 \end{aligned}$$

$$= \frac{1}{2}MR^2\omega^2.$$

Comparing this with $K = \frac{1}{2}I\omega^2$, we get $I = MR^2$ for the ring.

- 242.** A heavy radium (Ra) nucleus, initially at rest in the laboratory frame, undergoes radioactive decay and splits into a lighter radon (Rn) nucleus and an alpha particle (nucleus of a helium atom). The forces leading to the decay are internal to the system.

Questions:

1. What is the law of conservation of linear momentum for a system of particles?
2. If the radium nucleus is initially at rest, what is the total linear momentum of the system before decay?
3. After the decay, if the radon nucleus and the alpha particle move with momenta $\mathbf{P}_{Rn}$ and $\mathbf{P}_\alpha$ respectively, what is the relation between $\mathbf{P}_{Rn}$ and $\mathbf{P}_\alpha$ according to the conservation of linear momentum?
4. In which directions must the radon nucleus and the alpha particle move relative to each other if the original nucleus was at rest?

Ans :

1. When the total external force acting on a system of particles is zero, the total linear momentum of the system is constant (conserved).
2. If the radium nucleus is initially at rest, its momentum is zero. Thus, the total linear momentum of the system before decay is zero.
3. According to the conservation of linear momentum, the total momentum after decay must also be zero. So, $\mathbf{P}_{Rn} + \mathbf{P}_\alpha = \mathbf{0}$, which implies $\mathbf{P}_{Rn} = -\mathbf{P}_\alpha$.
4. If the original nucleus was at rest, the two product particles (radon nucleus and alpha particle) must move in opposite directions (back to back) with their centre of mass remaining at rest.

- 243.** A metal bar, 70 cm long and 4.00 kg in mass, is supported on two knife-edges placed 10 cm from each end. A 6.00 kg load is suspended at 30 cm from one end. The bar is of uniform cross-section and homogeneous, so its centre of gravity (G) is at its geometric centre. We need to find the reactions at the knife-edges. (Let $g = 9.8\text{m/s}^2$).

Questions:

1. For the bar to be in translational equilibrium, what condition must the forces acting on it satisfy?
2. For the bar to be in rotational equilibrium, what condition must the torques acting on it satisfy?

5. GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy at any point can be given defined as the amount of work done in bringing a mass from infinity to that point without acceleration.

$$U(at\ r) = -\frac{GMm}{r}$$

If a body is taken from the surface of earth to height h , the gravitational potential energy is given by

$$\text{Potential energy} = mgh$$

6. GRAVITATIONAL FIELD

The space surrounding a material body in which its gravitational force of attraction can be detected is called gravitational field.

7. KEPLER'S LAWS OF PLANETARY MOTION

Kepler formulated three laws of planetary motion.

- (i) **Kepler's First Law (law of orbits)** : It state that every planet revolves around the sun in an elliptical orbit with sun at one of the foci.
- (ii) **Kepler's Second Law (law of area)** : It state that the line joining the sun to the planet sweeps out equal areas in equal times i.e. areal velocity of the planet around the sun is constant.
- (iii) **Kepler's third law (law of periods)** : This law states that the square of the time period T of the revolving planet is directly proportional to the cube of the mean radius r of the orbit of planet, that is

$$T^2 \propto r^3$$

8. ESCAPE VELOCITY (v_e)

Escape velocity of a body is the minimum velocity with which it is to be projected so that it escapes from the gravitational field of earth or that of any other planet.

Escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

The escape velocity of a body from a planet depends upon the mass and radius (i.e. size) of the planet and hence the value of acceleration due to gravity on its surface.

9. SATELLITE

A body which is revolving in an orbit around a comparatively much heavier body is called a satellite e.g., moon is natural satellite of earth.

9.1 Natural Satellite

A natural satellite is a heavenly body revolving around a planet in a close and stable orbit. e.g.

1. Moon is natural satellite of earth.
2. Nine planets are natural satellites of sun.

9.2 Artificial Satellite

Man-made satellite is called artificial satellite. e.g.

1. First man-made satellite was Sputnik-1.
2. India's first man-made satellite was Aryabhata.

Uses of artificial satellites :

1. They are used in communicating radio, TV and telephone signals to distant places.
2. They are used to study the upper regions of atmosphere.
3. They are used in forecasting weather.
4. They are used to determine the exact shape of earth.
5. They are used to study cosmic rays and solar radiations.

9.3 Polar Satellite

A satellite that revolves in an orbit that passes over the north and south poles of the earth is called a polar satellite. The polar satellite are low attitude satellite (about 500 to 800 km from the surface of earth) having time period about 100 minutes . Polar satellite are used for remote sensing and are also known as remote sensing satellites.

9.4 Orbital Velocity

Orbital velocity is the velocity with which the body must be thrown upwards so that it starts orbiting round the earth (say at a height h above the surface of the earth) and is given by

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{g(R+h)}$$

Time period of a satellite is the time taken by the satellite to go once around the earth.

$$\text{And } T = 2\pi\sqrt{\frac{(R+x)^3}{GM}} = \frac{2\pi}{R}\sqrt{\frac{(R+x)^3}{g}}$$

When the satellite is close to earth ($x \ll R$)

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$$

CHAPTER 7

Gravitation

SUMMARY

1. GRAVITY

Gravity is the attractive force exerted by the Earth on all objects, pulling them toward its centre. It is a specific case of gravitation, which acts between any two masses in the universe. Gravity governs motion, weight, and planetary orbits.

2. LAW OF GRAVITATION

It states that force of attraction between any two objects is directly proportional to product of their masses ($m_1 m_2$) and inversely proportional to square of their distances (r). i.e.,

$$F = G \frac{m_1 m_2}{r^2}$$

Where G is called the gravitational constant and its value is $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Universal gravitational constant G is numerically equal to the force of attraction F between two bodies of unit mass separated by a unit distance.

3. ACCELERATION DUE TO GRAVITY

The acceleration with which a body falls towards the earth's surface when dropped from certain height is called acceleration due to gravity.

It is given by,

$$g = \frac{GM}{R^2} = 9.8 \text{ m s}^{-2}$$

1. Mass of Earth

Mass of earth is given by

$$M = \frac{gR^2}{g} = 5.98 \times 10^{24} \text{ kg}$$

2. Density of Earth

Density of earth is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = 5.47 \times 10^3 \text{ kg m}^{-3}$$

3.1 Variation of Acceleration due to Gravity

The value of gravitational acceleration (g) is not constant across the Earth. It varies due to several factors, such as:

(i) Effect of altitude (Height)

$$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

and $g_h = g\left(1 - \frac{2h}{R}\right)$, when $h \ll R$

Where g_h is acceleration due to gravity at a height h above the surface of the earth of radius R

(ii) Effect of depth

$$g_d = g\left(1 - \frac{h}{R}\right)$$

g_d is the acceleration due to gravity at a depth h below the surface of the earth.

(iii) Effect of the shape of the earth

$$g_p > g_e$$

Where g_p and g_e are the values of acceleration due to gravity at the poles and the equator respectively.

(iv) Effect of rotation of the earth

If ω is the angular velocity of the earth, then acceleration due to gravity at a latitude λ is given by

$$g_\lambda = (g - \omega^2 R \cos^2 \lambda)$$

At, the equator, $\lambda = 0^\circ$ and at the poles, $\lambda = 90^\circ$.

4. GRAVITATIONAL POTENTIAL

The gravitational potential at a point in the gravitational field of a body is the work done in bringing a body of unit mass from infinity to that specific point without causing any acceleration.

$$V(\text{at } r) = -\frac{GM}{r}$$

5. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50000 km from galaxy centre take to complete one revolution ? Take the diameter of the milky way to be 10^5 ly.

Ans :

Time period is given by

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T^2 = \frac{4\pi^2 (50000 \times 9.467 \times 10^{15})^3}{6.67 \times 10^{-11} \times 2.5 \times 10^{11} \times 2 \times 10^{30}}$$

$$T^2 = 12527.5 \times 10^{28} s^2$$

So $T = 111.93 \times 10^{14} s = 1.12 \times 10^{16} s$

$$T = \frac{111.93 \times 10^{14}}{365 \times 24 \times 3600}$$

$$= 3.55 \times 10^8 \text{ years}$$

6. Choose the correct alternative :
- (a) If the zero of potential energy is at infinity the total energy of an orbiting satellite is negative of its kinetic/potential energy.
- (b) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Ans :

(a) kinetic energy (b) less.

7. Does the escape velocity of a body from the Earth depend on (a) the mass of the body (b) the location from where it is projected (c) the direction of projection, (d) the height of the location from where the body is launched ? Explain your answer.

Ans :

Since escape velocity,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}, \text{ hence}$$

- (a) The escape velocity is independent of mass of the body.
- (b) No, it is independent of location from where it is projected.
- (c) The escape velocity is independent of the direction of projection.
- (d) The escape velocity depends upon the gravitational potential at the point from where the body is launched. Since this potential depends slightly on the latitude and height of the point, the escape velocity depends slightly on height of location.

8. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed (b) angular speed (c) angular momentum (d) kinetic energy (e) potential energy (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Ans :

- (a) No, The comet does not have a constant linear speed throughout its orbit.
- (b) No, The comet does not have a constant angular speed.
- (c) Yes, The comet has a constant angular momentum.
- (d) No, The comet does not have a constant kinetic energy.
- (e) No, the comet does not have a constant potential energy.
- (f) Yes, The comet has a constant total energy throughout its orbit.

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9. Which of the following symptoms is likely to afflict an astronaut in space ?
- (a) Swollen feet
- (b) Swollen face
- (c) Headache
- (d) Orientation-al problem.

Ans :

(b) Swollen face

In space, astronauts often experience a swollen face due to fluid shifting from the legs to the upper body in microgravity. This fluid shift can also cause headaches by increasing pressure in the head. Additionally, the absence of gravity affects the inner ear, leading to orientation-al problems like dizziness and disorientation. However, swollen feet are uncommon as fluids move away from the lower body in space.

10. The gravitational intensity at the centre of the drum head defined by a hemispherical shell has the direction indicated by the arrow (as shown in the figure) (i) a, (ii) b, (iii) c, (iv) zero.

Height of satellite above the earth's surface

$$x = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{\frac{1}{3}} - R$$

NCERT TEXT-BOOK QUESTIONS

1. Answer the following :

- You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means ?
- An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity ?
- If you compare the gravitational force on the Earth due to the sun to that due to the moon you would find that the Sun's pull is greater than the Moon's pull. However, the tidal effect to the Moon's pull is greater than the tidal effect of the Sun. Why ?

Ans :

- No, because gravitational screens are not possible.
- Yes, if the size of the spaceship is large enough for him to detect the variation in g .
- Tidal effect depends inversely on the cube of the distance unlike gravitational force which depends inversely on the square of the distance.

2. Choose the correct alternative :

- Acceleration due to gravity increases/decreases with increasing altitude.
- Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).
- Acceleration due to gravity is independent of the mass of the Earth/mass of the body.
- The formula - $GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is more/less accurate

than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_1 and r_2 distance away from the centre of the Earth.

Ans :

- decreases (b) decreases (c) mass of the body (d) more.

3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

Ans :

As we know that,

$$\left(\frac{T_p}{T_e} \right)^2 = \left(\frac{r_p}{r_e} \right)^3$$

$$T_e = 1 \text{ year}, T_p = \frac{1}{2} = 0.5 \text{ year}$$

$$\left(\frac{0.5}{1} \right)^2 = \left(\frac{r_p}{r_e} \right)^3$$

$$r_p = r_e (0.5)^{\frac{2}{3}}$$

$$r_p = 0.63 r_e = 0.63 \text{ A.U.} \quad r_e = 1 \text{ A.U.}$$

4. I_o , One of satellite of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is $4.22 \times 10^8 \text{ m}$ Show that mass of the Jupiter is about thousandth that of sun.

Ans :

As we know that,

$$T = 2\pi \sqrt{\frac{R_J^3}{GM_J}}$$

$$M_J = \frac{4\pi^2 R_J^3}{T^2 G} \quad \dots(1)$$

$$M_J = \frac{4\pi^2 [4.22 \times 10^8]^3}{G (1.769)^2}$$

If M_s is the mass of sun, then from equation (1), we have

$$M_s = \frac{4\pi^2 R_s^3}{GT^2}$$

$$M_s = \frac{4\pi^2 [1.496 \times 10^{11}]^3}{G (365)^2}$$

$$\left[R_s = 1.496 \times 10^{11} \text{ m} \right. \\ \left. \text{and } T = 365 \text{ days} \right]$$

$$M_s = \frac{4\pi^2}{G} \times 2.5 \times 10^{28}$$

$$\frac{M_J}{M_s} = \frac{2.4 \times 10^{25}}{2.5 \times 10^{28}} = \frac{1}{1000}$$

$$M_J = \frac{1}{1000} M_s$$

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$$M_s = \frac{4\pi^2 \times (1.5 \times 10^{11})^3}{6.7 \times 10^{-11} \times 9365 \times 24 \times 3600)^2}$$

$$= 2.0 \times 10^{30} \text{ kg}$$

14. A saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.5×10^8 km away from the Sun ?

Ans :

As we know that,

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$(29.5)^2 = \left(\frac{R_1}{1.50 \times 10^{11}}\right)^3$$

$$29.5 \times 29.5 = \frac{R_1^3}{1.50 \times 1.50 \times 1.50 \times 10^{33}}$$

$$R_1^3 = [29.5 \times 29.5 \times 1.50 \times 1.50 \times 1.50 \times 10^{33}]^{\frac{1}{3}}$$

$$R_1 = [29.5 \times 29.5 \times 1.50 \times 1.50 \times 1.50 \times 10^{33}]^{\frac{1}{3}}$$

$$= 14.32 \times 10^{11} \text{ m}$$

$$= 1.43 \times 10^{12} \text{ m}$$

15. A body weighs 63 N on the surface of the earth. What is the gravitational force on it at a height equal to half the radius of the earth ?

Ans :

According to the question,

$$h = \frac{R}{2}$$

$$\frac{gh}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{(R+\frac{R}{2})^2}$$

$$g_h = \frac{4}{9}g$$

$$mg_h = \frac{4}{9} \times mg = 28 \text{ N}$$

16. Assuming the Earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the Earth if it weighed 250 N on the surface ?

Ans :

As we know that,

$$W = mg$$

$$250 \text{ N} = m \times 9.8$$

$$m = \frac{250}{9.8} \text{ kg}$$

Value of g half way down to the centre of the Earth i. e.

when $d = \frac{R}{2}$

$$g_d = \left(1 - \frac{d}{R}\right)g$$

$$g_d = \left(1 - \frac{1}{2}\right) \times 9.8$$

$$g_d = 4.9 \text{ ms}^{-2}$$

$$\text{Weight of the body} = mg_d = \frac{250}{9.8} \text{ kg} \times 4.9 \text{ ms}^{-2}$$

$$= 125 \text{ N}$$

17. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth = $6.0 \times 10^{24} \text{ kg}$, mean radius of the earth = $6.4 \times 10^6 \text{ m}$ $G = 6.67 \times 10^{11} \text{ Nm}^2 \text{ kg}^{-2}$.

Ans :

Let the rocket reach a height h from the surface of earth where its velocity vanishes.

If m be its mass, at the surface of the Earth its total energy

$$= K.E. + P.E.$$

$$= \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Its total energy at a height h is its potential energy

$$= -\frac{GMm}{(R+h)}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{(R+h)}$$

$$\frac{1}{2}v^2 = \frac{gR^2}{R} - \frac{gR^2}{(R+h)}$$

$$\frac{1}{2}v^2 = gR\left(1 - \frac{R}{R+h}\right)$$

$$\frac{1}{2}v^2 = gR\left(\frac{h}{R+h}\right)$$

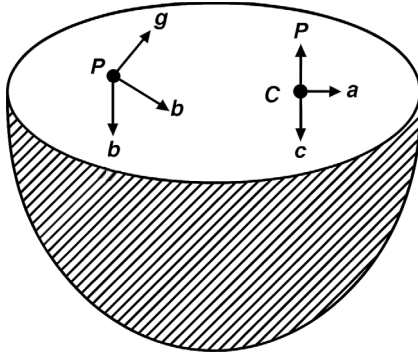
$$(R+h)v^2 = 2gRh$$

$$Rv^2 = 2gRh - hv^2 = h(2gR - v^2)$$

$$h = \frac{Rv^2}{2gR - v^2}$$

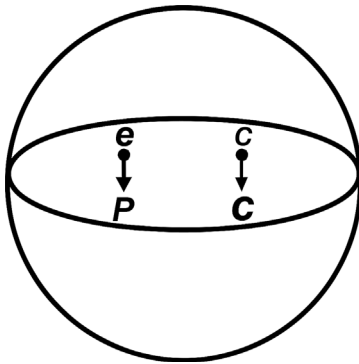
$$h = \frac{(64 \times 10^6)(5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$$

$$= \frac{6.4 \times 25 \times 10^{12}}{10^6(2 \times 9.8 \times 6.4 - 25)}$$



Ans :

On completing the hemisphere into sphere, we get the diagram as shown in the figure.



At point C, potential V is constant.

Hence intensity = 0

Answer (iii) c is correct.

11. For the above problem, the direction of the gravitational intensity at any arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Ans :

Since at point P , potential is constant, hence gravitational intensity = 0

Answer (ii) e is correct.

12. A rocket is fired from the Earth towards the Sun. At what point on its path is the gravitational force on the rocket zero? Mass of the Sun = 2×10^{30} kg. Mass of the Earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Ans :

Let x = distance between the earth and sun
 r = Radius of orbit.

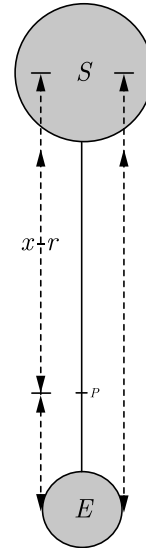
Let P be a point at a distance ' r ' from the earth where gravitation due to sun and earth is equal and give rise to zero gravitational force

$$\frac{GM_S m}{(x-r)^2} = \frac{GM_E m}{r^2},$$

where M_S is the mass of the sun M_E is the mass of the earth

$$\frac{M_S}{(x-r)^2} = \frac{M_E}{r^2}$$

$$\frac{2 \times 10^{30}}{(x-r)^2} = \frac{6 \times 10^{24}}{r^2}$$



$$\frac{x-r}{r} = \sqrt{\frac{2 \times 10^{30}}{6 \times 10^{24}}} = 577.35$$

$$x-r = 577.35 r$$

$$578.35 r = x$$

$$r = \frac{x}{578.35}$$

$$= \frac{15 \times 10^{11}}{578.35} = 2.594 \times 10^8 \text{ m}$$

13. How will you 'weigh the sun', i.e. estimate its mass? The mean orbital radius of the Earth around the Sun is 1.5×10^8 km.

Ans :

The mass M_S of the Sun can be found from the period of a satellite and its distance from the Sun. Consider the case of the Earth. Its period T is about 365 days or $365 \times 24 \times 3600$ seconds. Its distance R_s from the mass of the Earth is m , then for circular motion round the sun,

$$T = 2\pi \sqrt{\frac{R_s^3}{GM_S}}$$

$$M_S = \frac{4\pi^2 R_s^3}{GT^2}$$

21. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid-point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium or unstable?

Ans :

As we know that,

$$\text{Gravitational field} = -\frac{GM}{R^2}$$

Total gravitational field at the mid-point of line joining the centres of the sphere,

$$= -\frac{G \times 100}{\left(\frac{1}{2}\right)^2} + \frac{G \times 100}{\left(\frac{1}{2}\right)^2} = 0$$

Gravitational potential

$$= -\frac{GM}{R}$$

Total gravitational potential at the mid-point of line joining the centre of the sphere.

$$\begin{aligned} &= -\frac{100}{\left(\frac{1}{2}\right)} \times 6.67 \times 10^{-11} - \frac{100}{\left(\frac{1}{2}\right)} \times 6.67 \times 10^{-11} \\ &= -200 \times 6.67 \times 10^{-11} - 200 \times 6.67 \times 10^{-11} \\ &= -2[13.34 \times 10^{-9}] \\ &= -26.68 \times 10^{-9} \\ &= -2.7 \times 10^{-9} J kg^{-1} \\ &= 2.7 \times 10^{-8} J kg^{-1} \end{aligned}$$

An object placed at the mid-point is in unstable equilibrium.

2. Who was the first to recognise that all bodies, irrespective of their masses, are accelerated towards the Earth with a constant acceleration?

(A) Nicolas Copernicus
(B) Johannes Kepler
(C) Isaac Newton
(D) Galileo Galilei

Ans :

(D) Galileo Galilei

Galileo Galilei, an Italian physicist, recognized through experiments, such as those with bodies rolling down inclined planes, that all objects accelerate towards the Earth at a constant rate, regardless of their mass.

3. Which astronomer's compiled data on planetary observations was later analysed by Johannes Kepler to formulate his laws of planetary motion?

(A) Nicolas Copernicus
(B) Tycho Brahe
(C) Galileo Galilei
(D) Ptolemy

Ans :

(B) Tycho Brahe

Tycho Brahe, a Danish nobleman, spent his lifetime recording observations of the planets. His assistant, Johannes Kepler, later analyzed this extensive data to derive the three laws of planetary motion.

4. Kepler's first law, the law of orbits, states that all planets move in:

(A) Circular orbits with the Sun at the center.
(B) Elliptical orbits with the Sun at one of the foci.
(C) Parabolic orbits with the Sun at the focus.
(D) Hyperbolic orbits with the Sun at one focus.

Ans :

(B) Elliptical orbits with the Sun at one of the foci. Kepler's first law describes that planets follow elliptical paths, and the Sun is located at one of the two foci of the ellipse, not necessarily at the center.

5. According to Kepler's second law, the law of areas, the line that joins any planet to the Sun sweeps equal areas in equal intervals of time. This implies that a planet moves:

(A) Slower when it is farther from the Sun and faster when it is nearer.
(B) Faster when it is farther from the Sun and slower when it is nearer.
(C) With a constant speed throughout its orbit around the Sun always.

OBJECTIVE QUESTIONS

1. Which of the following ancient models of planetary motion proposed that all celestial objects, including stars, the sun, and the planets, revolved around the Earth?

(A) Heliocentric model
(B) Geocentric model
(C) Copernican model
(D) Kepler's model

Ans :

(B) Geocentric model

The geocentric model, proposed by Ptolemy, suggested that the Earth was the center of the universe and all celestial bodies revolved around it. This model was prevalent for about 2000 years.

$$h = \frac{6.4 \times 25}{100.44} \times 10^6$$

$$= 1.6 \times 10^6 \text{ m}$$

So height from the centre of earth.

$$= h + R$$

$$= 1.6 \times 10^6 + 6.4 \times 10^6$$

$$= 8.0 \times 10^6 \text{ m}$$

18. The escape speed of a projectile on the Earth's surface is 11.2 km s^{-1} . A body is projected out with thrice this speed. What is the speed of the body far away from the Earth? Ignore the presence of the Sun and other planets.

Ans :

Given, Escape velocity of the projectile on the Earth's surface

$$= 11.2 \text{ km s}^{-1}$$

According to the question, we can write,

$$\frac{1}{2}mv^2 = \frac{1}{2}m(3 \times 11.2)^2 - \frac{1}{2}m(11.2)^2$$

$$v^2 = 9 \times (11.2)^2 - (11.2)^2$$

$$= (11.2)^2(9 - 1)$$

$$v^2 = (11.2)^2 \times 8$$

$$v = 11.2\sqrt{8} = 11.2 \times 2.8$$

$$= 31.68 \text{ km s}^{-1}$$

19. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the Earth's gravitational influence? Mass of the satellite = 200 kg, mass of the Earth = $6.0 \times 10^{24} \text{ kg}$, radius of the Earth = $6.4 \times 10^6 \text{ m}$; $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Ans :

Total energy of the satellite when it is put into the orbit,

$$E = P.E. + K.E.$$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv_0^2$$

Since, $v_0 = \sqrt{\frac{GM}{(R+h)}}$

$$E = -\frac{GMm}{(R+h)} + \frac{1}{2}m\frac{GM}{(R+h)}$$

$$E = \frac{-GMm}{2(R+h)}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2[6.4 \times 10^6 + 0.4 \times 10^6]}$$

$$= \frac{6.67 \times 6.0 \times 2 \times 10^{15}}{2 \times 6.8 \times 10^6}$$

$$= -5.89 \times 10^9 \text{ J}$$

So the energy required to rocket the satellite out of earth's gravitational influence

$$= 0 - 5.89 \times 10^9 = 5.89 \times 10^9 \text{ J}$$

20. Two stars each of one solar mass ($= 2 \times 10^{30} \text{ kg}$) are approaching each other for a head-on collision. When they are at a distance of 10^9 km , their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km . Assume the stars to remain undistorted until they collide. (Use the known value of G).

Ans :

Given,

$$\text{Mass of each star, } M = 2 \times 10^{30} \text{ kg}$$

Initial distance between them,

$$r = 10^9 \text{ km} = 10^{12} \text{ m}$$

Final distance between them during collision

$$= R + R = 2R$$

$$= 2 \times 10^4 \text{ km}$$

$$= 2 \times 10^7 \text{ m}$$

Let speed with which the stars collide = v

Since,

Gain in kinetic energy = Loss in potential energy

$$\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right)$$

$$Mv^2 = -\left[\frac{GMM}{r} - \frac{GMM}{2R}\right]$$

$$v^2 = -GM\left[\frac{2R-r}{2Rr}\right]$$

$$= -6.67 \times 10^{-11} \times 2 \times 10^{30}$$

$$\times \left[\frac{2 \times 10^7 - 10^{12}}{2 \times 10^{19}}\right]$$

$$v^2 = 6.67 \times 10^{12}$$

$$v = 2.58 \times 10^6 \text{ ms}^{-1}$$

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- Set up the equation for translational equilibrium involving the reactions (R_1, R_2) at the knife-edges, the weight of the bar (W), and the suspended load (W_1).
- Choose a convenient point to take moments about (e.g., the centre of gravity G) and set up the equation for rotational equilibrium. How can these two equilibrium equations be used to find R_1 and R_2 ?

Ans :

- For translational equilibrium, the vector sum of all external forces acting on the bar must be zero.
- For rotational equilibrium, the vector sum of all external torques acting on the bar about any point must be zero.
- Let R_1 and R_2 be the upward reactions at the knife-edges. Let W be the weight of the bar and W_1 be the suspended load (both acting downwards). For translational equilibrium:

$$R_1 + R_2 - W - W_1 = 0.$$

- Taking moments about G : The weight W passes through G , so its torque is zero. Let K_1 and K_2 be the positions of the knife-edges and P be the position of the load. The equation for rotational equilibrium about G would be:

$$-R_1(K_1 G) + W_1(PG) + R_2(K_2 G) = 0$$

(assigning signs based on clockwise/anticlockwise rotation). These two equations (translational and rotational equilibrium) form a system of two linear equations with two unknowns (R_1 and R_2), which can be solved.

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- (D) With a speed that is independent of its distance from the Sun always.

Ans :

(A) Slower when it is farther from the Sun and faster when it is nearer.

The law of areas means that for the area swept by the line joining a planet to the Sun to be equal in equal time intervals, the planet must move faster when it is closer to the Sun and slower when it is farther away.

6. What is the term for the point in a planet's elliptical orbit where it is closest to the Sun?

- (A) Aphelion
(B) Perihelion
(C) Equinox
(D) Solstice

Ans :

(B) Perihelion

The perihelion is the point in the orbit of a planet, asteroid, or comet at which it is closest to the Sun.

7. Kepler's third law, the law of periods, states that the square of the time period of revolution of a planet (T^2) is proportional to:

- (A) The semi-major axis of the ellipse (a).
(B) The square of the semi-major axis of the ellipse (a^2).
(C) The cube of the semi-major axis of the ellipse (a^3).
(D) The inverse of the semi-major axis of the ellipse ($1/a$).

Ans :

(C) The cube of the semi-major axis of the ellipse (a^3).

Kepler's third law establishes a mathematical relationship where the square of a planet's orbital period (T) is directly proportional to the cube of the semi-major axis (a) of its elliptical orbit.

8. The law of areas is a consequence of the conservation of which physical quantity for a planet revolving around the Sun?

- (A) Linear momentum
(B) Angular momentum
(C) Kinetic energy
(D) Potential energy

Ans :

(B) Angular momentum

Kepler's second law, the law of areas, arises from the principle of conservation of angular momentum.

For a central force, like gravitation, the angular momentum of the planet remains constant.

9. Which of the following statements about Kepler's laws is incorrect?

- (A) All planets move in elliptical orbits with the Sun at one focus.
(B) A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.
(C) The square of the time period of revolution of a planet is proportional to the cube of its semi-minor axis.
(D) Kepler's laws were derived empirically from observational data before Newton's law of gravitation.

Ans :

(C) The square of the time period of revolution of a planet is proportional to the cube of its semi-minor axis.

Kepler's third law states that the square of the time period is proportional to the cube of the semi-major axis, not the semi-minor axis. The other statements correctly describe Kepler's laws.

10. A force is called a 'central force' if it:

- (A) Always acts towards a fixed point and its magnitude depends only on the distance from that point.
(B) Always acts perpendicular to the velocity of the object and its magnitude is constant always.
(C) Is always constant in magnitude and direction irrespective of the position of the object.
(D) Acts only when two objects are in direct physical contact with one another always.

Ans :

(A) Always acts towards a fixed point and its magnitude depends only on the distance from that point.

A central force is defined as a force that is always directed towards or away from a fixed point (the center of force) and whose magnitude depends only on the distance from this center. Gravitational force is an example.

Newton's Universal Law of Gravitation states that every body in the universe attracts every other body with a force that is:

- (A) Directly proportional to the product of their masses and directly proportional to the square of the distance between them.
(B) Directly proportional to the sum of their masses and inversely proportional to the square of the distance between them.

- (C) Directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- (D) Inversely proportional to the product of their masses and directly proportional to the square of the distance between them.

Ans :

(C) Directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Newton's law of gravitation quantifies the attractive force between two masses as being directly proportional to the product of their masses and inversely proportional to the square of the distance separating their centers.

12. What is the currently accepted value of the universal gravitational constant, G ?
- (A) $6.67 \times 10^{11} \text{Nm}^2/\text{kg}^2$
- (B) $6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$
- (C) $9.81 \times 10^{-11} \text{Nm}^2/\text{kg}^2$
- (D) $9.81 \times 10^{11} \text{Nm}^2/\text{kg}^2$

Ans :

(B) $6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$

The universal gravitational constant, G , is an empirical physical constant involved in the calculation of gravitational effects. Its experimentally determined value is approximately $6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$.

13. The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is as if:
- (A) The entire mass of the shell is concentrated at its closest point to the point mass.
- (B) The entire mass of the shell is concentrated at the centre of the shell always.
- (C) The mass of the shell is uniformly distributed on its surface, attracting the point mass.
- (D) The shell exerts no gravitational force on the point mass situated outside it.

Ans :

(B) The entire mass of the shell is concentrated at the centre of the shell always.

For a point mass outside a uniform spherical shell, the gravitational force exerted by the shell is the same as if the entire mass of the shell were concentrated at its geometric center.

14. Which of the following statements regarding the gravitational force between an extended object and a point mass is incorrect?

- (A) For a point mass inside a uniform spherical shell, the gravitational force exerted by the shell is zero.
- (B) For a point mass outside a uniform spherical shell, the force is as if the shell's mass is at its center.
- (C) The gravitational force between an extended object and a point mass is always directly along the line joining their centers of mass.
- (D) To find the total gravitational force by an extended object, one must vectorially sum forces from all its point masses.

Ans :

(C) The gravitational force between an extended object and a point mass is always directly along the line joining their centers of mass.

While true for spherically symmetric bodies or point masses, the gravitational force between an arbitrarily shaped extended object and a point mass is not necessarily along the line joining their centers of mass.

15. The acceleration due to gravity (g) on the surface of the Earth is related to the Earth's mass (M_E) and radius (R_E) by the expression:
- (A) $g = GM_E R_E^2$
- (B) $g = GM_E / R_E^2$
- (C) $g = GR_E^2 / M_E$
- (D) $g = GM_E / R_E$

Ans :

(B) $g = GM_E / R_E^2$

The acceleration due to gravity on the Earth's surface is derived from Newton's law of gravitation, where the force $F = mg$ is also $GM_E m / R_E^2$. This simplifies to $g = GM_E / R_E^2$.

16. How does the acceleration due to gravity $g(h)$ at a height h above the Earth's surface compare to the acceleration due to gravity g on the surface, assuming $h \ll R_E$?
- (A) $g(h) \approx g(1 + 2h/R_E)$
- (B) $g(h) \approx g(1 - 2h/R_E)$
- (C) $g(h) \approx g(1 + h/R_E)$
- (D) $g(h) \approx g(1 - h/R_E)$

Ans :

(B) $g(h) \approx g(1 - 2h/R_E)$

For small heights h compared to the Earth's radius R_E , the acceleration due to gravity decreases approximately linearly with height, given by the expression $g(h) \approx g(1 - 2h/R_E)$.

17. What happens to the acceleration due to gravity as one goes down from the surface of the Earth into a mine?
- It increases linearly with depth and is maximum at the center.
 - It decreases linearly with depth and is zero at the center.
 - It remains constant regardless of the depth below the surface.
 - It decreases quadratically with depth below the surface.

Ans :

(B) It decreases linearly with depth and is zero at the center.

As one goes deeper below the Earth's surface, the mass of the Earth contributing to the gravitational pull at that point decreases. Assuming uniform density, the acceleration due to gravity decreases linearly with depth, becoming zero at the Earth's center.

18. Gravitational potential energy of an object at a height h above the Earth's surface is conventionally taken as $V(h) = mgh$. This expression is valid when:
- The height h is very large compared to the Earth's radius R_E only.
 - The gravitational force is considered to vary inversely with height h .
 - The gravitational force is considered practically constant near the Earth's surface.
 - The object is in a circular orbit around the Earth at height h .

Ans :

(C) The gravitational force is considered practically constant near the Earth's surface.

The formula $V(h) = mgh$ for gravitational potential energy is an approximation valid for heights h much smaller than the Earth's radius, where the acceleration due to gravity g can be considered constant.

19. If the gravitational potential energy of a particle of mass m at a distance R from the center of the Earth (mass M_E) is defined to be zero at infinity, then its value at distance R (where $r > R_E$) is:
- $GM_E m/r$
 - $-GM_E m/r$
 - $GM_E m/r^2$
 - $-GM_E m/r^2$

Ans :

(B) $-GM_E m/r$

By convention, the gravitational potential energy is taken as zero at an infinite distance. The work done to bring a mass m from infinity to a distance R from a mass M_E is $-GM_E m/r$.

20. The escape speed from the surface of the Earth depends on:
- The mass of the body escaping and the Earth's mass only.
 - The Earth's mass and radius only.
 - The mass of the body and the Earth's radius only.
 - The Earth's mass, radius, and the mass of the body escaping always.

Ans :

(B) The Earth's mass and radius only.

The escape speed from the surface of a celestial body like Earth is given by the formula $V_e = \sqrt{2GM_E/R_E}$, which depends only on the mass (M_E) and radius (R_E) of the Earth, and the gravitational constant (G). It is independent of the mass of the escaping body.

21. What is the approximate value of the escape speed from the Earth's surface?
- 7.9 km/s
 - 11.2 km/s
 - 2.3 km/s
 - 9.8 km/s

Ans :

(B) 11.2 km/s

Using the values for the Earth's mass and radius, and the gravitational constant, the escape speed from the Earth's surface is calculated to be approximately 11.2 km/s.

22. Why does the Moon have no atmosphere?

- The Moon's temperature is too high for gases to exist.
- The Moon's gravitational pull is too strong, crushing any atmosphere.
- The Moon's escape speed is low, allowing gas molecules to escape.
- The Moon is too close to the Earth, and Earth's gravity strips its atmosphere.

Ans :

(C) The Moon's escape speed is low, allowing gas molecules to escape.

The Moon's escape speed is significantly lower than Earth's (about 2.3 km/s). Gas molecules, if present, would have thermal velocities exceeding this escape speed, causing them to escape into space.

47. Match the quantity in Column I with its correct SI unit or characteristic in Column II.

	Column I		Column II
1.	Gravitational Constant (G)	P.	m/s^2
2.	Escape Speed (V_e)	Q.	Joule (J)
3.	Gravitational Potential Energy (V)	R.	$\text{N m}^2/\text{kg}^2$
4.	Acceleration due to gravity (g)	S.	km/s

- (A) 1-R, 2-S, 3-Q, 4-P
 (B) 1-S, 2-R, 3-Q, 4-P
 (C) 1-R, 2-P, 3-Q, 4-S
 (D) 1-P, 2-S, 3-Q, 4-R

Ans :

- (A) 1-R, 2-S, 3-Q, 4-P

The SI unit for G is $\text{N m}^2/\text{kg}^2$ (1-R). Escape speed is typically measured in km/s (2-S). Gravitational Potential Energy is measured in Joules (3-Q). Acceleration due to gravity is measured in m/s^2 (4-P).

48. Match the phenomenon in Column I with the primary principle or law it illustrates from Column II.

	Column I		Column II
1.	Motion of planets around the Sun	P.	Newton's Law of Gravitation & Kepler's Laws
2.	Weightlessness in an orbiting satellite	Q.	Conservation of Mechanical Energy
3.	A planet moving faster near perihelion	R.	Free fall / Centripetal acceleration due to gravity
4.	An object thrown upwards and falling back	S.	Kepler's Law of Areas (Angular Momentum Conservation)

- (A) 1-P, 2-S, 3-R, 4-Q
 (B) 1-P, 2-R, 3-S, 4-Q
 (C) 1-Q, 2-R, 3-S, 4-P
 (D) 1-S, 2-Q, 3-P, 4-R

Ans :

- (B) 1-P, 2-R, 3-S, 4-Q

Planetary motion is described by Newton's and Kepler's laws (1-P). Weightlessness is due to free fall (2-R). Planets speeding up at perihelion illustrates

the law of areas (3-S). An object thrown up and falling back demonstrates conservation of mechanical energy (if air resistance is ignored) (4-Q).

49. Which of the following pairs correctly matches a scientist with their significant contribution to the understanding of gravitation or planetary motion?
 (A) Ptolemy - Heliocentric model
 (B) Nicolas Copernicus - Law of universal gravitation
 (C) Johannes Kepler - Three laws of planetary motion
 (D) Henry Cavendish - Discovery of Neptune

Ans :

- (C) Johannes Kepler - Three laws of planetary motion

Johannes Kepler formulated the three empirical laws describing planetary motion based on Tycho Brahe's observations. Ptolemy proposed the geocentric model. Newton formulated the law of universal gravitation. Cavendish measured G.

50. Select the correct pair of a gravitational concept and its mathematical representation or definition.
 (A) Escape Speed - $\sqrt{GM_E/R_E}$
 (B) Gravitational Force - Gm_1m_2/r
 (C) Time period of a satellite near Earth - $2\pi\sqrt{R_E^2/g}$
 (D) Acceleration due to gravity at height h (for $h \ll R_E$) - $g(1 - 2h/R_E)$

Ans :

- (D) Acceleration due to gravity at height h (for $h \ll R_E$) - $g(1 - 2h/R_E)$

The formula for acceleration due to gravity at a small height h is approximately $g(1 - 2h/R_E)$. Escape speed is $\sqrt{2GM_E/R_E}$. Gravitational force is Gm_1m_2/r^2 . Time period near Earth is $2\pi\sqrt{R_E/g}$.

51. Identify the incorrect pairing of a term related to elliptical orbits and its description.
 (A) Foci - Two fixed points within the ellipse, the sum of distances from which to any point on ellipse is constant.
 (B) Semi-major axis - Half the length of the longest diameter of the ellipse.
 (C) Perihelion - The point in the orbit where the planet is farthest from the Sun.
 (D) Law of Areas - A line joining a planet to the Sun sweeps out equal areas in equal intervals of time.

Ans :

- (C) Perihelion - The point in the orbit where the planet is farthest from the Sun.

- 30.** How does the value of acceleration due to gravity $g(h)$ at a height h compare with $g(d)$ at a depth d , assuming $h=d$ and both are much smaller than the Earth's radius?

(A) $g(h)$ is greater than $g(d)$.
 (B) $g(h)$ is less than $g(d)$.
 (C) $g(h)$ is equal to $g(d)$.
 (D) Their relationship depends on the Earth's density profile.

Ans :

(B) $g(h)$ is less than $g(d)$.
 For $h \ll R_E$, $g(h) \approx g(1 - 2h/R_E)$.
 For $d \ll R_E$, $g(d) = g(1 - d/R_E)$.

If $h=d$, then $(1 - 2h/R_E)$ is smaller than $(1 - d/R_E)$, meaning the decrease in g with height is more significant than with depth for the same small distance.

- 31.** A planet is orbiting the Sun in an elliptical path. At which point in its orbit is its speed the greatest?

(A) At aphelion
 (B) At perihelion
 (C) When its distance from the Sun is equal to the semi-minor axis
 (D) The speed is constant throughout the orbit

Ans :

(B) At perihelion
 According to Kepler's second law (law of areas), a planet moves faster when it is closer to the Sun and slower when it is farther away. Perihelion is the point closest to the Sun.

- 32.** If the Earth were a hollow spherical shell of uniform density, what would be the gravitational force on a point mass placed inside the shell?

(A) Directed towards the center of the shell.
 (B) Directed away from the center of the shell.
 (C) Zero.
 (D) Dependent on the mass's distance from the center.

Ans :

(C) Zero.
 A key result in gravitation states that the gravitational force on a point mass situated anywhere inside a uniform hollow spherical shell is zero. The forces from different parts of the shell cancel out.

- 33.** Which of the following statements about gravitational potential energy is INCORRECT?

(A) It is a scalar quantity.
 (B) Its value depends on the choice of the zero

reference point.

- (C) For a system of two particles, it is always positive if the zero is at infinity.
 (D) It is the energy stored in an object due to its position in a gravitational field.

Ans :

(C) For a system of two particles, it is always positive if the zero is at infinity.
 Gravitational potential energy for a system of two particles, with the zero reference at infinity, is given by $V = -Gm_1m_2/r$, which is always negative due to the attractive nature of gravity.

- 34.** The time period (t) of a satellite revolving very close to the Earth's surface (radius R_E , acceleration due to gravity g) is given by:

(A) $T = 2\pi\sqrt{R_E/g}$
 (B) $T = 2\pi\sqrt{g/R_E}$
 (C) $T = 2\pi R_E/g$
 (D) $T = 2\pi g/R_E$

Ans :

(A) $T = 2\pi\sqrt{R_E/g}$
 For a satellite very close to the Earth's surface, its orbital speed is $V = \sqrt{gR_E}$. The time period is $T = 2\pi R_E/V$, which simplifies to $T = 2\pi\sqrt{R_E/g}$.

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- 35.** What does the term 'aphelion' refer to in the context of planetary orbits?

(A) The point where a planet is closest to the Sun.
 (B) The point where a planet is farthest from the Sun.
 (C) The point where a planet crosses the Earth's orbital plane.
 (D) The average distance of a planet from the Sun in its orbit.

Ans :

(B) The point where a planet is farthest from the Sun.
 Aphelion is the point in the orbit of a planet or other astronomical body where it is farthest from the Sun.

- 36.** The value of the gravitational constant G was first experimentally determined by:

(A) Isaac Newton
 (B) Galileo Galilei
 (C) Johannes Kepler
 (D) Henry Cavendish

Ans :

(D) Henry Cavendish

57. Which of Kepler's laws implies that planets sweep out equal areas in equal times?
- (A) First Law (Law of Orbits)
 (B) Second Law (Law of Areas)
 (C) Third Law (Law of Periods)
 (D) None of the above named laws

Ans :

(B) Second Law (Law of Areas)

Kepler's Second Law, also known as the Law of Areas, states that the line joining a planet to the Sun sweeps out equal areas in equal intervals of time.

58. The point in a planet's orbit that is farthest from the Sun is called the:
- (A) Perihelion
 (B) Apogee
 (C) Aphelion
 (D) Perigee

Ans :

(C) Aphelion

Aphelion is the specific term for the point in an orbit around the Sun where the orbiting body (like a planet) is farthest from the Sun.

59. The universal law of gravitation proposed by Newton explained:
- (A) Only terrestrial gravitation and falling objects.
 (B) Only the motion of planets around the Sun.
 (C) Both terrestrial gravitation and Kepler's laws of planetary motion.
 (D) Only the tides experienced on Earth due to Moon and Sun.

Ans :

(C) Both terrestrial gravitation and Kepler's laws of planetary motion.

Newton's universal law of gravitation provided a unified framework that could explain both the falling of objects on Earth (terrestrial gravitation) and the orbital motions described by Kepler's laws.

60. The value of acceleration due to gravity (g) is generally taken as 9.8 m/s^2 on the Earth's surface. This value:
- (A) Is constant everywhere on Earth.
 (B) Varies slightly with latitude and altitude.
 (C) Is dependent on the mass of the object.
 (D) Is higher at the equator than at the poles.

Ans :

(B) Varies slightly with latitude and altitude.

While 9.8 m/s^2 is an average value, the actual acceleration due to gravity varies slightly depending on factors such as altitude, latitude (due to Earth's rotation and equatorial bulge), and local geological formations.

61. If the distance between the sun and the earth is increased by three times then attraction between two will :

- (a) remain constant
 (b) decreases by 63%
 (c) decreases by 83%
 (d) decreases by 89%

Ans :

(d) decreases by 89%

$$F = \frac{Gm_1m_2}{r^2} \text{ and } F_1 = \frac{Gm_1m_2}{(3r)^2} = \frac{F}{9}$$

% decreases in

$$F_1 = \left(\frac{F - F_1}{F} \right) \times 100 = \frac{F - \frac{F}{9}}{F} \times 100$$

$$= \frac{8}{9} \times 100 = 89\%$$

62. Two planets have same density but different radii. The acceleration due to gravity would be :
- (a) same on both the planets
 (b) greater on the smaller planet
 (c) greater on the larger planet
 (d) dependent of the distance of planet from the sun

Ans :

(c) greater on the larger planet

$$g = \frac{GM}{R^2}$$

$$= G \frac{4}{3} \pi R^3 \rho \frac{1}{R^2} = \frac{3}{4} G \rho R$$

63. If the radius of the earth were increased by a factor of 2 keeping the mass constant, by what factors would its density have to be changed to keep g same?

- (a) 4
 (b) $\frac{1}{2}$
 (c) $\frac{1}{4}$
 (d) $\frac{1}{8}$

Ans :

(b) $\frac{1}{2}$

Since,

$$g = \frac{4}{3} \pi G \rho R$$

Kepler's law of areas is a consequence of the conservation of angular momentum, not linear momentum. The gravitational force being a central force is what leads to the conservation of angular momentum.

43. Consider the following statements regarding planetary motion:

1. Planets move faster at aphelion than at perihelion.
2. The orbital period of a planet is independent of its mass.
3. All planets move in orbits with the Sun at the geometric center of the ellipse.
4. The gravitational force on a planet by the Sun provides the necessary centripetal force for its orbit.

Which of the above statements are correct?

- (A) 1 and 3 only
(B) 2 and 4 only
(C) 1, 2, and 4 only
(D) 2, 3, and 4 only

Ans :

(B) 2 and 4 only

Statement 1 is incorrect; planets move slower at aphelion. Statement 3 is incorrect; the Sun is at one of the foci, not necessarily the geometric center. Statements 2 and 4 are correct descriptions of planetary motion.

44. Which of the following statements concerning the acceleration due to gravity (g) are true?

1. g is a vector quantity always directed towards the center of the Earth.
2. The value of g is constant everywhere on the Earth's surface.
3. g decreases as one moves to higher altitudes from the Earth's surface.
4. g increases as one moves deeper into the Earth from its surface.

- (A) 1 and 3 only
(B) 1, 2, and 3 only
(C) 1, 3, and 4 only
(D) All four statements are true.

Ans :

(A) 1 and 3 only

Statement 2 is false; g varies due to Earth's rotation and non-spherical shape. Statement 4 is false; g decreases as one moves deeper into the Earth (assuming uniform density for simplicity, it becomes zero at the center). Statements 1 and 3 are true.

45. Identify the correct statements about Earth satellites:

1. The speed of a satellite in a circular orbit is inversely proportional to the square root of its orbital radius.
2. Geostationary satellites orbit the Earth with a period of approximately 24 hours.
3. The total mechanical energy of a satellite in a stable orbit is always positive.
4. Kepler's laws of planetary motion do not apply to artificial satellites orbiting the Earth.

- (A) 1 and 2 only
(B) 2 and 3 only
(C) 1, 2, and 4 only
(D) 1, 2, 3 and 4

Ans :

(A) 1 and 2 only

Statement 3 is incorrect; the total mechanical energy of an orbiting satellite is negative. Statement 4 is incorrect; Kepler's laws apply to satellites as well. Statements 1 and 2 are correct.

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46. Match the physical concept in Column I with its corresponding description or law in Column II.

	Column I		Column II
1.	Law of Orbits	P.	Speed of efflux from an open tank
2.	Law of Areas	Q.	All planets move in elliptical orbits
3.	Law of Periods	R.	Conservation of angular momentum
4.	Torricelli's Law	S.	$T^2 \propto a^3$

- (A) 1-Q, 2-R, 3-S, 4-P
(B) 1-R, 2-Q, 3-S, 4-P
(C) 1-Q, 2-S, 3-R, 4-P
(D) 1-S, 2-R, 3-Q, 4-P

Ans :

(A) 1-Q, 2-R, 3-S, 4-P

Kepler's Law of Orbits states planets move in elliptical orbits (1-Q). The Law of Areas is a consequence of angular momentum conservation (2-R). The Law of Periods relates the square of the period to the cube of the semi-major axis (3-S). Torricelli's Law describes the speed of efflux (4-P).

23. What is the only natural satellite of the Earth?

- (A) Phobos
- (B) Deimos
- (C) Titan
- (D) Moon

Ans :

(D) Moon

The Moon is Earth's only natural satellite. It orbits the Earth with a period of approximately 27.3 days, which is also roughly its rotational period.

24. For a satellite revolving in a circular orbit of radius ($R_E + h$) around the Earth, the necessary centripetal force is provided by:

- (A) The Sun's gravitational force on the satellite.
- (B) The Earth's gravitational force on the satellite.
- (C) The atmospheric drag on the satellite always.
- (D) The thrust from the satellite's engines.

Ans :

(B) The Earth's gravitational force on the satellite. A satellite in orbit around the Earth requires a centripetal force to maintain its circular path. This force is supplied by the Earth's gravitational attraction on the satellite.

25. The speed (v) of a satellite in a circular orbit of radius ($R_E + h$) around the Earth (mass M_E) is given by:

- (A) $V^2 = GM_E(R_E + h)$
- (B) $V^2 = GM_E/(R_E + h)$
- (C) $V^2 = G(R_E + h)/M_E$
- (D) $V = GM_E/(R_E + h)^2$

Ans :

(B) $V^2 = GM_E/(R_E + h)$

By equating the centripetal force ($mV^2/(R_E + h)$) with the gravitational force ($GM_E m/(R_E + h)^2$), we find that the square of the orbital speed is $V^2 = GM_E/(R_E + h)$.

26. How does the orbital speed of a satellite change as its height (h) above the Earth's surface increases?

- (A) It increases.
- (B) It decreases.
- (C) It remains constant.
- (D) It first increases then decreases.

Ans :

(B) It decreases.

From the formula $V^2 = GM_E/(R_E + h)$, as the height h increases, the denominator $R_E + h$ increases. Consequently, the orbital speed v decreases.

27. Which of the following is NOT a direct application or field related to artificial Earth satellites?

- (A) Telecommunication
- (B) Geophysics
- (C) Meteorology
- (D) Deep-sea exploration

Ans :

(D) Deep-sea exploration

Artificial Earth satellites are widely used for telecommunication, studying Earth's physical properties (geophysics), and weather forecasting (meteorology). While satellites can observe ocean surfaces, direct deep-sea exploration typically requires submersibles.

28. Which of the following statements about the total energy of an orbiting satellite in a circular orbit is correct?

- (A) The total energy is positive and constant throughout the orbit.
- (B) The total energy is negative, with kinetic energy being twice the magnitude of potential energy.
- (C) The total energy is negative, with potential energy being negative and twice the magnitude of kinetic energy.
- (D) The total energy is zero, as kinetic and potential energies cancel each other completely.

Ans :

(C) The total energy is negative, with potential energy being negative and twice the magnitude of kinetic energy.

The total mechanical energy of a satellite in a circular orbit is negative, indicating a bound system. The potential energy is negative, and its magnitude is twice that of the positive kinetic energy, resulting in $E = -GM_E m/(2(R_E + h))$.

29. The term 'geocentric model' refers to a model of the universe where:

- (A) The Sun is at the center.
- (B) The Earth is at the center.
- (C) The Milky Way galaxy is at the center.
- (D) All stars are fixed relative to each other.

Ans :

(B) The Earth is at the center.

The geocentric model, historically advocated by Ptolemy, places the Earth at the center of the universe, with all other celestial bodies revolving around it. This contrasts with the heliocentric model.

Perihelion is the point in the orbit where the planet is closest to the Sun. Aphelion is the point where it is farthest. The other pairs are correctly described.

52. Which of the following pairs of a quantity and its dependence is INCORRECT?

- (A) Escape speed from a planet - Depends on the planet's mass and radius.
- (B) Period of a satellite in orbit - Depends on the orbital radius and mass of the central body.
- (C) Acceleration due to gravity on a planet's surface - Depends on the planet's mass and radius.
- (D) Gravitational potential energy between two masses - Depends only on the product of their masses.

Ans :

(D) Gravitational potential energy between two masses - Depends only on the product of their masses.

Gravitational potential energy between two masses ($V = -Gm_1m_2/r$) depends on the product of their masses AND the distance between them, as well as the gravitational constant G. The other dependencies are correctly stated.

53. Arrange the following astronomers/physicists in chronological order of their major contributions to the understanding of planetary motion and gravitation:

1. Isaac Newton
2. Johannes Kepler
3. Ptolemy
4. Nicolas Copernicus

- (A) 3, 4, 2, 1
- (B) 4, 3, 2, 1
- (C) 3, 2, 4, 1
- (D) 2, 3, 4, 1

Ans :

- (A) 3, 4, 2, 1

Ptolemy (geocentric model, ~2000 years ago), Nicolas Copernicus (heliocentric model, 1473-1543), Johannes Kepler (laws of planetary motion, 1571-1640), Isaac Newton (law of universal gravitation, 1643-1727).

54. Consider the journey of an object thrown vertically upwards from the Earth's surface until it reaches its maximum height and then falls back. Assuming only gravitational force acts on it, arrange the following quantities in order of their values at (I) the moment of throw, (II) the maximum height, and (III) just before hitting the ground:
(Kinetic Energy, Gravitational Potential Energy - assuming zero at surface)

- (A) $K E (I) > K E (II) > K E (III)$; $PE(I) < PE(II) < PE(III)$
- (B) $K E (I) = K E (III) > K E (II)$; $PE(II) > PE(I) = PE(III)$
- (C) $K E (I) < K E (II) < K E (III)$; $PE(I) > PE(II) > PE(III)$
- (D) $K E (II) > K E (I) = K E (III)$; $PE(I) = PE(III) > PE(II)$

Ans :

(B) $KE(I) = KE(III) > KE(II)$; $PE(II) > PE(I) = PE(III)$
Kinetic energy is maximum at throw (I) and just before impact (III) (ideally equal if thrown from surface), and zero at maximum height (II). Potential energy is zero at surface (I and III, by choice of reference) and maximum at maximum height (II).

55. To launch a satellite into a stable circular orbit around the Earth, the initial velocity imparted to it must be:

- (A) Greater than the escape velocity at that altitude.
- (B) Exactly equal to the escape velocity at that altitude.
- (C) A specific horizontal velocity (orbital velocity) at that altitude.
- (D) Any velocity as long as it is pointed away from Earth.

Ans :

(C) A specific horizontal velocity (orbital velocity) at that altitude.

To achieve a stable circular orbit, the satellite must be given a specific horizontal velocity (orbital velocity) at the desired altitude. This velocity ensures the gravitational force provides the necessary centripetal force.

56. If the Earth's radius were to shrink by 1%, its mass remaining the same, the acceleration due to gravity on its surface would:

- (A) Decrease by approximately 1%
- (B) Increase by approximately 1%
- (C) Decrease by approximately 2%
- (D) Increase by approximately 2%

Ans :

(D) Increase by approximately 2%
Acceleration due to gravity $g = GM_E/R_E^2$. If R_E decreases by 1% to $0.99R_E$, then the new g' would be

$$\begin{aligned} GM_E/(0.99R_E)^2 &= g/(0.99)^2 \\ &\approx g/(0.98) \\ &\approx 1.02g \end{aligned}$$

This is an increase of approximately 2%.

Henry Cavendish, an English scientist, was the first to experimentally determine the value of the universal gravitational constant G in 1798 using a torsion balance.

37. If the Earth's density were perfectly uniform, at what location would the acceleration due to gravity be maximum?

(A) At the center of the Earth
(B) At the surface of the Earth
(C) Halfway between the surface and the center
(D) At a height equal to the Earth's radius above surface

Ans :

(B) At the surface of the Earth

Assuming uniform density, the acceleration due to gravity increases linearly from zero at the center to a maximum value at the surface, and then decreases with increasing height above the surface.

38. All of the following are consequences of Kepler's laws EXCEPT:

(A) Planets move faster when closer to the Sun.
(B) The orbits of planets are perfect circles.
(C) The square of a planet's orbital period is proportional to the cube of its semi-major axis.
(D) A line joining a planet and the Sun sweeps out equal areas in equal times.

Ans :

(B) The orbits of planets are perfect circles.

Kepler's first law states that planets move in elliptical orbits, with the Sun at one focus. A circle is a special case of an ellipse, but planetary orbits are generally not perfect circles.

39. Compare the total mechanical energy of a satellite in a circular orbit of radius R_1 with that of an identical satellite in a circular orbit of radius R_2 , where $R_1 < R_2$.

(A) The energy in orbit R_1 is greater (less negative) than in orbit R_2 .
(B) The energy in orbit R_1 is less (more negative) than in orbit R_2 .
(C) The energies are equal in both orbits since mass is same.
(D) The energy in orbit R_1 is positive, while in orbit R_2 it is negative.

Ans :

(B) The energy in orbit R_1 is less (more negative) than in orbit R_2 .

The total energy of a satellite in a circular orbit is $E = -GM_E m / (2R)$. Since $R_1 < R_2$, $1/R_1 > 1/R_2$.

Therefore, $-1/R_1 < -1/R_2$, making the energy in the smaller orbit R_1 more negative (less) than in the larger orbit R_2 .

40. Assertion: An astronaut in an orbiting satellite experiences weightlessness.

Reason: The gravitational force on the astronaut is zero in space.

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(C) Assertion is true, but Reason is false.
(D) Both Assertion and Reason are false.

Ans :

(C) Assertion is true, but Reason is false.

An astronaut in an orbiting satellite experiences weightlessness because both the astronaut and the satellite are in a state of continuous free fall towards the Earth. The gravitational force is not zero; it provides the centripetal force for the orbit.

41. Assertion: The escape speed from the Moon is less than that from the Earth.

Reason: The Moon has a smaller mass and a smaller radius compared to the Earth.

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(C) Assertion is true, but Reason is false.
(D) Both Assertion and Reason are false.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

Escape speed is given by $V_e = \sqrt{2GM/R}$. Since the Moon has significantly less mass (M) and a smaller radius (R) than Earth, its escape speed is lower.

42. Assertion: Kepler's law of areas is a consequence of the conservation of linear momentum.

Reason: The gravitational force exerted by the Sun on a planet is a central force.

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(C) Assertion is false, but Reason is true.
(D) Both Assertion and Reason are false.

Ans :

(C) Assertion is false, but Reason is true.

$$g \propto \rho R$$

So if the radius is increased by a factor 2, in order to keep g constant, the values of ρ has to be changed by a factor $\frac{1}{2}$.

64. Two spheres of equal radius r are touching each other. The force of attraction between them is proportional to
- (a) r^6 (b) r^4
- (c) r^2 (d) r^{-2}

Ans :

- (b) r^4

$$\begin{aligned} \frac{Gm_1m_2}{(2r)^2} &= \frac{G\frac{4}{3}\pi r^3 \times \frac{4}{3}\pi r^3 \rho}{4r^3} \\ &= G\frac{4}{9}\pi^2 \rho^2 r^4 \end{aligned}$$

65. If earth revolves round the sun in one year. If the distance between them become double, the new period of revolution will be:
- (a) $\left(\frac{1}{2}\right)$ year (b) $2\sqrt{2}$ years
- (c) 4 years (d) 8 years

Ans :

- (b) $2\sqrt{2}$ years

$$T^2 \propto r^3$$

So $\left(\frac{T_1}{T}\right)^2 = \left(\frac{r_1}{r}\right)^3$

$$T_1 = T\left(\frac{r_1}{r}\right)^{\frac{3}{2}} = 2\sqrt{2} \text{ years}$$

66. Which of the following statements is correct in respect of a geostationary satellite ?
- (a) It moves in a plane containing the Greenwich meridian.
- (b) It moves in a plane perpendicular to the celestial equatorial plane.
- (c) Its height above the earth's surface is about the same as the radius of the earth.
- (d) Its height above the earth's surface is about six times the radius of the earth.

Ans :

- (d) Its height above the earth's surface is about six times the radius of the earth.

The height of geostationary satellite is about 36000 km, which is nearly six times the radius of the earth.

67. Radius of the earth is 6400 km. The radius of the orbit of a stationary satellite is about:
- (a) 36000 km (b) 29600 km
- (c) 42400 km (d) Infinity

Ans :

- (a) 36000 km

Radius of the orbit of a stationary satellite is about 36000 km.

68. How many times is escape velocity (v_e) or orbital velocity (v_0) for a satellite revolving near earth?
- (a) $\sqrt{2}$ times (b) 2 times
- (c) 3 times (d) 4 times

Ans :

- (a) $\sqrt{2}$ times

69. The period of a satellite in a circular orbit around a planet is independent of
- (a) the mass of the planet
- (b) the radius of the planet
- (c) the mass of the satellite
- (d) all the three parameters (a), (b) and (c)

Ans :

- (c) the mass of the satellite

70. A satellite is orbiting round the earth at a height h above the surface of the earth. If the distance h is increased, the time period of satellite will
- (a) decrease
- (b) increase
- (c) remain unaffected
- (d) become zero

Ans :

- (b) increase

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}}$$

$$T \propto r^{\frac{3}{2}}$$

So as h increases, r also increases, hence T increases.

71. A spaceship entering the earth's atmosphere is likely to catch fire. This is due to
- (a) the surface tension of air
- (b) the viscosity of air
- (c) the high temperature of upper atmosphere
- (d) the greater portion of oxygen in the atmosphere at higher height

- 147.** Describe the gravitational force exerted by a hollow spherical shell of uniform density on a point mass outside it.

Ans :

The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is as if the entire mass of the shell is concentrated at its center. The force follows the inverse square law with distance from the center.

- 148.** What is the gravitational force on a point mass situated inside a hollow spherical shell of uniform density? Explain.

Ans :

The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside it is zero. This is because the gravitational forces exerted by various regions of the shell on the internal point mass cancel each other out completely.

- 149.** How is the acceleration due to gravity (g) on Earth's surface related to Earth's mass (M_E) and radius (R_E)?

Ans :

The acceleration due to gravity (g) on Earth's surface is given by the formula $g = \frac{GM_E}{R_E^2}$, where G is the universal gravitational constant. This shows g depends on Earth's mass and radius.

- 150.** Explain how the acceleration due to gravity varies with height (h) above Earth's surface.

Ans :

The acceleration due to gravity $g(h)$ at a height h is given by $g(h) = \frac{GM_E}{(R_E + h)^2}$. For small heights ($h \ll R_E$), it can be approximated as $g(h) \approx g(1 - \frac{2h}{R_E})$, indicating it decreases with height.

- 151.** Explain how the acceleration due to gravity varies with depth (d) below Earth's surface.

Ans :

The acceleration due to gravity $g(d)$ at a depth d below Earth's surface is given by $g(d) = g(1 - \frac{d}{R_E})$, assuming uniform density. This shows that g decreases linearly with depth, becoming zero at the Earth's center.

- 152.** Define gravitational potential energy. Why is it considered a negative quantity conventionally?

Ans :

Gravitational potential energy is the energy stored in a body by virtue of its position in a gravitational field. It is conventionally taken as negative because

the zero of potential energy is set at infinity; work is done by the gravitational force when masses are brought from infinity.

- 153.** What is escape speed? On what factors does it depend for a planet?

Ans :

Escape speed is the minimum initial speed an object must have to escape completely from the gravitational influence of a planet without any further propulsion. It depends on the mass (M) and radius (R) of the planet ($V_e = \sqrt{2GM/R}$).

- 154.** Explain why the Moon has no atmosphere while Earth does, relating it to escape speed.

Ans :

The Moon has no atmosphere because its escape speed (approximately 2.3 km/s) is much smaller than Earth's (11.2 km/s). Gas molecules on the Moon, if formed, would easily achieve speeds greater than this and escape its gravitational pull.

- 155.** Describe the motion of Earth satellites. Are Kepler's laws applicable to them?

Ans :

Earth satellites are objects that revolve around the Earth in circular or elliptical orbits. Their motion is very similar to planetary motion, and hence Kepler's laws of planetary motion are equally applicable to them.

- 156.** How is the orbital speed of a satellite related to its height above the Earth?

Ans :

The orbital speed V of a satellite at a height h above Earth is given by

$$V^2 = \frac{GM_E}{(R_E + h)}.$$

This shows that the orbital speed V decreases as the height h increases.

- 157.** Derive the expression for the time period of a satellite orbiting the Earth in a circular orbit.

Ans :

The time period T of a satellite is the distance $2\pi(R_E + h)$ divided by its speed V .

Substituting

$$V = \sqrt{\frac{GM_E}{(R_E + h)}},$$

We get

$$T = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{GM_E}}.$$

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Since, $g' = g - R\omega^2 \cos \lambda$

As latitude λ of a place increases, the value of $\cos \lambda$ decreases and hence g' increases. So we find that acceleration varies directly with latitude.

80. The gravitational potential energy of a body (in earth's field) is minimum
- on the surface of the earth
 - at infinity
 - below the earth's surface
 - between the earth's surface and infinity.

Ans :

- on the surface of the earth

On the surface of earth,

$$U = -\frac{GMm}{R}$$

Inside the earth's surface,

$$U = -\frac{GM(3R^2 - r^2)}{2R^3}$$

So U is minimum on the surface of earth.

81. Gravitational mass is proportional to gravitational:
- field
 - forces
 - intensity
 - all of these

Ans :

- all of these

Gravitational mass,

$$m = \frac{\text{gravitational force}}{\text{gravitational intensity}}$$

ONE MARK QUESTIONS

82. What are units and dimensions of G ?

Ans :

$$G = \frac{Fd^2}{m_1 m_2} = \frac{\text{Nm}^2}{\text{kg}^2} = \text{Nm}^2 \text{ kg}^{-2}$$

83. At what place on earth, the centripetal force is maximum ?

Ans :

At the equator.

84. Which one is India's first geostationary satellite ?

Ans :

APPLE which stands for Aeriane Passenger Pay Load Experiment.

85. According to Kepler's second law the earth travels fastest when it is closest to the sun. Is this consistent with the law of gravitation ? Explain.

Ans :

The gravitational force of the sun increases when the earth is closest to it according to the inverse square law. The earth must travel faster to counteract the greater gravitational force to the sun.

86. Is the sun's gravitational pull on the earth the same in all seasons of the year ? Explain.

Ans :

We have, gravitational force $F \propto \frac{1}{R^2}$.

The sun's gravitational pull on the earth varies in all seasons of the year because the distance R from the earth to the sun varies.

87. Why is G called universal gravitational constant ?

Ans :

It is so called because it does not depend on the nature of the medium between two bodies. It has universal value of $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.

88. Which of the two, mass or weight, is the essential property of matter ?

Ans :

Mass is essential property of matter because weight of a body can be zero (at the centre of earth) but mass of a body remains unaffected.

89. Earth is continuously pulling moon towards its centre. Why does not moon fall on to earth ?

Ans :

Because, earth provides the necessary centripetal force required by the moon for its orbital motion in the form of gravitational force of attraction. So the moon does not fall on to earth and revolve around the earth.

90. Why we can jump higher on the surface of moon than on the earth ?

Ans :

Gravitational field strength on the moon is nearly $\frac{1}{6}$ th of that at earth, hence we can jump higher on the moon than on the earth.

$$g = \frac{GM}{R^2}$$

Thus the acceleration g depends on the mass of earth and independent of the mass of the body.

- 165.** What do you mean by natural and artificial satellite ? Explain with examples.

Ans :

Natural satellite

A natural satellite is a heavenly body revolving around a planet in a close and stable orbit.

- e.g. 1. Moon is natural satellite of earth.
2. Nine planets are natural satellites of sun.

Artificial satellite

Man-made satellite is called artificial satellite. e.g.

1. First man-made satellite was Sputnik-1.
2. India's first man made satellite was Aryabhata.

- 166.** What are the uses of artificial satellites ?

Ans :

The uses of artificial satellites are as follows :

1. They are used in communicating radio, TV and telephone signals to distant places.
2. They are used to study the upper regions of atmosphere.
3. They are used in forecasting weather.
4. They are used to determine the exact shape of earth.
5. They are used to study cosmic rays and solar radiations.

- 167.** Compare the inertial and gravitational masses.

Ans :

1. Both are scalar quantities.
2. Both have the same units.
3. Both are independent of shape or state of the body.
4. Inertial mass is independent of the presence of other bodies near it and gravitational mass is affected by the presence of other bodies near it.
5. Inertial mass is determined by using acceleration produced by force and gravitational mass is determined by using gravitational pull acting on it.
6. Both are equivalent to each other.

- 168.** Derive Newton's law of gravitation from Kepler's law.

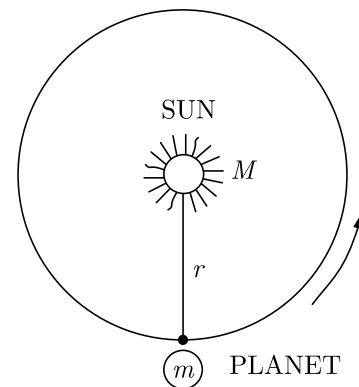
Ans :

The centripetal force (F) required by a planet of mass m revolving around the sun in an orbit of radius r with angular velocity ω is given by

$$\begin{aligned} F &= mr \omega^2 \\ &= mr \left(\frac{2\pi}{T} \right)^2 \quad \left[\omega = \frac{2\pi}{T} \right] \\ F &= \frac{4\pi^2 mr}{T^2} \quad \dots(1) \end{aligned}$$

From Kepler's law.

$$\begin{aligned} T^2 &= kr^3 \\ F &= \frac{4\pi^2 mr}{kr^3} \\ F &= \left(\frac{4\pi^2}{k} \right) \frac{m}{r^2} \quad \dots(2) \end{aligned}$$



The source of this force lies in the sun and hence the factor should depend only on some property of the sun. Let $F \propto m$ (mass of sun)

So Eq (2) become

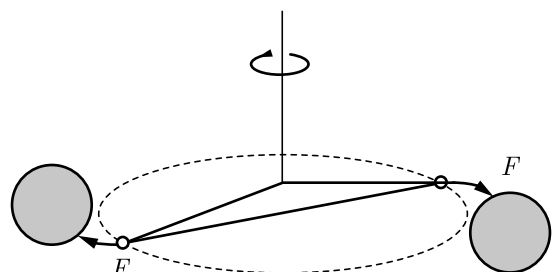
$$F = \frac{Mm}{r^2}$$

This is Newton's law of gravitation.

- 169.** Discuss Cavendish torsion balance to determine G .

Ans :

Two small lead balls are attached to the ends of bar which is supported by a fine wire.



- 109.** Does the value of ' g ' vary, as we move on the surface of earth ?

Ans :

Yes, its value is minimum at equator and maximum at poles.

- 110.** Gravitational force is a weak force but still it is considered the most important force. Why ?

Ans :

Gravitational force is long range force and plays an important role for initiating the birth of stars and controlling the entire structure of the universe.

- 111.** Why is a body weightless at the centre of earth ? Explain.

Ans :

At the centre of earth, $g = 0$ and as such at that place the weight (mg) of a body is zero.

- 112.** What would happen if gravity suddenly disappears?

Ans :

Weight of all bodies will become zero and all bodies will be thrown into space due to centrifugal force.

- 113.** An elephant and an ant are to be projected far away into space. Do we need different velocities for their projection ?

Ans :

No, the escape velocity does not depend upon the mass of the body to be projected.

- 114.** How do we choose zero level of gravitational potential energy ?

Ans :

The zero level of potential energy corresponds to infinite separation between two interacting masses.

- 115.** What does low values of escape velocity indicate ?

Ans :

Little probability of atmosphere on the planet.

- 116.** What is the unit of intensity of gravitational field ?

Ans :

N kg^{-2}

- 117.** What is the value of ' g ' and ' G ' at the centre of earth?

Ans :

$g = 0$ and $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

- 118.** What is satellite ?

Ans :

A body revolving around a heavenly body is called its satellite.

- 119.** Does the speed of a satellite remain constant in a particular orbit.

Ans :

Since, $v = \sqrt{\frac{GM}{r}}$, v depends only upon r , For a particular orbit, r is constant and so is v

- 120.** What is a parking orbit ?

Ans :

The orbit of a geostationary orbit is called parking orbit. The height of such orbit is 36000 km.

- 121.** What is geostationary satellite ?

Ans :

A satellite which revolves around the earth with same angular speed and in the direction as that of the earth around its axis is called geostationary satellite or geosynchronous satellite.

- 122.** What is the full form of geostationary satellite "APPLE" ?

Ans :

APPLE stands for "Aeriane Passenger Pay Load Experiment."

- 123.** Give two uses of polar satellite.

Ans :

- (i) Polar satellites are used to record the land and sea temperatures.
- (ii) They are used for spying and surveillance.

- 124.** Give two uses of geostationary satellite.

Ans :

- (i) It is used in communicating radio, TV and telephone signals across the oceans.
- (ii) It is used in studying the upper region of atmosphere.

- 125.** What is weightlessness ?

Ans :

A body is said to be in a state of weightlessness when the reaction of the supporting surface is zero.

91. How does earth retain its atmosphere ?

Ans :

Earth retain its atmosphere because the escape velocity from earth is more than the thermal speed of the gas molecules.

92. If earth suddenly stops rotating about its axis, what would the effect on g ? Would this effect be same at all places ?

Ans :

If earth suddenly stops rotating, the value of g increases.

This effect will not be the same at all places. It would be maximum at equator.

93. Is the value of ' g ' same or different for different objects at a given place ?

Ans :

The value of g at a given place is same for different objects.

94. Does inertial mass of a body depend upon the speed of body ?

Ans :

Yes

95. Are inertial mass and gravitational mass of a body different ?

Ans :

No, they are identical.

96. What are units of gravitational field intensity ?

Ans :

It is force on a unit mass and hence its unit in SI is N kg^{-1} or ms^{-2} .

97. Name one factor on which the period of revolution of a planet around the sun depends ?

Ans :

Mean distance of the planet from the sun.

98. Is the density of earth uniform throughout ?

Ans :

No, the density of upper layers of earth is less than that of lowers.

99. What is escape velocity for the sun ?

Ans :

It is 817 km s^{-1} .

100. Can gravitational potential be positive ?

Ans :

No

101. What is the value of G in SI ?

Ans :

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

102. What is the difference between g and G ?

Ans :

' g ' is acceleration due to gravity and $g = 9.8 \text{ ms}^{-2}$. 'G' is universal gravitational constant and $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

103. What is acceleration due to gravity ?

Ans :

Acceleration due to gravity is the acceleration produced in the motion of a body under the effect of gravity.

104. Which is greater-the attraction of earth for 1 kg of iron or attraction of 1 kg iron for earth ?

Ans :

It is same for both the cases, because gravitational force of attraction is due to the product of their masses i.e.,

$$F = G \frac{m_1 m_2}{r^2}$$

105. What is the value of G in SI ?

Ans :

$$\text{In SI, } G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

106. Who discovered universal law of gravitation ?

Ans :

Sir Issac Newton.

107. What will happen to the value of ' g ' as we go below the surface of the earth ?

Ans :

The value of ' g ' goes on decreasing.

108. What will happen to the value of ' g ' as we go above the surface of earth ?

Ans :

The value of ' g ' goes on decreasing.

- 137.** When we move our finger, whole of the universe is disturbed. Explain.

Ans :

According to Newton's law of gravitation, the attraction between two bodies depends on mass and square of the distance between them. When we move our finger, the distance of all the objects in the universe changes w.r.t. the finger, hence the force of attraction also change

- 138.** Imagine a spacecraft going from earth to the moon. How does its weight vary as it goes from the earth to the moon ? Will there be any change in the mass?

Ans :

The weight of spacecraft will decrease in the beginning then becomes zero, where the force of attraction on the spacecraft due to moon and earth become equal and opposite and then it will again increase as the spacecraft moves towards the moon. There will be no change in the mass of spacecraft.

- 139.** Newton's law of gravitation states that everybody exerts a gravitational force on every other body. If this is true, why for example two boys sitting in the examination hall do not move towards each other due to this force ?

Ans :

The gravitational attraction is observed only when (or both) of the masses are large. There is a gravitational attraction between the two boys sitting in the examination hall, but as the masses are so small, this attraction has no noticeable effect.

- 140.** Why has moon no atmosphere ?

Ans :

A planet or satellite will have atmosphere if the molecules of gases is less than the escape velocity. In case of moon, the escape velocity on the surfaces of moon is 2.38 km s^{-1} which is smaller than the r.m.s. velocity of gas molecules on the surface of moon, so all gases have escaped from the gravitational field of the moon and thus moon has no atmosphere.

- 141.** What is geocentric and heliocentric theories of universe?

Ans :

Geocentric theory : This theory was given by Ptolemy. According to this theory, earth is considered to be at rest and all planets, the sun and stars are supposed to be rotating around the earth in different fixed orbits.

Heliocentric theory : This theory was given by Copernicus. According to this theory, sun is considered to be at rest and all other planets including earth are rotating around the sun in different fixed orbital.

- 142.** Explain Kepler's law of orbits regarding planetary motion.

Ans :

Kepler's law of orbits states that all planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse. This means a planet's distance from the Sun varies as it orbits, unlike in a circular model.

- 143.** Describe Kepler's law of areas and its implication for a planet's speed.

Ans :

The law of areas states that the line joining any planet to the Sun sweeps out equal areas in equal intervals of time. This implies that planets move faster when they are closer to the Sun and slower when they are farther away.

- 144.** State Kepler's law of periods. How does it relate a planet's orbital period to its semi-major axis?

Ans :

The law of periods states that the square of the time period of revolution (T^2) of a planet is proportional to the cube of the semi-major axis (a^3) of its elliptical orbit. This establishes a mathematical relationship between a planet's orbital duration and its average distance from the Sun.

- 145.** How is the universal law of gravitation stated mathematically by Newton?

Ans :

Newton's law of gravitation states that the force (F) between two point masses (m_1 and m_2) is $F = G \frac{m_1 m_2}{r^2}$, where G is the universal gravitational constant and r is the distance between them. The force is attractive and directed along the line joining the masses.

- 146.** Explain the principle of superposition for gravitational forces.

Ans :

If we have a collection of point masses, the total gravitational force on any one of them is the vector sum of the gravitational forces exerted by all the other individual point masses. Each force acts independently.

Ans :

(b) the viscosity of air

72. The escape velocity of a particle of mass m varies directly as

(a) m^2 (b) m
(c) m^0 (d) m^{-1}

Ans :

(c) m^0

Escape velocity is independent of the mass m of the particle to be escaped. So $v_e \propto m^0$.

73. If a satellite is orbiting the earth very close to its surface, then the orbital velocity mainly depends upon

(a) the mass of the satellite only
(b) the radius of the earth only
(c) the orbital radius only
(d) the mass of the earth only.

Ans :

(b) the radius of the earth only

When satellite is orbiting very close to the surface of earth, then. $v_0 = \sqrt{gR}$ i.e., v_0 depends upon R .

74. If g_e and g_p denote the acceleration due to gravity on the surface of earth and another planet whose mass and radius are twice that of earth, then the ratio $\frac{g_p}{g_e}$ is

(a) $\frac{1}{2}$ (b) $\sqrt{2}$
(c) 1 (d) 2

Ans :

(a) $\frac{1}{2}$

$$\frac{g_p}{g_e} = \frac{M_p R_e^2}{M_e R_p^2} = \frac{2M_e R_e^2}{M_e (2R_e)^2} = \frac{1}{2}$$

75. Escape velocity on a planet is v_e . If the radius of the planet remains same and mass becomes four times, the escape velocity becomes

(a) $4v_e$ (b) $2v_e$
(c) v_e (d) $\frac{1}{2}v_e$

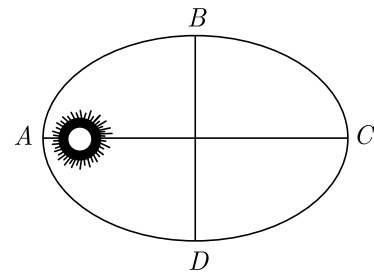
Ans :

(b) $2v_e$

$$v_e = \sqrt{\frac{2GM}{R}}$$

So
$$v'_e = \sqrt{\frac{2 \times G 4M}{R}} = 2v_e$$

76. The earth rotates about the sun as shown in the figure is an elliptical orbit. At which point the velocity will be maximum.



(a) At A (b) At B
(c) At C (d) At D

Ans :

(a) At A

The velocity of earth will be maximum when its distance is least from the sun (Kepler's second law) i.e., at point A.

77. If the earth suddenly shrinks to half of its present radius, the acceleration due to gravity will be

(a) $\frac{g}{2}$ (b) $4g$
(c) $\frac{g}{4}$ (d) $2g$

Ans :

(b) $4g$

Since,
$$g = \frac{GM}{R^2}$$

So, if R is made $2R$, g becomes $4g$.

78. If mass of a body is M on the earth surface, then the mass of the same body on the moon surface is

(a) $\frac{M}{6}$ (b) Zero
(c) M (d) None of these

Ans :

(c) M

Mass of a body is always constant and does not change from one place to another.

79. The acceleration on the surface of earth varies:

(a) directly with longitude
(b) directly with latitude
(c) inversely with longitude
(d) inversely with latitude

Ans :

(b) directly with latitude

Squaring gives

$$T^2 = \frac{4\pi^2}{GM_E} (R_E + h)^3,$$

Which is Kepler's third law.

THREE MARKS QUESTIONS

- 158.** A man can jump six times as high on moon as that on earth. Explain.

Ans :

We know that, when a body is thrown upward with initial velocity u and it attains height h ($v=0$) we can write

$$v^2 - u^2 = 2(-g)h$$

$$0 - u^2 = -2gh$$

$$h = \frac{u^2}{2g}$$

The value of g on moon $g' = \frac{g}{6}$, so height attained on moon will be

$$h' = \frac{u^2}{2g'} = \frac{6u^2}{2g} = 6h$$

and the whole universe is disturbed.

- 159.** Does acceleration due to gravity depend upon the mass of the planet? Explain.

Ans :

Yes, the acceleration due to gravity depends upon the mass of the planet. The relation between g and G is given by

$$g = \frac{Gm}{R^2}$$

where m is the mass of the planet and R is the radius of the planet

$$g \propto M$$

i.e. the acceleration due to gravity is proportional to the mass of the planet.

- 160.** (a) Define gravity.
(b) Explain that gravitation has universal character.

Ans :

- (a) Gravity is defined as the force of attraction between a body and a planet (earth, say).

The uniform acceleration produced in a freely falling body is called acceleration due to gravity, denoted by ' g '

- (b) Gravitational force of attraction has a universal character as the force between any two objects and lying anywhere in the universe, is the same.

- 161.** g_e and g_p denote the accelerations due to gravity on the surface of earth and another planet whose mass and radius are twice that of earth, then show that $g_p = \frac{g_e}{2}$.

Ans :

For earth, $g_e = \frac{GM}{R^2}$

For another planet,

$$g_p = \frac{G2M}{(2R)^2} = \frac{GM}{2R^2} = \frac{g_e}{2}$$

- 162.** Molecules in air in the atmosphere are attracted by gravitational force of the earth. Explain why all of them do not fall on to the earth just like an apple falling from a tree.

Ans :

No doubt the molecules in air in the atmosphere are attracted by gravitational force of earth but the thermal motion of molecule which is random do not all the velocities of the molecules in the vertical direction. Also due to the gravitational force, the density of air in the atmosphere is more close to the earth as compared to the density as we go up.

- 163.** Explain why an apple should fall down to meet the earth instead of earth moving upwards to meet the apple, when the gravitational pull is the same in both cases.

Ans :

The mass of the earth is very great as compared to the mass of the apple.

$$\text{Thus the acceleration} = \frac{\text{Force or weight}}{\text{mass}}$$

produced by the earth is too small to be detected. Therefore, the apple is attracted towards the centre of the earth. The distance through which the earth moves is very small.

- 164.** Lighter the body, smaller is the acceleration with which it falls towards the earth. Is this statement true? Explain.

Ans :

This statement is false, because the acceleration with which a body falls towards the earth is given by

TWO MARKS QUESTIONS

- 126.** An artificial satellite is revolving around the earth without using any fuel. On the other hand an aeroplane requires fuel to fly. Why ?

Ans :

Aeroplanes move in air and to overcome viscous drag they need fuel. But satellites moves in nearly viscous free conditions and need no fuel. It is the gravitational force that keeps them in orbit around the earth.

- 127.** What is apogee and perigee ?

Ans :

Apogee is the maximum distance of planet from the sun.

Perigee is the minimum distance of planet from the sun.

- 128.** The astronauts on landing on moon's surface found difficulty in moving about. Explain.

Ans :

This is because value of g on moon is about $\frac{1}{6}$ th that on earth. So, the weight of astronauts becomes $\frac{1}{6}$ th of their weights on earth. They found that they could not move about steadily. Their walking was made possible by lead shoes.

- 129.** What is the weight of a body at the centre of the earth ? Will your answer change if the earth were a hollow sphere ?

Ans :

The weight of a body at the centre of the earth

$$= m \times g_{\text{centre}}$$

$$= m \times 0 = \text{zero}$$

No, the weight of a body is zero in a hollow sphere as the gravitational force inside a hollow sphere is zero.

- 130.** What is gravitational potential ? What are its units and dimensions ?

Ans :

Gravitational potential at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.

Its unit is J kg^{-1} and dimensions $[\text{M}^0\text{L}^2\text{T}^{-2}]$.

- 131.** Give one example each of central force and non-central force.

Ans :

The gravitational force on a point mass and electrostatic force due to a point charge are central forces. The magnetic force between two current loops or the nuclear forces are non-central forces.

- 132.** Why gravitational force between two bodies is usually unnoticeable ?

Ans :

Since it is too small for small sized bodies, hence it is not noticeable. However, if masses of one of the bodies is large, it is noticeable. For example force between earth and body is noticeable.

- 133.** Mass of a body is 20 kg at the surface of earth. At what depth does its mass reduce to 5 kg ?

Ans :

As mass of a body is a constant quantity and does not change with the change of acceleration due to gravity, so it will never be 5 kg.

- 134.** Give some evidences in support of Newton's law of gravitation.

Ans :

There are many evidences which support Newton's law of gravitation like predictions about solar and lunar eclipses, tides in the ocean, rotation of earth around the sun etc.

- 135.** Why weight of body becomes zero at the centre of earth ?

Ans :

Since weight of a body is the force with which the body is attracted towards the centre of earth. Now at the centre of earth, the body is no longer attracted by any force, hence its weight becomes zero at the centre of earth.

- 136.** Define gravitational field and intensity of gravitational field. Give its SI unit.

Ans :

Gravitational field is the field around a material body in which its gravitational force of attraction can be measured.

Intensity of gravitational field (I) of a material body is the force experienced by a unit mass placed at the point.

Unit : In SI the unit of I is N kg^{-1} .

When two large lead balls are brought near the smaller ones the latter moves under the gravitational attraction. The wire is twisted by a small amount. By separate experiments using known forces the amount of twist is calibrated so that the force required to produce any given twist is known. Thus the gravitational force is known. By measuring the separation between the small and the large mass and substituting this in force equation along with the values of the small and large masses G the only unknown quantity, can be determined.

The value of G now accepted is $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

- 170.** (a) Define acceleration due to gravity. What is its value ?
 (b) Show that value of ' g ' is independent of mass of body.

Ans :

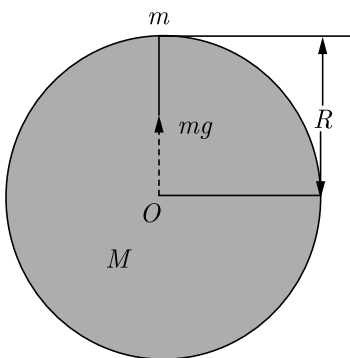
- (a) The uniform acceleration produced in a freely falling body due to the gravitational pull of the earth is known as the acceleration due to gravity. It is denoted by ' g '. Its value is 9.8 m s^{-2}
 (b) Let earth be a perfect sphere of radius R and mass M . Consider a body of mass m placed on its surface. If the mass of earth is supposed to be concentrated at the centre, the force exerted by the body, according to Newton's law of gravitation is

$$F = \frac{GMm}{R^2}$$

$$F = mg$$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$



Clearly, the value of ' g ' is independent of mass of body and depends upon the mass and radius of earth.

- 171.** Define orbital velocity. How is it related with escape velocity ?

Ans :

Orbital velocity of a satellite is the minimum velocity required to put the satellite into given orbit round the earth.

$$v_0 = \sqrt{\frac{gR^2}{R+h}}$$

where h is the height of the satellite above the earth. If $h \ll R$ i.e., h is less than 10 km, then

$$v_0 = \sqrt{\frac{gR^2}{R+0}} = \sqrt{gR} \quad \dots(1)$$

Escape velocity, $v_e = \sqrt{2gR}$

$$\frac{v_e}{v_0} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \sqrt{2} v_0$$

where $h \ll R$

- 172.** A satellite of mass ' m ' is revolving in a circular orbit of radius ' r ' around the earth of mass M . What is the total energy of the satellite ?

Ans :

Total energy of the satellite is the sum of kinetic energy and potential energy.

i.e., $E = \text{K.E.} + \text{P.E.}$

$$E = \frac{1}{2}mv_0^2 + \left(-\frac{GmM}{r}\right) \quad \dots(1)$$

Since orbital velocity of the satellite is given by

$$v_0 = \sqrt{\frac{GM}{r}}$$

$$v_0^2 = \frac{GM}{r}$$

Equation (1) becomes

$$E = \frac{1}{2} \frac{mGM}{r} - \frac{GmM}{r}$$

$$= \frac{GmM}{r} \left[\frac{1}{2} - 1 \right] = -\frac{GmM}{2r}$$

- 173.** What is geostationary satellite ? Name such Indian satellites.

Ans :

Geostationary or synchronous satellite is one which revolves around the earth with same angular speed in the same direction as done by earth around its axis.

We know $h = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{\frac{1}{3}} - R$

Here, $g = 9.8 \text{ ms}^{-2}$, $R = 6.38 \times 10^6 \text{ m}$

Centripetal force required to keep it in orbit,

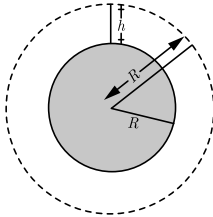
$$F = \frac{mv_0^2}{R}$$

Force of gravitational pull is given by

$$F = \frac{GMm}{(R+h)^2}$$

$$\frac{mv_0^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$v_0 = \sqrt{\frac{GM}{R+h}} \quad \dots(1)$$



Again,

$$g = \frac{GM}{R^2}$$

$$GM = gR^2$$

$$v_0 = \sqrt{\frac{gR^2}{R+h}}$$

$$v_0 = R\sqrt{\frac{g}{R+h}} \quad \dots(2)$$

We find that orbital velocity is independent of the mass of satellite and decreases with the increase of radius of orbit

If $h \ll R$, then

$$v_0 = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

which is nearly $\sqrt{2}$ times less than escape velocity.

Time period of satellite is the time taken by satellite to complete one revolution around the earth.

$$T = \frac{2\pi R_0}{v_0} = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T^2 = \frac{4\pi^2(R+h)^3}{GM}$$

This is called equation of satellite and is used to determine the mass of the earth/sun

$$\text{Since, } GM = gR^2 \quad \dots(3)$$

Special case

If the satellite is orbiting very close to the surface of earth i.e. $h \ll R$, so $R+h = R$

So equation (3) becomes

$$T = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}} \quad \dots(4)$$

Height of Satellite

Squaring both sides of equation (3), we get

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$(R+h)^3 = \frac{gR^2 T^2}{4\pi^2}$$

$$R+h = \left(\frac{gR^2 T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$h = \left(\frac{gR^2 T^2}{4\pi^2}\right)^{\frac{1}{3}} - R$$

Knowing the value of T , R and g , we can find the height h of the satellite above the surface of earth.

- 186.** What is a geostationary satellite ? Calculate radius of orbit and height of geostationary satellite.

Ans :

A satellite which revolves around the earth with the same angular speed and in the same direction as that of earth around its axis is called Geostationary satellite or Geosynchronous satellite.

The relative velocity of such a satellite with respect to earth is zero. Its time period is 24 hours.

Suppose a satellite of mass m revolves around with uniform orbital velocity v at a height h such that

$$R_0 = R + h$$

where

$$R = \text{radius of earth.}$$

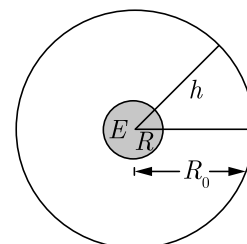
$$R_0 = \text{radius of orbit of satellite}$$

The centripetal force mv^2/R_0 on the satellite is provided by the force of attraction between the satellite and earth.

Let

$$T = \text{Time of revolution of planet}$$

$$\text{Orbital velocity, } v_0 = \frac{2\pi R_0}{T}$$



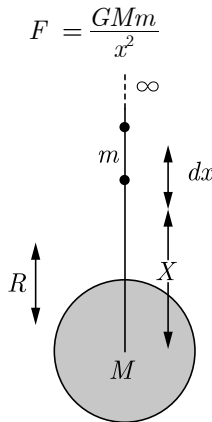
177. Show that velocity of escape is proportional to the square root of the product of acceleration due to gravity and the diameter of the earth.

or

Find the expression for escape velocity of earth.

Ans :

Let M be the mass of the earth and x be the distance of a body of mass m from its centre. The force of attraction between the two is



To displace it by dx , work done

$$dW = Fdx = \frac{GmM}{x^2} dx$$

In order that a body is to escape the work done on body to move it from the surface, of the earth i.e. at $x = R$ to $x = \infty$ is to be calculated.

$$W = \int_R^\infty \frac{GmM}{x^2} dx$$

$$W = GmM \int_R^\infty x^{-2} dx$$

$$= GmM \left[\frac{x^{-2+1}}{-2+1} \right]_R^\infty$$

$$W = \frac{GmM}{-1} \left[\frac{1}{x} \right]_R^\infty$$

$$W = \frac{GmM}{-1} \left[\frac{1}{\infty} - \frac{1}{R} \right] = \frac{GmM}{R}$$

To do this work, some kinetic energy is required. The minimum kinetic energy must be equal to $\frac{GmM}{R}$ i.e.

$$\frac{1}{2}mv_e^2 = \frac{GmM}{R}$$

$$v_e^2 = \frac{2GM}{R}$$

But $GM = gR^2$

$$v_e^2 = \frac{2gR^2}{R} = 2gR = g(2R)$$

Square of Escape velocity = g (Diameter of the earth)

$$v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= 11.2 \times 10^3 \text{ ms}^{-1} = 11.2 \text{ km s}^{-1}$$

178. Define gravitational field and gravitational intensity.

Ans :

Gravitational field : Gravitational field is defined as the space around a body such that due to its influence another mass particle when brought in this space experiences force of gravitational attraction.

Gravitational intensity at a point : Gravitational intensity at a point due to mass particle is defined as the force experienced by a unit mass placed at that point and acts radially inward to the mass particle.

Gravitational Intensity,

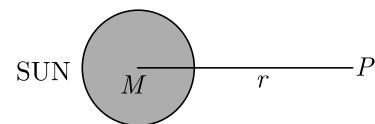
$$\vec{I} = \frac{\vec{F}}{m}$$

Expression for gravitational intensity : Let P be a point distant ' r ' from the mass particle of mass M due to which the gravitational intensity is to be determined at point P .

Consider a test mass m placed at point P at a remains constant and negative. If the total energy distance r According to Newton's law of gravitation,

$$F = \frac{GMm}{r^2}$$

$$\frac{F}{m} = \frac{GM}{r^2}$$



Gravitational intensity,

$$I = \frac{F}{m} = \frac{GM}{r^2} = \text{acceleration}$$

The intensity of gravitational field at a point is equal to free acceleration produced in the mass placed at that point.

On the surface of earth $r = R$. Thus gravitational intensity is numerically equal to acceleration due to gravity.

$$I = \frac{GM}{R^2} = g$$

It is a vector quantity and SI unit is m s^{-2} .

Substituting in above, we have

$$h = 3.6 \times 10^7 \text{ m} = 36000 \text{ km}$$

Geostationary satellite are used for the purpose of communication as they act as reflectors of suitable waves carrying messages.

Indian satellite in space are APPLE, INSAT-I series (A, B, C and D); INSAT-II series (A, B, C and D)

FIVE MARKS QUESTIONS

174. What is Newton's law of gravitation ? Define G . Give its units and dimensions.

Ans :

Newton's law of gravitation states that everybody in this universe attracts every other body with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distances between them.

If m_1, m_2 are the two masses distance r apart, then

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{G m_1 m_2}{r^2}$$

where G is a constant called universal gravitational constant.

If $m_1 = m_2 = 1, r = 1$, then $G = F$

Universal gravitational constant is the force of attraction between two unit masses placed at a unit distance apart.

Unit $G = \frac{F r^2}{m_1 m_2} = \frac{\text{Nm}^2}{\text{kg}^2} = \text{Nm}^2 \text{ kg}^{-2}$

Dimension, $G = \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{M}][\text{M}]} = [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$

175. Which is greater the attraction of earth for 1 kg of lead or attraction of 1 kg of lead for earth ? Calculate the acceleration in them due to these attractions.

Ans :

Let the distance between the centres of the earth and lead = x and mass of the earth = M

the force of attraction of earth for 1 kg of lead

$$= \frac{GM \times 1}{x^2} = \frac{GM}{x^2}$$

The force of attraction of 1 kg of lead for earth

$$= \frac{GM \times 1}{x^2} = \frac{GM}{x^2}$$

Hence, the forces of attraction are equal but act in the opposite directions.

Also acceleration produced in 1 kg of lead

$$= \frac{GM}{x^2} = \frac{GM}{x^2} \quad [F = ma]$$

Acceleration produced in the earth

$$= \frac{GM}{M} = \frac{G}{x^2}$$

176. Show that for same change in acceleration due to gravity, the depth to which a body is to be taken is twice the height ?

Ans :

We know that the either one goes above the surface of earth or below the surface of earth, the value of acceleration due to gravity goes on decreasing. This shows that its value is maximum on the surface of earth.

At height h , The value of g is given by

$$g_h = g \left(1 - \frac{2h}{R} \right) = g - \frac{2h}{R} g$$

$$g - g_h = \frac{2h}{R} g \quad \dots(1)$$

At depth d , the value of g is given by

Again $g_d = g \left(1 - \frac{d}{R} \right) g$

$$= g - \frac{d}{R} g$$

$$g - g_d = \frac{d}{R} g \quad \dots(2)$$

For same change in value of g

$$\frac{d}{R} g = \frac{2h}{R} g$$

$$d = 2h$$

For same change in value of g

$$\frac{d}{R} g = \frac{2h}{R} g$$

$$d = 2h$$

This shows that for same change in acceleration due to gravity, the depth to which a body is to be taken is twice the height.

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where G is called gravitational constant. It is called universal constant because its value is independent of place.

(b) Definition of G.F. $= G \frac{m_1 m_2}{r^2}$

If $m_1 = m_2 = 1$
 $r = 1$

Then, $G = F$

Hence, gravitational constant G is numerically equal to the gravitational force of attraction between two unit masses separated by a unit distance.

Unit of G

In S.I., $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

In cgs system,

$$G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$$

Dimensional formula of G

Since, $G = \frac{Fr^2}{m_1 m_2}$

$$\text{Dimensional formula of } G = \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{M}^2]}$$

$$= [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$$

- 182.** Derive Newton's law of gravitation in vector form. What conclusion do you draw from this law ?

Ans :

Consider two particles of masses m_1 and m_2 separated by a distance r as shown in figure.

Let $\vec{F}_{12}$ = Gravitational force exerted on mass

m_1 by mass m_2

$\vec{F}_{21}$ = Gravitational force exerted on mass

m_2 by mass m_1

$\hat{r}_{12}$ = Unit vector directed from m_1 to m_2

$\hat{r}_{21}$ = Unit vector directed from m_2 to m_1

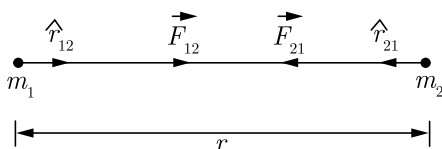
From Newton's law of gravitational, we have

$$\vec{F}_{12} = G \frac{m_1 m_2}{r^2} \hat{r}_{12} \quad \dots(1)$$

Similarly, $\vec{F}_{21} = G \frac{m_1 m_2}{r^2} \hat{r}_{21}$

$$\vec{F}_{21} = - G \frac{m_1 m_2}{r^2} \hat{r}_{12} \quad \dots(2)$$

$[\hat{r}_{12} = -\hat{r}_{21}]$



Adding equations (1) and (2), we get

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Conclusions

The gravitational force between two particles is equal in magnitude and opposite in direction.

Gravitational force obeys Newton's 3rd law.

Gravitational force is central force.

Gravitational force is conservation force.

- 183.** (a) Derive expression to find the mass of earth.
 (b) Derive expression to find the density of earth.

Ans :

- (a) **Mass of earth :** The force of gravity acting on mass m on the surface of the earth is

$$F = GM \frac{m}{R^2} \quad \dots(1)$$

where M is the mass of the earth. At the same time we also have

$$F = mg \quad \dots(2)$$

From equations (1) and (2), we get

$$mg = \frac{GMm}{R^2}$$

$$M = \frac{gR^2}{G}$$

$$g = 9.8 \text{ ms}^{-2},$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^{-2} \text{ kg}^{-2}$$

$$R = 6.38 \times 10^6 \text{ m}$$

Substituting the known values of the quantities on the right hand side, we get

$$M = \frac{(9.8) \times (6.38 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.91 \times 10^{24} \text{ kg}$$

- (b) Density of earth (ρ)

We know that,

$$g = \frac{GM}{R^2}$$

(where M = mass of earth, R = radius of earth)

$$M = \frac{gR^2}{G} \quad \dots(1)$$

Assuming earth to be a homogeneous sphere of radius R , its density ρ will be

$$\rho = \frac{\text{Mass of earth}}{\text{Volume of earth}}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3} \quad \dots(2)$$

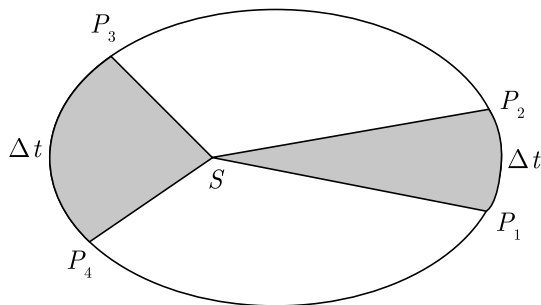
179. State and explain Kepler's laws of planetary motion

Ans :

Kepler's laws of planetary motion are as follows :

Kepler's First Law (Law of orbits) : All planets revolve around sun in elliptic orbits with sun at one of foci.

Kepler's Second Law (Law of area) : The line joining the sun to the planet sweeps out equal areas in equal times.



Explanation : Area $S P_1 P_2$ swept in time Δt = area $S P_3 P_4$ swept in same time Δt

It is clear from the figure that planet has to cover a greater distance $P_3 P_4$ in Δt when it is near the sun then $P_1 P_2$ when it is farther away in the same time. This means that planet travels faster near the sun as compared to when it is at a large distance.

Kepler's Third Law (Law of periods) : The square of time of revolution of a planet is proportional to cube of mean distance between the sun and the planet.

Explanation : Let us assume that planet revolves around the sun in a circle of radius R .

If m = mass of planet.

v = velocity of planet.

M_s = mass of sun.

The gravitational pull provides the centripetal force

$$\frac{GM_s m}{R^2} = \frac{mv^2}{R}$$

$$\frac{GM_s}{R} = v^2$$

Conclusions

1. The gravitational force between two particles is equal in magnitude and opposite in direction.
2. Gravitational force obeys Newton's 3rd law.
3. Gravitational force is central force.
4. Gravitational force is conservative force.

180. Derive an expression for energy of an orbiting satellite.

Ans :

The velocity of a satellite of mass m rotating at a height h from the surface of earth having mass M and radius R is given as

$$v = \sqrt{\frac{GM}{R+h}}$$

Kinetic energy of the satellite

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{GMm}{2(R+h)} \quad \dots(1)$$

The potential energy at a distance $(R+h)$ from the centre of earth is given by

$$\text{Potential energy} = -\frac{GMm}{(R+h)} \quad \dots(2)$$

Total energy of the orbiting satellite will be

$$\begin{aligned} E &= \text{K.E.} + \text{P.E.} \\ &= \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)} \\ &= -\frac{GMm}{2(R+h)} \end{aligned}$$

Thus total energy of a rotating satellite is negative (magnitude of kinetic energy is half the potential energy).

If the satellite is orbiting in elliptical path, both are kinetic energy and potential energy vary from point to point but total energy remain constant and negative. If the total energy becomes positive or zero, the object escapes to infinity.

181. State Universal Law of gravitation and hence define Gravitational constant and dimensional formula of G .

Ans :

It states that everyday in this universe attracts every other body with a force which is directly proportional to product of their masses and inversely proportional to square of distance between their centres.

Consider two particular A and B having masses m_1 and m_2 separated by a distance ' r '

According to Newton's law of gravitation, the force of attraction between the masses

$$(a) \quad F \propto m_1 m_2 \quad \dots(1)$$

$$\text{and} \quad F \propto \frac{1}{r^2} \quad \dots(2)$$

Combining (1) and (2), we get

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$

from the surface of earth will be

$$g_d = \frac{4}{3}\pi G(R-d)\rho \quad \dots(3)$$

Dividing eq.(3) by (2), we get

$$\begin{aligned} \frac{g_d}{g} &= \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi GR\rho} \\ \frac{g_d}{g} &= \frac{R-d}{R} \\ g_d &= g\left(1 - \frac{d}{R}\right) \end{aligned}$$

This shows that as depth below the surface of earth goes on increasing the value of acceleration due to gravity goes on decreasing.

At the centre of earth,

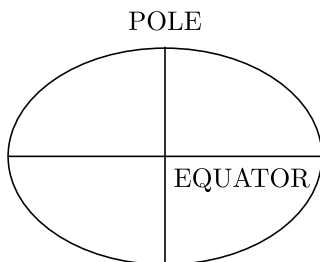
$$g_{\text{centre}} = g\left(1 - \frac{R}{R}\right) = 0$$

Value of acceleration due to gravity at the centre of earth is zero i.e. a body feels weightlessness at the centre of the earth.

(iii) Variation of 'g' due to shape of earth :

The value of acceleration due to gravity on the surface of earth is

$$g = \frac{GM}{R^2}$$



Because G and M are constant, therefore

$$g \propto \frac{1}{R^2}$$

Again the polar radius is less than the equatorial radius of earth, therefore, the value of acceleration due to gravity is more at poles than at equator.

$$g_{\text{pole}} > g_{\text{equator}}$$

(iv) Effect of rotation of earth :

The latitude at a place on the earth is defined as the angle which the line joining the place to the centre of the earth makes with the equatorial plane.

Let a body of mass m be lying at point P at latitude λ on the surface of earth. If the earth is at rest, the weight of the body mg will act along PO , but the earth is rotating about an axis N-S with angular velocity ω .

Due to rotation of earth, the body at P is rotating in a circle of radius $PC = r$ ($r = R \cos \lambda$) and will experience a centrifugal force in outward direction along PD as shown in figure. This force can be resolved into two components.

(i) $mr\omega^2 \cos \lambda$ along PM

(ii) $mr\omega^2 \sin \lambda$ along PN

From the figure, we find that $mr\omega^2 \cos \lambda$ component is opposite to the weight mg_λ of the body.

Resultant weight mg_λ of the body will be

$$mg_\lambda = mg - mr\omega^2 \cos \lambda$$

$$g_\lambda = g - r\omega^2 \cos \lambda$$

Since,

$$r = R \cos \lambda$$

$$g_\lambda = g - R\omega^2 \cos^2 \lambda \quad \dots(1)$$

At the equator $\lambda = 0^\circ$

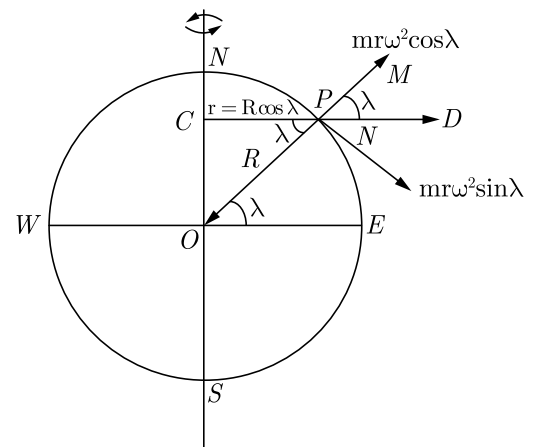
$$g_{\text{pole}} = g \quad \dots(2)$$

$$[\cos 0^\circ = 1]$$

At the pole $\lambda = 90^\circ$

$$g_{\text{pole}} = g \quad \dots(3)$$

$$[\cos 90^\circ = 0]$$



Putting equations (2) in (3), we get

$$g_{\text{equator}} = g_{\text{pole}} - R\omega^2$$

$$\text{i.e.,} \quad g_{\text{equator}} < g_{\text{pole}}$$

i.e. value of g decreases with decrease in latitude.

185. What is meant by orbital velocity ? Determine its value. Derive its time period and height.

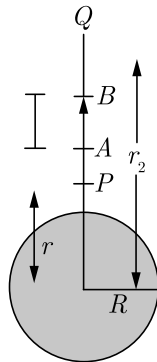
Ans :

Orbital velocity of a satellite is the velocity required to put the satellite into given orbit around the earth. Let its height be h above earth.

$$F = \frac{GMm}{r^2}$$

Small amount of work done in moving through infinitely small displacement dr is given by

$$\begin{aligned} W &= \int_{r_1}^{r_2} \frac{GMm}{r^2} dr \\ &= GMm \int_{r_1}^{r_2} r^{-2} dr \end{aligned}$$



$$\begin{aligned} W &= -GMm \left[\frac{1}{r_2} - \frac{1}{r_1} \right] \\ &= GMm \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \end{aligned}$$

If earth is zero level to potential energy and a body is taken to the surface of earth to distance r_1 from earth's centre, therefore, gain in potential energy is

$$= GMm \left[\frac{1}{R} - \frac{1}{r} \right] \quad \dots(1)$$

$$(P.E.)_{at(r=r_1)} = GMm \left[\frac{1}{R} - \frac{1}{r_1} \right] \quad \dots(2)$$

Similarly

$$(P.E.)_{at(r=r_2)} = GMm \left[\frac{1}{R} - \frac{1}{r_1} \right]$$

$$\begin{aligned} (P.E.)_{at(r=r_1)} &= -(P.E.)_{r=r_2} \\ &= -GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \end{aligned}$$

Zero level potential energy is arbitrary.

$$r_1 = r \text{ and } r_2 = \infty \quad \dots(1)$$

$$\text{Potential energy} = \frac{GMm}{r} \quad \dots(2)$$

This is true for any mass present at any distance potential energy, near the surface of the earth. Suppose the body is taken from surface of earth to height ' h ' above surface where value of ' g ' is constant such that $r_1 = R$ and $r_2 = R + h$, where R = radius of the earth.

Gravitation potential energy

$$\begin{aligned} &= \left(\frac{1}{R} - \frac{1}{R+h} \right) \\ &= GMm \left(\frac{R+h-R}{R(R+h)} \right) \\ &= GMm \left(\frac{h}{R(R+h)} \right) \end{aligned}$$

R = Radius of earth

M = Mass of earth

If $h \ll R$, then neglecting h in denominator

$$\text{Potential energy} = GMm \frac{h}{R^2} = \frac{GM}{R^2} mh$$

$$\text{Since, } g = \frac{GM}{R^2}$$

$$\text{Potential energy} = mgh$$

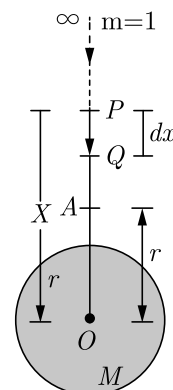
189. What do you understand by gravitational potential at a point ? Derive expression for it. How gravitational potential on the surface of earth is obtained from the above expression ? Give its units and dimensional formula.

Ans :

Gravitational potential at a point due to mass M is defined as the amount of work done in bringing a unit mass from ∞ to that point in the gravitational field without acceleration.

Let M = mass of a material body.

Let us find gravitational potential V at point A . At a distance r from the centre O of the material body as shown in figure.



Let us move a unit mass from ∞ towards point A . Gravitational force on the body at point P is given by

$$F = \frac{G.I.M.}{x^2} = \frac{GM}{x^2}$$

The centripetal acceleration,

$$a_c = \frac{v^2}{R_0} = \frac{4\pi^2 R_0^2}{T^2 R_0} = \frac{4\pi^2 R_0}{T^2} \dots (1)$$

Again according to Newton's law of gravitation

$$F = \frac{GMm}{R_0^2}$$

$$\text{Acceleration } a = \frac{F}{m} = \frac{GM}{R_0^2} \dots (2)$$

Further assuming that earth is a sphere

$$R_0 \left[\frac{(86400)^2 \times 9.8 \times (6.38 \times 10^6)^2}{4\pi^2} \right]^{\frac{1}{3}}$$

$$R_0 = 42,250 \text{ km}$$

Since,

$$R_0 = R + h$$

$$h = R_0 - R$$

$$= 42250 - 6380 = 35870 \text{ km}$$

If a satellite is placed in orbit at this height above the equator, it will appear to be stationary. It is also clear from equation (4) that

$$T^2 \propto R_0^3$$

i.e., square of time of revolution is proportional to cube of radius of orbit of satellite.

- 187.** Define inertial mass and gravitational mass. Show their equivalence.

Ans :

Inertial mass is defined as the ratio of force applied on a body placed on a smooth horizontal surface to the acceleration produced in it.

Suppose a force of magnitude F produces an acceleration of magnitude a in the body. If m is the mass of body then, according to Newton's second law of motion,

$$F = ma$$

$$m = \frac{F}{a}$$

This inertial mass of a body is a measure of inertia of the body. Greater the inertial mass of a body, greater the force required to produce a given acceleration. Gravity has no effect on the inertial mass of a body.

Gravitational mass is determined by measuring the gravitation force acting on it due to earth. The mass measured in this way is known as gravitational mass.

Suppose the earth attracts a body of mass m with a force F

According to law of gravitation,

$$F = \frac{GMm}{R^2}$$

where M = mass of earth and R = Radius of earth.

$$m = \frac{FR^2}{GM}$$

Equivalence of inertial mass and Gravitational mass-

Suppose a body of mass m_g called gravitational mass be lying on the surface of earth of mass M and radius R .

According to Newton's law of gravitation,

$$F = \frac{GMm_g}{R^2} \dots (1)$$

when the same body is allowed to fall freely, then by Newton's 2nd law of motion

$$F = m_i g$$

where m_i = inertial mass and g is according due to gravity

$$m_i g = \frac{GMm_g}{R^2}$$

$$\frac{m_i}{m_g} = \frac{GM}{R^2 g}$$

Since all quantities on right hand side are constants, this means that $\frac{m_i}{m_g} = \text{constant}$

i.e.,

$$m_i = \text{constant} \times m_g$$

$$m_i \propto m_g$$

$$\frac{m_i}{m_g} = k$$

If

$$k = 1, \text{ then } m_i = m_g \text{ Thus}$$

Inertial mass = Gravitational mass.

- 188.** (a) Define gravitational potential energy.
 (b) Find the expression for gravitational potential energy at any point.
 Deduce the gravitational potential energy near the surface of earth.
 (c) Deduce an expression for potential at a point due to gravitational field.

Ans :

- (a) Gravitational potential energy of a body in the gravitational field is defined as the amount of work done in bringing the body from infinity to that point.
 (b) Consider a body of mass m lying in the gravitational of earth at point A at a distance from the centre of earth.
 Gravitational force on the body at point P is

Putting eq. (1) in (2), we get

$$\rho = \frac{3gR^2}{4\pi R^3 G} = \frac{3g}{4\pi RG}$$

$$\rho = \frac{3 \times 9.8}{4 \times 3.142 \times 6.4 \times 10^6 \times 6.67 \times 10^{-11}}$$

$$\rho = 5.47 \times 10^3 \text{ kg m}^{-3}$$

- 184.** How does value of acceleration due to gravity vary with : (i) altitude (ii) depth (iii) shape of earth (iv) rotation of earth ?

Ans :

(i) Variation of 'g' with altitude

Let

m = mass of a body lying on the surface of earth.

M = Mass of earth

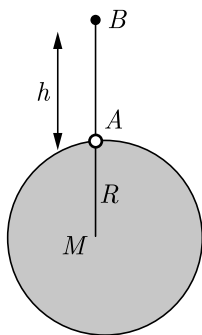
According to Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

But $F = mg$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} \quad \dots(1)$$



The body is taken to altitude h from point A to point B above the surface of earth. If g_h = value of acceleration due to gravity at altitude h .

$$g_h = \frac{GM}{(R+h)^2} \quad \dots(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2(1+\frac{h}{R})^2}$$

$$\frac{g_h}{g} = \left(1 + \frac{h}{R}\right)^{-2}$$

If $h \ll R$ or $\frac{h}{R} \ll 1$, then applying binomial theorem,

$$[(1+x)^n = 1 + nx \text{ provided } x \ll 1]$$

$$\frac{g_h}{g} = \left(1 - \frac{2h}{R}\right)$$

$$\frac{g_h}{g} = g\left(1 - \frac{2h}{R}\right)$$

As he goes on increasing, the value of acceleration due to gravity goes on decreasing.

(ii) Variation of 'g' with depth

Let

m = mass of body lying on the surface of earth.

M = mass of earth.

ρ = density of material of earth.

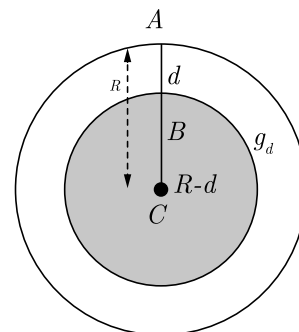
Earth is considered to be a solid sphere of homogeneous medium.

$$\text{Volume of earth} = \frac{4}{3}\pi R^3$$

Mass of the earth,

$$M = \text{Volume} \times \text{density}$$

$$= \frac{4}{3}\pi R^3 \times \rho \quad \dots(1)$$



Acceleration due to gravity on the surface of earth is given by

$$g = \frac{GM}{R^2}$$

Using equation (1), we get

$$g = \frac{4}{3}\pi \frac{R^3 \rho}{R^2}$$

$$g = \frac{4}{3}\pi GR\rho \quad \dots(2)$$

Suppose the body is taken to depth ' d ' below the surface of earth, so that now it lies on a sphere of radius $(R-d)$. Hence mass of earth of sphere $(R-d)$ is only effective for gravitational pull.

Acceleration due to gravity at point B at depth d

Small work done in moving through small displacement dx is given by

$$dW = Fdx = \frac{GM}{x^2} dx$$

Total amount of work done in moving the unit mass from ∞ to point A will be

$$\begin{aligned} W &= \int dW = \int_{\infty}^r \frac{GM}{x^2} dx \\ &= GM \int_{\infty}^r \frac{1}{x^2} dx = GM \left[-\frac{1}{x} \right]_{\infty}^r \\ &= -GM \left[\frac{1}{r} - \frac{1}{\infty} \right] = -\frac{GM}{r} \end{aligned}$$

This work done will be stored in the form of gravitational potential V .

$$V = -\frac{GM}{r} \quad \dots(1)$$

Gravitational potential on the surface of earth On the surface of earth, $r = R$

So, equation (1) becomes

$$V = -\frac{GM}{R}$$

Negative sign means that work is done by the gravitational field of earth.

Units : In SI. the unit. of gravitational potential is J kg^{-1} .

Dimensional formula of gravitational potential.

$$\begin{aligned} V &= \frac{\text{Work done}}{\text{Mass}} \\ &= \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{M}]} = [\text{L}^2\text{T}^{-2}] \end{aligned}$$

NUMERICAL QUESTIONS

- 190.** What is the force between two spheres weighing 40 kg each and placed 5 m apart ?

Ans :

Given, $m_1 = m_2 = 40 \text{ kg}$

$$r = 5 \text{ m}$$

As we know that,

$$\begin{aligned} F &= G \frac{m_1 m_2}{r^2} \\ &= 6.67 \times 10^{-11} \times \frac{40 \times 40}{5 \times 5} \\ &= 4.27 \times 10^{-9} \text{ N} \end{aligned}$$

- 191.** Calculate the value of acceleration due to gravity on the surface of moon. Given mass of moon $7.4 \times 10^{22} \text{ kg}$ and its radius 1740 km.

Ans :

Given,

Mass of moon, $m = 7.4 \times 10^{22} \text{ kg}$

Radius of moon, $r = 1740 \text{ km}$
 $= 1740 \times 10^3 \text{ m}$

$$\begin{aligned} \text{Since,} \quad g &= \frac{Gm}{r^2} \\ g &= \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1740 \times 1740 \times 10^6} \\ &= 1.64 \text{ ms}^{-2} \end{aligned}$$

- 192.** If the earth stops to revolve about its own axis then how much acceleration will change at equator ?

Ans :

If the earth stops to revolve, about its own axis, the acceleration will increase by $R\omega^2$

$$\begin{aligned} \Delta g &= R\omega^2 \\ &= 6.4 \times 10^6 \times \left(\frac{2\pi}{86400} \right)^2 \\ &= \frac{6.4 \times \pi^2 \times 10^6}{86400 \times 86400} \\ &= 3.36 \times 10^{-2} \text{ ms}^{-2} \end{aligned}$$

- 193.** At what height acceleration due to gravity is three-fourths of that at the surface of earth ?

Ans :

Let at height h the acceleration due to gravity is three-fourths of that at the surface of earth.

$$\text{i.e.,} \quad g_h = \frac{3}{4}g \quad \dots(1)$$

$$\text{Since,} \quad g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

So equation (1) becomes

$$\begin{aligned} \frac{g}{\left(1 + \frac{h}{R}\right)^2} &= \frac{3}{4}g \\ \left(1 + \frac{h}{R}\right)^2 &= \frac{4}{3} \\ \frac{h}{R} &= \frac{2}{\sqrt{3}} - 1 \\ h &= \left(\frac{2}{\sqrt{3}} - 1 \right) R \end{aligned}$$

Ans :

1. Kepler's second law (Law of areas) states: The line that joins any planet (or comet) to the sun sweeps equal areas in equal intervals of time.
2. No, the linear speed of the comet is not constant. According to Kepler's second law, to sweep equal areas in equal times, the comet must move faster when it is closer to the Sun and slower when it is farther from the Sun.
3. The law of areas can be understood as a consequence of conservation of angular momentum, which is valid for any central force.
4. No, the comet will take less time to traverse a path segment near the perihelion compared to a path segment of the same length near the aphelion because its speed is greater near perihelion. (The example 7.1 states it will take longer to traverse BAC than CPB, implying different times for different sections related to areas).

206. The planet Mars has two moons, Phobos and Deimos. Phobos has an orbital period of 7 hours, 39 minutes (which is $459 \cdot 60$ s) and an orbital radius of $9.4 \cdot 10^3$ km. Scientists use this data to estimate the mass of Mars.

Questions:

1. State Kepler's third law of periods.
2. The formula

$$T^2 = k(R_E + h)^3$$

$$\text{(or } T^2 = (\frac{4\pi^2}{GM_s})R^3\text{)}$$

relates the period (T) and orbital radius (R) of a satellite around a central body of mass M. How can this be used to find the mass of the central body?

3. Using the data for Phobos, set up the equation to calculate the mass of Mars (M_m).
4. If another moon of Mars had a larger orbital radius than Phobos, would its orbital period be longer or shorter, assuming a circular orbit?

Ans :

1. Kepler's third law (Law of periods) states: The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.
2. By rearranging Kepler's third law in the form

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

if T, R, and the universal gravitational constant G are known, the mass M of the central body can be calculated as

$$M = \frac{4\pi^2 R^3}{GT^2}$$

$$3. \quad \text{Using } M_m = \frac{4\pi^2 R^3}{GT^2},$$

where R is Phobos's orbital radius ($9.4 \cdot 10^6$ m) and T is its period ($459 \cdot 60$ s). So,

$$M_m = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{G(459 \times 60 \text{ s})^2}$$

4. According to Kepler's third law ($T^2 \propto R^3$), if another moon had a larger orbital radius, its orbital period would be longer.

207. A 400 kg satellite is initially in a circular orbit of radius $2R_E$ about the Earth (where R_E is the radius of the Earth). Ground control decides to move it to a higher circular orbit of radius $4R_E$. This maneuver requires energy.

Questions:

1. What is the formula for the total energy (E) of a satellite of mass 'm' in a circular orbit of radius 'a' around a central body of mass 'M'? Is this energy positive or negative?
2. How is the kinetic energy of the satellite related to its potential energy in a circular orbit?
3. Calculate the initial total energy (E_i) of the satellite in the orbit of radius $2R_E$.
4. How much energy is required to transfer the satellite from the orbit of radius $2R_E$ to the orbit of radius $4R_E$? Does the kinetic energy of the satellite increase or decrease in this transfer?

Ans :

1. The total energy E of a satellite in a circular orbit of radius 'a' is $E = -\frac{GMm}{2a}$.

This total energy is negative for any bound system.

2. The kinetic energy is positive, the potential energy is negative, and in magnitude, the kinetic energy is half the potential energy

$$K.E. = \frac{GmM_E}{2a}$$

3. Initially, radius $a_i = 2R_E$. So,

$$E_i = -\frac{GM_E m}{2(2R_E)} = -\frac{GM_E m}{4R_E}$$

4. Finally, radius $a_f = 4R_E$. So,

$$E_f = -\frac{GM_E m}{2(4R_E)} = -\frac{GM_E m}{8R_E}$$

Energy required

$$\begin{aligned} \Delta E &= E_f - E_i \\ &= -\frac{GM_E m}{8R_E} - \left(-\frac{GM_E m}{4R_E}\right) \\ &= \frac{GM_E m}{8R_E} \end{aligned}$$

The kinetic energy

$$K = \frac{GM_E m}{2a}$$

The satellite will appear stationary in the sky if its period of revolution round the earth is equal to period of revolution of the earth round its own axis (24 hours). Let us find the height of the satellite with this time period. Now,

From Eq (i), we have

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$\frac{T^2 g R^2}{4\pi^2} = (R+h)^3$$

$$(R+h) = \left[\frac{T^2 g R^2}{4\pi^2} \right]^{\frac{1}{3}}$$

Hence, $h = \left[\frac{T^2 g R^2}{4\pi^2} \right]^{\frac{1}{3}} - R$

So,
$$h = \left[\frac{(24 \times 3600 \text{ sec})^2 \times (9.8) \times (6400 \times 10^3 \text{ m})^2}{4 \times (3.14)^2} \right]^{\frac{1}{3}} - (6400 \times 10^3 \text{ m})$$

$$= 4.23 \times 10^7 \text{ metre} - 0.64 \times 10^7 \text{ m}$$

$$= 3.59 \times 10^7 \text{ metre}$$

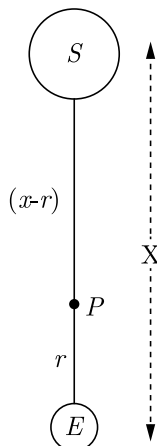
$$= 3.59 \times 10^4 \text{ km.}$$

- 197.** A rocket is fired from earth towards sun. At what point on its path is the gravitational force on the rocket zero? Mass of sun = 2×10^{30} kg. Mass of earth = 6×10^{24} kg. Neglect the effect of other planets etc. (Orbital radius = 1.5×10^{11} m).

Ans :

Let P be a point at a distance, r from earth where gravitational force due to sun and earth is equal and give rise to zero gravitational force. Distance between sun and earth = x .

This is the radius of orbit of earth.



Hence,
$$\frac{GM_s m}{(x-r)^2} = \frac{GM_e m}{r^2}$$

where M_s is mass of the sun, M_e is the mass of earth and m , is mass of rocket.

$$\frac{2 \times 10^{30}}{(x-r)^2} = \frac{6 \times 10^{24}}{r^2}$$

$$(x-r) = \frac{10^3 r}{\sqrt{3}}$$

$$x\sqrt{3} = (10^3 + \sqrt{3})r$$

$$r = \frac{\sqrt{3} \times 1.5 \times 10^{11}}{10^3 + \sqrt{3}}$$

$$= \frac{1.732 \times 1.5 \times 10^{11}}{1001.732}$$

$$r = 2.5 \times 10^2 \text{ m from earth.}$$

- 198.** If the earth be one-half its present distance from the sun, how many days will be in one year?

Ans :

Let the radius of the earth's orbit round the sun be R . When it is reduced to half, the new distance becomes $\frac{R}{2}$. As the earth is moving round the sun, centripetal force is provided by the gravitational attraction of the sun on the earth. Hence,

$$\frac{mv^2}{R} = \frac{GM_m}{R^2} \quad \dots(1)$$

Where the symbols have their usual meanings.

Hence,
$$v^2 = \frac{Gm}{R} \quad \dots(2)$$

From equations (1) and (2), we have

$$\frac{4\pi^2 R^2}{T^2} = \frac{GM}{R} \text{ or } T^2 = \frac{4\pi^2 R^3}{GM} \quad \dots(3)$$

Let T_1 be the time period of revolution of earth round the sun when the distance between the two is $\frac{R}{2}$. Hence,

Dividing equation (4) by equation (3), we have

$$T_1^2 = \frac{T^2}{8} \text{ or } T_1 = \frac{T}{2\sqrt{2}}$$

Hence,
$$T_1 = \frac{365}{2\sqrt{2}}$$

(Hence, 1 year = 365 days)

$$T_1 = 129 \text{ days}$$

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By measuring M , m , d , L , θ , and independently determining τ_{twist} (restoring couple per unit angle, denoted as τ), G can be calculated.

- 211.** Consider two planets, Planet X with mass M_X and Planet Y with mass M_Y . The distance between their centers is ' r '. Gravitational potential energy is defined to be zero when the distance between objects is infinite.

Questions:

1. What is the gravitational potential energy V associated with two particles of masses m_1 and m_2 separated by a distance r , if V is taken to be zero at infinite separation?
2. Is gravitational force a conservative or non-conservative force? What does this imply about the work done by it?
3. If Planet X has a mass of 6×10^{24} kg (like Earth) and Planet Y has a mass of 2×10^{30} kg (like the Sun), and they are 1.5×10^{11} m apart, calculate their mutual gravitational potential energy. (Use $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$).
4. If these two planets were to move further apart, would their mutual gravitational potential energy increase (become less negative) or decrease (become more negative)?

Ans :

1. The gravitational potential energy is given by

$$V = -\frac{Gm_1m_2}{r}$$

2. The force of gravity is a conservative force. This implies that the work done by it is independent of the path taken and depends only on the initial and final positions.

$$3. \quad V = -\frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6 \times 10^{24} \text{ kg})(2 \times 10^{30} \text{ kg})}{1.5 \times 10^{11} \text{ m}}$$

$$V = -\frac{6.67 \times 12 \times 10^{43}}{1.5 \times 10^{11}} \text{ J}$$

$$= -\frac{80.04}{1.5} \times 10^{32} \text{ J}$$

$$\approx -5.336 \times 10^{33} \text{ J.}$$

4. If the planets move further apart, ' r ' increases. Since V is $-\frac{Gm_1m_2}{r}$,

as ' r ' increases, the magnitude of V decreases, meaning V becomes less negative (increases towards zero). So, their mutual gravitational potential energy would increase.

- 212.** Consider a hollow spherical shell of uniform density. A point mass ' m ' is placed at two locations: first, outside the shell, and second, inside the shell.

Questions:

1. How does a hollow spherical shell of uniform density attract a point mass situated outside it?
2. Recall: What is the force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside it?
3. If the mass of the shell is M and its radius is R , and a point mass m is at a distance $r > R$ from its centre, what is the magnitude of the gravitational force on m ?
4. If an astronaut is inside a large, hollow, spherical spacecraft far from other celestial bodies, would she experience a gravitational pull from the spacecraft's shell itself? Explain.

Ans :

1. The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.
2. The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.
3. The magnitude of the gravitational force on m would be

$$F = G\frac{Mm}{r^2}$$

4. No, the astronaut would experience zero net gravitational pull from the spacecraft's shell itself because she is inside a hollow spherical shell (assuming uniform density).

- 213.** An artificial Earth satellite is in a stable circular orbit at a distance $(R_E + h)$ from the centre of the Earth, where R_E is the radius of the Earth and ' h ' is its altitude above the surface. The mass of the satellite is ' m ' and the mass of the Earth is M_E .

Questions:

1. What force provides the necessary centripetal force for the satellite to maintain its orbit?
2. How does the orbital speed V of the satellite depend on the altitude ' h '? Does it increase or decrease as ' h ' increases?
3. Derive an expression for the orbital speed V of the satellite in terms of G , M_E , R_E , and h .
4. Using Kepler's third law for satellites ($T^2 = k(R_E + h)^3$), explain how the period T changes if the altitude ' h ' of a circular orbit is increased.

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- 203.** A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg. Mass of the Sun = 2×10^{30} kg, Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km, radius of the orbit of Mars = 2.28×10^8 km; $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

Ans :

As we know that,

$$\text{Total energy} = [\text{P.E.}]_{\text{Mars}} + [\text{P.E.}]_{\text{Sun}}$$

$$\begin{aligned} U &= -\frac{GM_m m}{R_p} - \frac{GM_s m}{R_s} \\ &= -GM \left[\frac{M_m}{R_p} + \frac{M_s}{R_s} \right] \\ &= -6.67 \times 10^{-8} \left[\frac{6.4 \times 10^{23} \times 1000}{3.395 \times 10^6} + \frac{2 \times 10^{30} \times 1000}{2.28 \times 10^{11}} \right] \end{aligned}$$

$$U = -5.98 \times 10^{11} \text{ J}$$

So energy required to rocket the spaceship out of the solar system

$$\begin{aligned} &= 0 - (-5.98 \times 10^{11}) \\ &= 5.98 \times 10^{11} \text{ J} \end{aligned}$$

- 204.** A rocket is fired 'vertically' from the surface of Mars with a speed of 2 km s^{-1} . If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it? Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg m}^{-2}$.

Ans :

As we know that,

$$\text{Total energy} = \text{K.E.} + \text{P.E.}$$

$$g = \frac{GM}{R^2} \frac{1}{2} mv^2 - \frac{GMm}{R}$$

20 % initial energy is lost, only 80 % remains to reach the height

$$\text{Energy remained, } E = \left[\frac{1}{2} mv^2 \times \frac{80}{100} \right]$$

So total initial energy available

$$\begin{aligned} &= \frac{80}{100} \times \frac{1}{2} mv^2 - \frac{GMm}{R} \\ &= 0.4mv^2 - \frac{GMm}{R} \end{aligned}$$

At height h , the potential energy of the rocket

$$= -\frac{GmM}{R+h}$$

From law of conservation of energy

$$0.4mv^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}$$

$$0.4v^2 - \frac{GM}{R} = -\frac{GM}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - 0.4v^2$$

$$\frac{GM}{R+h} = \frac{1}{R} [GM - 0.4Rv^2]$$

$$\frac{GM}{Rm - 0.4Rv^2} = \frac{R+h}{R} = 1 + \frac{h}{R}$$

$$\frac{h}{R} = \frac{GM}{Gm - 0.4Rv^2} - 1$$

$$\frac{h}{R} = \frac{GM - GM + 0.4Rv^2}{Gm - 0.4Rv^2}$$

$$h = \frac{0.4R^2 v^2}{Gm - 0.4Rv^2}$$

$$= \frac{0.4 \times (2 \times 10^3)^2 \times (3395 \times 10^3)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times 3395 \times 10^3 \times (2 \times 10^3)^2}$$

$$= \frac{18.4416}{37.254} \times 10^3$$

$$= 495 \times 10^3 \text{ m}$$

$$h = 495 \text{ km.}$$

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CASE BASED QUESTIONS

- 205.** A comet orbits the Sun in a highly elliptical orbit. Its distance from the Sun varies greatly during its orbit; sometimes it is very close to the Sun (perihelion), and other times it is very far (aphelion). The gravitational force exerted by the Sun on the comet is always directed towards the Sun.

Questions:

1. State Kepler's second law, also known as the law of areas.
2. Is the linear speed of the comet constant throughout its orbit? Explain why or why not, based on Kepler's laws or conservation principles.
3. The law of areas is a consequence of which conservation principle valid for any central force?
4. Will the comet take equal times to traverse a path segment near the perihelion compared to a path segment of the same length near the aphelion? Explain.

- 199.** An object of mass m is raised from the surface of the earth to a height equal to the radius of the earth, that is, taken from a distance R to $2R$ from the centre of the earth. What is the gain in its potential energy ?

Ans :

Let M = mass of earth.

So Gain in Potential energy $= -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Here $r_1 = 2R$ and $r_2 = R$

So, Gain in Potential energy

$$\begin{aligned} &= -GMm\left(\frac{1}{2R} - \frac{1}{R}\right) \\ &= -GMm\left(\frac{R-2R}{2R^2}\right) \\ &= -GMm\left[\frac{-R}{2R^2}\right] \end{aligned}$$

Gain in Potential energy

$$= \frac{GMm}{2R} \quad \dots(1)$$

Since $\frac{GM}{R^2} = g$

$$\frac{GM}{R} = gR$$

So, Equation (1) becomes

Gain in Potential energy

$$= \frac{gRm}{2} = \frac{mgR}{2}$$

- 200.** Find the expression for mass of earth.

Ans :

Since, $g = \frac{GM}{R^2}$

$$M = \frac{gR^2}{G}$$

For earth, $g = 9.8 \text{ m s}^{-2}$; $R = 6.38 \times 10^6 \text{ m}$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

So $M = \frac{9.8(6.38 \times 10^6)^2}{6.67 \times 10^{-11}}$

$$= 5.99 \times 10^{24} \text{ kg}$$

$$= 6 \times 10^{24} \text{ kg}$$

- 201.** A geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the Earth. What is the potential due to earth's gravity at the site of this satellite ? (Take the potential energy at infinity to be zero.) Mass of the Earth = 6.0×10^{24} kg, radius = 6400 km.

Ans :

Given, Mass of the Earth = $6.0 \times 10^{24} \text{ kg}$

Radius of the earth, $R = 6400 \text{ km}$

Distance of satellite from the surface of the Earth

$$h = 36,000 \text{ km}$$

The potential due to earth's gravity at the site of the satellite.

$$\begin{aligned} V &= -\frac{GM}{(R+h)} \\ &= \frac{-6 \times 10^{24} \times 6.67 \times 10^{-11}}{(6400 + 36,000) \times 10^3} \\ &= \frac{-6 \times 6.67 \times 10^{24} \times 10^{-11}}{42400 \times 10^3} \\ &= \frac{-6 \times 667 \times 10^7}{424} \\ &= -9.44 \times 10^6 \text{ J kg}^{-1} \end{aligned}$$

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- 202.** A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev per second. (Extremely compact stars of this kind are known as neutron stars. Certain observed stellar objects called pulsars are believed to belong to this category).

Will an object placed on its equator remain stuck to its surface due to gravity ? (Mass of the Sun = $2 \times 10^{30} \text{ kg}$).

Ans :

Acceleration due to gravity acting inward is given by

$$\begin{aligned} g &= \frac{GM}{R^2} \\ &= \frac{6.67 \times 10^{-11} \times 2.5 \times 2 \times 10^{30}}{(12 \times 10^3)^2} \end{aligned}$$

Outward acceleration due to centrifugal force is given by

$$\begin{aligned} \omega^2 R &= 4\pi^2 v^2 R \\ &= 4 \times \left(\frac{22}{7}\right)^2 \times (1.5)^2 \times 12000 \end{aligned}$$

$$\omega^2 R = 1.1 \times 10^6 \text{ ms}^{-2}$$

Thus in the rotating frame of the star, the inward force is much greater than the outward centrifugal force at its equator. The object will remain stuck and does not fly off due to centrifugal force.

Since 'a' increases from $2R_E$ to $4R_E$, the kinetic energy decreases. Example 7.8 shows ΔK is negative.

- 208.** The acceleration due to gravity 'g' on the Earth's surface is approximately 9.8 m/s^2 . An explorer measures 'g' at the top of a high mountain (height 'h' above the surface) and then deep inside a mine (depth 'd' below the surface). Assume the Earth is a sphere of uniform density.

Questions:

1. How does the acceleration due to gravity $g(h)$ at a height 'h' above the Earth's surface vary for $h \ll R_E$?
2. Recall: How does the acceleration due to gravity $g(d)$ at a depth 'd' below the Earth's surface vary?
3. Where is the acceleration due to Earth's gravity maximum?
4. If the explorer finds that the acceleration due to gravity is less than 9.8 m/s^2 at both locations (mountaintop and mine), what does this imply about how 'g' changes as one moves away from the Earth's surface or towards its centre?

Ans :

1. For $h \ll R_E$, the acceleration due to gravity at height h is

$$g(h) \approx g\left(1 - \frac{2h}{R_E}\right),$$

where g is the acceleration on the surface.

2. The acceleration due to gravity at depth d is

$$g(d) = g\left(1 - \frac{d}{R_E}\right).$$

3. The acceleration due to Earth's gravity is maximum on its surface.
4. This implies that the acceleration due to gravity decreases whether you go up from the Earth's surface or down below the Earth's surface.

- 209.** The Moon has a significantly smaller mass and radius compared to Earth. Its acceleration due to gravity on the surface is also much smaller. This has implications for launching objects from its surface to escape its gravitational pull. The escape speed from Earth is about 11.2 km/s .

Questions:

1. What is the formula for the escape speed $(V_i)_{min}$ from the surface of a celestial body of mass M and radius R ?
2. Does the escape speed of a body depend on the mass of the body being launched?
3. The escape speed for the Moon is about 2.3 km/s . If an astronaut on the moon throws a rock upwards with a speed of 10 m/s , will it escape the Moon's gravity?

4. What is a significant consequence of the Moon's low escape speed?

Ans :

1. The escape speed is

$$(V_i)_{min} = \sqrt{\frac{2GM}{R}}.$$

2. No, the escape speed does not depend on the mass of the body being launched (m cancels out in the derivation).
3. $10 \text{ m/s} = 0.01 \text{ km/s}$. Since 0.01 km/s is much less than the escape speed of 2.3 km/s , the rock will not escape the Moon's gravity and will fall back down.
4. A significant consequence of the Moon's low escape speed is that the Moon has no atmosphere, as gas molecules, if formed, would likely have speeds greater than the escape speed and escape the Moon's gravitational pull.

- 210.** In 1798, Henry Cavendish conducted an experiment to determine the value of the universal gravitational constant G . His apparatus involved a bar with two small lead spheres suspended by a fine wire. Two large lead spheres were brought close to the small ones, causing a gravitational attraction and a measurable twist in the wire due to the torque.

Questions:

1. What is the universal law of gravitation stated mathematically for the magnitude of force F between two point masses m_1 and m_2 separated by a distance r ?
2. In Cavendish's experiment, what provides the restoring torque that balances the gravitational torque when the wire is twisted?
3. If F is the gravitational force between a big sphere and its neighboring small sphere, and L is the length of the bar AB supporting the small spheres, what is the gravitational torque causing the twist?
4. How does the measurement of the angle of twist (θ) and the restoring couple per unit angle of twist (τ_{twist}) allow for the calculation of G ?

Ans :

1. The magnitude of the force F is

$$|F| = G \frac{m_1 m_2}{r^2}.$$

2. The restoring torque of the wire itself (due to its elasticity) balances the gravitational torque.
3. The torque arising out of F is F multiplied by L (the length of the bar AB).
4. At equilibrium, the gravitational torque equals the restoring torque:

$$G \frac{Mm}{d^2} L = \tau_{twist} \theta.$$

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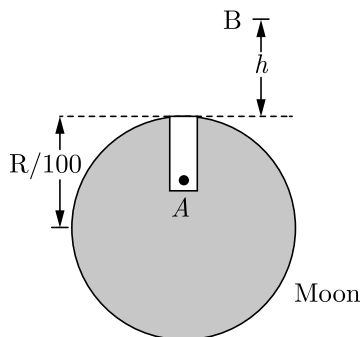
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- 194.** There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upwards from the crater with velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile.

Ans :

Let the projectile be projected from the point A and it reaches to point B at a height h from the surface of the moon. Also let M and m be the masses of the moon and the projectile respectively. It is projected with the escape velocity

$$v = \frac{\sqrt{2GM}}{R}$$



Energy at A = Energy at B

$$\text{K.E.} + \text{P.E. (at } A) = \text{P.E. at } B \quad [\text{at } B, v = 0]$$

$$\begin{aligned} \frac{1}{2} m \frac{2GM}{R} - \frac{GmM}{2R} \left[3R^2 - \left(\frac{99}{100} \right)^2 R^2 \right] \\ = - \frac{GMm}{(R+h)} \\ \frac{1}{2} m \frac{2GM}{R} - \frac{2.0199 GmM}{2R} \\ = - \frac{GMm}{(R+h)} \\ \frac{-0.0199 GmM}{2R} = \frac{-GMm}{(R+h)} \\ R+h = \frac{2R}{0.0199} = 100.5R \\ h = 100R \end{aligned}$$

- 195.** A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the Earth's gravitational influence? Mass of the satellite = 200 kg, Mass of the Earth = 6.0×10^{24} kg, Radius of the Earth = 6.4×10^6 m, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Ans :

Total energy of the satellite when it is put into the orbit,

$$E = PE + KE$$

$$E = -\frac{GM_E m}{R_E + h} + \frac{1}{2} m v_0^2$$

Since,

$$\begin{aligned} v_0 &= \sqrt{\frac{GM_E}{R_E + h}} \\ &= -\frac{GM_E m}{R_E + h} + \frac{1}{2} m \frac{GM_E}{R_E + h} \\ E &= -\frac{GM_E m}{2(R_E + h)} \end{aligned}$$

Total work to send the rocket to ∞ i.e.,

$$W = \frac{GM_E m}{R_E}$$

Diff. of work done

$$\begin{aligned} &= \frac{GM_E m}{R_E} - \frac{GM_E m}{2(R_E + h)} \\ &= GM_E m \left[\frac{1}{R_E} - \frac{1}{2(R_E + h)} \right] \\ &= GM_E m \left[\frac{R_E + 2h}{2R_E(R_E + h)} \right] \\ &= 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200 \\ &\quad \times \left[\frac{6.4 \times 10^6 + 2(400 \times 10^3)}{2(6.4 \times 10^6)(6.4 \times 10^6 + 400 \times 10^3)} \right] \\ &= 6.67 \times 12 \times 10^{15} \times \frac{64 \times 10^5 + 8 \times 10^5}{12.8 \times 10^6(64 \times 10^5 + 4 \times 10^5)} \\ &= 6.67 \times 12 \times 10^{15} \times \frac{12.8 \times 10^6 \times 68 \times 10^5}{72 \times 10} \\ &= \frac{6.67 \times 12 \times 10^{15} \times 72}{12.8 \times 68 \times 10^6} \\ &= 6.62 \times 10^9 \text{ J} \end{aligned}$$

- 196.** A satellite is launched into a circular orbit 1600 km above the surface of the earth. Find the period of revolution, the radius of the earth $R = 6400$ km and the acceleration due to gravity is 9.8 m/sec^2 . At what height from the ground should it be launched so that it may appear stationary over a point on the earth's equator?

Ans :

Time of revolution of satellite at height h is given by

$$T = 2\pi \sqrt{\left\{ \frac{(R+h)^3}{gR^2} \right\}} \quad \dots(1)$$

Substituting the given values, we have

$$\begin{aligned} T &= 2 \times 3.14 \times \left[\frac{\{(6400 + 1600) \times 10^3 \text{ m}\}^3}{(9.8 \text{ m/s}^2)(6400 \times 10^3 \text{ m})^2} \right]^{\frac{1}{2}} \\ &= 7090 \text{ sec.} \\ &= 1.97 \text{ hour.} \end{aligned}$$

Ans :

1. The centripetal force is provided by the gravitational force between the Earth and the satellite.
2. The orbital speed V decreases as the altitude ' h ' increases.
3. The centripetal force is $\frac{mV^2}{(R_E + h)}$
And the gravitational force is $\frac{GmM_E}{(R_E + h)^2}$.
Equating these gives

$$V^2 = \frac{GM_E}{(R_E + h)},$$

$$V = \sqrt{\frac{GM_E}{(R_E + h)}}.$$
4. According to Kepler's third law for satellites, T^2 is proportional to $(R_E + h)^3$. If the altitude ' h ' is increased, $(R_E + h)$ increases, and therefore $(R_E + h)^3$ increases, leading to an increase in T^2 and thus an increase in the orbital period T .

3. Given

$$M_E = \frac{4}{3}\pi R_E^3 \rho \text{ and}$$

$$M_r = \frac{4}{3}\pi r^3 \rho.$$

$$M_r = M_E \left(\frac{r^3}{R_E^3}\right).$$

Substituting M_r into the force equation:

$$F = \frac{Gm}{r^2} \left(M_E \frac{r^3}{R_E^3} \right)$$

$$= \frac{GmM_E}{R_E^3} r.$$

Since G , m , M_E , and R_E are constants, $F \propto r$.

4. Since the force is proportional to the displacement ' r ' from the centre and directed towards the centre (a restoring force), the mass ' m ' would execute Simple Harmonic Motion (SHM).

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- 214.** Imagine a tunnel is drilled straight through the Earth from one side to the other, passing through its centre. A small mass ' m ' is dropped into this tunnel. Assume the Earth has uniform density ρ and radius R_E . Consider the force on ' m ' when it is at a distance ' r ' from the centre ($r < R_E$).

Questions:

1. When the mass ' m ' is at a distance ' r ' from the centre (inside the Earth), which part of the Earth exerts a net gravitational force on it?
2. If M_r is the mass of the Earth contained within a sphere of radius ' r ', write down the magnitude of the gravitational force on ' m ' due to this mass M_r .
3. Show that the force on the mass ' m ' is proportional to ' r ' (i.e., $F \propto r$) when it is inside the Earth. (Recall $M_E = \frac{4}{3}\pi R_E^3 \rho$ and $M_r = \frac{4}{3}\pi r^3 \rho$).
4. What kind of motion would the mass ' m ' execute if it were dropped into such a tunnel (neglecting air resistance and rotational effects)?

Ans :

1. The shells of the Earth with radius greater than ' r ' exert no net gravitational force on the mass ' m '. Only the smaller sphere of radius ' r ' (with mass M_r) exerts a net force as if its mass is concentrated at the centre.
2. The magnitude of the gravitational force on ' m ' due to M_r is

$$F = \frac{GmM_r}{r^2}.$$

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CHAPTER 8

Mechanical Properties of Solid

SUMMARY

1. SOLID

A solid is a substance that exists in the solid state, which is one of the four fundamental states of matter. In solids, atoms are closely packed, and their kinetic energies are much lower than those in liquids and gases. Solids have a rigid structure, which resists external forces. They possess a fixed, definite shape, unlike liquids and gases, which take the shape of their containers.

1.1 Type of Solids

The solid are classified in the following parts :

1. Crystalline Solids : A crystalline solid is that in which there is regular and repeated arrangement of atoms or molecules in three dimensions.
2. Amorphous Solids : An amorphous solid is that which has random but rigid arrangement of atoms.

2. ELASTICITY

The property of body by virtue of which it regains its original shape or size when the deforming forces acting on it are removed, is called elasticity.

2.1 Plasticity

The inability of a body to return to its original shape and size, when the deforming forces are removed, is called plasticity.

2.2 Stress (σ)

The restoring force acting per unit area of the surface when a deforming force is applied is called stress. The unit is N m^{-2}

2.3 Strain (E)

The fraction change produced in the body as a result of deforming force is called strain.

3. HOOKE'S LAW

Within the elastic limit the strain produced in a body is directly proportional to stress acting on it. Mathematically,

$$\text{Stress} \propto \text{strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

3.1 Young's Modulus

The ratio of stress to the longitudinal strain is called Young's modulus of elasticity (Y).

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}} = \frac{F.L}{a.l}$$

3.2 Bulk Modulus

The ratio of hydrostatic stress to volume strain is called bulk modulus of elasticity (K)

$$K = -\frac{\Delta P}{(v - V)} = -V \left(\frac{\Delta P}{v} \right)$$

3.3 Rigidity Modulus

The ratio of shearing stress to shearing strain is called rigidity modulus (η).

$$\eta = \frac{\frac{F}{A}}{\theta}$$

3.4 Elastic Limit

The maximum value of stress when permanent deformation begins to occur is called elastic limit.

3.5 Ductile Materials

Those materials in which there is large gap between the elastic limit and the breaking point are called ductile materials.

3.6 Brittle Materials

Those materials in which the breaking point is very close to the elastic limit are called brittle materials.

3.7 Elastomers

The materials which can be pulled to several times their original length and still return to their original value after the removal of deforming force are called elastomers.

3.8 Elastic Fatigue

The loss of elastic strength of the material due to repeated alternating stress to which the material is subjected is called elastic fatigue.

NCERT TEXT-BOOK QUESTIONS

1. A steel wire of length 4.7 m and cross section $3 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper ?

Ans :

Given,

For steel wire, $L = 4.7 \text{ m}$

$$A = 3 \times 10^{-5} \text{ m}^2$$

For copper wire, $L = 3.5 \text{ m}$

$$A = 4.0 \times 10^{-5} \text{ m}^2$$

$$\text{As we know that, } Y = \frac{\left(\frac{F}{A}\right)L}{\Delta L} = \frac{F \times l}{A \times \Delta l}$$

$$\text{Now, } Y_s = \frac{F \times l_1}{A_1 \times \Delta l} \quad \dots(1)$$

$$Y_C = \frac{F \times l_2}{A_2 \times \Delta l} \quad \dots(2)$$

From equation (1) and (2), we get

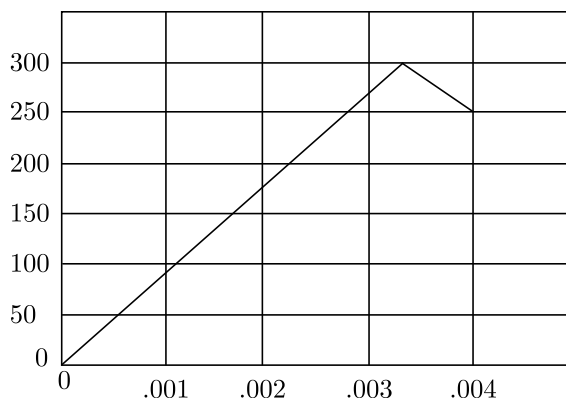
$$\begin{aligned} \frac{Y_s}{Y_C} &= \frac{l_1}{l_2} \times \frac{A_2}{A_1} \\ &= \frac{4.7}{3.5} \times \frac{4.0 \times 10^{-5}}{3.0 \times 10^{-5}} \\ &= \frac{18.8}{10.5} = 1.8 \end{aligned}$$

2. The given figure shows the strain stress curve for a given material. What are
(a) Young's modulus and
(b) approximate yield strength for this material ?

Ans :

From the figure we find that

- (a) When stress is $150 \times 10^6 \text{ N m}^{-2}$, strain is $= 0.002$



Young's modulus,

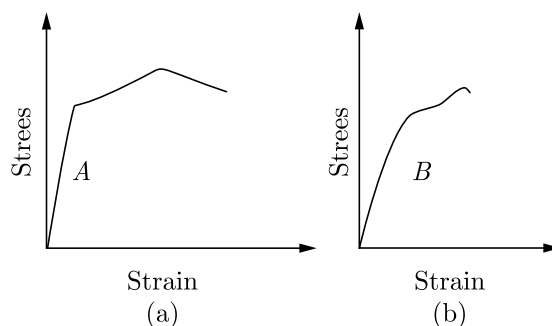
$$Y = \frac{\text{stress}}{\text{strain}} = \frac{150 \times 10^6}{0.002}$$

$$Y = 7.5 \times 10^{10} \text{ N m}^{-2}$$

- (b) Approximate yield strength of the material
 $= 300 \times 10^6 \text{ N m}^{-2}$

3. The stress-strain graphs for materials A and B for two materials are shown in figure. (The graphs are to the same scale).

- (a) Which material has greater Young's modulus ?
(b) Which of the two is the stronger material ?



Ans :

- (a) A material has greater Young's modulus.
(b) Strength of a material is determined by the amount of stress required to cause fracture. Material (A) is stronger than (B).

4. Read each statement below carefully and state, with reason, if it is true or false ?
(a) The Young's modulus of elasticity of rubber is greater than that of steel.
(b) The stretching of a coil spring is determined by its shear modulus.

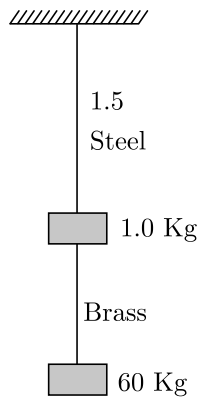
Ans :

- (a) False, because more stress is required to produce same strain in steel wire in comparison to a

rubber. Therefore young's modulus of steel is greater than that of rubber.

- (b) True, because stress produces, relative displacement between threads of coil.

5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass and loaded as shown in the figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.



Ans :

Given,

For steel, $F = 4.0 \times 9.8 \text{ N}$

$$a = \pi r^2 = 3.14 \times 0.125 \times 0.125$$

$$= 3.14 \times 0.125 \times 0.125 \times 10^{-4} \text{ m}^2$$

$$L = 1.5 \text{ m}, l = ?$$

$$Y = 2.0 \times 10^{11} \text{ Pa}$$

$$\text{So } 2.0 \times 10^{11} = \frac{F \times L}{a \times l}$$

$$= \frac{4.0 \times 9.8 \times 1.5}{3.14 \times 0.125 \times 0.125 \times 10^{-4} \times l}$$

$$l = \frac{4.0 \times 9.8 \times 1.5}{2.0 \times 10^{11} \times 3.14 \times 0.125 \times 0.125 \times 10^{-4}}$$

$$= 1.3 \times 10^{-4} \text{ m}$$

For Brass,

$$F = 6.0 \times 9.8 \text{ N}$$

$$a = \pi r^2 = 3.14 \times 0.125 \times 0.125 \text{ cm}^2$$

$$= 3.14 \times 0.125 \times 0.125 \times 10^{-4} \text{ m}^2$$

$$L = 1.0 \text{ m}, l = ?$$

$$Y = 0.91 \times 10^{11} \text{ Pa}$$

So

$$0.91 \times 10^{11} = \frac{F \times L}{a \times l}$$

$$= \frac{6.0 \times 9.8 \times 1}{3.14 \times 0.125 \times 0.1235 \times 10^{-4} \times l}$$

$$l = \frac{6.0 \times 9.8 \times 1}{0.91 \times 10^{11} \times 3.14 \times 0.125 \times 0.125 \times 10^{-4}}$$

$$= 1.3 \times 10^{-4} \text{ m}$$

6. The edges of an aluminium cube are 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 1000 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is vertical deflection of this face ? ($1 \text{ Pa} = 1 \text{ N m}^{-2}$)

Ans :

Given, $l = 10 \text{ cm} = 0.1 \text{ m}$

$$G = 25 \text{ GPa} = 25 \times 10^{10} \text{ Pa}$$

$$= 25 \times 10^{10} \text{ N m}^{-2}$$

Area, $A = l^2 = 0.01 \text{ m}^2$, $m = 100 \text{ kg}$

Force, $F = mg = 100 \times 10 = 1000 \text{ N}$

$$\text{Stress} = \frac{F}{A} = \frac{1000}{0.01} = 10^5 \text{ N m}^{-2}$$

Vertical deflection,

$$\Delta l = \frac{\text{stress}}{G} \times l$$

$$= \frac{10^5}{25 \times 10^{10}} \times 0.01$$

$$\Delta l = 4 \times 10^{-7} \text{ m}$$

7. Four identical hollow cylindrical columns of steel support a big structure of mass 50000 kg the inner and outer radii of each column are 30 cm and 60 cm respectively. Assume the load distribution to be uniform. Calculate the compressional strain of each column.

Ans :

Given, Inner radius = 0.3 m

Outer radius = 0.6 m

$$\text{Area of each cylinder} = \pi (0.6)^2 - \pi (0.3)^2$$

$$= \pi \times 0.27$$

$$\text{Total area of four cylinders} = 4 \times \pi \times 0.27$$

Original length = Thickness of each cylinder

$$= 0.6 - 0.3 = 0.3 \text{ m}$$

Here $Y = 2 \times 10^{11} \text{ Pa}$

and $F = 50000 \times 9.8$

$$\text{Compression Strain} = \frac{\frac{F}{a}}{Y} = \frac{F}{Ya}$$

$$= \frac{50000 \times 9.8}{2 \times 10^{11} \times 4 \times \pi \times 0.27}$$

$$= 7.2 \times 10^{-7} \text{ m}$$

Compression strain of each column

$$= 7.2 \times 10^{-7}$$

8. A piece of copper having cross-section of 15.2 mm $\times$ 19.2 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.

Ans :

Given area, $A = 15.2 \times 19.1 \times 10^{-6}$
 $= 290.32 \times 10^{-6} \text{ m}^2$

Tension of force, $F = 44,500 \text{ N}$

For copper, $Y = 1.1 \times 10^{11} \text{ Pa}$

Since, $Y = \frac{\text{stress}}{\text{strain}}$

$$\begin{aligned} \text{Strain} &= \frac{\text{Stress}}{Y} = \frac{F}{AY} \\ &= \frac{44,500 \times 10^6}{290.32 \times 1.1 \times 10^{11}} \\ &= 139.34 \times 10^{-5} \end{aligned}$$

9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2} , what is the maximum load the cable can support ?

Ans :

Given, $r = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$

Total force that can be applied without breaking.

$$\begin{aligned} &= \text{Breaking stress} \times \text{area of cross-section} \\ &= 10^8 \times \pi r^2 \\ &= 10^8 \times \frac{22}{7} \times (1.5 \times 10^{-2})^2 \\ &= \frac{22}{7} \times 2.25 \times 10^{-4} \times 10^8 \\ &= 7.07 \times 10^4 \text{ N} \end{aligned}$$

Maximum load the cable can support

$$\begin{aligned} &= \frac{7.07 \times 10^4}{9.8} \text{ kg} = 0.72 \times 10^4 \text{ kg} \\ &= 7200 \text{ kg.} \end{aligned}$$

10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Ans :

Here L and l for three wires is same.

Since, $Y = \frac{4FL}{\pi D^2 l}$

$$D^2 = \frac{4FL}{\pi Y l}$$

$$D \propto \sqrt{\frac{1}{Y}}$$

Now, $\frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}} = \sqrt{\frac{1.9 \times 10^{11}}{1.1 \times 10^{11}}}$
 $= \sqrt{1.727} = 1.31$

11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2 . Calculate the elongation of the wire when mass is at the lowest point of its path.

Ans :

Given, $m = 14.5 \text{ kg}$, $L = 1 \text{ m}$

$$v = 2 \text{ rev/s}$$

$$\omega = 2\pi v = 2\pi \times 2 = 4\pi \text{ rad s}^{-1}$$

Area of cross-section of the wire

$$\begin{aligned} a &= 0.065 \text{ cm}^2 = 0.065 \times 10^{-4} \text{ m}^2 \\ &= 65 \times 10^{-7} \text{ m}^2 \end{aligned}$$

$$Y_{\text{steel}} = 2 \times 10^{11} \text{ pa}$$

Due to rotation of wire, the stretching force developed at the lowest point is given by

$$T - mg = m\omega^2 l$$

where T is the tension in the wire

$$\begin{aligned} T &= mg + m\omega^2 l \\ &= 14.5 \times 9.8 + 14.5 \times 1 \times (4\pi)^2 \\ &= 2431.9 \text{ N} \end{aligned}$$

$$\text{Stress} = \frac{F}{a} = \frac{2431.9}{65 \times 10^{-7}}$$

$$\text{Strain} = \frac{l}{L} = l \quad [L = 1]$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{2431.9}{65 \times 10^{-7} \times l}$$

$$\begin{aligned} l &= \frac{2431.9}{65 \times 10^{-7} \times Y} = \frac{2431.9 \times 10^{-7}}{65 \times 2 \times 10^{11}} \\ &= 0.187 \times 10^{-2} \text{ m} \end{aligned}$$

$$l = 0.187 \text{ cm} = 0.19 \text{ cm}$$

- (D) Permanent set : Non-zero strain when stress is removed after plastic deformation.

Ans :

(C) Elastomer : Obeys Hooke's law over a very large strain range.

Elastomers can be stretched to cause large strains and return to their original shape, but they typically do not obey Hooke's law over most of this large strain region; their stress-strain curve is non-linear. The other pairs are correctly matched.

26. Arrange the following points on a typical stress-strain curve for a metal in the order they are encountered as stress increases from zero:

1. Fracture point (E)
2. Yield point(B) or (Elastic limit)
3. Proportional limit (A)
4. Ultimate tensile strength (D)

(A) 3, 2, 4, 1

(B) 2, 3, 1, 4

(C) 3, 4, 2, 1

(D) 2, 4, 3, 1

Ans :

(A) 3, 2, 4, 1

As stress increases from zero on a typical stress-strain curve for a metal, the sequence is: Proportional limit (A) where Hooke's law is obeyed, then Yield point (B) or elastic limit, followed by the Ultimate tensile strength (D), and finally the Fracture point (E).

27. Consider the process of stretching a wire within its elastic limit and then releasing the force. What is the correct sequence of energy transformations?

1. Elastic potential energy is stored in the wire.
2. Work is done by the external pulling force.
3. The wire returns to its original length, releasing potential energy.

(A) 1, 2, 3

(B) 2, 1, 3

(C) 2, 3, 1

(D) 1, 3, 2

Ans :

(B) 2, 1, 3

First, an external force does work on the wire to stretch it. This work done is then stored as elastic potential energy within the wire (within the elastic limit). When the external force is removed, this stored potential energy is released as the wire returns to its original length.

28. Why are steel ropes in cranes made of a number of thin wires braided together instead of a single solid wire of the same radius?

(A) To increase the overall Young's modulus of the rope material significantly.

(B) To make the rope more brittle and prone to sudden fracture for safety.

(C) To ensure the rope is rigid and does not bend under any load.

(D) For ease in manufacture, flexibility, and increased overall strength.

Ans :

(D) For ease in manufacture, flexibility, and increased overall strength.

Braiding thin wires together provides flexibility, makes manufacturing easier for thick ropes, and helps distribute stress, thereby increasing the overall strength and resistance to catastrophic failure compared to a single solid wire.

29. The beams used in the construction of bridges often have a cross-section of the type 'I'. What is the primary reason for this specific shape?

(A) To maximize the weight of the beam for greater stability.

(B) To reduce the bending under load while minimizing material usage.

(C) To make the beam more susceptible to buckling under load.

(D) To decrease the overall depth of the beam significantly.

Ans :

(B) To reduce the bending under load while minimizing material usage.

The 'I' shape provides a large load-bearing surface and significant depth to resist bending, concentrating material where it is most effective against bending stresses, thus optimizing strength while minimizing weight and material cost.

30. What is the term for the restoring force per unit area developed in a body when it is subjected to a deforming force?

(A) Strain

(B) Pressure

(C) Stress

(D) Elastic Modulus

Ans :

(C) Stress

Stress is defined as the internal restoring force developed per unit area of a body when a deforming

$$\frac{\Delta V}{V} = \frac{P}{K} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9}$$

$$\begin{aligned}\frac{\Delta V}{V} &= \frac{1.013}{37 \times 10^3} \\ &= 0.0274 \times 10^{-3} \\ &= 2.74 \times 10^{-5}\end{aligned}$$

15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7×10^6 Pa.

Ans :

For copper, $K = 140 \times 10^9$ Pa
 $L = 10 \text{ cm} = 0.1 \text{ m} = 10^{-1} \text{ m}$

As we know that, $V = L^3 = 10^{-3} \text{ m}^3$

$$\begin{aligned}P &= 7 \times 10^6 \text{ pa} \\ K &= \frac{pV}{\Delta V} \\ \Delta V &= \frac{pV}{K} \\ &= \frac{7 \times 10^6 \times 10^{-3}}{140 \times 10^9} \\ &= 0.05 \times 10^{-6} \text{ m}^3\end{aligned}$$

16. How much should the pressure on a litre of water be changed to compress it by 0.10%. Carry one quarter of the load.

Ans :

Given, $V = 1 \text{ litre}$
 $\Delta V = -0.10\% \text{ of } V$
 $= -\frac{0.10}{100} \times 1 = -\frac{1}{1000} \text{ litre}$

Let Δp = Change in pressure required for compression of 1 litre of water.

$$\begin{aligned}K &= \text{bulk modulus of water} \\ &= 2.2 \times 10^9 \text{ N m}^{-2}\end{aligned}$$

$$K = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}, \text{ we get}$$

$$\Delta p = -K \cdot \frac{\Delta V}{V}$$

$$\Delta p = 2.2 \times 10^9 \times \frac{\left(\frac{1}{1000}\right)}{1}$$

$$\Delta p = \frac{2.2 \times 10^9}{1000} = 2.2 \times 10^6 \text{ N m}^{-2}$$

OBJECTIVE QUESTIONS

1. What term describes the property of a body that causes it to regain its original size and shape after an applied force is removed?
 (A) Plasticity
 (B) Elasticity
 (C) Deformity
 (D) Rigidity

Ans :

(B) Elasticity
 Elasticity is the property of a body by virtue of which it tends to regain its original size and shape when the applied deforming force is removed. Plasticity, on the other hand, describes materials that permanently deform.

2. What is the SI unit of stress?

- (A) N m
 (B) Pa s
 (C) N m⁻²
 (D) J kg⁻¹

Ans :

(C) N m⁻²
 Stress is defined as the restoring force per unit area. The SI unit of force is Newton (N) and area is meter squared (m²), so the SI unit of stress is N m⁻², which is also known as Pascal (Pa).

3. Which type of stress occurs when two equal and opposite deforming forces are applied parallel to the cross-sectional area of a cylinder, causing relative displacement between opposite faces?

- (A) Tensile stress
 (B) Compressive stress
 (C) Hydraulic stress
 (D) Shearing stress

Ans :

(D) Shearing stress
 Shearing stress occurs when deforming forces are applied parallel to the cross-sectional area, causing a relative displacement of the object's faces. Tensile and compressive stresses act perpendicular to the area.

4. Longitudinal strain is defined as the ratio of:

- (A) Change in volume to original volume
 (B) Change in length to original length
 (C) Applied force to cross-sectional area
 (D) Relative displacement of faces to length

Ans :

(B) Change in length to original length
Longitudinal strain is a measure of the deformation of a material in the direction of the applied force. It is specifically defined as the ratio of the change in length (ΔL) to the original length (L) of the body.

5. According to Hooke's law, for small deformations, stress is directly proportional to what?

- (A) Strain
- (B) Force
- (C) Area
- (D) Modulus

Ans :

(A) Strain
Hooke's law states that for small deformations, the stress developed in a material is directly proportional to the strain produced. The constant of proportionality is known as the modulus of elasticity.

6. In a typical stress-strain curve for a metal, what is the point B, beyond which stress and strain are no longer proportional but the body still returns to its original dimension when the load is removed, known as ?

- (A) Fracture point
- (B) Yield point
- (C) Proportional limit
- (D) Ultimate strength

Ans :

(B) Yield point
The yield point (also known as the elastic limit) on a stress-strain curve marks the end of the elastic region where the material will return to its original shape. Beyond this point, plastic deformation occurs.

7. What is Young's modulus (Y) defined as the ratio of?

- (A) Shearing stress to shearing strain
- (B) Hydraulic stress to volume strain
- (C) Tensile stress to longitudinal strain
- (D) Compressive stress to volume strain

Ans :

(C) Tensile stress to longitudinal strain
Young's modulus is a measure of the stiffness of a solid material. It is defined as the ratio of tensile stress (force per unit area) to longitudinal strain (proportional deformation along the length).

8. Which of the following statements about Young's modulus is incorrect?

- (A) Its SI unit is N m^{-2} or Pa.
- (B) It is a characteristic of the material.
- (C) A material with a larger Young's modulus is less elastic.
- (D) Metals generally have large Young's moduli.

Ans :

(C) A material with a larger Young's modulus is less elastic.

A material with a larger Young's modulus is actually more elastic (stiffer), meaning it requires a larger force to produce a small change in length. The other statements are correct descriptions of Young's modulus.

9. Which of the following statements correctly describes elastomers?

- (A) They strictly obey Hooke's law over their entire elastic region.
- (B) They have a very well-defined plastic deformation region.
- (C) They can be stretched to cause large strains and still return to their original shape.
- (D) They possess very high Young's moduli compared to metals.

Ans :

(C) They can be stretched to cause large strains and still return to their original shape.

Elastomers, like rubber, are materials that can undergo very large strains (deformations) when stretched and can return to their original shape when the stress is removed. They do not typically obey Hooke's law strictly over large strains.

10. If a material is said to be brittle, what does this imply about its stress-strain curve?

- (A) The ultimate strength and fracture points are far apart.
- (B) The material undergoes significant plastic deformation before fracture.
- (C) The ultimate strength and fracture points are close together.
- (D) The material has a very low yield strength.

Ans :

(C) The ultimate strength and fracture points are close together.

A brittle material is one that fractures with little to no plastic deformation. On a stress-strain curve, this means the ultimate tensile strength (point D)

and the fracture point (point E) are very close to each other.

11. All of the following are examples of contact forces in mechanics EXCEPT:

(A) Frictional force
(B) Normal reaction
(C) Gravitational force
(D) Viscous force

Ans :

(C) Gravitational force

Gravitational force is a non-contact force that acts at a distance between objects with mass. Frictional force, normal reaction, and viscous force are all examples of contact forces that arise from the physical contact between objects or between an object and a fluid.

12. How does static friction differ from kinetic friction?

(A) Static friction is always less than kinetic friction.
(B) Static friction opposes impending motion, while kinetic friction opposes actual relative motion.
(C) Static friction is independent of the normal force, unlike kinetic friction.
(D) Static friction comes into play only when an object is already moving.

Ans :

(B) Static friction opposes impending motion, while kinetic friction opposes actual relative motion.

Static friction (f_s) is the force that prevents an object from starting to move when an external force is applied, opposing impending motion. Kinetic friction (f_k) is the force that opposes the relative motion between surfaces already in contact and moving past each other.

13. Assertion (A): The centre of gravity of a body coincides with its centre of mass if the gravitational field is uniform.

Reason (R): The centre of mass of a body depends only on the distribution of mass of the body and is independent of gravity.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(B) Both A and R are true but R is NOT the correct explanation of A.

The assertion is true; the center of gravity and center of mass coincide when the gravitational field is uniform across the body. The reason is also true; the center of mass is purely a geometric property related to mass distribution. However, the reason doesn't fully explain why they coincide under uniform gravity, which is because the gravitational force on each particle is then proportional to its mass, making the weighted average of positions for gravity align with that for mass.

14. Assertion (A): A pair of equal and opposite forces with different lines of action is known as a couple. Reason (R): A couple produces rotation without translation.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The assertion correctly defines a couple. The reason correctly states the effect of a couple. Since the forces are equal and opposite, the net force is zero, resulting in no translational acceleration. However, because their lines of action are different, they produce a net torque, causing rotation.

15. Assertion (A): The stress-strain curve for elastomers does not obey Hooke's law over most of its region. Reason (R): Elastomers can be stretched to cause large strains and still return to their original shape.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

Elastomers, like rubber, exhibit non-linear elastic behavior (Assertion A is true). They are characterized by their ability to undergo large deformations and recover their original shape (Reason R is true). This large elastic deformation capability is why their stress-strain relationship is not linear like materials that strictly obey Hooke's law.

16. Consider the following statements regarding stress and strain:

- I. Stress is the restoring force per unit area.
- II. Strain is the fractional change in dimension.
- III. The SI unit of stress is Pascal.
- IV. Strain has no units or dimensional formula.

Which of the above statements are correct?

- (A) I and II only
- (B) I, II, and III only
- (C) I, II, III, and IV
- (D) III and IV only

Ans :

- (C) I, II, III, and IV

Statement I correctly defines stress. Statement II correctly defines strain. Statement III is correct as Pascal (Pa) or N m^{-2} is the SI unit of stress. Statement IV is correct as strain is a ratio of similar quantities.

17. Identify the correct statements about elastic moduli:

- I. Young's modulus relates tensile stress to longitudinal strain.
- II. Shear modulus is also known as the modulus of rigidity.
- III. Bulk modulus relates hydraulic stress to volume strain.
- IV. The SI unit for all elastic moduli is N m^{-1} .

- (A) I and II only
- (B) I, II, and III only
- (C) II, III, and IV only
- (D) I, II, III and IV

Ans :

- (B) I, II, and III only

Statements I, II, and III correctly define Young's modulus, shear modulus, and bulk modulus respectively. Statement IV is incorrect; the SI unit for all elastic moduli is N m^{-2} or Pascal (Pa), as they represent stress per unit strain (and strain is dimensionless).

18. Which of the following describe characteristics of the yield point on a stress-strain curve?

- I. It is also known as the elastic limit.
- II. Beyond this point, the material undergoes plastic deformation.
- III. If the load is removed before this point, the body regains its original dimensions.
- IV. It represents the maximum stress a material can withstand before fracturing.

- (A) I and II only
- (B) I, II, and III only
- (C) II, III, and IV only

- (D) I, II, III, and IV

Ans :

- (B) I, II, and III only

The yield point (elastic limit) marks the boundary between elastic and plastic deformation (Statements I and II). If stress is below this point, the material behaves elastically (Statement III). Statement IV describes the ultimate tensile strength, not the yield point.

19. Match the type of stress in Column I with its corresponding strain in Column II.

	Column I		Column II
1.	Tensile Stress	P.	Change in volume / Original volume
2.	Shearing Stress	Q.	Change in length / Original length
3.	Hydraulic Stress	R.	Relative displacement of faces / Length
4.	Compressive Stress	S.	Angular displacement of the surface

- (A) 1-Q, 2-R, 3-P, 4-Q
- (B) 1-P, 2-S, 3-Q, 4-R
- (C) 1-Q, 2-R, 3-P, 4-S
- (D) 1-S, 2-P, 3-R, 4-Q

Ans :

- (A) 1-Q, 2-R, 3-P, 4-Q

Tensile and compressive stresses cause longitudinal strain (change in length/original length). Shearing stress causes shearing strain (relative displacement of faces/length, which is also related to angular displacement). Hydraulic stress causes volume strain (change in volume/original volume).

20. Match the material property in Column I with its description in Column II.

	Column I		Column II
1.	Elasticity	P.	Substance deforms permanently under load
2.	Plasticity	Q.	Material breaks with little plastic deformation
3.	Ductility	R.	Body regains original shape after force removal
4.	Brittleness	S.	Material can be drawn into wires (large plastic deformation before fracture)

- (A) 1-R, 2-P, 3-S, 4-Q
- (B) 1-P, 2-R, 3-Q, 4-S

(C) 1-S, 2-Q, 3-P, 4-R

(D) 1-Q, 2-S, 3-R, 4-P

Ans :

(A) 1-R, 2-P, 3-S, 4-Q

Elasticity is the ability to regain original shape. Plasticity is permanent deformation. Ductility allows materials to be drawn into wires (undergoes significant plastic deformation). Brittleness means the material fractures with little plastic deformation.

- 21.** Match the elastic modulus in Column I with the type of deformation it relates to in Column II.

	Column I		Column II
1.	Young's Modulus (Y)	P.	Change in volume without change in shape
2.	Shear Modulus (G)	Q.	Change in shape without change in volume
3.	Bulk Modulus (B)	R.	Change in length along the direction of force
4.	Compressibility (k)	S.	Reciprocal of resistance to uniform compression

(A) 1-R, 2-Q, 3-P, 4-S

(B) 1-P, 2-R, 3-S, 4-Q

(C) 1-Q, 2-P, 3-R, 4-S

(D) 1-S, 2-R, 3-Q, 4-P

Ans :

(A) 1-R, 2-Q, 3-P, 4-S

Young's modulus (Y) relates to changes in length (longitudinal deformation). Shear modulus (G) relates to changes in shape (shearing deformation). Bulk modulus (B) relates to changes in volume under hydraulic stress. Compressibility (k) is the reciprocal of the bulk modulus.

- 22.** Which of the following pairs correctly matches a term with its definition?

(A) Stress : The fractional change in the dimension of a body.

(B) Strain : The internal restoring force per unit area of a body.

(C) Elastic limit : The point beyond which a material fractures.

(D) Plastic deformation : Permanent deformation of a material under stress.

Ans :

(D) Plastic deformation : Permanent deformation of a material under stress.

Plastic deformation refers to the permanent change in shape or size of a material that occurs when it is subjected to a stress exceeding its elastic limit. Stress is force/area, strain is fractional change, and elastic limit is the point beyond which permanent deformation begins, not necessarily fracture.

- 23.** Identify the correctly matched pair regarding types of stress and strain.

(A) Tensile stress : Results in a decrease in the length of the body.

(B) Shearing strain : Defined as the ratio of change in volume to original volume.

(C) Hydraulic stress : Leads to a change in shape without a change in volume.

(D) Longitudinal strain : Ratio of change in length to the original length.

Ans :

(D) Longitudinal strain : Ratio of change in length to the original length.

Longitudinal strain is correctly defined as the ratio of the change in length to the original length. Tensile stress leads to an increase in length. Shearing strain is related to the relative displacement of faces. Hydraulic stress causes a change in volume.

- 24.** Which of the following pairs is INCORRECTLY matched?

(A) Hooke's Law : Stress is proportional to strain within the elastic limit.

(B) Young's Modulus : Ratio of shearing stress to shearing strain.

(C) Bulk Modulus : Ratio of hydraulic stress to volume strain.

(D) Yield Strength : Stress at which plastic deformation begins.

Ans :

(B) Young's Modulus : Ratio of shearing stress to shearing strain.

Young's Modulus is the ratio of tensile (or compressive) stress to longitudinal strain. The ratio of shearing stress to shearing strain is the Shear Modulus (or Modulus of Rigidity).

- 25.** Identify the INCORRECTLY matched pair from the following:

(A) Ductile material : Fracture point and ultimate strength point are far apart.

(B) Brittle material : Undergoes little plastic deformation before fracture.

(C) Elastomer : Obeys Hooke's law over a very large strain range.

- 12.** Compute the bulk modulus of water from the following data : initial volume = 100.0 litre, pressure increase = 100.0 atm (1 atm = 1.013×10^5 pa). Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large ?

Ans :

Given, $P = 100$ atmosphere
 $= 100 \times 1.013 \times 10^5$ pa
 (1 atm = 1.013×10^5) pa

Initial volume,

$$V_1 = 100 \text{ litre} = 100 \times 10^{-3} \text{ m}^3$$

Final volume,

$$V_2 = 100.5 \text{ litre} = 100.5 \times 10^{-3} \text{ m}^3$$

Change in volume,

$$\begin{aligned} \Delta V &= V_2 - V_1 \\ &= (100.5 - 100) \times 10^{-3} \text{ m}^3 \\ &= 0.5 \times 10^{-3} \text{ m}^3 \end{aligned}$$

If $K_W =$ bulk module of water = ?

$$\begin{aligned} K_W &= \frac{P}{\Delta V} \text{ we get} \\ &= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}} \\ &\quad (V = V_1) \end{aligned}$$

$$K_W = 2.026 \times 10^9 \text{ pa}$$

Also we know that the bulk modulus of air at S.T.P. is given by

$$\begin{aligned} K_{air} &= 1.0 \times 10^5 \text{ pa} = 10^5 \text{ pa} \\ \frac{K_W}{K_{air}} &= \frac{2.026 \times 10^9}{10^5} = 2.026 \times 10^4 \\ &= 20260 \end{aligned}$$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger inter atomic forces in liquids than in gases.

- 13.** What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{ kg m}^{-3}$, compressibility of water is $45.8 \times 10^{-11} \text{ pa}^{-1}$

Ans :

Given, $P = 80.0 \text{ atm}$
 $= 80 \times 1.013 \times 10^5 \text{ pa}$
 $\frac{1}{K} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$

Density of water at the surface, $\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$. Let ρ' be the density of water at a given depth. If V and V' be the volumes of certain mass M of the water at the surface and at a given depth, Then

$$V = \frac{M}{\rho} \text{ and } V' = \frac{M}{\rho'}$$

Change in volume,

$$\frac{\Delta V}{V} = V - V' = M \left(\frac{1}{\rho} - \frac{1}{\rho'} \right)$$

Volumetric strain,

$$\begin{aligned} \frac{\Delta V}{V} &= M \left(\frac{1}{\rho} - \frac{1}{\rho'} \right) \times \frac{\rho}{M} \\ &= \left(1 - \frac{\rho}{\rho'} \right) \end{aligned}$$

$$\frac{\Delta V}{V} = 1 - \frac{1.03 \times 10^3}{\rho'}$$

Also we know that Bulk modulus of water is given by the formula

$$K = \frac{p}{\left(\frac{\Delta V}{V} \right)} = \frac{pV}{\Delta V}$$

$$\text{Compressibility} = \frac{1}{K} = \frac{\Delta V}{pV} = \frac{1}{p} \left(\frac{\Delta V}{V} \right)$$

$$45.8 \times 10^{-11} = \frac{1}{80 \times 1.013 \times 10^5} \times \left(1 - \frac{1.03 \times 10^3}{\rho'} \right)$$

$$\begin{aligned} 1 - \frac{1.03 \times 10^3}{\rho'} &= 45.8 \times 10^{-11} \times 80 \times 1.013 \times 10^5 \\ &= 3.712 \times 10^{-3} \end{aligned}$$

$$\frac{1.03 \times 10^3}{\rho'} = 1 - 3.712 \times 10^{-3} = 0.996288$$

$$\rho' = \frac{1.03 \times 10^3}{0.996288} = 1034 \text{ kg m}^{-3}$$

$$\rho' = 1.034 \times 10^3 \text{ kg m}^{-3}$$

- 14.** Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans :

Given, $P = 10 \times 1.013 \times 10^5 \text{ pa}$

Bulk modulus of glass,

$$K = 37 \times 10^9 \text{ Nm}^{-2}$$

As,

$$K = \frac{P}{\left(\frac{\Delta V}{V} \right)} \text{ we get}$$

force is applied to it. Strain is the fractional deformation, pressure is force per unit area (often external), and elastic modulus is the ratio of stress to strain.

31. What does the term 'plasticity'?

- (A) The property of a body to regain its original shape after force removal.
- (B) The property of a body to undergo permanent deformation under load.
- (C) The stress level at which a material fractures or breaks.
- (D) The ratio of stress to strain within the elastic limit of a material.

Ans :

(B) The property of a body to undergo permanent deformation under load.

Plasticity is the property of a material by which it undergoes permanent deformation (does not return to its original shape) when subjected to an external force beyond its elastic limit.

32. If a cylinder is stretched by two equal forces applied normal to its cross-sectional area, the stress developed is known as:

- (A) Shearing stress
- (B) Hydraulic stress
- (C) Tensile stress
- (D) Volumetric stress

Ans :

(C) Tensile stress

When a body, like a cylinder, is stretched by forces applied normal (perpendicular) to its cross-sectional area, the internal restoring force per unit area is termed tensile stress, leading to an elongation of the body.

33. The strain produced by a hydraulic pressure, defined as the ratio of change in volume to the original volume, is called:

- (A) Longitudinal strain
- (B) Shearing strain
- (C) Volume strain
- (D) Tensile strain

Ans :

(C) Volume strain

Volume strain is the measure of deformation caused by hydraulic pressure, defined as the ratio of the change in volume (ΔV) to the original volume (V) of the body.

34. The constant of proportionality 'k' in Hooke's law (stress = $k \times$ strain) is known as:

- (A) Yield strength
- (B) Modulus of elasticity
- (C) Ultimate tensile strength
- (D) Coefficient of friction

Ans :

(B) Modulus of elasticity

In Hooke's law, the constant of proportionality 'k' that relates stress to strain within the elastic limit is known as the modulus of elasticity. It is a measure of the material's stiffness.

35. What does the region O to A in a typical stress-strain curve for a metal represent, where the curve is linear?

- (A) Plastic deformation region
- (B) Region where Hooke's law is obeyed
- (C) Region of necking and fracture
- (D) Region of permanent set

Ans :

(B) Region where Hooke's law is obeyed

The initial linear portion of the stress-strain curve, from the origin (O) to the proportional limit (A), represents the region where stress is directly proportional to strain, and thus Hooke's law is obeyed.

36. The ultimate tensile strength (σ_u) of a material, represented by point D on the stress-strain curve, is the point:

- (A) Where the material begins to deform plastically.
- (B) Up to which the material behaves as an elastic body.
- (C) Where the material experiences maximum stress before necking.
- (D) Where the material fractures or breaks.

Ans :

(C) Where the material experiences maximum stress before necking.

The ultimate tensile strength (σ_u) is the maximum stress that a material can withstand while being stretched or pulled before necking (localized reduction in cross-sectional area) begins, which eventually leads to fracture.

37. Which of the following statements about the stress-strain curve is incorrect?

- (A) The yield point marks the limit of elastic behavior.
- (B) In the plastic region, the material does not regain its original shape upon load removal.

THREE MARKS QUESTIONS

115. Why does a body regain its shape or size when deforming forces are removed ? Explain it on the basis of atomic view of elasticity.

or

What is the cause of elasticity ?

Ans :

When a body is compressed, the atoms come close to each other on deformation. Due to this, the mean distance between atoms decreases and a restoring force comes into play. On the other hand, in case of extension, the distance between atoms increases which results in an attractive restoring force. In both the cases, the atoms will try to regain their original positions on removal of deforming force.

116. A cable is replaced by another of the same length and material but of twice diameter.

- How does this affect elongation under a given load ?
- How many times will be the maximum load supported by the latter as compared to the former?

Ans :

$$(a) \quad Y = \frac{FL}{al} = \frac{4FL}{\pi d^2 l}$$

where d is the diameter of the cable

$$\text{Now,} \quad l = \frac{4Fl}{\pi d^2 Y}$$

As $l \propto \frac{1}{d^2}$ the extension will become one-fourth on replacing by the cable of double the diameter.

$$(b) \quad F = \frac{\pi d^2 l Y}{4L}$$

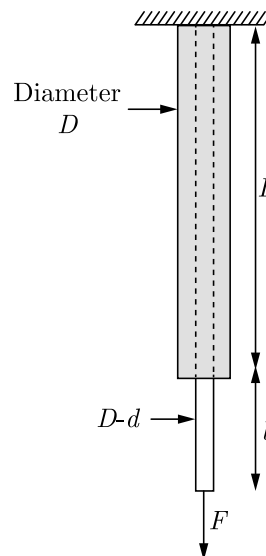
As $F \propto d^2$, the cable of double the diameter will be able to support a load four times as much as the former cable can support.

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117. What is Poisson's ratio ? Derive its expression. What are its units ?

Ans :

Poisson's ratio may be defined as the ratio of lateral strain to longitudinal strain.



$$\text{i.e.,} \quad \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \quad \dots(1)$$

If D = diameter of a wire

L = Length of the wire

F = force applied

l = increase in length of the wire

d = decrease in diameter of the wire

$$\text{Then, Lateral strain} = -\frac{d}{D} \quad \dots(2)$$

$$\text{Longitudinal strain} = \frac{l}{L} \quad \dots(3)$$

After putting equations (2) and (3) in (1), we get

$$\sigma = \frac{-\frac{d}{D}}{\frac{l}{L}} = -\frac{L}{D} \frac{d}{l}$$

Negative sign means that if l increases, D decreases and vice versa.

Since σ is the ratio of same quantities, so it has no units and no dimensions.

118. What do you mean by elastomers ? How does stress, strain graph take shape in case of rubber ?

Ans :

Some substances can be pulled to several times their original lengths and still return to their original value after the removal of deforming force (within elastic limits), such substances are called elastomer. e.g. rubber and aorta (large vessels carrying blood to the heart).

The graph between strain and stress relation for rubber like material, elastic tissue of aorta is as shown.

43. Consider the following regarding the properties of solids and fluids:

- I. Solids can withstand shearing stress, while ideal fluids cannot.
- II. Both solids and liquids have definite volumes at a given temperature and pressure.
- III. Gases have a definite shape and expand to fill their container.
- IV. The bulk modulus is a relevant elastic property for solids, liquids, and gases.

Which statements are correct?

- (A) I and II only
- (B) I, II, and IV only
- (C) II, III, and IV only
- (D) I, III, and IV only

Ans :

(B) I, II, and IV only

Statement I is correct. Statement II is correct. Statement III is incorrect; gases have no definite shape and fill their container. Statement IV is correct, as bulk modulus describes resistance to volume change.

44. Match the physical quantity in Column I with its dimensional formula in Column II.

	Column I		Column II
1.	Stress	P.	Dimensionless
2.	Strain	Q.	$[ML^{-1} T^{-2}]$
3.	Young's Modulus	R.	$[ML^2 T^{-2}]$
4.	Elastic Potential Energy per unit volume	S.	$[ML^{-1} T^{-2}]$

- (A) 1-Q, 2-P, 3-Q, 4-S
- (B) 1-Q, 2-P, 3-S, 4-Q
- (C) 1-S, 2-P, 3-S, 4-R
- (D) 1-P, 2-Q, 3-P, 4-S

Ans :

(A) 1-Q, 2-P, 3-Q, 4-S

Stress (Force/Area) has dimensions $[MLT^{-2}]/[L^2] = [ML^{-1} T^{-2}]$. Strain (Change in dimension/Original dimension) is dimensionless. Young's Modulus (Stress/Strain) has the same dimensions as stress, $[ML^{-1} T^{-2}]$. Elastic potential energy per unit volume (Energy/Volume) also has dimensions $[ML^2 T^{-2}]/[L^3] = [ML^{-1} T^{-2}]$, same as stress. Thus S is a duplicate for Q.

45. Which of the following is a correct pair regarding elastic behavior?

- (A) Poisson's Ratio : Ratio of longitudinal strain to lateral strain.

- (B) Regelation : Refreezing of water under increased pressure after melting.
- (C) Thermal Stress : Stress developed due to change in volume only.
- (D) Elastic Hysteresis : Region within the proportional limit of the stress-strain curve.

Ans :

(B) Regelation : Refreezing of water under increased pressure after melting.

Regelation is the phenomenon where ice melts under pressure and refreezes when the pressure is removed. Poisson's ratio is lateral strain / longitudinal strain. Thermal stress arises from prevented thermal expansion/contraction (change in length or volume). Elastic hysteresis is related to energy loss in cyclic loading, not the proportional limit.

46. Which of the following descriptions is INCORRECTLY paired with the term?

- (A) Elastic Limit : The maximum stress a material can withstand without permanent deformation.
- (B) Fracture Point : The point on the stress-strain curve where the material breaks.
- (C) Proportional Limit : The point up to which stress is directly proportional to strain.
- (D) Modulus of Rigidity : The ratio of tensile stress to shearing strain.

Ans :

(D) Modulus of Rigidity : The ratio of tensile stress to shearing strain.

Modulus of Rigidity (or Shear Modulus) is the ratio of shearing stress to shearing strain. Tensile stress is related to Young's modulus, not directly to shear modulus or shearing strain.

47. A metal wire is stretched by a gradually increasing load. Arrange the following phenomena in the order they occur:

1. Permanent set is observed.
2. Strain is directly proportional to stress.
3. Fracture occurs.
4. Yielding begins.

- (A) 2, 4, 1, 3
- (B) 2, 1, 4, 3
- (C) 4, 2, 3, 1
- (D) 1, 2, 3, 4

Ans :

(A) 2, 4, 1, 3

Initially, strain is proportional to stress (Hooke's law). Then, yielding begins (elastic limit/yield point is reached). If stressed further, a permanent set occurs upon unloading. Finally, with enough stress, fracture occurs.

48. The design of a bridge requires materials that can withstand various loads without excessive bending or breaking. Which elastic property is most crucial for selecting the material for the main support beams to minimize sagging?

(A) High coefficient of thermal expansion
(B) Low bulk modulus
(C) High Young's modulus
(D) High Poisson's ratio

Ans :

(C) High Young's modulus

Sagging or bending resistance is primarily determined by the material's Young's modulus (Y). A higher Young's modulus means the material is stiffer and will deform less (sag less) under a given load.

49. When a wire is stretched, work is done by the applied force. If the deformation is within the elastic limit, this work done is stored as:

(A) Kinetic energy of the atoms
(B) Heat energy dissipated
(C) Elastic potential energy
(D) Chemical energy

Ans :

(C) Elastic potential energy

When a wire is stretched elastically, the work done by the applied force against the inter-atomic restoring forces is stored within the material as elastic potential energy. This energy is recovered when the force is removed.

50. The phenomenon where ice melts at a lower temperature due to an increase in pressure, and then refreezes when the pressure is removed, is known as:

(A) Sublimation
(B) Regelation
(C) Vaporisation
(D) Fusion

Ans :

(B) Regelation

Regelation is the process where the freezing point of water is lowered by the application of pressure, causing ice to melt, and then refreeze once the pressure is removed. This is distinct from sublimation (solid to gas), vaporisation (liquid to gas), or simple fusion (melting).

51. With the rise of temperature the Young's modulus of elasticity:
(a) decreases

(b) increases
(c) changes erratically
(d) remains unchanged.

Ans :

(a) decreases

When temperature increases, the strain increases hence Y decreases.

52. The bulk modulus of a perfectly rigid body, is equal to :

(a) infinity
(b) zero
(c) some finite value
(d) non zero constant

Ans :

(a) infinity

For perfectly rigid body, $\Delta V = 0$

So, $k = \frac{pV}{\Delta V} = \alpha$

53. The shear modulus of elasticity of a liquid is:

(a) Infinity
(b) Unity
(c) Some finite, none zero constant
(d) Zero

Ans :

(d) Zero

Since the liquid does not resist change in its shape, so shear modulus is zero.

54. The reciprocal of bulk modulus of a substance is called its :

(a) Compressibility
(b) Rigidity
(c) Viscosity
(d) Modulus of elasticity

Ans :

(a) Compressibility

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}}.$$

55. If a metal wire is stretched a little beyond its elastic limit (or yield point), and released, it will :

(a) not contract
(b) lose its elastic property completely
(c) contract only up its length at the elastic limit
(d) contract, but its final length will be greater than its initial length.

Ans :

(d) contract, but its final length will be greater than its initial length

On stretching a wire a little beyond its elastic limit, there develops elastic and plastic properties and on removing the force, the wire is unable to return to its original length.

56. A copper wire and steel wire of same diameter and length are connected end to end and a force is applied which stretches their combined length by 1 cm. The two wires will have :

- (a) same stress and strain
- (b) different stress and strain
- (c) different stress and same strain
- (d) same stress and different strain.

Ans :

(d) same stress and different strain.

Since F and a are same, so stress $= \frac{F}{a}$ is same. But strain $\propto \frac{1}{Y}$ as Y is different for different wires, so strain is also different.

57. The potential energy of a stretched spring is proportional to:

- (a) the square of the force constant
- (b) the square of amount of stretch
- (c) the square of the original length
- (d) None of these.

Ans :

(b) the square of amount of stretch

The potential energy of a stretched spring

$$U = \frac{1}{2} kx^2 \text{ or } U \propto x^2.$$

58. The volume elasticity is possessed by:

- (a) solids only
- (b) liquids only
- (c) gases only
- (d) All the three states of matter.

Ans :

(d) All the three states of matter.

Volume elasticity (also called bulk modulus) refers to the ability of a substance to resist a change in volume under pressure. All three states of matter—solids, liquids, and gases—can experience changes in volume when subjected to pressure, though the degree varies. Hence, volume elasticity is a property possessed by all three states of matter.

59. The breaking stress of a wire depends upon

- (a) length of the wire
- (b) radius of the wire
- (c) material of the wire
- (d) shape of cross-section

Ans :

(c) material of the wire

Breaking stress is the maximum stress a material can withstand before breaking. It is an intrinsic property of the material and does not depend on the wire's length, radius, or shape of cross-section. Therefore, it depends only on the material of the wire.

60. Which of the following affects the elasticity of a substance ?

- (a) Hammering and annealing
- (b) Change in temperature
- (c) Impurity in substance
- (d) All of these

Ans :

(d) All of these

Elasticity of a substance depends on all the factors given in (a), (b) and (c).

Elasticity of a substance is affected by hammering and annealing, temperature changes, and the presence of impurities. Hammering aligns particles, annealing relieves stress, temperature alters molecular motion, and impurities disturb internal structure. Thus, all these factors influence elasticity.

ONE MARK QUESTIONS

61. What is the value of Young's modulus for a perfectly rigid body ?

Ans :

Infinite

62. What is meant by elasticity ?

Ans :

Elasticity is the property of a material body by virtue of which it regains its original configuration (or dimensions) after the removal of deforming forces.

63. What is a plastic body ?

Ans :

Plastic body is a body which does not regain its original dimensions after the removal of deforming forces.

64. State Hooke's law.

Ans :

Hooke's law states that stress is directly proportional to strain within the elastic limits.

65. Which of the two forces deforming or restoring is responsible for elastic behaviour of a substance ?

Ans :

Restoring forces are responsible for elastic behaviour of a substance.

66. Define Young's modulus of elasticity.

Ans :

Young's modulus is the ratio of normal stress to the longitudinal strain.

$$\text{i.e., } Y = \frac{\text{Normal stress}}{\text{Longitudinal Strain}}$$

67. On what factors, the modulus of elasticity depends?

Ans :

- (i) The nature of material used.
- (ii) Type of stress used in producing strain.

68. On what factors, the elastic limit of a material depends ?

Ans :

- (i) The nature of material used.
- (ii) Radius of wire.

69. Our knowledge about crystalline solids be better than amorphous solids. Why ?

Ans :

As crystalline solids possess long range and regular arrangement of atoms, hence their behaviour can be easily understood.

70. When does a body acquire a permanent set ?

Ans :

When deforming force exceeds the elastic limit, the body acquires the permanent set.

71. Which of the three elastic moduli possible in all the three states of matter ?

Ans :

Bulk modulus (K).

72. Which of the solids are the hardest ?

Ans :

Ionic solids.

73. Write copper, steel, rubber and glass in the increasing order of their elasticity.

Ans :

Rubber, glass, copper and steel.

74. An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support ?

Ans :

The maximum load that the wire can support will remain the same.

75. What is deforming force ?

Ans :

Deforming force is an external force which produces a change in the shape or size of a body.

76. What is a plasticity ?

Ans :

Plasticity is reverse of elasticity. It is the property of not regaining the original position after deforming force is removed.

77. "Stress is the force per unit area." Is it correct ?

Ans :

No, Stress is the restoring force per unit area of deformed body.

78. What are ductile materials ?

Ans :

Ductile substances are those substances which show large plastic range beyond elastic limit.

79. What are brittle materials ?

Ans :

Brittle materials are those which have small plastic range beyond elastic limit.

80. What are elastomers ?

Ans :

Elastomers are those substance for which stress is not proportional to strain. Strain varies much faster than stress

- 81.** A heavy wire is suspended from a roof but no weight is attached to its lower end. Is it under stress. Explain?

Ans :

Yes, the heavy wire is under stress, because the weight of the heavy wire acts as the deforming force.

- 82.** What is the expression for Young's modulus of elasticity ?

Ans :

$$Y = \frac{MgL}{\pi r^2 l},$$

where the symbols have their usual meanings.

- 83.** What is more elastic water or air, why ?

Ans :

Water is more elastic than air, because bulk modulus of elasticity is reciprocal of compressibility and air is more compressible than water.

- 84.** Some work has to be done in stretching a wire, where is this work done ?

Ans :

To stretch a wire, work has to be done against interatomic forces. This work is stored in the wire in the form of elastic potential energy.

- 85.** Why are electric poles given hollow structure ?

Ans :

Because hollow shaft is stronger than a solid shaft made from the same and equal amounts of material.

- 86.** Name the factors, which affect the property of elasticity of a solid.

Ans :

Presence of impurities, change in temperatures and effect of hammering, rolling and annealing.

TWO MARKS QUESTIONS

- 87.** Why springs are made of steel and not of copper ?

or

Why do we prefer steel to copper in the manufacture of spring ?

Ans :

A better spring is one, in which large restoring force is developed on being deformed, which in

turn depends upon elasticity of the material of the spring. As Young's modulus for steel is more than that of copper, steel is preferred to manufacture a spring.

- 88.** What is stress ? Give its S.I. unit.

Ans :

Stress is the restoring force per unit area in a body.

$$\text{i.e.,} \quad \text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

At equilibrium,

$$\text{Restoring force} = \text{Deforming force}$$

So stress may also be defined as the external force per unit area.

In S.I., the unit of stress is N m^{-2}

- 89.** Some work has to be done in stretching a wire. Why? What happens to the energy given to the wire in this process ?

Ans :

When a wire is stretched, its atoms are displaced from their respective positions and as such work is done against inter-atomic force of attraction. This work is stored up in the wire in the form of elastic potential energy.

- 90.** Why do spring balances show wrong readings after they have been used for a long time ?

Ans :

When spring balances are used for long time, elastic fatigue is developed in them and the balances take some time to regain their original configuration and hence they show wrong readings.

- 91.** What is elastic limit ?

Ans :

Elastic limit is that value of stress beyond which the body loses its property of elasticity and it does not regain its original length, shape or volume when the deforming force is removed.

It is the maximum stress that a body can bear and yet returns to its original state.

It is the characteristic of material of body.

- 92.** What is compressibility ? Give its units and dimensions.

Ans :

Compressibility is the reciprocal of bulk modulus of elasticity

- (C) Brittle materials show a large plastic deformation region before fracture.
 (D) Ductile materials show significant strain beyond the yield point before fracture.

Ans :

(C) Brittle materials show a large plastic deformation region before fracture.

Brittle materials fracture with very little or no plastic deformation. Ductile materials, on the other hand, exhibit a significant plastic deformation region before they fracture.

38. Identify the correct statement regarding the properties of materials from their stress-strain curves.

- (A) A material with a higher Young's modulus is more easily deformed.
 (B) The area under the stress-strain curve up to the fracture point represents the energy per unit volume required to fracture the material.
 (C) A ductile material will fracture very shortly after its elastic limit is reached.
 (D) For all materials, the proportional limit and the elastic limit are the same point.

Ans :

(B) The area under the stress-strain curve up to the fracture point represents the energy per unit volume required to fracture the material.

The area under the stress-strain curve represents the energy absorbed per unit volume (toughness). A higher Young's modulus means less deformation for a given stress. Ductile materials deform significantly after the elastic limit. Proportional and elastic limits can be different, though often close.

39. When a load is removed from a material that has been stressed beyond its yield point but before its fracture point, the material:

- (A) Returns completely to its original dimensions.
 (B) Fractures immediately upon removal of the load.
 (C) Retains some permanent deformation (permanent set).
 (D) Shows an increase in its Young's modulus value.

Ans :

(C) Retains some permanent deformation (permanent set).

If a material is stressed beyond its yield point, it enters the plastic deformation region. When the load is removed, the material will not return to its original dimensions but will exhibit a permanent set or residual strain.

40. All of the following are types of elastic moduli EXCEPT:

- (A) Young's Modulus
 (B) Shear Modulus
 (C) Stress Modulus
 (D) Bulk Modulus

Ans :

(C) Stress Modulus

Young's Modulus, Shear Modulus (Modulus of Rigidity), and Bulk Modulus are the three primary types of elastic moduli. "Stress Modulus" is not a standard term for an elastic modulus.

41. How does the shear modulus (G) generally compare to Young's modulus (Y) for most materials?

- (A) G is generally greater than Y.
 (B) G is generally equal to Y.
 (C) G is generally less than Y (often around $Y/3$).
 (D) There is no consistent relationship between G and Y.

Ans :

(C) G is generally less than Y (often around $Y/3$). For most materials, the shear modulus (G) is less than Young's modulus (Y). A common empirical relationship is that G is approximately $Y/3$, though this can vary.

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42. Assertion (A): The compressibility of gases is much larger than that of solids.

Reason (R): The bulk modulus of gases is very small compared to solids.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is NOT the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

Compressibility is the reciprocal of the bulk modulus. Gases have very low bulk moduli, meaning they are easily compressed (high compressibility). Solids have high bulk moduli, meaning they are difficult to compress (low compressibility). Therefore, the reason correctly explains the assertion.

- 104.** What are elastomers? Give an example and mention a characteristic from their stress-strain curve.

Ans :

Elastomers are substances that can be stretched to cause large strains and still return to their original shape, like rubber or the elastic tissue of the aorta. Their stress-strain curve shows a large elastic region but does not obey Hooke's law over most of it.

- 105.** Define Young's Modulus (Y) and state its significance.

Ans :

Young's modulus (Y) is the ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ϵ). It is a measure of a material's stiffness or resistance to elastic deformation under load; materials with large Y require large forces for small length changes.

- 106.** How does steel compare in elasticity to copper, brass, and aluminum? What is an implication?

Ans :

Steel has a larger Young's modulus than copper, brass, and aluminum, meaning it is more elastic. For this reason, steel is preferred in heavy-duty machines and structural designs where minimal deformation under load is critical.

- 107.** Define Shear Modulus (G). What is another name for it?

Ans :

The shear modulus (G), also called the modulus of rigidity, is the ratio of shearing stress to the corresponding shearing strain. It quantifies a material's resistance to shearing deformation, where layers slide relative to each other.

- 108.** How is Bulk Modulus (B) defined? What does a large bulk modulus indicate?

Ans :

Bulk modulus (B) is the ratio of hydraulic stress (pressure, p) to the corresponding hydraulic strain ($\Delta V/V$). A large bulk modulus indicates that the material is resistant to changes in volume under uniform pressure, meaning it has low compressibility.

- 109.** What is compressibility (k) and how does it relate to bulk modulus?

Ans :

Compressibility (k) is the reciprocal of the bulk modulus (B). It is defined as the fractional change in volume per unit increase in pressure, representing

how easily a material's volume can be reduced under pressure.

- 110.** Explain Poisson's Ratio and its nature.

Ans :

Poisson's ratio is the ratio of lateral strain (change in diameter/original diameter) to longitudinal strain (change in length/original length) in a stretched wire within the elastic limit. It is a dimensionless pure number that depends only on the material's nature.

- 111.** How is elastic potential energy stored in a stretched wire? Give the formula per unit volume.

Ans :

When a wire is put under tensile stress, work is done against inter-atomic forces, and this work is stored as elastic potential energy. The elastic potential energy per unit volume (u) is given by

$$u = \frac{1}{2} \times \text{stress} \times \text{strain}.$$

- 112.** Why is knowledge of material strength important in designing structures like buildings or bridges?

Ans :

Knowledge of material strength, related to elastic properties like yield strength and Young's modulus, is crucial to ensure that structural components like columns and beams can withstand expected loads without excessive deformation or failure, ensuring safety and durability.

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- 113.** Explain the importance of a material's yield strength when selecting it for a crane rope.

Ans :

The yield strength (σ_y) dictates the maximum stress a material can endure before permanent deformation occurs. For a crane rope, the stress due to the load (Mg/A) must not exceed this yield strength to prevent permanent stretching and potential failure.

- 114.** How does the depth of a beam affect its sagging under a load, according to the given formula?

Ans :

According to the formula $\delta = Wl^3 / (4bd^3 Y)$, the sag (δ) is inversely proportional to the cube of the depth (d^3). Therefore, increasing the depth of a beam is highly effective in reducing its bending or sagging under a given load.

Ans :

$$\text{Compressibility} = \frac{v}{pV} \text{ (in magnitude).}$$

$$\text{Units} = \frac{m^3}{Nm^{-2}m^3} = N^{-1}m^2$$

Dimensions.

$$\frac{[L^3]}{[ML^{-1}T^{-2}L^3]} = [M^{-1}L^1T^2]$$

93. Explain the property of elasticity in solid bodies.

Ans :

Elasticity is the property of a body by virtue of which it tends to regain its original size and shape when an applied deforming force is removed. This deformation is known as elastic deformation, and solids exhibiting this are crucial in engineering designs.

94. Differentiate between elastic and plastic deformation. Provide an example for plasticity.

Ans :

Elastic deformation is temporary; the body regains its original shape when the force is removed. Plastic deformation is permanent; the body does not regain its previous shape. Putty and mud are examples of materials exhibiting plasticity.

95. Define stress and state its SI unit.

Ans :

Stress is the restoring force per unit area developed within a body when it is subjected to a deforming force. If F is the force applied normal to cross-section A , stress is F/A . Its SI unit is $N m^{-2}$ or pascal (Pa).

96. Describe longitudinal stress and longitudinal strain.

Ans :

Longitudinal stress occurs when a cylinder is stretched or compressed by forces normal to its cross-sectional area. Longitudinal strain is the ratio of the change in length (ΔL) to the original length (L) of the body.

97. Explain shearing stress and shearing strain with an example.

Ans :

Shearing stress is the restoring force per unit area developed due to an applied tangential force on a cylinder's cross-section. Shearing strain is the ratio of the relative displacement (Δx) between opposite faces to the cylinder's length (L), like when a book is pushed horizontally.

98. What is hydraulic stress and volume strain? When do they occur?

Ans :

Hydraulic stress is the internal restoring force per unit area in a body compressed uniformly by a fluid, equal to the hydraulic pressure. Volume strain is the ratio of change in volume (ΔV) to the original volume (V) due to this pressure.

99. State Hooke's law and define the modulus of elasticity.

Ans :

Hooke's law states that for small deformations, stress is directly proportional to strain. The constant of proportionality, k , in the equation stress = $k \times$ strain, is known as the modulus of elasticity.

100. Describe the elastic behavior of a material in the linear region of its stress-strain curve.

Ans :

In the linear region of a stress-strain curve, Hooke's law is obeyed, meaning stress is proportional to strain. The material behaves as an elastic body and regains its original dimensions when the applied force is removed.

101. What is the yield point (elastic limit) and yield strength of a material?

Ans :

The yield point (point B in Fig. 8.2) is the point on the stress-strain curve beyond which stress and strain are not proportional, but the body still returns to its original dimension if the load is removed. The corresponding stress is yield strength (σ_y).

102. Explain what happens when a material is loaded beyond its yield strength.

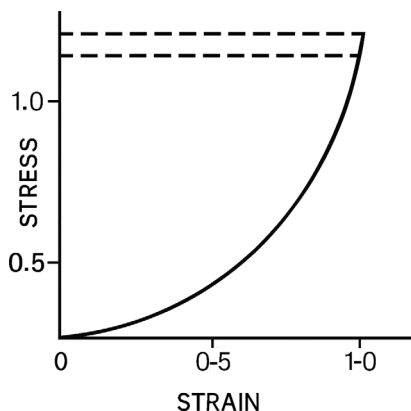
Ans :

If the load increases such that the stress exceeds the yield strength, strain increases rapidly even for a small change in stress. The material undergoes plastic deformation and does not regain its original dimensions when the load is removed, resulting in a permanent set.

103. Define ultimate tensile strength and fracture point from a stress-strain curve.

Ans :

The ultimate tensile strength is the maximum stress a material can withstand before additional strain is produced even by a reduced applied force. Fracture occurs at point E, where the material breaks.



There exists no well defined plastic flow region rubber just breaks when pulled beyond a certain limit. The Young's modulus is very small about $3 \times 10^5 \text{ N m}^{-2}$. Elastic region is very small and material does not obey Hooke's law at all.

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119. What are ductile and brittle materials ?

Ans :

Those materials in which there is large gap between the elastic limit and breaking point are called ductile materials, as shown in figure (a). So these materials can be converted into different shapes i.e. Au, Ag, Fe, Cu, quartz etc.

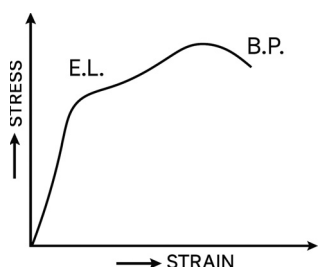


Figure: (a)

Those materials in which breaking point is very close to the elastic limit are called brittle materials as shown in figure (b). If the stress is increased beyond the elastic limit they are broken e.g. glass, cast iron etc.

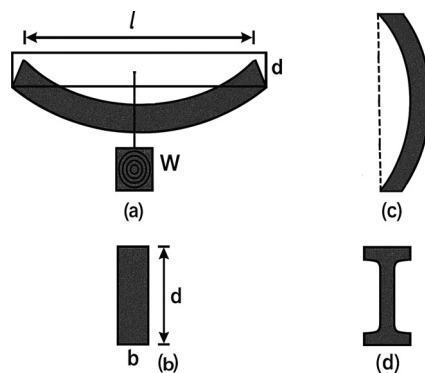


Figure: (b)

120. What is elastic after effect and elastic fatigue ?

Ans :

Elastic after effect is found that few solids take time to regain original shape or size even within the elastic limit. After the removal of deforming forces. The delay in regaining the original configuration (dimensions) of a body after the removal of deforming forces is called elastic after effect. Quartz regains its original configuration immediately and glass fibre may take hours to come back to its original position after the removal of deforming forces.

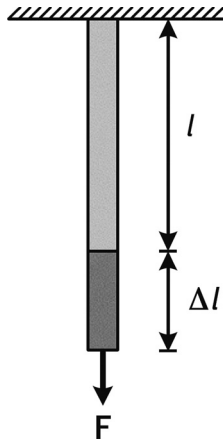
When a wire is made to vibrate for a long time, it is found that the time of vibration is much larger than when the wire is fresh. Lord Kelvin suggested that the wire gets tired due to continuous vibrations which is called elastic fatigue. Thus elastic fatigue is the loss of strength of the material due to repeated alternating stress to which the material is subjected is called elastic fatigue. This phenomenon is of great importance in cases like those of the piston and the connecting rods used in a locomotive. Bridges are declared unfit after certain time due to elastic fatigue.

121. Detail the key regions and points on a typical stress-strain curve for a metal.

Ans :

A typical stress-strain curve for a metal reveals its behavior under tensile stress. Key regions and points include:

- **Elastic Region (O-A):** The curve is linear, and Hooke's law is obeyed; the material returns to its original shape when the load is removed.
- **Yield Point (B):** Also known as the elastic limit; beyond this point, permanent deformation (plastic deformation) begins. The corresponding stress is the yield strength (σ_y).



Longitudinal Strain,

$$\varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

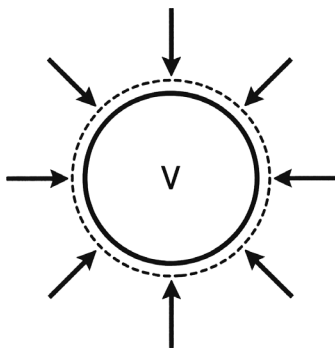
It has no units as it is the ratio of lengths.

2. **Volumetric strain** : It is defined as change in volume per unit volume

Let V = Original volume of a ball

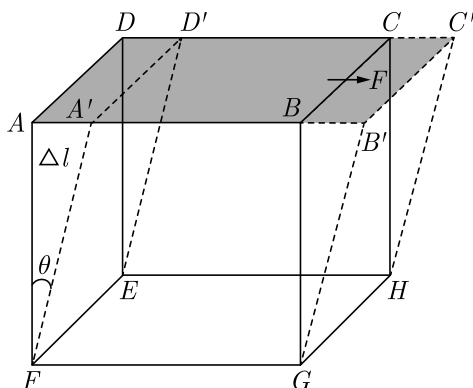
v = Change in volume of ball

$$\text{Volume strain} = \frac{v}{V}$$



It is a ratio of volumes, so it has no units

3. **Shearing strain** : Shearing strain may be defined as the angle through which a line originally perpendicular to the fixed face is turned.



Suppose face EFGH of a rectangular block is kept fixed and a tangential force F be applied along its upper face in the direction as shown. Various layers displace in their own planes angle θ . They are said to be sheared, θ is called angle of shear

$$\begin{aligned}\theta &= \tan \theta \\ &= \frac{AA'}{AD} \\ &= \frac{\text{Relative displacement between two } \parallel \text{ planes}}{\text{Distance of separation between them}}\end{aligned}$$

Shearing strain may also be defined as the relative displacement between two parallel planes at a unit distance apart.

132. Define modulus of elasticity. Explain its kinds.

Ans :

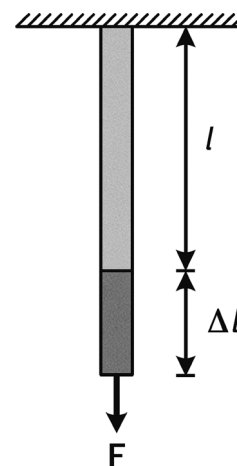
Modulus of elasticity is also called coefficient of elasticity. It is called as the ratio of stress to strain within elastic limits.

$$\text{Modulus of elasticity, } E = \frac{\text{Stress}}{\text{Strain}}$$

Types of modulus of elasticity

- (i) **Young's Modulus (Y)** : It is defined as ratio of normal stress to longitudinal strain within elastic limits.

Consider a wire of length L and area of cross-section ' a ' suspended from a rigid support.



Let

F = Stretching force and

l = Change in length

$$\text{Normal strain, } \sigma = \frac{F}{a}$$

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- A large load-bearing surface to distribute the applied forces.
- Sufficient depth (d) which is crucial in reducing bending, as sag is inversely proportional to d^3 .
- Reduced weight compared to a solid rectangular beam of similar depth and load-bearing capacity, which reduces material cost.
- This shape also helps prevent buckling, which can occur in very deep, narrow bars.

129. Explain the concept of thermal stress developed in a rod whose ends are rigidly fixed.

Ans :

Thermal stress arises when a material is prevented from expanding or contracting due to temperature changes.

- If a rod's ends are rigidly fixed, an increase in temperature will cause it to try to expand.
- Since expansion is prevented by the rigid supports, the rod acquires a compressive strain.
- This strain leads to an internal stress, known as thermal stress, which can be calculated using Young's modulus and the coefficient of linear expansion.

130. What is plasticity? How does it differ from elasticity?

Ans :

Plasticity is a property of materials where they undergo permanent deformation when subjected to an applied force, meaning they do not regain their original shape or size after the force is removed.

- This contrasts with elasticity, where a material tends to return to its original dimensions after the deforming force is removed.
- Putty and mud are examples of plastic materials.

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FIVE MARKS QUESTIONS

- 131.** (a) Define the term elasticity in physics.
(b) Define stress and strain. What are the units in which these quantities are measured.

Ans :

- (a) The property of matter due to which it regains its original length, shape or volume when the deforming forces have been removed is called Elasticity.

When a force is applied on a body, it is deformed or a change appears either in length, shape or vol-ume. The atomic forces of attraction inside the body immediately set up restoring forces which offer resistance to deformation and try to bring the body back to its original length, shape or volume when deforming force is removed.

A body which completely recovers its original length, shape or volume is called a perfectly elastic body.

Quartz is the nearest approach to perfectly elastic body.

- (b) The restoring force set up in the body itself per unit area is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

Since restoring force is a reaction of deforming force, it is numerically equal to it.

Stress is measured as deforming force per unit area.

$$\text{Stress} = \frac{\text{Deforming force}}{\text{Area}} = \frac{F}{A}$$

where F = weight pulling the wire

- (i) **Normal Stress :** If the deforming forces are acting normal to the surface of the body, the stress is named as normal stress.

If the normal causes increases in length it is called Tensile stress but if causes decreases in length it called compressional stress.

- (ii) **Tangential Stress :** If the deforming forces are tangential to the surface, the stress is named as tangential stress.

Units of stress :

$$\text{Stress} = \frac{F}{A}$$

SI unit of stress is newton m^{-2} (N m^{-2})

Strain : Whenever an external force is applied on a body, a change in configuration of the body takes place and the body is said to be deformed.

Strain may be defined as the ratio of change produced in certain configuration to the original configuration of the body.

i.e.,
$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

There are three kinds of strain

1. **Longitudinal strain :** It is defined as the change in length per unit original length

If L = Original length of a wire.

l = Change in length when subjected to force.

NUMERICAL QUESTIONS

- 137.** What force is required to stretch a steel wire 1 sq. cm in cross-section to double its length ?
($Y = 2 \times 10^{11} \text{ N m}^{-2}$)

Ans :

Given,

$$a = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2,$$

$$F = ? \quad L = x \quad l = x$$

$$Y = 2 \times 10^{11} \text{ N m}^{-2}$$

$$Y = \frac{F \times L}{a \times l} = \frac{F \times x}{ax}$$

$$Y = \frac{F}{a}$$

$$F = Ya$$

$$= 2 \times 10^{11} \times 10^{-4}$$

$$= 2 \times 10^7 \text{ N}$$

- 138.** A one metre long wire having area of cross-section 1 mm^2 is stretched by a mass of 3 kg. Calculate the increase in length. Take $Y = 2 \times 10^{11} \text{ N m}^{-2}$

Ans :

Given, $L = 1 \text{ m}$, $a = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$, $m = 3 \text{ kg}$

$$Y = 2 \times 10^{11} \text{ N m}^{-2}, \quad l = ?$$

Since, $Y = \frac{F \times L}{a \times l} = \frac{mgL}{al}$

$$l = \frac{mgL}{aY}$$

$$l = \frac{3 \times 9.8 \times 1}{10^{-6} \times 2 \times 10^{11}}$$

$$= 14.7 \times 10^{-5} \text{ m}$$

- 139.** A stress of 10^7 N m^{-2} is required to break a material of density $4 \times 10^4 \text{ kg m}^{-3}$. Calculate the length of a wire of this material, which can break under its own weight.

Ans :

Given,

$$\text{Stress} = 10^7 \text{ N m}^{-2}, \quad \rho = 4 \times 10^4 \text{ kg m}^{-3}$$

Since, $\text{stress} = \frac{\text{Force}}{\text{area}} = \frac{mg}{a} = \frac{V\rho g}{a}$

$$\text{Stress} = \frac{(al)\rho g}{a} = l\rho g$$

$$l = \frac{\text{stress}}{\rho g} = \frac{10^7}{4 \times 10^4 \times 9.8} = 25.5 \text{ m}$$

- 140.** The length of a metallic wire is L_1 when tension on the wire is T_1 and length is L_2 when the tension in the wire is T_2 . Find original length of the wire.

Ans :

If L is the length and a is the area of cross-section of the wire and l is the extension produced when a force F is applied,

then, $Y = \frac{\frac{F}{a}}{\frac{l}{L}} = \frac{FL}{al}$

In 1st case

$$F = T_1 \text{ and } l = L_1 - L$$

$$Y = \frac{T_1 L}{a(L_1 - L)} \quad \dots(1)$$

In 2nd case

$$F = T_2 \text{ and } l = L_2 - L$$

$$Y = \frac{T_2 L}{a(L_2 - L)} \quad \dots(2)$$

From (1) and (2), we have

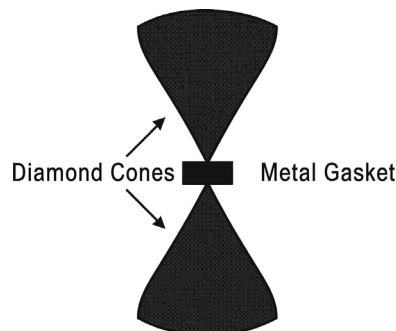
$$\frac{T_1 L}{a(L_1 - L)} = \frac{T_2 L}{a(L_2 - L)}$$

$$T_1(L_2 - L) = T_2(L_1 - L)$$

$$L(T_2 - T_1) = T_2 L_1 - T_1 L_2$$

$$L = \frac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$$

- 141.** Anvils made of single crystals of diamond, with the shape as shown in the figure are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvils have a diameter of 0.50 mm and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?



Ans :

Given, $F = 50,000 \text{ N} = 5 \times 10^4 \text{ N}$

$$A = \frac{\pi d^2}{4} = \frac{3.14}{4} \times (0.5 \times 10^{-3})^2$$

Longitudinal Strain,

$$\varepsilon = \frac{l}{L}$$

Young's modulus, $Y = \frac{\frac{F}{a}}{\frac{l}{L}} = \frac{\sigma}{\varepsilon}$

$$\sigma = \varepsilon Y$$

If

r = radius of wire

$$a = \pi r^2 = \pi \frac{D^2}{4}$$

$$Y = \frac{4F \times L}{\pi D^2 l}$$

In S.I Units, Y , is expressed in newton m^{-2} (N m^{-2})

(ii) **Bulk Modulus (K)** : It is defined as the ratio of normal stress to volumetric strain.

Suppose,

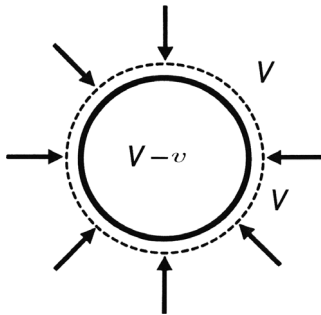
V = Volume of a sphere of area of surface.

v = Change in volume which is caused

due. to normal force F

$$\text{Normal stress} = \frac{F}{a}$$

$$= \Delta p, \text{ pressure on the body.}$$



$$\text{Volumetric strain} = \frac{v}{V}$$

Bulk modulus of elasticity,

$$B = -\frac{\Delta p}{\frac{v}{V}} = \frac{\Delta p \times V}{v}$$

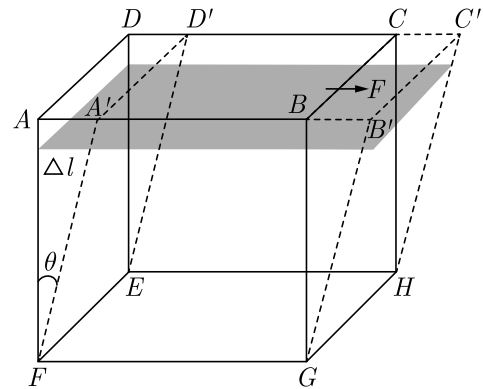
For a stable system negative sign indicates that increases in pressure leads to decreases in volume. S.I. unit of B is newton m^{-2} . B is also called incompressibility.

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility

$$\text{Compressibility} = \frac{1}{K}$$

(iii) **Modulus of rigidity (η)** : It is defined as the ratio of tangential stress to the shearing strain, within elastic limits.

Consider a cube $ABCDEFGH$. Keeping the lower face $EFGH$ fixed, if a tangential force F be applied to surface $ABCD$, the cube is said to be under shearing strain.



$$\text{Shearing strain} = \theta = \frac{l}{L}$$

If θ is small, tangential stress or shearing stress is given by $\sigma_s = \frac{F}{a}$ where a is the area of each face of cube.

Modulus of elasticity,

$$\eta = \frac{\sigma_s}{\theta} = \frac{\frac{F}{a}}{\frac{l}{L}}$$

$$\eta = \frac{F \times L}{a \times l}$$

133. (a) What is elastic limit ?
(b) State Hooke's law. Verify it experimentally.

Ans :

- (a) Elastic limit is that value of stress beyond which the body loses its property of elasticity and it does not regain its original length, shape or volume when the deforming force is removed. It is the maximum stress that a body can bear and yet returns to its original state. It is the characteristic of material of body.
(b) Within elastic limits, stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = E$$

Experimental Verification of Hooke's law :

Suspend a spring balance from a rigid support. Attach a pointer to the spring which moves along a linear scale. Note the reading of pointer on scale when pan is empty. Add a weight of 5 g in the pan so that spring increases in length. Again note the reading of the pointer.

- **Ultimate Tensile Strength (D):** The maximum stress the material can withstand.
- **Fracture Point (E):** The point where the material breaks.

122. Define Young's modulus. Why is it an important property for materials like steel in engineering?

Ans :

Young's modulus (Y) is a measure of a material's stiffness or resistance to elastic deformation under tensile or compressive stress. It is defined as:

- The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ϵ).
- Materials like steel have large Young's moduli, meaning they require a large force to produce a small change in length. This makes steel highly desirable for structural designs and heavy-duty machines where maintaining shape under load is critical.

123. What is shear modulus? How is it defined and what is its general relation to Young's modulus?

Ans :

Shear modulus (G), also known as the modulus of rigidity, quantifies a material's resistance to shearing deformation.

- It is defined as the ratio of shearing stress (σ_s) to the corresponding shearing strain.
- The shearing strain can be expressed as $\Delta x/L$ or approximately θ , where Δx is the relative displacement and L is the length.
- For most materials, shear modulus is generally less than Young's modulus, often approximated as $G \approx Y/3$.

124. Explain bulk modulus and its significance for solids, liquids, and gases.

Ans :

Bulk modulus (B) measures a substance's resistance to uniform compression. It is defined as:

- The ratio of hydraulic stress (pressure, p) to the corresponding volumetric strain ($\Delta V/V$), given by $B = -p/(\Delta V/V)$.
- Solids have much larger bulk moduli than liquids, which in turn are much larger than those for gases.
- This signifies that solids are least compressible, while gases are most compressible due to the varying strength of intermolecular coupling.

125. Describe Poisson's ratio. What kind of strains does it relate?

Ans :

Poisson's ratio describes the phenomenon where a material tends to contract (or expand) in directions perpendicular to the direction of stretching (or compression).

- It is defined as the ratio of lateral strain to the longitudinal strain within the elastic limit of a stretched wire.
- Lateral strain is the fractional change in diameter ($\Delta d/d$), while longitudinal strain is the fractional change in length ($\Delta L/L$).
- Poisson's ratio is a dimensionless number dependent only on the material's nature.

126. Discuss how elastic potential energy is stored in a stretched wire.

Ans :

When a wire is subjected to a tensile stress and elongates, work is done against the inter-atomic forces within the material. This process involves:

- The applied force doing positive work to overcome the internal restoring forces of the spring.
- This work done is stored in the wire as elastic potential energy.
- The elastic potential energy per unit volume (u) can be expressed as

$$u = (1/2) \times \text{stress} \times \text{strain}.$$

127. Explain the factors considered when determining the thickness of a steel rope for a crane with a specific lifting capacity.

Ans :

When determining the thickness of a steel rope for a crane, several factors related to its elastic behavior are considered:

- The rope must withstand the load without permanent deformation, so the stress induced must not exceed the steel's yield strength (σ_y).
- The minimum cross-sectional area (A) required is calculated as $A \geq W/\sigma_y$, where W is the load (Mg).
- A significant margin of safety (often a factor of ten) is incorporated to account for dynamic loads and unforeseen stresses, leading to a thicker recommended rope.

128. Why are beams used in construction often given an I-shaped cross-section?

Ans :

Beams in construction are often I-shaped to optimize strength and weight. This design offers:

- (ii) **Designing of bridges :** A bridge should be designed such that under the load of traffic, the force of winds and under its own weight, it should not bend too much or break. A bar of length l , breadth b , and depth d sags by amount δ under a load W at its mid-point, where

$$\delta = \left(\frac{Wl^3}{4bd^3Y} \right)$$

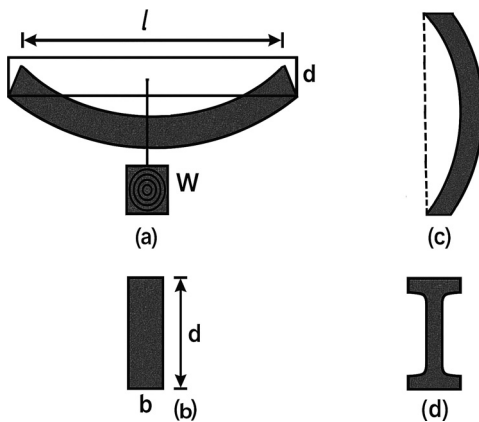


Figure shows the bar. From the above equation, we see that to reduce the bending for a given load, one should use a material with a large Young's modulus Y and for a given amount of material, increase the depth rather than the breadth the bar since δ is proportional to d^3 and only to b^{-1} . But on increasing the depth, unless the load is exactly at the right place, the deep bar may be as shown in figure (a). This is called buckling. A common compromise is the cross-sectional shape in figure (a), with a large load bearing surface to prevent buckling and enough depth, to prevent bending too much.

- (iii) **Bridges declared unfit after long use :** Bridges are made up of iron and other material and are under continuous stress and strain due to continuous flow of traffic, so the bridge material loses its elastic strength and large strains are developed corresponding to same stress and it may lead to collapse, hence the bridges are declared unfit after certain time.
- (iv) A hollow shaft is found to be stronger than a solid shaft made of same and equal material.
- (v) Any metallic part of a machinery is never subjected to a stress beyond the elastic limit of the material to avoid permanent deformation in the metallic parts.

136. What do you understand by potential energy in a stretched wire ? Find relation for it and hence determine the elastic potential energy per unit volume of the wire. (i.e. energy density).

Ans :

To produce strain in a body (longitudinal bulk or shear), deforming forces has to do work. This work done by deforming forces is stored in the form of elastic potential energy. When deforming forces are removed, this elastic potential energy appears in the form of heat. The potential energy per unit volume of the deformed body is called energy density.

Consider a wire of length L and area of cross-section ' a ' fixed at one end and stretched by a normal force F applied to the free end.

If l is the increase in the length of the wire, then

$$Y = \frac{FL}{al}$$

$$F = \frac{Ya}{L}l \quad \dots(1)$$

Let the wire be stretched further through an infinitesimally small distance dl under the constant force F . Then small amount of work done

$$dW = F \cdot dl$$

$$dW = \frac{Ya}{L}l \cdot dl$$

Total work done in producing extension l is given by

$$W = \int dW = \frac{Ya}{L} \int_0^l l \cdot dl$$

$$W = \frac{1}{2} \frac{Ya}{L} l^2$$

Using equation (1), we get

$$W = \frac{1}{2} F \cdot l$$

Work done per unit volume

$$= \frac{W}{\text{Volume}} = \frac{W}{a \cdot L}$$

$$= \frac{F \cdot l}{2 \cdot a \cdot L} \quad \dots(2)$$

Since, $\frac{F}{a} = \text{normal stress}$

and $\frac{l}{L} = \text{longitudinal strain}$

So equation (2) becomes

Work done per unit vol. (i.e. energy density)

$$= \frac{1}{2} \text{ normal stress} \times \text{longitudinal strain.}$$

$$7,644(94.81 - L) = 6,664(95 - L)$$

$$L = 93.52 \text{ cm}$$

- 145.** Compare the densities of water at the surface and bottom of a lake 100 m deep, given that the compressibility is $\frac{10^{-3}}{22}$ per atm and 1 atm = 1.015×10^5 Pa.

Ans :

$$\text{Since compressibility} = \frac{1}{K} = \frac{10^{-3}}{22} \text{ per atm.}$$

$$K = \frac{22}{10^{-3}} \text{ atm}$$

$$= 22 \times 10^3 \times 1.015 \times 10^5 \text{ Pa}$$

$$(1 \text{ atm} = 1.015 \times 10^5 \text{ pa})$$

Let V = Volume of 1 kg water at the surface.

V' = Volume of 1 kg water at the bottom
of lake 100 m deep

$$= V - \Delta V$$

where ΔV = decrease in volume.

Increase in pressure,

$$P = h\rho g$$

$$= 100 \times 10^3 \times 9.8 \text{ N m}^{-2}$$

From the relation, $K = \frac{P}{\left(\frac{\Delta V}{V}\right)}$, we get

$$\frac{\Delta V}{V} = \frac{P}{K}$$

$$= \frac{100 \times 10^3 \times 9.8}{22 \times 1.015 \times 10^8}$$

$$= \frac{9.8}{22 \times 1.015} \times 10^{-3}$$

If ρ_s and ρ_b be the densities of water at the surface and at the bottom of lake respectively, $\rho_s = \frac{1}{V}$, $\rho_b = \frac{1}{V'}$

Then, $\frac{\rho_s}{\rho_b} = \frac{\text{Density of water at the surface}}{\text{Density of water at the bottom}}$

$$\frac{\frac{1}{V}}{\frac{1}{V - \Delta V}} = \frac{V - \Delta V}{V}$$

$$1 - \frac{\Delta V}{V} = 1 - \frac{9.8 \times 10^{-3}}{22 \times 1.015}$$

$$= 0.99956$$

- 146.** A wire stretches a certain amount under a load. If the load and diameter are both increased to three times, find the stretched caused in the wire.

Ans :

$$\text{Since, } Y = \frac{FL}{Al}$$

$$l = \frac{FL}{AY}$$

$$l \propto \frac{F}{A}$$

$$\text{so } \frac{l'}{l} = \frac{3F}{F} \times \frac{A}{9A} = \frac{1}{3}$$

CASE BASED QUESTIONS

- 147.** Engineers are designing a crane for a construction site with a lifting capacity of 10 metric tons (10,000 kg). They need to select a steel rope of appropriate thickness. The yield strength (σ_y) for mild steel is approximately $300 \times 10^6 \text{ N/m}^2$. The design must ensure that the rope does not deform permanently under the maximum load, and a safety factor is usually included.

Questions:

- What is yield strength (σ_y) of a material?
- Why is it important that the stress in the crane rope does not exceed the elastic limit of the steel?
- Calculate the minimum cross-sectional area (A) of the steel rope required to lift 10 metric tons without exceeding its yield strength. (Use $g = 9.8 \text{ m/s}^2$).
- The ropes are made of a number of thin wires braided together. What are the advantages of this construction over a single solid wire of the same radius?

Ans :

- The yield point (or elastic limit) is the point on the stress-strain curve beyond which the body does not regain its original dimension when the load is removed, and the corresponding stress is known as yield strength (σ_y) of the material.
- If the stress exceeds the elastic limit (yield strength), the rope will undergo plastic deformation and will not return to its original dimensions when the load is removed, leading to permanent damage and potential failure.
- The maximum weight

$$W = Mg$$

$$= 10^4 \text{ kg} \times 9.8 \text{ m/s}^2$$

$$= 9.8 \times 10^4 \text{ N.}$$

$$= \frac{3.14 \times 0.25 \times 10^{-6}}{4}$$

$$= 0.196 \times 10^{-6} \text{ m}^2$$

Pressure at the top of the anvil,

$$P = \frac{F}{A} = \frac{5 \times 10^{-4}}{0.196 \times 10^{-6}}$$

$$= 2.55 \times 10^{11} \text{ Nm}^2$$

$$= 2.5 \times 10^{11} \text{ pa}$$

- 142.** Calculate the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg f. Y for the wire = $12.5 \times 10^{11} \text{ dyne cm}^{-2}$

Ans :

Given, $Y = 12.5 \times 10^{11} \text{ dyne cm}^{-2}$

$$= 12.5 \times 10^{10} \text{ N m}^{-2}$$

Diameter, $D = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$

Radius $r = \frac{D}{2} = 1.25 \times 10^{-3} \text{ m}$

$$F = 100 \text{ kg}$$

$$f = 100 \times 9.8 \text{ N} = 980 \text{ N}$$

$$\frac{\Delta L}{L} \times 100 = ?$$

$$A = \pi r^2 = \pi (1.25 \times 10^{-3})^2 \text{ m}^2$$

Using the relation

$$Y = \frac{FL}{A \Delta L}, \text{ we get}$$

Percentage increase in length

$$= \frac{\Delta L}{L} \times 100$$

$$= \frac{F}{AY} \times 100 = \frac{F}{\pi r^2 Y} \times 100$$

$$= \frac{980}{3.14 \times (1.25 \times 10^{-3})^2 \times (12.5 \times 10^{10})} \times 100$$

$$= 15.96 \times 10^{-2} = 0.16\%$$

- 143.** The Young's modulus for Steel is $2.0 \times 10^{11} \text{ N m}^{-2}$. If the interatomic spacing for the metal is $2.8 \text{ \AA}$, find the increase in the interatomic spacing for a force of 10^9 N m^{-2} and the force constant.

Ans :

Given, $Y = 2.0 \times 10^{11} \text{ N m}^{-2}$

$$l = 2.8 \times 10^{-10} \text{ m}, \frac{F}{A} = 10^9 \text{ N m}^{-2}$$

$$\Delta l = \frac{F}{A} \times \frac{l}{Y}$$

As $Y = \frac{F}{A} \times \frac{l}{Y} = 10^9 \times \frac{2.8 \times 10^{-10}}{2 \times 10^{11}}$

$$= 1.4 \times 10^{-12} \text{ m} = 0.014 \text{ \AA}$$

As distance between two atoms is l , then area of one chain of atoms, $A = l \times l = l^2$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l} = \frac{F}{\Delta l} \times \frac{l}{A}$$

$$= \frac{F}{\Delta l} \times \frac{l}{l^2} = \frac{F}{\Delta l} \times \frac{1}{l}$$

Force constant,

$$k = \frac{F}{\Delta l} = Yl$$

$$= 2.0 \times 10^{11} \times (2.8 \times 10^{-10})$$

$$= 56 \text{ N m}^{-1}$$

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- 144.** A wire loaded by a weight of density 7.8 g cm^{-3} is found to be of length 95 cm. On immersing the weight in water, the length decreased by 0.19 cm. Find the original length of the wire.

Ans :

Let length of wire = L

Area of cross-section of the wire = a

If V is the volume of the weight attached to the wire, then

$$F = V\rho g = V \times 7.8 \times 980$$

$$= 7,644 V \text{ dyne}$$

Also, extension in the wire, $l = 95 - L$

Now, $Y = \frac{FL}{a l}$

$$Y = \frac{7,644 V \times L}{a \times (95 - L)} \quad \dots(1)$$

When the weight is immersed in water :

Apparent weight,

$$F' = F - \text{weight of the water displaced}$$

$$F' = V\rho g - V\sigma g = V(\rho - \sigma)g$$

$$= V \times (7.8 - 1) \times 980$$

$$F' = 6,664 V \text{ dyne}$$

$$L = 95 - 0.19 = 94.81 \text{ cm and}$$

$$l' = (94.81 - L)$$

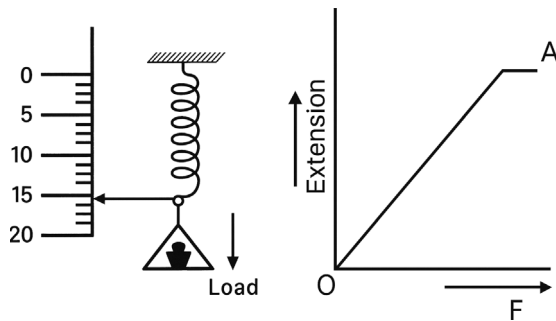
Now, $Y = \frac{F' L}{a l'}$

$$Y = \frac{6,664 V \times L}{a \times (94.81 - L)} \quad \dots(2)$$

From the equations (1) and (2), we have

$$\frac{7,644 V \times L}{a \times (95 - L)} = \frac{6,664 V \times L}{a \times (94.81 - L)}$$

The difference of these two readings will give extension for 5 g. Add a weight of 5 g more and find extension for 10 g and repeat the experiment for more weights. Draw a graph between force in g and extension in cm. The graph is a straight line OA .



The constant E is called modulus of elasticity or coefficient of elasticity of the given material.

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134. Discuss stress-strain graph.

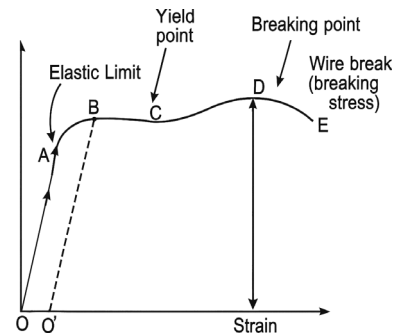
Ans :

Consider a wire, fixed at its upper end to a rigid support. Load the wire gradually at its lower end and note the values of extension produced in the wire for different loads.

The graph between strain and stress has the shape as shown in figure.

The straight portion OA of the curve shows that stress is directly proportional to strain. Therefore, Hooke's law is obeyed upto point A . On being unloaded at any point between O and A , the wire will come back to its original length. Point A corresponds to the elastic limit. So elastic limit of a material may also be defined as the maximum stress that may be applied to the material so that the strain produced in the material disappears when the stress is removed. This is called elastic behaviour.

Beyond the elastic limit (point A), the stress is no longer proportional to strain. The curve bends towards the strain axis. If the wire is unloaded after reaching a point B , the curve does not follow the path BA instead it follows the path BO . This shows that when stress becomes zero, strain does not become zero. A permanent strain OO' is left in the wire. Thus beyond point A , the wire shows the plastic behaviour.



At point C , the curve becomes almost parallel to the strain axis. This means that the wire would go on increasing in length even when almost no more load is applied. The point C on the curve is called the yield point. If the wire is to be prevented from breaking the load applied to it has to be decreased.

Once the yield point is crossed, the thinning of the wire no longer remains uniform, its cross-section decreases more rapidly at some points than at others. The wire ultimately breaks. This point D on the curve at which the wire breaks is called the breaking point.

Breaking Stress : The maximum stress which the material of the wire is able to bear without breaking it is called the breaking stress or ultimate strength of the material.

135. Discuss applications of elasticity.

Ans :

Elasticity is useful in designing things in such a way that they continue to function under a given load.

Examples

- (i) **Thickness of rope used for lifting :** Many cranes used for lifting and moving heavy loads from one place to another have a thick metal rope to which the load is attached. The rope is pulled up using pulleys and motors. Suppose we want to pull up (lift) a load of 10 metric tons. We know that elastic limit corresponds to a stress or load of $30 \times 10^7 \frac{N}{m^2}$

Thus the area of the rope should be at least

$$\begin{aligned} A &\geq W/\sigma_e = \frac{Mg}{\sigma_e} \\ &= \frac{(10^4 \times 10)}{(30 \times 10^7)} \\ &= 3.3 \times 10^{-4} \text{ m}^2 \end{aligned}$$

corresponding to a radius of about 1 cm for a rope of circular cross-section. Generally a large margin of safety is provided, so the rope is thicker.

The minimum area

$$\begin{aligned} A &\geq \frac{W}{\sigma_y} \\ &= \frac{9.8 \times 10^4 \text{ N}}{300 \times 10^6 \text{ N/m}^2} \\ &\approx 3.27 \times 10^{-4} \text{ m}^2. \end{aligned}$$

4. Ropes made of thin wires braided together provide flexibility and strength and are easier to manufacture compared to a single solid wire of large radius which would be practically rigid.

- 148.** A structural steel rod has a radius of 10 mm (0.01 m) and a length of 1.0 m. A force of 100 kN ($100 \times 10^3 \text{ N}$) stretches it along its length. The Young's modulus (Y) of structural steel is $2.0 \times 10^{11} \text{ N/m}^2$.

Questions:

1. Define Young's modulus (Y).
2. What is longitudinal stress and longitudinal strain?
3. Calculate the stress on the steel rod.
4. Determine the elongation (ΔL) of the rod.

Ans :

1. Young's modulus (Y) is defined as the ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ϵ).
2. Longitudinal stress is the restoring force per unit area when a force is applied normal to the cross-sectional area, causing a change in length. Longitudinal strain is the change in length (ΔL) to the original length (L) of the body.
3. Area

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times (10^{-2} \text{ m})^2 \\ &= 3.14 \times 10^{-4} \text{ m}^2. \end{aligned}$$

$$\begin{aligned} \text{Stress} &= \frac{F}{A} \\ &= \frac{100 \times 10^3 \text{ N}}{3.14 \times 10^{-4} \text{ m}^2} \\ &\approx 3.18 \times 10^8 \text{ N/m}^2. \end{aligned}$$

4. Elongation

$$\begin{aligned} \Delta L &= \frac{(\text{Stress}) \times L}{Y} \\ &= \frac{(3.18 \times 10^8 \text{ N/m}^2) \times (1.0 \text{ m})}{2.0 \times 10^{11} \text{ N/m}^2} \\ &= 1.59 \times 10^{-3} \text{ m or } 1.59 \text{ mm}. \end{aligned}$$

- 149.** In a human pyramid circus act, the entire weight of the balanced group is supported by the legs of a performer lying on his back. The combined mass of all persons and equipment above him is 220 kg. Each thighbone (femur) of this performer

has a length of 50 cm and an effective radius of 2.0 cm. The Young's modulus for bone is given as $9.4 \times 10^9 \text{ N/m}^2$.

Questions:

1. Which type of stress are the thighbones primarily subjected to in this scenario?
2. The weight is supported by two thighbones. How much weight does each thighbone support?
3. Calculate the cross-sectional area of one thighbone.
4. Determine the amount by which each thighbone gets compressed (ΔL) under this load. (Use $g = 9.8 \text{ m/s}^2$).

Ans :

1. The thighbones are primarily subjected to compressive stress.
2. Weight supported by both legs

$$= 220 \text{ kg} \times 9.8 \text{ m/s}^2 = 2156 \text{ N}.$$

Weight supported by each thighbone

$$= \frac{1}{2} \times 2156 \text{ N} = 1078 \text{ N}.$$

3. Radius $r = 2.0 \text{ cm} = 0.02 \text{ m}$. Area $A = \pi r^2$
4. Compression

$$\begin{aligned} \Delta L &= \frac{(F \times L)}{(Y \times A)} \\ &= \frac{(1078 \text{ N} \times 0.5 \text{ m})}{(9.4 \times 10^9 \text{ N/m}^2 \times 1.26 \times 10^{-3} \text{ m}^2)} \\ &\approx 4.55 \times 10^{-5} \text{ m}. \end{aligned}$$

- 150.** A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force of $9.0 \times 10^4 \text{ N}$ on its narrow face (area $50 \text{ cm} \times 10 \text{ cm}$). The lower edge of this face is riveted to the floor. The shear modulus (G) for lead is $5.6 \times 10^9 \text{ N/m}^2$.

Questions:

1. What is shearing stress?
2. How is shearing strain defined in terms of the relative displacement Δx and length L?
3. Calculate the shearing stress applied to the lead slab.
4. How much will the upper edge of the narrow face be displaced (Δx)?

Ans :

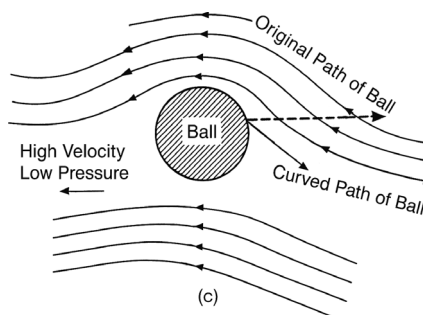
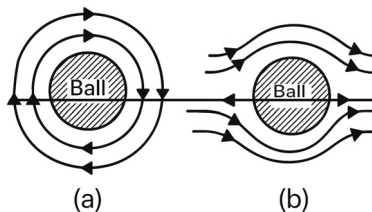
1. Shearing stress is the restoring force per unit area developed due to an applied tangential force.
2. Shearing strain is defined as the ratio of relative displacement of the faces (Δx) to the length of the cylinder (or height of the face here, L), i.e., $\frac{\Delta x}{L}$.

4. Explain why

- To keep a piece of paper horizontal, you should blow over not under it.
- When we try to close a water tap with our fingers, fast jets of water rush through the openings between our finger.
- The size of needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.
- A fluid flowing out of a small hole in a vessel results a backward thrust on the vessel.
- A spinning cricket ball in air does not follow a parabolic trajectory.

Ans :

- When a piece of paper is blown over it, according to Bernoulli's principle the pressure over is decreases than that of the pressure on lower surface, therefore it remains horizontal.
- Area of openings decreases thereby increasing the velocity.
- According to the equation of continuity, decrease in area in the part of syringe increases the velocity of liquid.
- Since area of hole is small therefore, velocity of liquid rushing out is large which result in backward thrust.
- Spinning, of a ball :** When a spherical ball is rotated about an axis perpendicular to the plane of paper, the surrounding air is set into motion in streamlines of concentric circles in a plane parallel to the plane of paper and the direction of rotation of the ball [as shown in the figure (a)]. The more rough the surface of the sphere, the thicker is the layer of air set in motion.



When the ball is given only translatory (linear) motion forward, the air in front is pushed aside. It flows backward on the sides and towards its rear end [as shown in the figure (b)]. However, if the ball possesses translational as well as rotatory motion, as in case of a spinning ball, [as shown in the figure (c)] then the two streams on the upper side are in opposite directions hence the resultant velocity of air is decreased and pressure increased. On its lower side the two sets of streamlines are in the same direction and help each other. Thus, the velocity of air is increased and pressure decreased. Hence the ball experiences a net lateral force in downward direction and it is for this reason that the ball describes a curved path, which is convex on the side of greater pressure. This is called Magnus effect and is exhibited by a tennis or golf ball when given spin.

5. A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?

Ans :

Given, $m = 50 \text{ kg}$, $d = 1.0 \text{ cm} = 10^{-2} \text{ m}$

$$\text{Hence, } P = \frac{F}{A} = \frac{4 \text{ mg}}{\pi d^2} = \frac{4 \times 50 \times 9.8}{3.14 \times 10^{-4}}$$

$$P = \frac{1960}{3.14} \times 10^4 = 6.24 \times 10^6 \text{ Pa} \\ = 6.2 \times 10^6 \text{ Pa.}$$

6. Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3} . Determine the height of wine column for normal atmospheric pressure.

Ans :

Given, $\rho = 984 \text{ kg m}^{-3}$, $h = ?$

Since, $P = h\rho g$

$$\text{So } h = \frac{P}{\rho g} = \frac{1.01 \times 10^5}{984 \times 9.8} \\ = 10.47 = 10.5 \text{ m}$$

7. A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa . Is the structure suitable for putting up of top of an oil well in the ocean ? Take the depth of ocean to be roughly 3 km and ignore ocean currents.

Ans :

Given, Stress = $10^9 \text{ Pa} = 10^9 \text{ Nm}^{-2}$,

$$h = 3 \text{ km} = 3000 \text{ m}$$

- 153.** A metal wire of original length L and cross-sectional area A is subjected to a deforming force F along its length, causing it to elongate by a small amount ' l '. The Young's modulus of the material is Y . Work is done against the inter-atomic forces, and this work is stored as elastic potential energy in the wire.

Questions:

1. When a wire is put under tensile stress, what happens to the work done against the inter-atomic forces?
2. How is the force F related to the elongation l , Young's modulus Y , area A , and original length L ?
3. What is the formula for the elastic potential energy per unit volume (u) stored in the stretched wire in terms of stress (σ) and strain (ϵ)?
4. If two wires, one of steel and another of copper, of the same dimensions are stretched by the same amount, which wire will store more elastic potential energy?

Ans :

1. When a wire is put under a tensile stress, work is done against the inter-atomic forces. This work is stored in the wire in the form of elastic potential energy.
2. The force $F = YA \times (l/L)$.
3. The elastic potential energy per unit volume

$$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \sigma \epsilon.$$

4. Elastic potential energy.

$$U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$$

Since the dimensions and elongation (and thus strain) are the same, the wire with the larger Young's modulus (Y) will store more energy. Steel has a larger Y than copper (Steel: $200 \times 10^9 \text{ N/m}^2$, Copper: $110 \times 10^9 \text{ N/m}^2$). Therefore, the steel wire will store more elastic potential energy.

- 154.** Beams used in the construction of bridges and as supports in buildings often have a cross-section of type 'I'. This shape is chosen to efficiently withstand loads and prevent bending or breaking. A bar of length l , breadth b , and depth d , when loaded at the centre by a load W , sags by an amount

$$\delta = Wl^3 / (4bd^3 Y)$$

Questions:

1. How is the sagging (δ) of a beam related to its depth (d) according to the given formula?
2. To reduce bending for a given load, is it more effective to increase the depth ' d ' or the breadth ' b ' of the beam? Why?

3. What problem can arise if the depth of a beam is significantly increased without appropriate design, especially if the load is not exactly at the right place? What is this phenomenon called?
4. How does the I-shaped cross-section address the issues of bending and buckling while also being economical?

Ans :

1. The sagging (δ) is inversely proportional to the cube of its depth (d^3) i.e., $\delta \propto d^{-3}$.
2. It is more effective to increase the depth ' d ' because the sagging (δ) is proportional to d^{-3} , whereas it is only proportional to b^{-1} .
3. If the depth is significantly increased, the deep bar may bend sideways or twist under the load. This is called buckling.
4. The I-shaped section provides a large load-bearing surface and enough depth to prevent bending. The distribution of material away from the neutral axis increases the moment of inertia (related to bending resistance). This shape also reduces the weight of the beam without sacrificing much strength, thus reducing cost.

- 155.** The maximum height of mountains on Earth is limited, around 10 km (like Mt. Everest). This limit can be understood by considering the elastic properties of rocks. The base of a mountain is under stress due to the weight of the material above it, which can cause shearing stress.

Questions:

1. What kind of stress at the base of the mountain is critical in determining its maximum possible height?
2. If ' h ' is the height of the mountain, ' ρ ' is the density of its material, and ' g ' is the acceleration due to gravity, what is the approximate force per unit area (stress) at the base due to the weight of the mountain?
3. The elastic limit (critical shearing stress) for a typical rock is $30 \times 10^7 \text{ N/m}^2$. Using a rock density of $\rho = 3 \times 10^3 \text{ kg/m}^3$ and $g \approx 10 \text{ m/s}^2$, estimate the maximum height ' h '.
4. Why can mountains on smaller celestial bodies (with lower ' g ') potentially be taller than on Earth, assuming similar rock properties?

Ans :

1. Shearing stress at the base of the mountain is critical. The stress due to all material on top should be less than the critical shearing stress at which rocks flow.
2. The force per unit area (stress) at the base is $h\rho g$.

3. Area of the narrow face

$$A = 50\text{cm} \times 10\text{cm} = 0.5\text{m} \times 0.1\text{m} \\ = 0.05\text{m}^2.$$

Shearing stress

$$\sigma_s = \frac{F}{A} \\ = \frac{9.0 \times 10^4 \text{N}}{0.05\text{m}^2} = 1.80 \times 10^6 \text{N/m}^2.$$

4. Shearing strain

$$\frac{\text{Stress}}{G} = \frac{\Delta x}{L}.$$

Here L is the side length of 50 cm (0.5 m) as the force is applied on the narrow face, and the displacement is relative to the fixed lower edge.

$$\Delta x = \frac{(\text{Stress} \times L)}{G} \\ = \frac{(1.8 \times 10^6 \text{N/m}^2 \times 0.5\text{m})}{(5.6 \times 10^9 \text{N/m}^2)} \\ \approx 1.6 \times 10^{-4} \text{m or } 0.16 \text{ mm}.$$

- 151.** The average depth of the Indian Ocean is about 3000 m. Water, though often considered incompressible, does compress slightly under immense pressure. The bulk modulus of water is $2.2 \times 10^9 \text{N/m}^2$. (Take density of water $\rho = 1000 \text{kg/m}^3$ and $g = 10 \text{m/s}^2$).

Questions:

1. How is bulk modulus (B) defined in terms of hydraulic stress and volume strain?
2. What is hydraulic stress essentially equal to in magnitude when a body is submerged in a fluid?
3. Calculate the hydraulic pressure (stress) at the bottom of the ocean at a depth of 3000 m.
4. Determine the fractional compression ($\Delta V/V$) of water at the bottom of the ocean.

Ans :

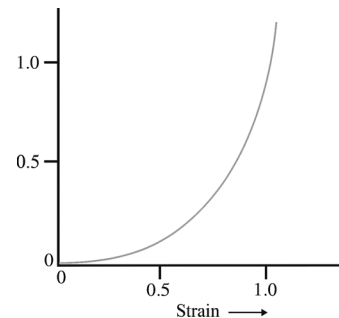
1. Bulk modulus (B) is the ratio of hydraulic stress (p) to the corresponding hydraulic strain ($\Delta V/V$), i.e., $B = -p/(\Delta V/V)$.
2. Hydraulic pressure is equal in magnitude to the hydraulic pressure.
3. Pressure

$$p = h\rho g \\ = 3000\text{m} \times 1000\text{kg/m}^3 \times 10\text{m/s}^2 \\ = 3 \times 10^7 \text{N/m}^2.$$

4. Fractional compression

$$\frac{\Delta V}{V} = \frac{p}{B} = \frac{(3 \times 10^7 \text{N/m}^2)}{(2.2 \times 10^9 \text{N/m}^2)} \\ \approx 1.36 \times 10^{-2} \text{ or } 1.36\%.$$

- 152.** The elastic tissue of the aorta, the large artery carrying blood from the heart, exhibits a specific stress-strain behavior. Unlike metals, it can be stretched to cause large strains and does not strictly obey Hooke's law over most of its elastic region. Its stress-strain curve (figure) is non-linear and shows no well-defined plastic region.



Questions:

1. What is Hooke's law?
2. What are materials called that can be stretched to cause large strains and still return to their original shape, like the tissue of the aorta?
3. Looking at figure (Stress-strain curve for aorta tissue), does the aorta tissue become stiffer or less stiff as the strain increases? Explain your reasoning based on the slope of the curve.
4. Why is the specific elastic behavior of the aorta important for its function in the cardiovascular system? .

Ans :

1. Hooke's law states that for small deformations, the stress and strain are proportional to each other.
2. Substances like the tissue of aorta, which can be stretched to cause large strains, are called elastomers.
3. The slope of the stress-strain curve represents the effective modulus of elasticity. For the aorta tissue in figure, the slope increases with increasing strain. This means the tissue becomes stiffer (its effective Young's modulus increases) as it is stretched more.
4. The aorta needs to be elastic to expand and accommodate the surge of blood pumped by the heart (systole) and then recoil to maintain blood pressure and flow during heart relaxation (diastole). Its ability to handle large strains and its non-linear stiffening help prevent overstretching and rupture while efficiently managing pressure changes.

CHAPTER 10

Mechanical Properties of Fluids

SUMMARY

1. PRESSURE

Pressure is defined as the force applied per unit area.

$$\text{Pressure } (P) = \frac{\text{Force } (F)}{\text{Area } (A)}$$

The unit of pressure in SI system is Nm^{-2} or Pascal (Pa).

2. PASCAL'S LAW

When pressure is applied to a fluid in a confined space, the pressure is transmitted equally and undiminished in all directions throughout the fluid.

3. ATMOSPHERIC PRESSURE

Air has weight owing to its weight, the atmosphere exerts a pressure at the surface of earth of approximately $100,000 \text{ Nm}^{-2}$ (or 100 kPa).

At sea level,

$1 \text{ atm} =$ pressure exerted by 0.76 cm of mercury column

$$\begin{aligned} &= h\rho g \\ &= 0.76 \times 13.6 \times 10^3 \times 9.8 \\ &= 1.013 \times 10^5 \text{ Nm}^{-2} \end{aligned}$$

Other unit of pressure are

$$1 \text{ torr} = 133.3 \text{ Pa}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

4. ARCHIMEDES' PRINCIPLE

When a body is immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid that is displaced by the body.

4.1 Law of Floatation

It states that a body will float, if the weight of liquid displaced by the immersed part is at least equal or greater than the weight of the body.

A body may sink in one liquid and float on another e.g.

- (a) a lump of iron sinks in water but floats on mercury.
- (b) a fresh egg sinks in ordinary water but floats in strong brine.
- (c) a fully loaded boat on sea is to be partly unloaded for safety before entering a river of fresh water.

As sea water is heavier than river water, the immersed portion of the body of a swimmer experiences greater buoyancy in sea water and so it is easier for the swimmer to swim in sea water than in river water.

5. FLUID

The term fluid refers to a substance that can flow and does not have shape of its own. e.g. liquids and gases.

6. TORRICELLIAN VACUUM

The empty space above mercury in the barometer tube is almost a perfect vacuum is called torricellian vacuum.

7. VISCOSITY

The property of fluid by virtue of which an opposing force (internal friction) comes into play, whenever there is relative motion between the different layers of the fluid is called viscosity.

7.1 Velocity Gradient

The change of velocity of liquid layer with distance from the solid surface is called velocity gradient.

7.2 Coefficient of Viscosity

Coefficient of viscosity of a liquid is defined as the viscous dragging force, which maintains a unit velocity gradient between two parallel layers each of unit area.

7.3 Stokes' Law

The backward dragging force acting on a small sphere of radius r moving with uniform velocity v through a medium of coefficient of viscosity η is given by

$$F = 6\pi\eta rv.$$

8. STREAMLINE FLOW

The flow of liquid in which each particle of the liquid follows the path of its proceeding particles with exactly the same velocity is called the streamline flow.

9. TURBULENT FLOW

The flow of liquid in which the velocity of all the particles of the fluid crossing a given point is not same is called turbulent flow.

10. REYNOLDS NUMBER

If a liquid of density ρ , coefficient of viscosity η and flowing through a tube of diameter D , then Reynolds number R is given by

$$R = \frac{v_c \rho D}{\eta}$$

(v_c is critical velocity of the liquid)

11. IDEAL LIQUID

A liquid which is incompressible, non-viscous and isotropic is called an ideal liquid.

12. EQUATION OF CONTINUITY

It expresses the law of conservation of mass in fluid dynamics. It states that if there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant.

13. BERNOULLI'S THEOREM

It states that for an ideal fluid having streamline flow the sum of pressure energy, kinetic energy and

potential energy is always constant.

Mathematically $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant}.$

14. SURFACE TENSION

The property of a liquid at rest by virtue of which its free surface behave like a stretched membrane with a tendency to contract to a minimum area is called surface tension.

Surface tension may be defined as the force acting per unit length on either side of imaginary line drawn on the liquid surface at rest. The units of surface tension in SI system is Nm⁻¹.

14.1 Force of Cohesion

The force of attraction between the molecules of the same substance is called the force of cohesion.

14.2 Force of Adhesion

The force of attraction between the molecules of different substances is called force of adhesion.

15. SURFACE ENERGY

The amount of work done per unit area in increasing the surface area of a liquid film is called surface energy.

16. ANGLE OF CONTACT

The angle, which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid, is called the angle of contact.

Excess pressure inside a liquid drop (or air bubble in a liquid). It is given by

$$p_i - p_o = \frac{2T}{r}$$

Excess pressure inside a liquid bubble (or soap bubble). It is given by

$$p_i - p_o = \frac{4T}{r}$$

17. CAPILLARITY

The phenomenon of the rise and fall of liquid in a capillary tube is called capillarity.

Effect of temperature on surface tension. Surface tension decreases with increase in temperature of liquid.

Surface tension acts over the free surface of a liquid only and not within the interior of the liquid.

Potential energy of a liquid surface film and its surface energy are different. Surface energy is equal to the potential energy per unit area of the surface 1 film under isothermal conditions. It is also called free surface energy density.

A wetting agent (like a detergent) greatly reduces θ , whereas a water proofing agent aims to prevent wetting by increasing θ .

The rise and fall of liquid in a capillary tube is according to the law of conservation of energy.

NCERT TEXT-BOOK QUESTIONS

1. Explain why

- The blood pressure in human is greater at feet than at brain.
- Atmospheric pressure at a height of about 6 km decreases nearly to half its value at the sea level though the height of the atmosphere is more than 1000 km.
- Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

Ans :

- The height of the blood column in the human body is more for feet than for brain. So the blood exerts more pressure at the feet than at the brain.
- Because density falls nearly half its value of surface and this height decreases rather very slowly.
- When a force is applied on a liquid, the pressure is transmitted equally in all directions inside the liquid. Therefore, hydrostatic pressure has not fixed direction and hence it is scalar quantity.

2. Explain why

- The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
- Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not.)
- The surface tension is independent of area of surface.
- Detergents should have small angle of contact.
- A drop of liquid under no external forces is always spherical in shape.

Ans :

- Water wets the surface of glass and rises along the surface. Hence its meniscus is concave upward and therefore the angle of contact is

acute. While mercury does not wet the surface of the glass, mercury is depressed. Its meniscus concave downward and the angle of contact is obtuse (140°).

- Water wets the walls of glass surface because water sticks with glass because force of adhesion between glass and water molecule is greater than force of cohesion between the water molecules. The mercury does not stick with the glass because force of cohesion between the mercury molecules is greater than the force of adhesion between the mercury and glass.
- The free surface of water behaves as a stretched membrane due to its surface tension. Surface tension is measured as force per unit length of imaginary line drawn on liquid surface.
- Because surface tension decreases.
- The surface of liquid drop tries to shrink in area and a sphere has a minimum surface area for a given volume. Hence liquid drop attains the shape of a sphere under no external forces.

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3. Fill in the blanks using the words from list appended with each statement :

- Surface tension of liquids generally with temperature (increases/decreases).
- Viscosity of gases with temperature, whereas viscosity of liquids with temperature (increases/decreases).
- For solids with elastic modulus of rigidity, the shearing force is proportional to while for liquids it is proportional to (shear strain/rate of shear strain).
- For a fluid in steady flow, the increase in flow speed at a constriction follows from while the decrease of pressure there follows from (conservation of mass/Bernoulli's equation).
- For the model of a plane in a wind tunnel, turbulence occurs at a speed than the critical speed for turbulence for an actual plane (greater/smaller).

Ans :

- decreases
- increases, decreases
- shear strain, rate of shear strain
- conservation of mass, Bernoulli's equation
- greater

3. Equating $h\rho g$ to the critical shearing stress:

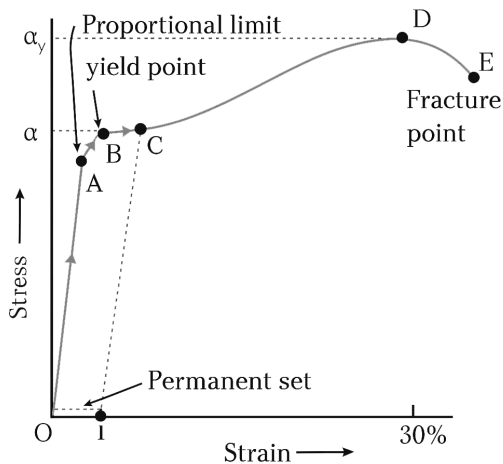
$$h(3 \times 10^3 \text{ kg/m}^3)(10 \text{ m/s}^2) = 30 \times 10^7 \text{ N/m}^2$$

$$h = \frac{30 \times 10^7 \text{ N/m}^2}{(3 \times 10^3 \text{ kg/m}^3 \times 10 \text{ m/s}^2)} = \frac{30 \times 10^7}{3 \times 10^4}$$

$$= 10 \times 10^3 \text{ m} = 10 \text{ km}.$$

4. The stress at the base is $h\rho g$. If 'g' is lower on another celestial body, then for the same rock properties (same critical shearing stress and density ρ), 'h' can be larger before the critical stress is reached. Thus, mountains could be taller.

156. The stress-strain curve for a material provides information about its mechanical properties. Point D on a typical curve for metal (Fig.) represents the ultimate tensile strength (σ_u), and point E represents the fracture point. The separation between these points differs for various materials.



Questions:

1. What is meant by the ultimate tensile strength (σ_u) of a material?
2. How is a brittle material characterized based on its stress-strain curve, specifically concerning points D and E?
3. How is a ductile material characterized based on its stress-strain curve regarding points D and E?
4. Glass is known to be brittle, while copper is ductile. How would their stress-strain curves differ in the region beyond the ultimate tensile strength?

Ans :

1. The point D on the stress-strain graph is the ultimate tensile strength (σ_u) of the material; beyond this point, additional strain is produced even by a reduced applied force.

2. If the ultimate strength and fracture points D and E are close, the material is said to be brittle.
3. If the ultimate strength and fracture points D and E are far apart, the material is said to be ductile.
4. For glass (brittle), the fracture point E would be very close to the ultimate tensile strength point D; it would break soon after reaching maximum stress. For copper (ductile), point E would be much further from point D, indicating it can undergo significant plastic deformation (necking) before fracturing.

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Since, $P = h\rho g$

$$P = 3000 \times 10^3 \times 9.8$$

$$= 2.94 \times 10^7 = 3 \times 10^7 \text{ Pa}$$

Since $3 \times 10^7 \text{ Pa} < 10^9 \text{ Pa}$ (maximum stress)

Thus we find that the structure is suitable because the stress applied ($3 \times 10^7 \text{ Pa}$) is much less than the maximum stress it can withstand.

- 8.** A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2 . What maximum pressure would the smaller piston have to bear ?

Ans :

Maximum pressure, the smaller piston would have to bear

$$P_{\text{maxi}} = \frac{3000 \times 9.8}{425 \times 10^{-4}} \text{ Pa.}$$

$$= 6.92 \times 10^5 \text{ Pa.}$$

- 9.** A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit ?

Ans :

As the mercury columns in the two arms are in level, therefore at the same pressure

$$h_1 \rho_1 g = h_2 \rho_2 g$$

where h_1 and h_2 are the heights of water and spirit above mercury and P_1 and P_2 are their densities respectively. Here $\rho_1 = 1$

$$\text{Hence, } 10 \times 1 \times g = 12.5 \times \rho_2 \times g$$

$$\rho_2 = \frac{10 \times 1 \times g}{12.5 \times g}$$

$$= 0.800 = 0.8 \text{ g cm}^{-3}$$

Hence, Relative density of spirit

$$= \frac{\rho_2}{\rho_1} = \frac{0.8}{1} = 0.8$$

- 10.** In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury, in the two arms ? (Relative density of mercury = 13.6).

Ans :

$$\text{For water, } h_1 \rho_1 g = h_2 \rho_2 g$$

$$\text{where } h_1 = 15.0 \text{ cm}$$

$$\text{Here, } \rho_1 = 1, \rho_2 = 13.6$$

$$\text{Hence, } 15 \times 1 \times g = h_2 \times 13.6 \times g$$

$$h_2 = \frac{15}{13.6} \text{ cm}$$

$$\text{For spirit, } h_1 \rho_1 g = h_2 \rho_2 g$$

$$h_1 = 15.0 \text{ cm}, \rho_1 = 0.8, \rho_2 = 13.6$$

$$h_2 = \frac{15 \times 0.8}{13.6} = \frac{12}{13.6} \text{ cm}$$

The difference in levels of mercury

$$= \frac{15}{13.6} - \frac{12}{13.6}$$

$$= \frac{3}{13.6} \text{ cm} = 0.221 \text{ cm}$$

Mercury will rise in the arm containing spirit = 0.221 cm and the difference in levels of mercury will be 0.221 cm.

- 11.** Can Bernoulli's equation be used to describe the flow of water through a rapid river ? Explain.

Ans :

No, Bernoulli's equation applies to streamline flow only and in a rapid river, the flow of water is not streamline.

- 12.** Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation ? Explain.

Ans :

No, unless the atmospheric pressure at the two points where Bernoulli's equation is applied are significantly different.

- 13.** Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} \text{ kg s}^{-1}$. What is the pressure difference between the two ends of the tube, (density of glycerine = $1.3 \times 10^3 \text{ kg m}^{-3}$ and viscosity of glycerine = 0.83 Pas) ? [You may also like to check if the assumption of laminar flow in the tube is correct].

Ans :

By Poissuille's formula,

Volume of liquid flowing in time t is given by

$$V = \frac{\pi}{8} \times \frac{r^4}{\eta} \left(\frac{\Delta P}{l} \right) t$$

$$\frac{4 \times 10^{-3}}{1.3 \times 10^3} = \frac{3.14}{8} \times \frac{(1 \times 10^{-2})^4}{0.83} \times \frac{(\Delta P)}{1.5} \times 1$$

$$\Delta P = \frac{4 \times 10^{-6} \times 8 \times 0.83 \times 1.5}{1.3 \times 3.14 \times 10^{-8} \times 1}$$

$$= \frac{4 \times 8 \times 83 \times 15}{13 \times 3.14} \times 10^2$$

$$= 9.8 \times 10^2 \text{ Pa}$$

14. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of wings are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2 ? Take the density of air to be 1.3 kg m^{-3} .

Ans :

$$P_1 V + \frac{1}{2} m v_1^2 = P_2 V + \frac{1}{2} m v_2^2$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

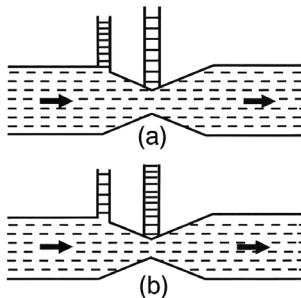
$$(P_1 - P_2) = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$= \frac{1 \times 7 \times 133 \times 1.3}{2}$$

$$F = \frac{1}{7} \times 7 \times 133 \times 1.3 \times 2.5$$

$$= 1.5 \times 10^3 \text{ N}$$

15. Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid, which of the two figures is incorrect? Why?



Ans :

Figure (a) is incorrect. At a constriction (i.e. where the area of cross-section of the tube is smaller), flow speed is larger due to mass conservation. Consequently pressure there is smaller according to Bernoulli's equation.

16. The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine hole each of diameter 1.0 mm . If the liquid flows inside the tube 1.5 m per minute, what is the speed of ejection of liquid through the holes?

Ans :

Using relation of continuity,

$$a_1 v_1 = a_2 v_2$$

$$8 \times 10^{-6} \times \frac{1.5}{60} = 40 \times \pi (0.5 \times 10^{-3})^2 \times v^2$$

$$v_2 = \frac{8 \times 10^{-6} \times 15 \times 2 \times 2 \times 7}{40 \times 22 \times 10^{-6} \times 1}$$

$$v_2 = 0.64 \text{ m s}^{-1}$$

17. A U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and a light slider supports a weight of $1.5 \times 10^{-2} \text{ N}$ (which includes the small weight of the slider). The length of the slider is 30 cm . What is the surface tension of the film?

Ans :

As we know that,

$$\text{Surface tension, } T = \frac{F}{2l}$$

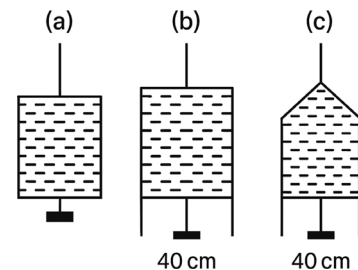
where l is the length of the wire slider.

$$\text{Hence, } T = \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}}$$

$$= \frac{15 \times 10^{-2}}{2 \times 30 \times 10^2 \times 10^{-2}}$$

$$= 2.5 \times 10^{-2} \text{ Nm}^{-1}$$

18. Figure (a) below shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2} \text{ N}$. What is the weight supported by a film of the same liquid at the same temperature in figures (b) and (c)? Explain your answer physically.



Ans :

The surface tension of a liquid surface in contact with its own vapour or with air is found to depend not only on the nature of liquid but also on the temperature.

$$\text{Surface tension, } T = \frac{F}{2l}$$

[Since, the film has two surfaces]

$$\text{Since } l \text{ is same, } T = \frac{4.5 \times 10^{-2}}{2 \times 0.4}$$

$$= 5.625 \times 10^{-2} \text{ Nm}^{-1}$$

Therefore, the weight supported by a film on the same liquid at the same temperature in figures (b) and (c) will be $4.5 \times 10^{-2} \text{ N}$.

19. What is the pressure inside a drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20°C) is $4.65 \times 10^{-1} \text{ Nm}^{-1}$. The atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$. Also give the excess pressure inside the drop.

Ans :

$$\begin{aligned}\text{Given, } S &= 4.65 \times 10^{-1} \text{ Nm}^{-1} \\ R &= 3 \text{ mm} = 3 \times 10^{-3} \text{ m} \\ P_{\text{outside}} &= 1.01 \times 10^5 \text{ Pa}\end{aligned}$$

Using the relation,

$$\text{Excess pressure} = \frac{2S}{r}$$

$$\text{We get, } P_{\text{outside}} - P_{\text{inside}} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

$$\begin{aligned}\text{Total pressure} &= 1.01 \times 10^5 + 310 \\ &= 1.0131 \times 10^5 \text{ Pa}\end{aligned}$$

20. What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20°C) is $2.50 \times 10^{-2} \text{ Nm}^{-1}$? If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? ($1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$).

Ans :

$$\text{Given, } T = 2.50 \times 10^{-2} \text{ Nm}^{-1}, R = 5 \times 10^{-3} \text{ m}$$

Excess pressure inside the air bubble

$$\text{Using, } P = \frac{2T}{R}$$

$$P = \frac{2 \times 2.50 \times 10^{-2} \text{ Nm}^{-1}}{5 \times 10^{-3} \text{ m}} = 10 \text{ Pa}$$

Excess pressure inside the soap solution

$$= \frac{4T}{R}$$

$$= \frac{4 \times 2.50 \times 10^{-2} \text{ Nm}^{-1}}{5 \times 10^{-3} \text{ m}}$$

$$= 20 \text{ Pa}$$

Pressure inside air bubble

$$\begin{aligned}&= 1.01 \times 10^5 + 0.4 \times 10^3 \times 9.8 \times 1.2 \\ &\quad \quad \quad [\text{Since, } p = h\rho g] \\ &= 1.01 \times 10^5 + 0.047 \times 10^5 \\ &= 1.057 \times 10^5 \\ &= 1.06 \times 10^5 \text{ Pa}\end{aligned}$$

The excess pressure is so small that upto three significant figures, total pressure inside the air bubble $= 1.06 \times 10^5 \text{ Pa}$.

OBJECTIVE QUESTIONS

- Which of the following best defines a fluid?
 - A substance that has a definite shape and volume.
 - A substance that can flow and has no definite shape.
 - A substance that resists shear stress with permanent deformation.
 - A substance that is compressible and has a fixed shape.

Ans :

(B) A substance that can flow and has no definite shape.

Fluids, which include liquids and gases, are characterized by their ability to flow and take the shape of their container, lacking a definite shape of their own. This property distinguishes them from solids.

- What distinguishes liquids and gases from solids in a fundamental way?
 - Their fixed volume.
 - Their ability to flow.
 - Their definite shape.
 - Their high compressibility.

Ans :

(B) Their ability to flow.

The primary characteristic that differentiates fluids (liquids and gases) from solids is their capacity to flow. Solids maintain a fixed shape, while fluids adapt to the shape of their container.

What is the approximate percentage of Earth's surface covered by water?

- One-third
- Half
- Two-thirds
- Three-fourths

Ans :

(C) Two-thirds

A significant portion of the Earth's surface, approximately two-thirds, is covered by water, which is a vital fluid for life and various geological processes.

4. Which of the following statements is true regarding the volume of solids and liquids under atmospheric pressure?
- (A) Solids have a fixed volume, but liquids do not.
 - (B) Liquids have a fixed volume, but solids do not.
 - (C) Both solids and liquids have a fixed volume.
 - (D) Neither solids nor liquids have a fixed volume.

Ans :

(C) Both solids and liquids have a fixed volume. Under atmospheric pressure, both solids and liquids maintain a fixed volume. Gases, however, expand to fill the entire volume of their container.

5. How does the compressibility of solids and liquids compare to that of gases?
- (A) Solids and liquids have much higher compressibility than gases.
 - (B) Solids and liquids have much lower compressibility than gases.
 - (C) Solids, liquids, and gases have nearly equal compressibility.
 - (D) Compressibility is not a relevant property for solids or liquids.

Ans :

(B) Solids and liquids have much lower compressibility than gases. Solids and liquids are significantly less compressible compared to gases. This means their volume changes very little with changes in external pressure.

6. What is the key property of fluids concerning shear stress?
- (A) They offer high resistance to shear stress.
 - (B) They offer very little resistance to shear stress.
 - (C) They become more rigid under shear stress.
 - (D) They are unaffected by shear stress at all.

Ans :

(B) They offer very little resistance to shear stress. A defining characteristic of fluids is their minimal resistance to shear stress, causing them to change shape easily when such stress is applied.

7. The impact of a force distributed over an area is known as what?
- (A) Stress
 - (B) Strain
 - (C) Pressure
 - (D) Density

Ans :

(C) Pressure

Pressure is defined as the force acting per unit area. A smaller area of application for the same force results in a greater pressure or impact.

8. Which of the following statements about the force exerted by a fluid at rest on a submerged object is incorrect?
- (A) The force is always normal to the object's surface.
 - (B) If there were a parallel component, the fluid would flow.
 - (C) The force can have components parallel to the surface.
 - (D) The fluid is at rest, so no parallel force component exists.

Ans :

(C) The force can have components parallel to the surface. A fluid at rest exerts a force that is always perpendicular (normal) to the surface of a submerged object. Any parallel component would cause the fluid to flow.

9. Which of the following accurately describes pressure?
- (A) Pressure is a vector quantity with a specific direction.
 - (B) Pressure is a scalar quantity and has no direction.
 - (C) Pressure always acts in the downward direction due to gravity.
 - (D) Pressure is the total force exerted by the fluid particles.

Ans :

(B) Pressure is a scalar quantity and has no direction. Pressure is a scalar quantity, meaning it only has magnitude and no specific direction, even though the force it originates from is normal to the area.

10. Why does a sharp needle pierce the skin more easily than a blunt object with the same applied force?
- (A) The needle material is harder than the blunt object.
 - (B) The needle exerts greater force due to its shape.
 - (C) The needle concentrates the force onto a smaller area.
 - (D) The blunt object distributes the force more effectively.

Ans :

(C) The needle concentrates the force onto a smaller area.

A sharp needle pierces the skin more easily because it applies the same force over a much smaller contact area, resulting in a significantly higher pressure.

11. All of the following are true about the force exerted by a fluid at rest on a surface EXCEPT:
- (A) It is always normal to the surface.
 - (B) It is responsible for buoyancy.
 - (C) It can cause the fluid to flow if parallel.
 - (D) Its magnitude per unit area defines pressure.

Ans :

(C) It can cause the fluid to flow if parallel.

If a fluid is at rest, there can be no component of force parallel to the surface it is in contact with, as this would induce flow. The force is always normal.

12. How does the density of liquids generally compare with the density of gases under similar conditions?
- (A) Liquids are generally much denser than gases.
 - (B) Gases are generally much denser than liquids.
 - (C) Liquids and gases have comparable densities.
 - (D) Density is not a distinguishing factor between them.

Ans :

(A) Liquids are generally much denser than gases. Liquids typically have significantly higher densities compared to gases under similar temperature and pressure conditions due to the closer packing of their molecules.

13. What is the SI unit of pressure?
- (A) Newton (N)
 - (B) Joule (J)
 - (C) Pascal (Pa)
 - (D) Watt (W)

Ans :

(C) Pascal (Pa)

The SI unit of pressure is the pascal (Pa), which is defined as one newton per square meter (N/m^2). This unit honors Blaise Pascal for his work on fluid pressure.

14. What is the dimensional formula for pressure?
- (A) $[MLT^{-2}]$
 - (B) $[ML^{-1}T^{-2}]$
 - (C) $[ME^2T^{-2}]$
 - (D) $[ML^{-1}T^{-1}]$

Ans :

(B) $[ML^{-1}T^{-2}]$

Pressure is force per unit area. Force has dimensions $[MLT^{-2}]$ and area has dimensions $[L^2]$. Therefore, the dimensions of pressure are $[MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$.

15. What is the standard atmospheric pressure at sea level in pascals?
- (A) $1.013 \times 10^3 \text{ Pa}$
 - (B) $1.013 \times 10^5 \text{ Pa}$
 - (C) $1.013 \times 10^4 \text{ Pa}$
 - (D) $1.013 \times 10^6 \text{ Pa}$

Ans :

(B) $1.013 \times 10^5 \text{ Pa}$

The standard atmospheric pressure at sea level is internationally defined as 1.013×10^5 pascals, which is also equivalent to one atmosphere (atm).

16. What is the density of water at 4°C ?
- (A) $1.0 \times 10^2 \text{ kg/m}^3$
 - (B) $1.0 \times 10^3 \text{ kg/m}^3$
 - (C) $1.0 \times 10^4 \text{ kg/m}^3$
 - (D) $1.0 \times 10^1 \text{ kg/m}^3$

Ans :

(B) $1.0 \times 10^3 \text{ kg/m}^3$

Water reaches its maximum density at 4°C , which is approximately 1000 kg/m^3 or $1.0 \times 10^3 \text{ kg/m}^3$. This value is a common reference point.

Relative density of a substance is the ratio of its density to the density of what substance at 4°C ?

- (A) Mercury
- (B) Air
- (C) Water
- (D) Oil

Ans :

(C) Water

Relative density, also known as specific gravity, is the ratio of the density of a substance to the density of a reference substance, which is typically water at 4°C .

18. Which scientist is credited with the observation that pressure in a fluid at rest is the same at all points if they are at the same height?
- (A) Isaac Newton
 - (B) Galileo Galilei
 - (C) Blaise Pascal
 - (D) Evangelista Torricelli

Ans :

(C) Blaise Pascal

Blaise Pascal, a French scientist, formulated the principle known as Pascal's Law, which states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. It also implies that pressure is the same at the same depth.

19. According to Pascal's Law, what happens when external pressure is applied to an enclosed fluid?
- (A) The pressure increases only at the point of application.
 - (B) The pressure is transmitted undiminished and equally in all directions.
 - (C) The pressure increases more at the bottom of the container.
 - (D) The pressure is absorbed by the fluid and decreases rapidly.

Ans :

(B) The pressure is transmitted undiminished and equally in all directions.
Pascal's Law states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

20. Which of the following statements about Pascal's Law is incorrect?
- (A) Pressure is the same at all points at the same depth in a fluid at rest.
 - (B) External pressure is transmitted equally throughout an enclosed fluid.
 - (C) Pressure in a fluid is a vector quantity, acting downwards.
 - (D) Hydraulic machines like lifts operate based on this principle.

Ans :

(C) Pressure in a fluid is a vector quantity, acting downwards.
Pressure is a scalar quantity, not a vector. While the force due to pressure acts perpendicularly to a surface, pressure itself does not have a direction.

21. Which of the following scenarios correctly illustrates Pascal's law?
- (A) The pressure at the bottom of a lake is greater than at the surface.
 - (B) A hydraulic jack lifts a heavy car using a small input force.
 - (C) A boat floats because the buoyant force equals its weight.
 - (D) Air pressure decreases with increasing altitude in the atmosphere.

Ans :

(B) A hydraulic jack lifts a heavy car using a small input force.

Hydraulic jacks operate on Pascal's law, where a small force applied to a small piston creates pressure that is transmitted to a larger piston, resulting in a larger output force.

22. Why is the pressure at a certain depth in a liquid independent of the shape of the container?
- (A) Because the liquid adjusts its density to maintain constant pressure.
 - (B) Because pressure only depends on the total volume of the liquid.
 - (C) Because pressure at a depth depends only on the height and density.
 - (D) Because the container material equalizes the pressure throughout.

Ans :

(C) Because pressure at a depth depends only on the height and density.

The pressure at a specific depth in a liquid ($P = P_a + \rho gh$) depends on the atmospheric pressure (P_a), the density of the liquid (ρ), the acceleration due to gravity (g), and the depth (h), not on the container's shape. This is known as the hydrostatic paradox.

23. Bernoulli's equation is applicable to all the following types of fluid flow EXCEPT:
- (A) Steady flow
 - (B) Incompressible flow
 - (C) Turbulent flow
 - (D) Non-viscous flow

Ans :

(C) Turbulent flow

Bernoulli's equation is derived assuming steady, incompressible, and non-viscous (ideal) fluid flow. It does not accurately describe turbulent flow, where velocity and pressure fluctuate erratically.

24. What is the main difference between gauge pressure and absolute pressure?
- (A) Gauge pressure is always greater than absolute pressure.
 - (B) Absolute pressure is the pressure relative to atmospheric pressure.
 - (C) Gauge pressure is the pressure relative to a perfect vacuum.
 - (D) Absolute pressure is the sum of gauge pressure and atmospheric pressure.

Ans :

(D) Absolute pressure is the sum of gauge pressure and atmospheric pressure.

Absolute pressure is the total pressure exerted on a surface, including atmospheric pressure. Gauge pressure is the difference between the absolute pressure and the local atmospheric pressure.

25. Assertion: The pressure exerted by a fluid is always normal to the surface of contact.

Reason: If there were a component of force parallel to the surface, the fluid would flow.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

For a fluid at rest, any force component parallel to a surface would cause the fluid to move along that surface. Therefore, the force exerted by a static fluid must be perpendicular (normal) to the surface.

26. Assertion: A spinning ball moving through air experiences a dynamic lift.

Reason: The speed of air is different above and below the spinning ball, causing a pressure difference.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

A spinning ball drags air with it. This results in different air speeds above and below the ball, leading to a pressure difference according to Bernoulli's principle, which creates a dynamic lift known as the Magnus effect.

27. Assertion: The viscosity of liquids generally decreases with an increase in temperature.

Reason: Increased thermal energy in liquids reduces the cohesive forces between molecules, allowing them to flow more easily.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

(C) Assertion is true, but Reason is false.

(D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

In liquids, viscosity arises from cohesive forces. As temperature increases, the increased kinetic energy of molecules overcomes these cohesive forces more effectively, leading to a decrease in viscosity.

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28. Consider the following statements regarding fluid pressure:

1. Pressure is a scalar quantity.
2. Pressure exerted by a fluid at rest is the same in all directions at a given depth.
3. The SI unit of pressure is Newton.
4. Pressure increases linearly with depth in an incompressible fluid.

Which of the above statements are correct?

- (A) 1, 2, and 3
 (B) 1, 2, and 4
 (C) 2, 3, and 4
 (D) 1, 3, and 4

Ans :

(B) 1, 2, and 4

Pressure is a scalar, is the same in all directions at a given depth (Pascal's Law for static fluids), and increases linearly with depth ($P = P_a + \rho gh$). The SI unit of pressure is Pascal (Pa), not Newton (N), which is the unit of force.

29. Which of the following statements correctly describe characteristics of an ideal fluid for applying Bernoulli's principle?

1. The fluid is compressible.
2. The flow is steady or streamline.
3. The fluid has significant viscosity.
4. The flow is irrotational.

Select the correct answer using the codes given below:

- (A) 1 and 3
 (B) 2 and 4
 (C) 1, 2, and 3
 (D) 2, 3, and 4

Ans :

(B) 2 and 4

Bernoulli's principle is ideally applied to fluids that are incompressible (constant density), in steady (streamline) flow, non-viscous (no internal friction), and irrotational (fluid elements do not rotate).

30. Consider the following regarding surface tension:
1. It is a force per unit area.
 2. It arises due to cohesive forces between liquid molecules.
 3. Liquids tend to minimize their surface area due to surface tension.
 4. Surface tension of a liquid generally increases with temperature.

Which of the above statements are correct?

- (A) 1 and 4
(B) 2 and 3
(C) 1, 2, and 3
(D) 2, 3, and 4

Ans :

- (B) 2 and 3

Surface tension is a force per unit length or surface energy per unit area. It arises from cohesive forces between liquid molecules, causing the liquid to minimize its surface area. Surface tension generally decreases with increasing temperature.

31. Match the physical quantity in Column I with its corresponding SI unit in Column II.

	Column I		Column II
1.	Pressure	a.	kg/m^3
2.	Density	b.	N s/m^2
3.	Viscosity	c.	Pa
4.	Surface Tension	d.	N/m

- (A) 1-c, 2-a, 3-b, 4-d
(B) 1-a, 2-c, 3-d, 4-b
(C) 1-c, 2-b, 3-a, 4-d
(D) 1-d, 2-a, 3-b, 4-c

Ans :

- (A) 1-c, 2-a, 3-b, 4-d

Pressure is measured in Pascals (Pa). Density is measured in kilograms per cubic meter (kg/m^3). Viscosity is measured in Pascal-seconds ($\text{Pa} \cdot \text{s}$) or Newton-seconds per square meter (N s/m^2). Surface tension is measured in Newtons per meter (N/m).

32. Match the principle/law in Column I with its associated phenomenon in Column II.

	Column I		Column II
1.	Pascal's Law	a.	Speed of efflux from an open tank
2.	Bernoulli's Principle	b.	Hydraulic lift operation
3.	Torricelli's Law	c.	Lift on an aircraft wing
4.	Stokes' Law	d.	Terminal velocity of a falling sphere

- (A) 1-b, 2-c, 3-a, 4-d
(B) 1-a, 2-d, 3-b, 4-c
(C) 1-b, 2-a, 3-d, 4-c
(D) 1-d, 2-c, 3-a, 4-b

Ans :

- (A) 1-b, 2-c, 3-a, 4-d

Pascal's Law explains hydraulic lift operation. Bernoulli's Principle explains lift on an aircraft wing. Torricelli's Law describes the speed of efflux from an open tank. Stokes' Law is used to determine the terminal velocity of a sphere falling through a viscous fluid.

33. Match the term in Column I with its defining characteristic in Column II.

	Column I		Column II
1.	Streamline	a.	Flow where velocity at a point varies
2.	Turbulent Flow	b.	Resistance to fluid motion
3.	Viscosity	c.	Path taken by a fluid particle in steady flow
4.	Critical Speed	d.	Speed beyond which flow becomes turbulent

- (A) 1-c, 2-a, 3-b, 4-d
(B) 1-a, 2-c, 3-d, 4-b
(C) 1-c, 2-d, 3-a, 4-b
(D) 1-d, 2-a, 3-b, 4-c

Ans :

- (A) 1-c, 2-a, 3-b, 4-d

A streamline is the path of a fluid particle in steady flow. Turbulent flow is characterized by chaotic and varying velocities. Viscosity is the internal friction or resistance to flow in a fluid. Critical speed is the speed at which flow transitions from laminar to turbulent.

34. Which of the following pairs of a fluid property and its characteristic is correct?

- (A) Compressibility : High for liquids, low for gases.
(B) Density : Constant for gases, varies significantly for liquids.
(C) Shear Stress Resistance : High for fluids.
(D) Pressure : Force per unit area normal to the surface.

Ans :

- (D) Pressure : Force per unit area normal to the surface.

Pressure is correctly defined as the normal force exerted per unit area. Liquids have low compressibility, gases have high density variation, and fluids offer little resistance to shear stress.

- 35.** Identify the correct pairing of a device and the scientific principle it primarily utilizes.

(A) Mercury Barometer : Archimedes' Principle
 (B) Hydraulic Brakes : Bernoulli's Principle
 (C) Aerofoil : Pascal's Law
 (D) Venturi Meter : Bernoulli's Principle

Ans :

(D) Venturi Meter : Bernoulli's Principle

A Venturi meter measures fluid flow rate based on the pressure drop that occurs as fluid passes through a constricted section, an application of Bernoulli's principle. Barometers measure atmospheric pressure (related to fluid statics), hydraulic brakes use Pascal's Law, and aerofoils generate lift due to Bernoulli's principle.

- 36.** Which of the following pairs related to fluid dynamics is incorrectly matched?

(A) Equation of Continuity : Conservation of mass
 (B) Stokes' Law : Viscous drag on a sphere
 (C) Magnus Effect : Dynamic lift on a non-spinning ball
 (D) Torricelli's Law : Speed of efflux from a tank

Ans :

(C) Magnus Effect : Dynamic lift on a non-spinning ball

The Magnus effect specifically refers to the dynamic lift experienced by a spinning ball moving through a fluid, due to the pressure differences created by the spin. A non-spinning ball generally does not experience this effect.

- 37.** Identify the incorrect pairing of a term and its description in the context of fluids.

(A) Capillary Rise : Ascent of liquid in a narrow tube due to surface tension.
 (B) Angle of Contact : Angle between liquid surface and solid surface inside the liquid.
 (C) Regelation : Refreezing of water after pressure is removed, related to melting point change.
 (D) Efflux : Tendency of a fluid to resist changes in its flow rate.

Ans :

(D) Efflux : Tendency of a fluid to resist changes in its flow rate.

Efflux refers to the outflow of a fluid from a container or opening. The tendency of a fluid to resist changes in its flow rate is more related to inertia and viscosity.

- 38.** Arrange the following steps in the correct order to measure atmospheric pressure using a mercury barometer as devised by Torricelli.

1. Invert the filled glass tube into a trough of mercury.
 2. Observe the height of the mercury column in the tube.
 3. Fill a long glass tube, closed at one end, with mercury.
 4. The space above the mercury column becomes a near vacuum.

(A) 3, 1, 4, 2

(B) 1, 3, 2, 4

(C) 3, 4, 1, 2

(D) 4, 3, 1, 2

Ans :

(A) 3, 1, 4, 2

Torricelli's method involves first filling a long glass tube (closed at one end) with mercury (3). Then, this tube is inverted into a trough of mercury (1). This creates a vacuum (Torricellian vacuum) in the space above the mercury column in the tube (4). Finally, the height of the mercury column is observed, which balances the atmospheric pressure (2).

- 39.** Consider the process of a raindrop falling through the air. Arrange the following stages in the typical sequence of its motion.

1. The raindrop accelerates due to gravity.
 2. The net force on the raindrop becomes zero.
 3. The viscous drag force increases with velocity.
 4. The raindrop descends with a constant terminal velocity.

(A) 1, 3, 2, 4

(B) 1, 2, 3, 4

(C) 3, 1, 4, 2

(D) 2, 1, 3, 4

Ans :

(A) 1, 3, 2, 4

Initially, the raindrop accelerates downwards due to gravity (1). As its velocity increases, the viscous drag force from the air also increases (3). Eventually, the drag force (plus buoyant force) equals the gravitational force, making the net force zero (2). At this point, the acceleration ceases, and the raindrop falls at a constant terminal velocity (4).

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40. Why are the wings of an airplane shaped as aerofoils?
- To minimize air resistance and increase fuel efficiency primarily.
 - To make the air flow faster over the lower surface than the upper surface.
 - To create a pressure difference, with higher pressure on the upper surface.
 - To make air flow faster over the upper surface, creating lower pressure there.

Ans :

(D) To make air flow faster over the upper surface, creating lower pressure there.

Aerofoil shape causes air to travel faster over the curved upper surface than the flatter lower surface. According to Bernoulli's principle, faster-moving air has lower pressure, creating a net upward force (lift).

41. The property of a fluid that describes its internal resistance to flow is known as:
- Density
 - Pressure
 - Viscosity
 - Surface Tension

Ans :

(C) Viscosity

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction between adjacent layers of the fluid as they move relative to each other.

42. What type of flow is characterized by smooth, parallel layers of fluid sliding past one another?
- Turbulent flow
 - Laminar flow
 - Compressible flow
 - Irrotational flow

Ans :

(B) Laminar flow

Laminar flow, also known as streamline flow, is characterized by fluid particles moving in smooth paths or layers, with each layer sliding smoothly past the adjacent layers.

43. What is the SI unit of the coefficient of viscosity?
- N/m^2
 - J/s
 - $\text{Pa} \cdot \text{s}$
 - kg/m^3

Ans :

(C) $\text{Pa} \cdot \text{s}$

The SI unit for the coefficient of viscosity is Pascal-second ($\text{Pa} \cdot \text{s}$), which is equivalent to Newton-second per square meter ($\text{N} \cdot \text{s/m}^2$). Another common unit is the poise.

44. According to Stokes' Law, the viscous drag force on a small sphere moving through a fluid is proportional to what power of its radius?
- Radius
 - Square of the radius
 - Cube of the radius
 - Independent of radius

Ans :

(A) Radius

Stokes' Law states that the viscous drag force (F) on a sphere of radius (a) moving with velocity (v) through a fluid of viscosity (η) is given by $F = 6\pi\eta av$. Thus, the force is directly proportional to the radius.

45. What happens to the viscosity of liquids as temperature increases?
- It increases significantly.
 - It decreases.
 - It remains constant.
 - It first increases, then decreases.

Ans :

(B) It decreases.

The viscosity of liquids generally decreases as temperature increases. This is because the increased thermal energy allows molecules to overcome cohesive forces more easily, reducing internal friction.

46. The phenomenon where a liquid tends to minimize its surface area is known as:
- Viscosity
 - Capillarity
 - Surface Tension
 - Buoyancy

Ans :

(C) Surface Tension

Surface tension is the property of a liquid surface that allows it to resist an external force, due to the cohesive nature of its molecules. This causes liquids to minimize their surface area.

47. What shape does a small liquid drop tend to assume in the absence of external forces?
- Cubical
 - Cylindrical

- (C) Spherical
(D) Flat disc

Ans :

(C) Spherical

Due to surface tension, liquids try to minimize their surface area for a given volume. A sphere has the smallest surface area for a given volume, so small drops tend to be spherical.

48. Which of the following statements about surface tension is incorrect?

- (A) It is the force per unit length acting in the plane of the liquid interface.
(B) It is the extra energy per unit area that molecules at the interface possess.
(C) Surface tension of most liquids increases with increasing temperature.
(D) It is responsible for the formation of spherical bubbles.

Ans :

(C) Surface tension of most liquids increases with increasing temperature.

The surface tension of most liquids decreases as the temperature increases. Increased thermal agitation weakens the cohesive forces between surface molecules.

49. Which statement accurately describes the angle of contact?

- (A) It is always an acute angle for all liquid-solid interfaces.
(B) It is the angle between the liquid surface and the solid surface outside the liquid.
(C) It determines whether a liquid will wet a solid surface or not.
(D) It is independent of the nature of the liquid and solid.

Ans :

(C) It determines whether a liquid will wet a solid surface or not.

The angle of contact, formed at the interface of a liquid, a solid, and a gas, indicates the wettability of the solid by the liquid. An acute angle implies wetting, while an obtuse angle implies non-wetting.

50. Why does water rise in a narrow glass capillary tube?

- (A) Due to the higher pressure inside the tube compared to outside.
(B) Because the adhesive forces between water and glass are weaker than cohesive forces in water.

(C) Due to surface tension and an acute angle of contact between water and glass.

(D) Because the density of water is less than the density of air.

Ans :

(C) Due to surface tension and an acute angle of contact between water and glass.

Capillary rise occurs when the adhesive forces between the liquid and the tube walls are stronger than the cohesive forces within the liquid, resulting in an acute angle of contact and a concave meniscus. Surface tension then pulls the liquid up the tube.

51. All the following phenomena are related to surface tension EXCEPT:

- (A) Formation of spherical soap bubbles.
(B) Insects walking on water.
(C) The rise of oil in a lamp wick.
(D) The buoyant force on a submerged object.

Ans :

(D) The buoyant force on a submerged object.

Buoyant force is due to the pressure difference in the fluid at different depths acting on a submerged object (Archimedes' Principle), not directly related to surface tension. The other phenomena are consequences of surface tension.

52. How does the pressure inside a soap bubble compare to the pressure inside a liquid drop of the same radius?

- (A) Pressure inside the bubble is half that of the drop.
(B) Pressure inside the bubble is the same as that of the drop.
(C) Pressure inside the bubble is twice that of the drop.
(D) Pressure inside the bubble is four times that of the drop.

Ans :

(C) Pressure inside the bubble is twice that of the drop.

A soap bubble has two liquid-air interfaces (inner and outer), while a liquid drop has only one. The excess pressure inside a bubble ($4S/r$) is therefore twice the excess pressure inside a drop ($2S/r$) of the same radius and surface tension.

53. The study of fluids in motion is known as:

- (A) Fluid statics
(B) Fluid dynamics
(C) Thermodynamics
(D) Kinematics

Ans :

(B) Fluid dynamics

Fluid dynamics is the branch of physics that deals with the study of fluids (liquids and gases) in motion and the forces acting on them.

54. What does the equation of continuity for incompressible fluid flow represent?

- (A) Conservation of energy
- (B) Conservation of momentum
- (C) Conservation of mass
- (D) Conservation of angular momentum

Ans :

(C) Conservation of mass

The equation of continuity ($A_1 v_1 = A_2 v_2$) for an incompressible fluid states that the volume flow rate (Av) is constant, which is a direct consequence of the principle of conservation of mass.

55. What is the term for the speed beyond which a steady fluid flow becomes turbulent?

- (A) Escape speed
- (B) Terminal speed
- (C) Critical speed
- (D) Sound speed

Ans :

(C) Critical speed

Critical speed is the threshold velocity above which a smooth, laminar flow of a fluid transitions into a chaotic, turbulent flow.

56. Bernoulli's principle relates pressure, kinetic energy per unit volume, and what other quantity per unit volume for a flowing ideal fluid?

- (A) Internal energy
- (B) Potential energy
- (C) Enthalpy
- (D) Entropy

Ans :

(B) Potential energy

Bernoulli's principle states that for an ideal fluid in steady flow, the sum of pressure, kinetic energy per unit volume ($\frac{1}{2}\rho v^2$), and potential energy per unit volume (ρgh) remains constant along a streamline.

57. Torricelli's law states that the speed of efflux from an open tank is identical to that of a body freely falling from what height?

- (A) The total height of the tank.
- (B) The height of the liquid surface from the hole.
- (C) Half the height of the liquid surface from the bottom.

(D) The depth of the hole from the bottom.

Ans :

(B) The height of the liquid surface from the hole. Torricelli's law ($v = \sqrt{2gh}$) gives the speed of efflux (v) from a hole at a depth (h) below the surface of the liquid in an open tank, which is the same speed a body would acquire falling freely through height (h).

58. The upward dynamic lift on an airplane wing is primarily a consequence of:

- (A) Newton's third law
- (B) Pascal's law
- (C) Archimedes' principle
- (D) Bernoulli's principle

Ans :

(D) Bernoulli's principle

The shape of an aerofoil (airplane wing) causes air to flow faster over its upper surface than its lower surface. According to Bernoulli's principle, this speed difference creates a pressure difference, resulting in an upward lift force.

59. Why are detergents added to water for cleaning clothes?

- (A) To increase the viscosity of water, making it more effective.
- (B) To increase the surface tension of water, allowing it to lift dirt.
- (C) To decrease the angle of contact, allowing water to wet fabric better.
- (D) To decrease the density of water, making it penetrate fibers easily.

Ans :

(C) To decrease the angle of contact, allowing water to wet fabric better.

Detergents are wetting agents that reduce the surface tension of water and decrease the angle of contact between water and fabric. This allows the water to spread out and penetrate the fabric more effectively, aiding in cleaning.

60. If two streamlines in a fluid flow were to cross each other, what would it imply about the flow at that point?

- (A) The fluid velocity is zero at that point.
- (B) The flow is necessarily turbulent at that point.
- (C) A fluid particle at that point could have two velocities.
- (D) The pressure at that point is extremely high.

Ans :

(C) A fluid particle at that point could have two velocities.

A streamline represents the direction of velocity of fluid particles. If two streamlines crossed, it would mean that at the point of intersection, a fluid particle could move in two different directions simultaneously, which contradicts the definition of steady (streamline) flow.

61. The most characteristic property of liquid is :

- (a) Elasticity
- (b) Fluidity
- (c) Formlessness
- (d) Volume conservation

Ans :

- (d) Volume conservation

The most characteristic property of a liquid is that it has a definite volume but no fixed shape. Liquids take the shape of their container but do not expand to fill it completely like gases. While properties like fluidity and formlessness are also features of liquids, the most defining characteristic is their ability to conserve volume under normal conditions.

62. 1 kg of cotton and iron are weighed in vacuum, then:

- (a) Cotton and iron will weigh same
- (b) Iron will weigh more than cotton
- (c) Cotton will weigh more than iron
- (d) Both have zero weight.

Ans :

- (a) Cotton and iron will weigh same

In vacuum, there is no buoyancy force on cotton and iron hence both will weigh same.

63. Small liquid drops assume spherical shape because:

- (a) Atmospheric pressure exerts a force on a liquid drop
- (b) Volume of a spherical drop is minimum
- (c) Gravitational force acts upon the drop
- (d) Liquid tends to have the minimum surface area due to surface tension.

Ans :

- (b) Volume of a spherical drop is minimum

The sphere has minimum surface area for a given volume of the liquid.

64. Bernoulli's equation for steady, non-viscous, incompressible flow expresses the :

- (a) Conservation of angular momentum
- (b) Conservation of density
- (c) Conservation of momentum
- (d) Conservation of energy.

Ans :

- (d) Conservation of energy.

Bernoulli's equation is derived from the principle of conservation of energy. It states that for a steady, incompressible, and non-viscous fluid, the sum of pressure energy, kinetic energy, and potential energy per unit volume remains constant along a streamline. This equation essentially shows how energy is conserved in fluid flow, converting between pressure, speed, and height.

65. If we dip capillary tubes of different radii r in water and water rises to different height h in them, then :

- (a) $\frac{h}{r^2} = \text{constant}$
- (b) $\frac{h}{r} = \text{constant}$
- (c) $hr = \text{constant}$
- (d) $hr^2 = \text{constant}$

Ans :

- (c) $hr = \text{constant}$

Since, $h = \frac{2T \cos \theta}{r \rho g}$

so $hr = \frac{2T \cos \theta}{\rho g} = \text{constant}$

66. In a streamline flow, if the gravitational head is h , the kinetic and pressure heads are:

- (a) $\frac{v^2}{g}$ and $\frac{P}{\rho}$
- (b) $\frac{v^2}{2g}$ and $\frac{P}{\rho g}$
- (c) $\frac{v^2}{2}$ and $\frac{P}{\rho}$
- (d) $\frac{v^2}{2}$ and $\frac{P}{\rho g}$

Ans :

- (b) $\frac{v^2}{2g}$ and $\frac{P}{\rho g}$

Kinetic head = $\frac{v^2}{2g}$

Pressure head = $\frac{P}{\rho g}$

67. A big drop of radius R is formed by 1000 small droplets of water, then radius of small drop is :

- (a) $\frac{R}{2}$
- (b) $\frac{R}{5}$
- (c) $\frac{R}{6}$
- (d) $\frac{R}{10}$

Ans :

- (d) $\frac{R}{10}$

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$R = 10r \text{ or } r = \frac{R}{10}$$

TWO MARKS QUESTIONS

- 125.** Why high viscosity liquids are used as buffers in trains?

Ans :

Because high viscosity liquids absorbs the shock.

- 126.** How does the ploughing of fields help in preservation of moisture in the soil?

Ans :

Ploughing destroys the capillaries in the field and hence water does not rise up and is retained by the soil.

- 127.** Why small drop of mercury is spherical but bigger drops are oval in shape ?

Ans :

Small drops are spherical due to large force of surface tension than the weight of the drop and bigger drops are oval due to large weight of the drop.

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- 128.** Why flags flutter on a windy day ?

Ans :

The wind velocity is different at different points on a flag and hence pressure is different at these points.

- 129.** If surface tension of water is 70 dyn cm^{-1} , what is its value in S.I. ?

Ans :

It is $70 \times 10^{-3} \text{ N m}^{-1}$.

- 130.** What is fluid friction ?

Ans :

Fluid friction or viscosity is the property of fluid by virtue of which an internal force comes into play when the fluid is in motion and opposes the relative motion of different layers.

- 131.** What is the physical significance of Reynolds number?

Ans :

Reynolds number describes the ratio of the internal force per unit area to the viscous force per unit area of the flowing fluid.

- 132.** The bags and suitcases are provided with broad handles. Explain why.

Ans :

Since pressure, $P = \frac{F}{A}$

Due to large area of the broad handles, the pressure exerted on the hand will be small to carry the bag or suitcase.

- 133.** On what factors, the atmospheric pressure at a place depends ?

Ans :

Atmospheric pressure at a place depends upon

- height of the atmosphere.
- density of the atmosphere.
- acceleration due to gravity.

- 134.** Straws are used to take soft drinks. Why ?

Ans :

With a straw we suck the air inside, so the pressure inside the straw becomes less than atmospheric pressure and the drink rises in the straw.

- 135.** Why two holes are made to empty on oil tin ?

Ans :

With one hole, the pressure inside the tin would be less than atmospheric pressure. So oil would not come out. With two holes, the pressure inside the tin is equal to atmospheric pressure.

- 136.** Why is it dangerous to stand on the edge of the platform near the railway line when the train is passing by ?

Ans :

When a train passes, the fast moving air between the person and the train produces a decrease in pressure and the excess pressure on the other side pushes the person towards the train.

- 137.** Why hot soup tastes better than cold soup ?

Ans :

Since surface tension decreases with rise in temperature so the surface tension of hot soup is less than cold soup and it spread over a large area of tongue and hence, hot soup tastes better than cold soup.

ONE MARK QUESTIONS

76. What is one torr of pressure ?

Ans :

It is the pressure exerted by 1 mm of mercury column.

77. What do you mean by gauge pressure ?

Ans :

The excess pressure above the atmospheric pressure is called gauge pressure.

78. What is buoyancy ?

Ans :

The upward thrust acting on the body immersed in a liquid is called buoyancy or buoyant force.

79. What is centre of buoyancy ?

Ans :

The centre of buoyancy is the centre of gravity of the displaced liquid by the body when immersed in a liquid.

80. Does Archimedes' principle hold in a vessel in free fall ?

Ans :

No, because during free fall the vessel is in weightlessness state, where the buoyant force accounting for Archimedes' principle does not exist.

81. What does the sudden fall, gradual fall and gradual increase in barometer level indicate ?

Ans :

Sudden fall indicates the coming of storm, Gradual fall indicates the possibility of rain, Gradual increase indicates dry weather.

82. If few drops of water are introduced in the barometer tube, what would be the effect on the barometric height ?

Ans :

Barometer height would decrease due to the pressure of water vapours.

83. Why water does not come out of a dropper unless its rubber head is pressed hard ?

Ans :

Water would come out only if the pressure exerted on the dropper head is greater than the atmospheric pressure.

84. Why mercury is used in barometer ?

Ans :

Due to large density of mercury, the barometer is of convenient size.

85. When does the flow of liquid become turbulent ?

Ans :

The flow of liquid becomes turbulent when its velocity becomes more than its critical velocity.

86. What is the value of Reynolds number for streamline flow ?

Ans :

$$R < 2000$$

87. What is continuity equation ?

Ans :

$$av = \text{constant.}$$

88. Write the dimensions of Reynolds number.

Ans :

$$[M^0 L^0 T^0]$$

89. In streamline flow, what is the velocity of the liquid in contact with the containing vessel ?

Ans :

Zero

90. Why is that a liquid set in motion comes to rest after some time ?

Ans :

It comes to rest due to internal friction between the layers of the liquid called viscosity.

91. Write two factors affecting viscosity. Which one is more viscous pure water or saline water ?

Ans :

Viscosity of a liquid depends upon its nature and its temperature.

Saline water is more viscous than pure water.

92. What is the effect of density on viscosity of liquids and gases ?

Ans :

In liquids, viscosity increases with increase in density and for gases, viscosity decreases with increase in density.

- 144.** Describe Poiseuille's formula for flow of liquid through a tube of narrow circular bore.

Ans :

Poiseuille proved from theoretical consideration that whenever a viscous liquid is flowing through a horizontal tube of uniform, narrow circular bore, and flow is steady, total volume of liquid flowing out per second V is

- (i) directly proportional to the pressure (P) across the two ends of the tube,

$$V \propto P \quad \dots(1)$$

- (ii) proportional to fourth power of the radius of tube i.e.,

$$V \propto r^4 \quad \dots(2)$$

- (iii) inversely proportional to the length (l) of the tube

$$\text{i.e.,} \quad V \propto \frac{1}{l} \quad \dots(3)$$

- (iv) inversely proportional to the coefficient of viscosity η of the liquid

$$\text{i.e.,} \quad V \propto \frac{1}{\eta} \quad \dots(4)$$

On combining all these four factors, we get

$$V \propto \frac{Pr^4}{\eta l}$$

$$V = \frac{KPr^4}{\eta l} = \frac{\pi Pr^4}{8\eta l}$$

where K is a dimensionless proportionality constant whose value was found by Poiseuille to be $\frac{\pi}{8}$.

- 145.** What is Poiseuille's formula ? Deduce it by means of dimensions.

Ans :

When a liquid flows through horizontal tube with a steady flow under external pressure, then volume of liquid flowing out per unit volume is given by

$$V = \frac{\pi \rho r^4}{8\eta l}$$

V depends upon

- (i) pressure gradient (p/l)
 (ii) radius of capillary tube r
 (iii) coefficient of viscosity

$$\text{Let} \quad V = k \left(\frac{P}{l} \right)^a (r)^b (\eta)^c \quad \dots(1)$$

Writing dimensions of both sides, we have

$$[L^3 T^{-1}] = [ML^{-2} T^{-2}]^a [L]^b [ML^{-1} T^{-1}]^c$$

$$[M^0 L^3 T^{-1}] = [M^{a+c} L^{-2a+b-c} T^{-2a-c}]$$

Applying principle of homogeneity of dimensions, we have

$$\begin{aligned} a + c &= 0 \\ 1 + c &= 0 \\ c &= -1 \\ -2a + b - c &= 3 \\ = -2 \times 1 + b - (-1) &= 3 \\ b - 1 &= 3 \\ b &= 4 \\ -2a - c &= -1 \\ -2a + 1 &= -1 \\ -a &= -1 \\ a &= 1 \end{aligned}$$

Substituting in equation (i), we have

$$V = k \left(\frac{P}{l} \right)^1 (r)^4 (\eta)^{-1}$$

$$= \frac{kpr^4}{\eta l}$$

where $k = \frac{\pi}{8}$ calculated experimentally.

$$\text{Hence,} \quad V = \frac{\pi pr^4}{8\eta l}$$

- 146.** What is Stokes's law ? Derive the relation by method of dimensions.

Ans :

Stokes derived the relation for backward force F acting on a small spherical body of radius r moving through medium of coefficient of viscosity η with velocity v and found that

$$F = 6\pi\eta rv$$

This is called Stoke's law.

$$\text{Let} \quad F = k \eta^a v^b r^c \quad \dots(1)$$

Taking dimensions on both sides, we have

$$[M^1 L^1 T^{-2}] = [ML^{-1} T^{-1}]^a [LT^{-1}]^b [L]^c$$

$$[M^1 L^1 T^{-2}] = [M^a L^{-a+b+c} T^{-a-b}]$$

Applying principle of homogeneity of dimensions, we have

$$\begin{aligned} a &= 1 \\ -a + b + c &= 1 \\ -1 + 1 + c &= 1 \\ c &= 1 \\ -a - b &= -2 \\ -1 - b &= -2 \end{aligned}$$

- 110.** What is meant by saying that atmospheric pressure is 75 cm of mercury ?

Ans :

It means that the air is applying the same pressure as applied by 75 cm of vertical column of mercury.

- 111.** What is surface tension ?

Ans :

Surface tension is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum area and behaves as a stretched membrane.

- 112.** What is one bar ?

Ans :

$$1 \text{ bar} = 10^5 \text{ Nm}^{-2}$$

- 113.** What are the dimensions of Reynolds number ?

Ans :

It is dimensionless.

- 114.** What is upthrust ?

Ans :

When a body is immersed in a liquid (partially or wholly), the upward force exerted by the liquid on the body is called upthrust.

- 115.** Will 2 litre of kerosene oil weigh more in winter or summer ?

Ans :

Since the density of kerosene oil is more in winter than in summer, so the same volume of kerosene oil weigh more in winter than in summer.

- 116.** How many 'pascal' are there in one atmosphere pressure ?

Ans :

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

- 117.** Why do liquids exert pressure on the walls of the containing vessels ?

Ans :

Liquids exert pressure equally in all directions.

- 118.** What are the factors, which affect the atmospheric pressure at a place ?

Ans :

(i) Height of atmosphere (ii) acceleration due to gravity and (iii) density of the atmosphere.

- 119.** What is Poisson's ratio ?

Ans :

Poisson's ratio is the ratio of lateral strain β to the longitudinal strain α .

Poisson's Ratio, $\sigma = \frac{\beta}{\alpha}$

$$\beta = \frac{\text{Change in diameter}}{\text{Original diameter}}$$

and

$$\alpha = \frac{\text{Change in length}}{\text{Original length}}$$

- 120.** Write down Poiseuille's equation in terms of velocity of flow of liquid.

Ans :

$$V = \frac{\pi Pr^4}{8\eta l} \quad \dots(1)$$

$$V = Av = \pi r^2 v$$

Now, $\pi r^2 v = \frac{\pi Pr^4}{8\eta l}$

$$v = \frac{Pr^2}{8\eta l}$$

- 121.** If honey and water are dropped out of a tube separately the honey comes out later than water. Why?

Ans :

Because, coefficient of viscosity of honey is large as compared to the coefficient of viscosity of water.

- 122.** What is streamline ?

Ans :

It is a path, straight or curved, the tangent to which inside a fluid at any point gives the direction of the flow of fluid at that point.

- 123.** Why liquids exert pressure on the walls of the containing vessel ?

Ans :

Because liquids exert pressure in all directions.

- 124.** Define one decapoise.

Ans :

The coefficient of viscosity of a liquid is said to be 1 decapoise if a tangential force of 1N maintains a velocity gradient of 1 ms^{-1} between two parallel layers each of 1 m^2 in area.

93. Why machines are sometimes jammed in winter ?

Ans :

Viscosity increases with decrease in temperature.

94. Why small air bubbles rises slowly through the liquid whereas the bigger one rises rapidly ?

Ans :

Terminal velocity is proportional to square of radius
 $v \propto r^2$.

95. Why does velocity increase when water flowing in a broader pipe enters narrower pipe ?

Ans :

When water enters into a narrower pipe, the area of cross-section (a) decreases and consequently velocity (v) increases as $av = \text{constant}$.

96. What are the dimensions of coefficient of viscosity ?

Ans :

$$[\eta] = [\text{ML}^{-1}\text{T}^{-1}]$$

97. Hotter liquids flow faster than cold liquids. Why ?

Ans :

Volume of liquid flowing per second through a pipe is given by

$$V = \frac{\pi pr^4}{8\eta l}$$

$$V \propto \frac{1}{\eta}$$

As the temperature of a liquid increases, the coefficient of viscosity decreases, so the rate of flow increases.

98. Why falling water drops are spherical in shape ?

Ans :

Due to surface tension of water.

99. Mercury does not cling to glass. Why ?
 or

Why mercury does not wet glass ?

Ans :

Because force of cohesion of molecules of mercury is more than the force of adhesion between the molecules of mercury and glass.

100. What happens to surface tension when impurity is mixed in a liquid ?

Ans :

The surface tension of the liquid decreases.

101. By which phenomenon water rises from the root to the leaves of plants ?

Ans :

It rises by capillary action.

102. Why the nib of a pen is split?

Ans :

For capillary action.

103. Why cotton dresses should be preferred in summer?

Ans :

Due to the reason that they have fine pores which act as capillaries for the sweat.

104. What is the shape of meniscus of mercury in contact with glass?

Ans :

Convex.

105. The shape of liquid meniscus is concave. Will it rise or fall in a capillary tube ?

Ans :

It will rise.

106. Why clothes become water-proof when wax is rubbed on them ?

Ans :

When wax is rubbed on the clothes, the capillaries in the threads disappear.

107. What do you understand by molecular range ?

Ans :

The maximum distance upto which the force of attraction between two molecules is appreciable is called molecular range.

108. How do tree draw water from ground ?

Ans :

By capillary action.

109. The radius of a capillary tube is doubled. What changes will take place in height of the capillary rise ?

Ans :

Height, $h \propto \frac{1}{r}$. So capillary rise will be halved.

- 138.** Two drops of mercury when brought together coalesce into one. Why ?

Ans :

When two drops of mercury are brought together, they coalesce into one due to strong forces of cohesion between the mercury molecules.

- 139.** What is the effect :

- of highly soluble impurities on the surface tension of a liquid
- of less soluble impurities on the surface tension of a liquid?

Ans :

- The surface tension increases
- The surface tension decreases

- 140.** Why antiseptics have low surface tension ?

Ans :

For the quick heal of cut or wound antiseptic is used. So antiseptic should be able to spread itself into thin layer on the cut or wound. A liquid spread more on the surface if its own surface tension is low. Hence antiseptics have low surface tension.

THREE MARKS QUESTIONS

- 141.** What do you mean by the terms fluids, hydrostatic and hydrodynamics ?

Ans :

The term fluid refers to a substance that can flow and does not have shape of its own. As liquids and gases have common properties, they are given a common name of fluids. A given quantity of liquid having same temperature fill a specific container to the same level. When the laws of mechanics are applied to fluids, generally, we do not consider the fluid as a whole but concentrate on some arbitrary position of it and we customarily refer all physical quantities to unit volume. In order to understand the various characteristics of fluid, we divide it into two parts, namely hydrostatic and hydrodynamics.

Hydrostatic is the study of fluid at rest and hydrodynamics is the study of fluid in motion.

- 142.** Derive an expression for the pressure exerted by a liquid column.

Ans :

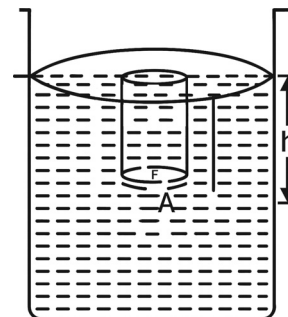
Let us suppose we wish to determine pressure around point A. Consider a small area say a round

this point forming a cylindrical vessel over this area.

Weight of cylindrical liquid vessel over a

$$= (\text{Area}) (\text{length}) (\text{density}) g$$

$$= ah\rho g$$



$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{ah\rho g}{a} = h\rho g$$

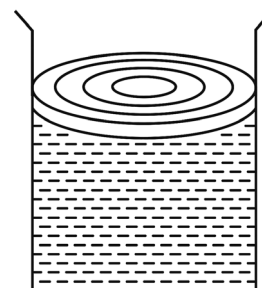
Hence, pressure is directly proportional to height of the liquid above the surface and density of liquid.

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- 143.** How does an actual liquid differ from an ideal liquid?

Ans :

An ideal liquid is that, which is completely frictionless and perfectly incompressible. However, in actual practice no liquid is ideal. Force of internal friction against the flow of liquid is present in all liquids. As an example, let us consider a cup of tea. We add teaspoonful of sugar into it and stir. Liquid molecules start moving in circular whirls, (as shown in the figure) but after few moments the motion of the liquid stops. This shows that internal viscous forces are present, which oppose the motion of liquid molecules.



Again all liquids are compressible. For moderate velocities of flow we may consider most of the liquids as perfectly incompressible but at very high velocities they can be compressed.

- (iii) inversely proportional to the diameter of the tube.

$$v_c \propto \frac{1}{D}$$

Combining the above three factors, we get

$$v_c \propto \frac{\eta}{\rho D}$$

$$v_c = \frac{\eta R}{\rho D}$$

where R is Reynolds number.

So
$$R = \frac{v_c \rho D}{\eta}$$

Importance of Reynolds number

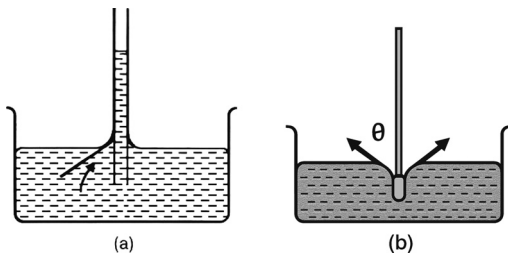
- (a) If $R < 2000$, the flow is said to be laminar or streamlined.
- (b) If $R > 3000$, the flow is said to be turbulent.
- (c) If $2000 < R < 3000$, the flow is unstable and may change from one form to another.

150. Define Angle of contact.

Ans :

Angle of contact between a liquid and a solid is defined as the angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid.

For example, when a glass strip is dipped in water or mercury, the angle of contact in both the cases is as shown in the figure.



The value of the angle of contact depends upon the nature of the liquid and solid in contact and is fixed for a given pair.

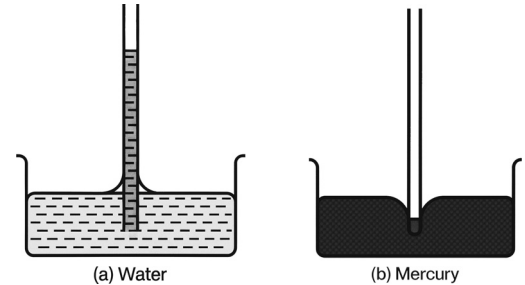
For instance, in case of liquids, which wet the walls of container e.g. water, angle of contact θ (with glass) is acute (i.e. $< 90^\circ$) whereas it is obtuse (i.e. $> 90^\circ$) for liquids which do not wet the sides of the container e.g. mercury and glass.

The angle of contact of mercury with glass is about 140° , whereas the angle of contact of water with glass is about 8° . However, for pure water in a glass tube $\theta = 0^\circ$.

151. What do you understand by capillarity ?

Ans :

Capillarity. The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surroundings is called capillarity.



All those liquids which wet the wall of container e.g. water, alcohol etc. rise in capillary tubes. But level of those liquids, which do not wet the walls of container are depressed in the capillary tube e.g. mercury, water taken in a tube coated with wax on its inner surface etc.

The phenomenon of capillarity is due to surface tension and force of adhesion between liquid and glass.

152. Explain what happens when length of a tube is less than the height upto which a liquid may rise in it ?

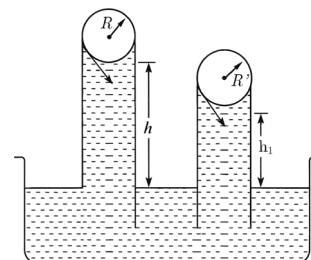
Ans :

If the length of a capillary tube dipped in a liquid is less than the height upto which liquid may normally rise in a tube of that much radius, the liquid does not overflow. To prevent the overflow, curvature of the liquid meniscus changes.

From simple geometry it can be shown that if the radius of curvature of meniscus be R , it is related with r the radius of the tube, by the formula

$$r = R \cos \theta$$

$$R = \frac{r}{\cos \theta}$$



Hence, Height to which liquid will rise normally in a tube

$$b = 1$$

Substituting in equation (1), we have

$$F = k \eta^1 v^1 r^1 = 6\pi\eta r v$$

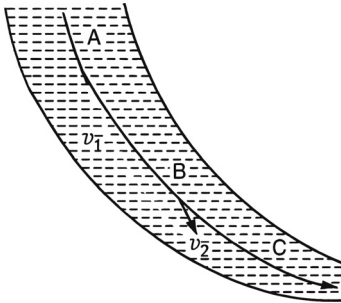
where $k = 6\pi$ found experimentally.

- 147.** What is streamline, streamline flow and turbulent flow ?

Ans :

Streamline may be defined as the path straight or curved, the tangent to which at any point gives the direction of flow of the liquid at that point e.g. ABC line in figure is streamline.

Streamline flow is defined as the motion in which flow every particle of the liquid follows the path of its proceeding particle with exactly the same velocity.



e.g. $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ are the velocities of a particle of fluid

(or liquid) at points A, B and C respectively, then the successive particles moving through these points will

possess velocities $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ at points A, B and C respectively.

Turbulent flow is defined as that motion in which the velocity of all the particles of the fluid crossing a given point is not the same i.e. disordered.

Streamline flow occurs only when the velocity of liquid is below a certain limiting value, termed as 'critical velocity' of the liquid. When the velocity of flow exceeds the critical value the liquid follows sinuous paths from side to side and flow becomes turbulent.

- 148.** What do you mean by critical velocity ? Establish an expression for it dimensionally.

or

Derive an expression for critical velocity using dimensional considerations.

Ans :

When a liquid moves through a tube having a comparatively small velocity, the flow is streamlined. If we gradually increase the velocity of flow of liquid, a limit comes beyond which streamlines are broken into abrupt curves and vertices and the motion becomes turbulent. The velocity of flow of a liquid at which the streamline flow just changes into turbulent flow is called critical velocity.

Let us assume that critical velocity of a liquid flowing through a tube depends on diameter of the tube D , density ρ and the coefficient of viscosity η as

$$v_c \propto (D)^a (\rho)^b (\eta)^c$$

$$v_c = R D^a \rho^b \eta^c$$

where R is a dimensionless constant.

We know that,

Dimensions of velocity $[v] [M^0 L T^{-1}]$,

Diameter, $[D] = [L]$,

Density, $[\rho] = [M L^{-3}]$

and viscosity $[\eta] = [M L^{-1} T^{-1}]$

Hence, $[M^0 L^1 T^{-1}] = L^a [M L^{-3}]^b [M L^{-1} T^{-1}]^c$

$$[M^0 L^1 T^{-1}] = [M^{b+c} L^{a-3b-c} T^{-c}]$$

Comparing powers of M , L and T on both sides, we have

$$b + c = 0$$

$$a - 3b - c = 1$$

$$-c = -1$$

On simplifying these, we get

$$a = -1, b = -1 \text{ and } c = 1$$

Therefore, $v_c = \frac{R\eta}{D\rho}$

Here R is a dimensionless constant and is called Reynolds number.

- 149.** What is Reynolds number ? What is its importance?

Ans :

Reynolds number is a pure number and it determines the nature of flow of liquid through a pipe.

According to Reynolds, the critical velocity v_c of a liquid flowing through a pipe is

- (i) directly proportional to the coefficient of viscosity of the liquid.

$$\text{i.e., } v_c \propto \eta$$

- (ii) inversely proportional to the density of the liquid

$$\text{i.e., } v_c \propto \frac{1}{\rho}$$

68. Surface tension of water is 0.072 Nm^{-1} . The excess pressure inside a water drop of diameter 1.2 mm is :

(a) 240 Nm^{-2} (b) 120 Nm^{-2}
(c) 0.06 Nm^{-2} (d) 72 Nm^{-2}

Ans :

(a) 240 Nm^{-2}

$$p = \frac{2T}{r} = \frac{4T}{D}$$

$$= \frac{4 \times 0.072}{1.2 \times 10^{-3}} = 240 \text{ Nm}^{-2}$$

69. What will be angle of contact of a liquid relative to solid, if forces of adhesion and cohesion are equal ?

(a) 0° (b) 30°
(c) 90° (d) None of these

Ans :

(c) 90°

When force of cohesion = Force of adhesion, the angle of contact of a liquid relative to solid is 90° .

70. In a container having water filled upto a height h , a hole is made in the bottom. The velocity of the water flowing out of the hole is :

(a) Independent of h
(b) Proportional to $h^{\frac{1}{2}}$
(c) Proportional to h
(d) Proportional to h^2

Ans :

(b) Proportional to $h^{\frac{1}{2}}$

Since, $v = \sqrt{2gh}$

So $v \propto \sqrt{h}$

71. The flow of liquid is laminar or streamline is determined by:

(a) Rate of flow of liquid
(b) Density of fluid
(c) Radius of the tube
(d) Coefficient of viscosity of liquid.

Ans :

(a) Rate of flow of liquid

For laminar flow, the mass of liquid entering the tube per second is equal to the mass of the liquid leaving the tube per second, hence the answer is (a).

72. For flow of a fluid to be turbulent:

(a) fluid should have high density
(b) critical velocity should be large

(c) Reynold number should be less than 2000
(d) All the above.

Ans :

(a) fluid should have high density

For turbulent flow, the Reynold number $R > 3000$

and $R = \frac{v_0 \rho D}{\eta}$ i.e. $R \propto \rho$

73. If two soap bubbles of different radii are connected by a tube:

(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from smaller to the bigger
(d) there is no flow of air

Ans :

(c) air flows from smaller to the bigger

Since, $p \propto \frac{1}{r}$, so air flows from smaller to bigger bubble.

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74. The spherical shape of a rain drop is due to:

(a) density of water
(b) surface tension
(c) acceleration due to gravity
(d) viscosity

Ans :

(b) Surface tension.

Pressure is higher in the smaller bubble due to its smaller radius. Air flows from high to low pressure, so it moves from the smaller to the bigger bubble.

75. In cold countries, water pipes sometimes burst, because :

(a) pipe contracts
(b) water expands on freezing
(c) when water freezes, pressure increased
(d) when water freezes, it takes heat from pipes

Ans :

(b) water expands on freezing

Water expands when it freezes, increasing its volume. This expansion exerts pressure on the walls of the pipe, often causing it to burst in cold countries.

$$h = \frac{2T \cos \theta}{r \rho g} = \frac{2T}{R \rho g} \quad \dots(1)$$

Now if the length of the tube is less than this expected height h , meniscus will change its radius of curvature to R' such that

$$h' = \frac{2T}{R' \rho g} \quad \dots(2)$$

Comparing (1) and (2), we have

$$Rh = R'h'$$

Hence if the tube is of insufficient length, radius of curvature of the liquid meniscus goes on increasing, making it more and more flat till liquid may be in equilibrium. Thus the liquid does not over flow.

- 153.** (a) How does surface tension change with temperature?
 (b) How do detergents change surface tension of water?

Ans :

- (a) The surface tension of a liquid decreases with rise of temperature. When the temperature is about to approach critical temperature, the surface tension reduces to zero. For small temperature differences, the variation in surface tension is linear. It is given by the relation

$$T = T_0(1 - \alpha t)$$

where T is the surface tension at $t^\circ\text{C}$ and T_0 is the surface tension at 0°C , α is temperature coefficient of surface tension.

- (b) Dirty clothes may contain grease and oil stains. Washing with water does not remove grease stains because water does not wet greasy dirt. When detergent is added to water the molecules of a detergent which are like hair pin-shaped, have their one end attracted to water and other end to molecules of grease oil. This forms water oil interface and surface tension is drastically reduced. It may even become energetically favourable to form such interfaces i.e. globules of dirt surrounded by detergent and then by water. This kind of process using surface active detergents is important not only for cleaning, but also recovering oil, mineral ores etc.

- 154.** Define surface tension. Illustrate with examples.

Ans :

Surface tension is the property of a liquid due to which the free surface of the liquid at rest tries to have minimum area, and as such it behaves like a stretched membrane.

Rain drops are spherical in shape. We know that for a given volume, the surface area of a sphere is minimum. Therefore, the spherical shape of rain drops is because the free surface of water in the rain drops try to have minimum surface area.

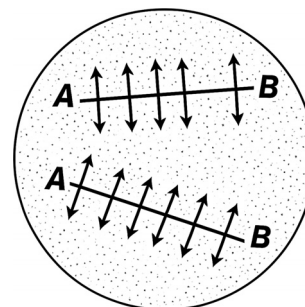
- 155.** How will you measure surface tension ?

Ans :

Surface tension of a liquid is measured as the force acting on unit length on either side of a line imagined to be drawn anywhere on the free surface of the liquid at rest. It acts at right angle to this line on both the sides and along tangent to the liquid surface.

If we imagine a line AB drawn anywhere on the liquid surface, the force of surface tension acts at right angles to this line on both of its sides and also along the tangent to the liquid surface. The force acting per unit length of such a line gives the quantitative measure of surface tension.

If F is the total force acting on an imaginary line of length l , drawn on the liquid surface, the force of surface tension T , by definition, is



$$T = \frac{F}{l}$$

SI unit of T is Nm^{-1} .

- 156.** How do detergents change surface tension of water ?

Ans :

Dirty clothes may contain grease and oil stains. Washing with water, does not remove grease stains because water does not wet greasy dirt. When detergent is added to water, the molecules of a detergent which are like hair pin-shaped, have their one end attracted to water and other end to molecules of grease oil. This forms water oil interface and surface tension is drastically reduced. It may even become energetically favourable to form such interfaces i.e. globules of dirt surrounded by detergent and then by water. This kind of process using surface active detergents is important not

only for cleaning, but also recovering oil, mineral ores etc.

- 157.** Water rises in capillary tube, whereas mercury falls in the same tube. Explain.

Ans :

The height upto which a liquid will rise in a capillary tube is given by

$$h = \frac{2T \cos \theta}{r \rho g}$$

For water, θ is acute and therefore $\cos \theta$ and hence h is positive. Due to this, water rises in a capillary tube. For mercury θ is obtuse and therefore $\cos \theta$ and hence h is negative. For this reason, mercury gets depressed in the capillary tube.

- 158.** What is equation of continuity ? Derive it.

Ans :

Consider a tube AB of variable area of cross-section. Let it be a_1 and a_2 at A and B and the respective velocities of liquid be v_1 and v_2

Mass of liquid entering A in 1 second = $a_1 v_1 \rho$

Mass of liquid entering B in 1 second = $a_2 v_2 \rho$

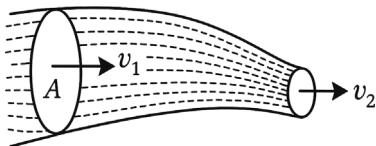
There is no loss of liquid, therefore

$$a_1 v_1 \rho = a_2 v_2 \rho$$

$$a_1 v_1 = a_2 v_2$$

$$av = \text{a constant.}$$

Which is called equation of continuity.



- 159.** How is the velocity of liquid flowing through capillary tube depends ?

Ans :

According to Poiseuille's formula,

$$V = \frac{\pi p r^4}{8 \eta l}$$

According to equation of continuity,

$$V = Av = \pi r^2 v,$$

where v is the velocity of liquid.

$$\pi r^2 v = \frac{\pi p r^4}{8 \eta l}$$

$$v = \frac{p r^2}{8 \eta l}$$

- 160.** Define capillary tube and capillarity. Give few examples of capillarity.

Ans :

A tube with a fine uniform bore throughout its length is called a capillary tube. The tube is as narrow as hair.

When a capillary tube is dipped in a liquid like water, liquid level in the tube rises and in case of mercury (which does not wet) the level falls in tube. This phenomenon of rise or fall of a liquid in a capillary tube is called capillarity.

Examples :

- (i) Boorg in towel soak water as they act as small capillaries.
- (ii) Oil rises through wick of lamp as small holes acts as capillary tubes.
- (iii) The fine pores of a blotting paper act like capillary tubes.
- (iv) Farmers plough their fields. By ploughing the fine capillaries in the soil, are broken, water from within the soil shall not rise and evaporate.

- 161.** What is the cause of special shape of bullets ?

Ans :

Spherical bullets do not hold on to a straight path due to 'magnus effect'. For this reason, bullets are invariably made cylindrical and additional gyrostatic stability is provided by giving them a spinning motion with an axis along the direction of flight by rifling the barrel. The bullet moves in a straight path undeviated without suffering from magnus effect as there is no lateral pressure acting on the bullet.

- 162.** If a person stands near a fast moving train, there is a possibility of his falling towards it. Why ?

Ans :

If a train is moving at a fast speed through air medium it drives a part of it to the sides and carries another part along with it in the front. This results in increased pressure at the head called head pressure and the air streamlines set up, go along the sides in backward direction. Thus, on the sides of a fast moving railway train whirls or eddies may be formed tending to push a person standing near train forward in the direction of motion and the person may fall.

- 163.** Bernoulli derived the relation by assuming the liquid to be non-viscous. How does the relation will change if fluid is viscous ?

Ans :

For non-viscous liquid having streamline flow, the Bernoulli equation is given by

$$\frac{P}{\rho} + gh + \frac{v^2}{2} = \text{constant}$$

For viscous liquid, the quantity $\frac{P}{\rho} + gh + \frac{v^2}{2}$ is not constant and will decrease along the direction of flow of liquid.

- 164.** Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water than the second vessel requires to fill upto a particular common height. Is the force exerted by water on the base of the vessel the same in the two cases ? If so, why do the vessel filled with water to that same height give different readings on a weighing scale ?

Ans :

Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has a non-zero vertical component which the sides of the vessel are not perfectly normal to the base. The net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence the vessels weigh different even when the force on the base is the same in the two cases.

- 165.** In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy.

- (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present ?
 (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.

Ans :

- (a) The pressure drop is greater.
 (b) Yes, the dissipative forces become more important with increasing flow velocity.

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FIVE MARKS QUESTIONS

- 166.** (a) Define thrust and pressure. Give their units. Show that when a liquid is in equilibrium, forces acting on the liquid must be perpendicular to its free surface.
 (b) What is the difference between pressure and stress?

Ans :

- (a) The total normal force exerted by a liquid on any surface in contact with it is called thrust of liquid pressure. The normal force (or thrust) exerted by a liquid at rest per unit area of the surface in contact with it is called pressure of liquid.

If F is the magnitude of normal force acting on area A , then

$$\text{Pressure, } P = \frac{F}{A}$$

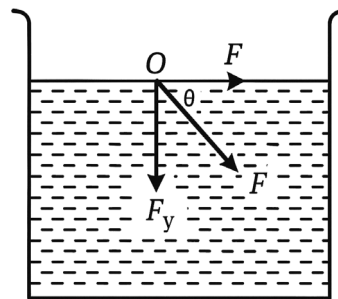
SI unit of pressure is newton metre⁻² (Nm⁻²) or pascal (Pa).

The value of atmospheric pressure is about 10⁵ Pa.

Pressure is a scalar quantity because the direction of force acting on a liquid in equilibrium is always specified, the force is completely described by its magnitude.

Force acting on a liquid at rest is perpendicular to its surface.

Consider a container containing a liquid in equilibrium state of rest. Let F be force acting on the liquid is not perpendicular but makes an angle θ with the horizontal as shown in figure.



Force F can be resolved in two rectangular components.

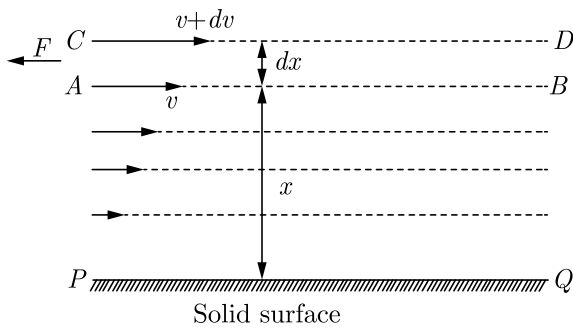
$$F_x = F \cos \theta \text{ along horizontal}$$

$$F_y = F \sin \theta \text{ along vertical}$$

As the liquid is at rest, so

$$F_x = 0$$

$$F \cos \theta = 0$$



Since the layer CD is moving faster than layer AB it tends to accelerate the motion of liquid, molecules lying in layer AB. On the other hand, molecules of layer AB tend to retard the motion of molecules of layer CD. Consequently, the two layers tend to destroy their relative motion and a viscous force acts between these layers.

According to Newton, the viscous drag on certain layer of a liquid is

- (i) directly proportional to area of layers in contact i.e.

$$F \propto A$$

- (ii) directly proportional to velocity gradient between the layers

$$F \propto \frac{dv}{dx}$$

Combining two factors,

$$F \propto A \frac{dv}{dx}$$

$$F = -\eta A \frac{dv}{dx}$$

where η is called coefficient of viscosity and depends upon the nature of liquid. The negative sign shows that viscous force is opposite to flow of liquid.

If $A = 1$, $\frac{dv}{dx} = 1$ then

$$\eta = F \quad (\text{numerically})$$

Thus coefficient of viscosity of a liquid is the force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of liquid.

Unit of coefficient of viscosity : S.I. unit of viscosity is Newton second per square metre or poiseuille (P_1) or decapoise.

To define one poiseuille, we consider

$$F = \eta A \frac{dv}{dx}$$

$$\eta = \frac{F}{A \times \frac{dv}{dx}}$$

then, $\eta = 1$ poiseuille.

The coefficient of viscosity of a liquid is said to be one poiseuille if a tangential force of 1 newton maintains a velocity gradient of 1 ms^{-1} between two parallel layers each 1 m^2 in area.

- (c) Differences between viscosity and friction are as follows :

	Viscosity	Friction
1.	Viscosity depends directly on relative velocity between layers.	Friction is independent of relative velocity of moving bodies.
2.	Viscosity is directly proportional to area of layers.	Friction is independent of areas of solid surfaces in contact.
3.	Velocity is due to cohesion.	Friction can be both due to cohesion or adhesion.
4.	It is a property of fluids.	It is a property of solids.
5.	Viscosity cannot be reduced by methods like polishing.	Friction can be reduced by polishing.

Similarities between viscosity and friction :

- Both tend to oppose relative motion.
- Both come into play whenever there is relative motion and act tangential to the surface in a direction opposite to that of motion.
- Both the forces are due to molecular attractions.

- 173.** Derive an expression for the terminal velocity of a sphere falling through a viscous fluid.

Ans :

Terminal velocity is the maximum constant velocity acquired by the body while falling freely in a viscous medium.

The various forces acting on the sphere of radius r falling in a liquid of viscosity η is

- Weight W of body acting vertically downwards.
- Upward force due to buoyancy (F_1) which is equal to the weight of liquid displaced.
- Force due to viscosity F_v , acting in a direction opposite to direction of motion of the body.

As the body falls through medium, its velocity goes on increasing due to gravity. An opposing viscous

- 169.** Obtain the expression for Pascal's law in the presence of gravity.

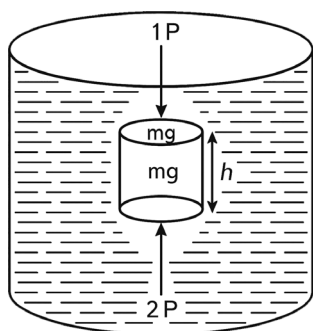
Ans :

Consider a vessel with a fluid in it. Imagine a cylinder of liquid of height h . The pressure at a point 1 and at another point 2 vertically a distance h' below it should be the same if forces due to gravity are neglected. Suppose they are included. A cylinder of fluid in equilibrium. The force acting on it in the vertical direction are due to fluid pressure at the top and at the bottom and weight $m_f g$ of the fluid

$$P_1 A + m_f g - P_2 A = 0$$

$$(P_2 - P_1) A = m_f g$$

where A is the area of base of cylinder. For liquid, the density is uniform and does not depend on pressure.



$$m_f = \text{Volume} \times \text{density} = Ah \times \rho$$

$$(P_2 - P_1) A = Ah \rho g$$

$$(P_2 - P_1) = h \rho g$$

This is modified Pascal's law and the liquid pressure depends upon

- (i) h , vertical distance
- (ii) m , Mass of liquid
- (iii) ρ , density of liquid.

Special cases :

- (i) If the effect of gravity is neglected i.e. $g = 0$

$$\text{Then, } P_2 - P_1 = 0$$

$$P_2 = P_1$$

- (ii) If point 1 lies on the surface of the liquid i.e.

$$P_1 = 0$$

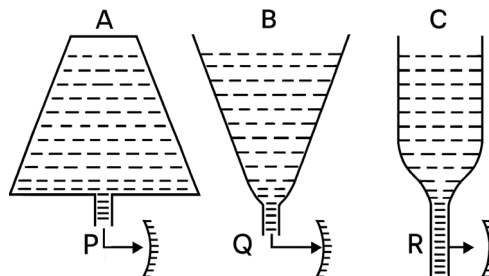
$$P_2 = h \rho g$$

Generally the pressure exerted is taken as P ,

$$P = h \rho g$$

Illustration : Consider three vessels, containing water at equal heights as shown in the figure and are connected to aneroid barometer. As long as

the heights of water in three vessels is same, the pressure at the bottom is same. Vessel A has much more water than C but the pressure at P is same as that of at point R and Q.



Suppose the vessels A and C are connected by a tube, no water will flow from A to C because the pressure is same even though the vessel A has much more water.

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- 170.** (a) What is atmospheric pressure ? Explain.
 (b) What is its value ? Give its units.
 (c) How does pressure vary with height ?
 (d) Why don't we feel effect of atmospheric pressure?

Ans :

- (a) The atmospheric pressure is due to weight of the air above.

The pressure exerted by the atmospheric air at any point is equal to weight of air contained in a column of unit cross-sectional area and extending upto top of atmosphere.

The instrument used for measuring the atmospheric pressure is known as Barometer. It was devised by Torricelli.

Take a glass tube T one metre long which is closed at one end and open at other end. The glass tube is filled completely with mercury. The opening of the tube is closed with thumb. The tube is inverted in a vessel V containing mercury in such a way that open end of the tube is inside the mercury surface. Care should be taken to see that no air enters the tube. On removing the thumb, the level of mercury in the tube starts falling and stands when a certain height is reached. The space above the mercury column does not contain any air and is known as Torricellian vacuum. The height of mercury column above the surface in the vessel V, is noted. The height of mercury column is found to be 76 cm.

Since, $F \neq 0$

Hence, $\cos \theta = 0$ or $\theta = 90^\circ$

i.e. the force acting on a liquid in equilibrium state of rest is always perpendicular to its surface.

- (b) Pressure is force per unit area acting on the surface of a system, the force being every where perpendicular to the surface and force per unit area being the same for constant pressure.

Pressure is taken to be positive if the force is directed inward and is taken to be negative if it is directed outwards.

Stress is also force per unit area, but the force may be different in magnitude on different surfaces and may not be directed perpendicular to surface.

Pressure is a particular kind of stress, one which changes only volume of substance and not its shape.

167. Discuss Hydraulic lift.

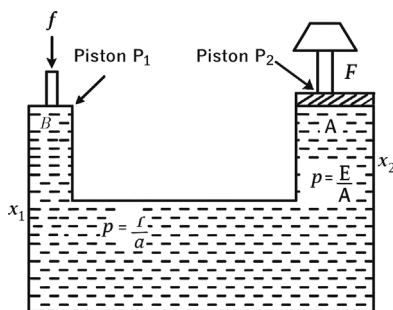
Ans :

Hydraulic lift is a device which is used to lift heavy objects such as cars and other vehicles. It multiplies forces.

It is based on Pascal law. i.e. a liquid exerts pressure equally in all directions.

It consists of two cylinder X_1 and X_2 connected to each other with a pipe. Let a and A be area of cross-section of cylinders X_1 and X_2 respectively.

Let P_1 and P_2 be two points on cylinder X_1 and X_2 . Fill the whole cylinder with a liquid as shown in the figure.



Let f = force applied on piston P_1

Hence, Pressure exerted by piston P_1 on liquid

$$p = \frac{f}{a} \quad \dots(1)$$

According to Pascal's law this pressure is transmitted to piston P_2

Hence, Force transmitted to bigger piston

$$F = pA$$

Using equation (1), we get

$$F = f \frac{A}{a} = f \left(\frac{A}{a} \right)$$

As $A \gg a$

Hence, $F \gg f$

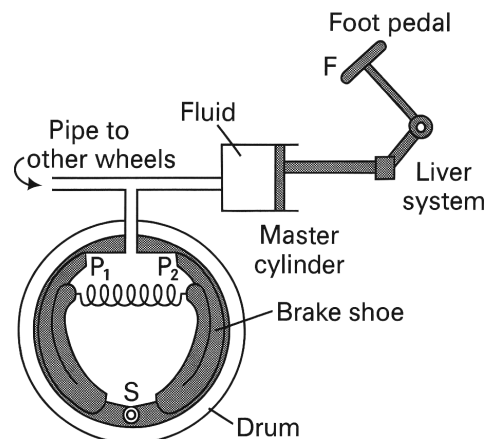
Thus by increasing the area A of piston P_2 , the force exerted on it can be increased, hence a heavy load placed on the bigger piston can be lifted easily.

168. Discuss Hydraulic brakes with the diagram.

Ans :

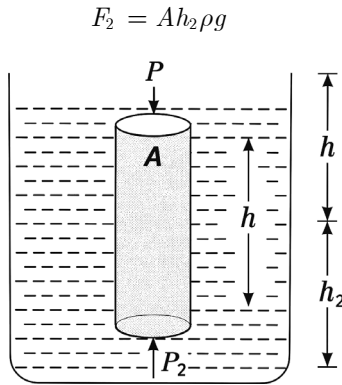
Hydraulic brakes works on the principle on Pascal's law.

It consists of a master cylinder C containing fluid and a frictionless piston P . The piston is connected to foot pedal and the fluid can pass through strong pipes to cylinders fitted with pistons in different wheels. Thus a force of smaller magnitude applied over the brake pedal is transmitted as large force to piston for each brake through the brake fluid. The wheel cylinder contains two piston P_1 and P_2 which are connected to brake shoes as shown in the figure. Similar system is connected to all the wheels of a vehicle.



When the driver of the vehicle presses the foot pedal, the lever system moves the piston into cylinder C . The fluid from master cylinder is led through strong pipe to cylinder of larger area of

cross-section fitted with piston P_1 and P_2 . Due to the pressure of fluid, the piston P_1 and P_2 move outwards and brake shoes open up wider and press against the inner rim of the wheel and stops it. On releasing the pedal, the brake shoes and brake fluid returns to their normal position with the help of spring S .



Since $F_2 > F_1$ therefore the net force upward acting on cylinder is

$$\begin{aligned}
 F &= F_2 - F_1 = Ah_2\rho g - Ah_1\rho g \\
 &= A\rho g(h_2 - h_1) \\
 &= A\rho gh = (Ah)\rho g \\
 &= (\text{volume}) (\text{density of liquid}) g \\
 &= \text{mass of liquid displaced} \times g \\
 &= \text{weight of liquid displaced} \\
 &= mg
 \end{aligned}$$

True weight of body,

$$W = Mg$$

Upward force on the body

$$F = mg$$

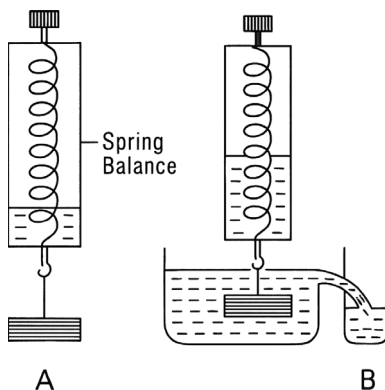
Observed weight of the body in liquid

$$= W - F = Mg - mg$$

i.e. whenever a body is immersed in a liquid, it loses its weight equal to the weight of the liquid displaced by the immersed portion of the body which is Archimedes' principle.

Experimental Verification : Suspend a solid to the hook of a spring balance. Reading of the balance gives the actual weight.

Fill a vessel having a side tube with water. Dip the solid body in water and note the spring balance reading.



This gives the weight of body when dipped in water. The displaced water collected in beaker B is weighed. The difference in two readings of spring balance gives the buoyancy force and is equal to weight of water displaced.

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172. (a) Define viscosity. What is its cause ? Define velocity gradient.
 (b) Define coefficient of viscosity. Give its unit.
 (c) What is difference between viscosity and friction? What are the similarities between viscosity and friction ?

Ans :

- (a) Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its different layers.

If a liquid in a vessel is stirred and left to itself, the liquid comes to rest after some time. This indicates that there is some internal friction in the liquid. This internal friction is called viscosity.

The force opposing the flow of liquid is called viscous force or viscous drag.

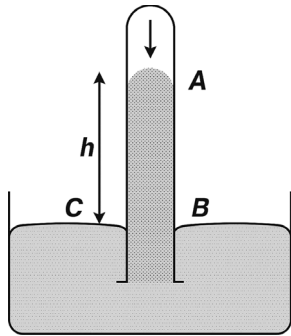
Cause of Viscosity : It is due to force of cohesion between the layers of a fluid. Consider a liquid flowing steadily over a solid horizontal surface. The layer of liquid in contact with the solid horizontal surface is at rest. The velocities of other layer increase uniformly with the increase in distance from fixed layer. The layer with lesser velocity drags the layer with larger velocity in backward direction and this backward (due to cohesion) is the cause of viscosity.

Velocity gradient : The change of velocity of liquid layer with distance from "the solid surface is called velocity gradient.

If v and $v + dv$ be velocities of two layers of a liquid at distance x and $x + dx$ from the solid surface, then

$$\text{Velocity gradient} = \frac{dv}{dx}$$

- (b) Consider a liquid flowing steadily over a fixed horizontal surface PQ. Let us consider a layer AB situated at a distance x from the fixed solid surface PQ. If this layer be moving with a velocity v the layer CD, immediately above it is moving slightly faster with a velocity $v + dv$.



The mercury is kept standing in the tube by the atmospheric pressure, which acts on the surface of mercury contained in the vessel and is transmitted through the mercury in the vessel. The atmospheric pressure pushes the mercury into the tube but the pressure due to weight of the mercury column tries to push it out. When the height of mercury column is about 76 cm then the pressure due to the weight of mercury becomes just equal to outside the atmospheric pressure and we can say that in a mercury barometer height of mercury column supported by the atmosphere pressure is used to measure atmospheric pressure.

(b) Calculation of atmospheric pressure

$$\begin{aligned}
 \text{Pressure at point B} &= \text{Pressure at A} \\
 &+ \text{Pressure due to mercury} \\
 &= 0 + h\rho g \quad \text{Since, Pressure at A} = 0 \\
 &= h\rho g \\
 &= \frac{76}{100} \times 13.6 \times 10^3 \times 9.8 \\
 &= 1.013 \times 10^5 \text{ Nm}^{-2}
 \end{aligned}$$

Units of pressure : S.I. unit of Pressure is Nm^{-2} Or pascal (Pa). Another unit of pressure is torr. A pressure equivalent to 1 mm of Hg is called torr.

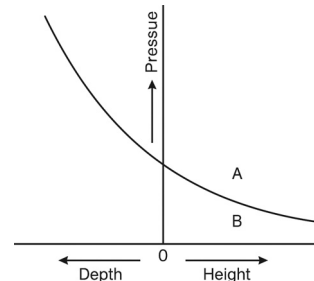
$$\begin{aligned}
 1 \text{ torr} &= 1 \text{ mm of Hg} \\
 &= 10^{-3} \times 13.6 \times 10^3 \times 9.8 \\
 &= 133.28 \text{ Nm}^{-2}
 \end{aligned}$$

(c) **Variation of pressure with height :** Height of the column of mercury is a measure of atmospheric pressure. When the height of Hg decreases, the pressure decreases. It means that as one goes up a mountain where the air pressure is less, the height of mercury column is less.

The variation of pressure with increase in altitude above the earth's surface is as shown in the figure.

Realistic model of atmosphere is shown in the figure. From the figure, we find that the

atmospheric pressure decreases with height above the earth's surface. Full line graph shows the variation of atmospheric pressure with height assuming the temperature and value of g constant and dotted line graph shows the variation of atmospheric pressure under real conditions.



- (d) We don't feel atmospheric pressure because
- At any depth, air pressure is same in all directions. These pressures balance each other and hence we do not feel uneasy.
 - The internal organs of our body are connected to the atmospheric pressure through nose and mouth. So, the pressure acting on our body from inside is nearly the same as that acting from outside.
 - The atmospheric pressure is balanced by our blood pressure and we do not feel uneasy.

171. (a) Define buoyancy.
(b) State and deduce Archimedes' principle. Illustrate an experiment to verify it.

Ans :

- (a) When a body is immersed in a liquid, it experiences an upward force known as Buoyant force or Buoyancy.
(b) Archimedes' principle states that when a body is wholly or partially immersed in a liquid, it experiences a loss in weight and this loss in weight is equal to the weight of liquid displaced by it.

Apparent loss in weight of a body in a liquid = Weight of liquid displaced by it.

Archimedes' principle is applicable to objects in liquids as well as gases.

Consider a cylinder of length h and area A . Pressure P_1 acting on top face is given by

$$P_1 = h_1 \rho g$$

Force (downward) on top face = $P_1 A$

$$F_1 = Ah_1 \rho g$$

Pressure on lower face = $P_2 A$

$$P_2 = h_2 \rho g$$

Force (upward) on lower face = $P_2 A$

drag acting in the direction opposite to direction of motion (upward) also increases. At a certain stage weight of the body is equal to the sum of the upward thrust due to buoyancy and upward viscous drag. There is no net force to accelerate the body and hence it starts falling with a constant velocity called terminal velocity.

Now, $W = \text{volume of sphere} \times \text{density} \times g$

$$= \frac{4}{3}\pi r^2 \times \rho \times g,$$

where ρ is the density of the body.

Upward force due to buoyancy,

$$F_1 = \frac{4}{3}\pi r^2 \sigma g$$

where σ is the density of liquid/gas through which the body is falling down.

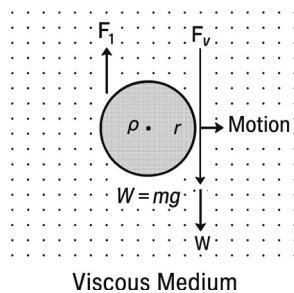
If v is the terminal velocity, then

$$F_v = 6\pi\eta r v$$

When terminal velocity is attained

$$W = F_1 + F_v$$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v$$



$$\frac{4}{3}\pi r^3 g(\rho - \sigma) = 6\pi\eta r v$$

$$\frac{2}{9}r^2 g(\rho - \sigma) = \eta v$$

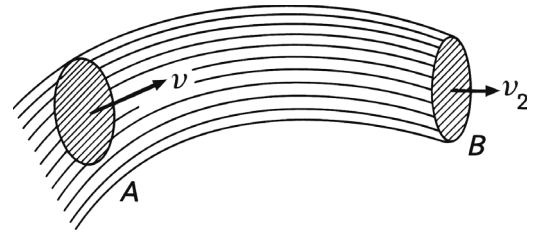
$$v = \frac{2r^2 g(\rho - \sigma)}{9\eta}$$

which is the required equation.

174. (a) What is an ideal liquid ?
 (b) Derive the equation of continuity for steady flow of an ideal liquid.

Ans :

- (a) A liquid which is incompressible, non-viscous and isotropic is called an ideal liquid.
 (b) Consider a liquid flowing through a tube AB of varying area of cross-section with a steady flow as shown in the figure.



Let $a_1 = \text{area of cross-section of the tube at A,}$

$a_2 = \text{area of cross-section of the tube at B,}$

$v_1 = \text{velocity of flow of the liquid at A,}$

$v_2 = \text{velocity of flow of the liquid at B,}$

$\rho_1 = \text{density of the liquid at A,}$

$\rho_2 = \text{density of the liquid at B,}$

Hence, Volume of the liquid flowing across A per sec.

$$= a_1 v_1$$

Mass of the liquid flowing across A per sec.

$$= a_1 v_1 \rho_1$$

Similarly, mass of the liquid flowing across B per sec.

$$= a_2 v_2 \rho_2$$

In case of a steady flow,

mass of liquid flowing across A per sec

$$= \text{mass of liquid flowing across B per sec.}$$

$$\text{Hence, } a_1 v_1 \rho_1 = a_2 v_2 \rho_2 \quad \dots(1)$$

As the liquid is assumed to be incompressible.

$$\text{Hence, } \rho_1 = \rho_2$$

$$\text{From (1) } a_1 v_1 = a_2 v_2 \quad \dots(2)$$

$$\text{or } av = \text{constant} \quad \dots(3)$$

This is known as the equation of continuity.

So, if a is large, v is small and vice versa.

i.e. larger the area of cross-section, smaller would be the rate of flow. This is shown in the figure. The reverse is also true. It is due to this reason that (a) the deep water runs slow (b) Jet of falling water becomes narrow as it goes downwards.

175. What are the various forms of energy possessed by a flowing liquid ? Find their expressions.

Ans :

Energy possessed by a liquid in motion is of three forms namely (i) pressure energy (ii) gravitational potential energy and (iii) kinetic energy.

These are as follows :

- (i) **Pressure energy** : Let an ideal liquid of density ρ be contained in a rectangular vessel, provided with a side tube at a depth h below the free liquid surface. At this level liquid pressure $P = h\rho g$.

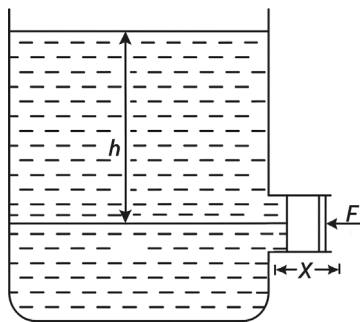
If we want to introduce more liquid into the vessel at this very pressure, we can do so by forcing it through the side pipe. For this work has to be done in forcing piston inwards.

If a be the area of cross-section of the piston, then

Force acting on the piston $= Pa$

and work done in moving the piston through a distance x .

$$W = Pax$$



As a result of motion of piston the mass of the liquid forced in the vessel $= \rho ax$

This work done is stored up in the liquid in the form of pressure energy because it is capable of doing some work in pushing the piston back while escaping from the tank.

Hence, Pressure energy of liquid per unit mass

$$= \frac{Pax}{\rho ax} = \frac{P}{\rho}$$

- (ii) **Gravitational potential energy** : Let at any stage of its flow a liquid element of mass m is situated at a height h from the surface of earth, then its gravitational potential energy in that position is mgh .

$$\text{Potential energy per unit mass} = \frac{mgh}{m} = gh$$

- (iii) **Kinetic energy** : Let at any stage of its flow a liquid element of mass m is moving with a velocity v , then the kinetic energy of this liquid element

$$= \frac{1}{2}mv^2$$

Kinetic energy per unit mass

$$= \frac{1}{2} \frac{mv^2}{m} = \frac{1}{2}v^2$$

176. State and prove Bernoulli's theorem of a liquid having streamline flow.

Ans :

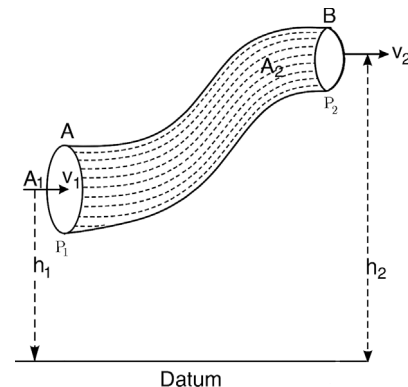
Bernoulli's theorem states that for the streamline flow of an ideal liquid, the sum of pressure, potential and kinetic energy per unit mass remains constant at every cross-section throughout the flow.

$$\text{i.e., } \frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

(An ideal liquid is one which is incompressible and non-viscous.)

Consider a tube of variable cross-section through which an ideal liquid is in streamline flow.

Let $P_1, P_2, A_1, A_2, h_1, h_2, v_1, v_2$ be the pressures, areas, heights and velocities at A and B respectively. Further let ρ be the density of liquid throughout the tube.



Since liquid flows from A to B, therefore $P_1 > P_2$.

The mass m of the liquid crossing per second through any section of the tube is

$$A_1 v_1 \rho = A_2 v_2 \rho = m,$$

According to equation of continuity.

$$\text{Hence, } A_1 v_1 = A_2 v_2$$

$$\text{Since } A_1 > A_2, \quad \text{Hence, } v_2 > v_1$$

$$\text{Force on A} = P_1 A_1$$

$$\text{Force on B} = P_2 A_2$$

Work done per second on liquid at A

$$= P_1 A_1 \times v_1 = P_1 (A_1 v_1) = P_1 V$$

and work done per second by the liquid at B

$$= P_2 A_2 \times v_2 = P_2 (A_2 v_2) = P_2 V$$

Hence, Net work done on the liquid by the pressure energy in moving the liquid from A to B

$$= P_1 V - P_2 V$$

Increase of P.E. of liquid from A to B

$$= mgh_2 - mgh_1$$

Increase of K.E. of liquid from A to B

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

According to work energy theorem,

Work done by pressure energy

$$= \text{Increase of P.E.} + \text{Increase of K.E.}$$

$$P_1 V - P_2 V = (mgh_2 - mgh_1) + \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$

Dividing throughout by m , we have

$$\frac{P_1 V}{m} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2 V}{m} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant} \quad \dots(1)$$

Hence, Sum of pressure energy per unit mass, potential energy per unit mass, kinetic energy per unit mass is a constant quantity.

Multiplying above equation by ρ , we have

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant} \quad \dots(2)$$

For horizontal flow,

$$h = h_2 - h_1 = 0$$

So, equation (1) becomes

$$\frac{P}{\rho} + \frac{1}{2}v^2 = \text{constant}$$

i.e., when the velocity of the flow of liquid is large, the pressure is low and vice-versa.

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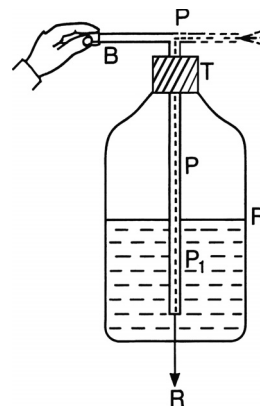
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177. Give some practical applications of Bernoulli's theorem.

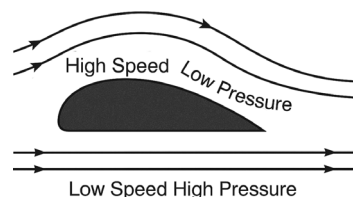
Ans :

Applications of Bernoulli's theorem are as follows :

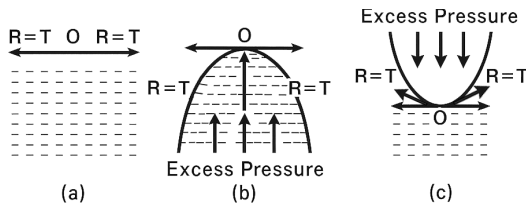
- (i) **Atomiser** : The common form of the atomiser is shown in the figure. When its rubber bulb B is squeezed, the air blow in the tube with high speed. According to Bernoulli's theorem, it creates a low pressure inside T which is much less than atmospheric pressure P_1 on liquid surface. Due to it the liquid rises up in the tube T and is pushed out with air through nozzle in the form of a spray. This principle is used in paint, oil, scent and nozzle sprayers.



- (ii) **Lift on an Aeroplane Wing** : The shape of the aeroplane wing (i.e. aerofoil) is peculiar. Its upper surface is more curved than its lower surface and its leading edge (i.e. head) is more thick than its trailing edge (i.e. trail). As the aeroplane moves forward the air blows in the form of streamlines over the wings of aeroplane. As the upper surface of wing is more curved than its lower surface, therefore, the speed of the air below the wing is less than that on the upper surface of wing. According to Bernoulli's theorem, the pressure above the wings becomes less than the pressure below the wings. Due to this difference of pressure on the two sides of the wings, the aeroplane is lifted up.



- (iii) **Blowing off the roofs during storms**. During storms or cyclones, the roofs of the huts or tinned roofs are generally blown off without causing any damage to the house. This is so because during the storms, the wind blow with very high speed over the top side of the roof. Due to it, the pressure (P_1) becomes low. Below the roof, inside the room the wind is not blowing. The pressure is P_2 , where $P_2 > P_1$. It means there is a difference of pressure acting upwards. Due to it, the roof of a hut is lifted up and is blown off by the wind.

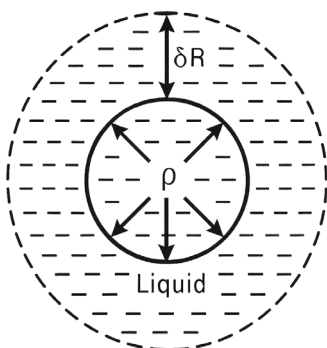


If a liquid surface is curved, the molecule O , on the liquid surface is acted upon, by the forces (a, a) due to surface tension, along the tangent to the surface [as shown in the figure (a), (b) and (c)]. Resolving these forces into horizontal and vertical components, we note that the horizontal components neutralise each other whereas vertical components are added up. Thus a resultant force would be acting on the curved surface of the liquid. It is directed downwards in case of convex surface and is directed upwards in case of concave surface.

Thus for a curved liquid surface, there is a resultant force acting on the liquid surface due to forces of surface tension. In order that the curved surface of liquid may be in equilibrium there must be an excess of pressure on its concave side so that the forces of excess pressure may balance the resultant forces due to surface tension.

Hence the pressure on the concave side of the liquid surface will be greater than pressure on convex side.

- (b) (i) **Excess pressure inside a drop :** Consider a liquid drop of radius R . The molecules lying on the surface of the liquid drop, due to surface tension will experience a resultant force acting inwards, perpendicular to the surface. As a result of it, the pressure inside the drop must be greater than the pressure outside it. This excess of pressure inside the drop will provide a force acting outwards perpendicular to the surface, to counterbalance the resultant force due to surface tension.



Due to it, let there be increase in the radius of

the drop from R to $R + \delta R$, where δR is very small quantity.

Let T = surface tension of the liquid drop.

p = excess of pressure inside the drop.

Then work done by the excess of pressure

$$= \text{Force} \times \text{distance}$$

$$= (\text{Excess of pressure} \times \text{area}) \times \text{distance}$$

$$= p \times 4\pi R^2 \times \delta R$$

Increase in surface area of the drop

$$= \text{Final surface area} - \text{Initial surface area}$$

$$= 4\pi (R + \delta R)^2 - 4\pi R^2$$

$$= 4\pi [R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$$

$$= 8\pi R(\delta R)$$

Increase in surface energy

$$= \text{Increase in surface area} \times \text{Surface tension}$$

$$= 8\pi R\delta R \times T$$

As the increase in surface energy is at the cost of excess of pressure, therefore

$$p \times 4\pi R^2 \times \delta R = 8\pi R\delta R \times T$$

$$p = \frac{2T}{R}$$

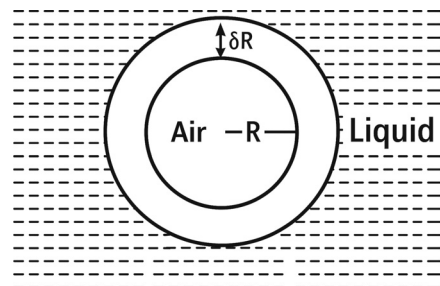
- (ii) **Excess of pressure inside an air bubble in a liquid:**

Let R = Radius of air bubble

T = Surface tension of liquid.

Proceed as in case of liquid drop.

Then,
$$p = \frac{2T}{R}$$



- (iii) **Excess of pressure inside a soap bubble :**

Let R = Radius of soap solution bubble.

The excess pressure inside the bubble will provide a force acting outwards perpendicular to the surface to counterbalance the resultant force due to surface tension.

Due to it, let there be increase in radius of bubble from R to $R + \delta R$

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According to equation of continuity,

$$V = a_1 v_1 = a_2 v_2 \quad \dots(1)$$

Since the tube is horizontal the potential energy of the liquid through A and B is the same.

According to Bernoulli's theorem,

Total energy volume of liquid through tube A =
Total energy volume of the liquid through tube B

$$\text{Hence, } P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad \dots(2)$$

But $P_1 = P_2 = h\rho g$, where h is the difference in levels of liquid in tubes D and E.

$$h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$hg = \frac{1}{2}(v_2^2 - v_1^2)$$

From equation (1), $v_1 = \frac{V}{a_1}$ and $v_2 = \frac{V}{a_2}$

Substituting in (1),

$$hg = \frac{1}{2}\left(\frac{V^2}{a_2^2} - \frac{V^2}{a_1^2}\right) = \frac{V^2(a_1^2 - a_2^2)}{2a_1^2 a_2^2}$$

$$V^2 = \frac{2a_1^2 a_2^2 hg}{a_1^2 - a_2^2}$$

$$V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$$

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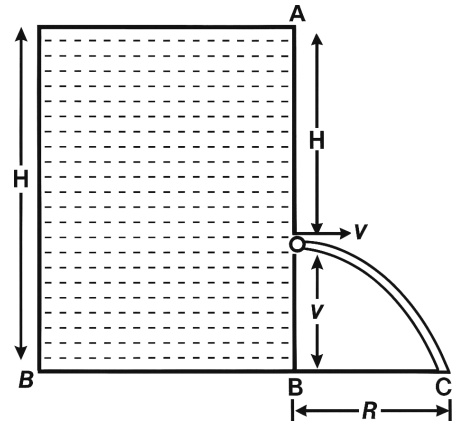
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179. Water stands at a depth H in a tank whose side walls are vertical. A hole is made on one of the walls at a depth h below the water surface.

- What is the velocity of efflux ?
- At what distance R from the foot of the wall does the emerging stream of water strike the floor ?
- For what value of h this range is maximum ?

Ans :

- Velocity of Efflux :** Let there be a tank containing water upto a height H and let side walls of the tank are vertical. Let O be orifice at a depth h below the free surface of water. The velocity v with which water comes out of the orifice is called its "velocity of efflux."



Obviously the pressure at the free surface of the water A and the orifice O, both is equal to the atmospheric pressure P and the water at free surface is at rest. Thus, according to Bernoulli's theorem, we have

Total energy at A = Total energy at O

$$\frac{P}{\rho} + 0 + hg = \frac{P}{\rho} + \frac{1}{2}v^2 + 0$$

[Here ρ = density of water]

$$hg = \frac{1}{2}v^2 \text{ or } v^2 = 2gh$$

$$v = \sqrt{2gh} \quad \dots(1)$$

Obviously, velocity of efflux of a liquid is equal to velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called Toricellian Theorem.

- Range (R) :** Water jet coming out of orifice O with a horizontal velocity will flow in a parabolic path and fall on the floor at a point C situated at a horizontal distance R (called range) from the foot of the wall, i.e. $BC = R$. Now as the vertical height covered by this flowing water = $(H - h)$, the time of fall t is given by

$$(H - h) = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H - h)}{g}}$$

Now horizontal range,

$$R = vt = v \times \sqrt{\frac{2(H - h)}{g}}$$

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H - h)}{g}}$$

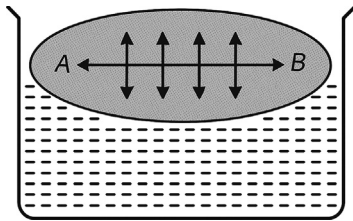
$$R = 2\sqrt{h(H - h)} \quad \dots(2)$$

- Condition for maximum range
Squaring equation (2), we have

$$R^2 = 4h(H - h) = 4Hh - 4h^2$$

into the pores of the cloth and remove the dirt, grease etc. from the clothings.

- (b) **Measurement of Surface Tension :** Surface tension of a liquid is measured as the force acting on unit length on either side of a line imagined to be drawn anywhere on the free surface of the liquid at rest. It acts at right angle to this line on both the sides and along tangent to the liquid surface.



If we imagine a line AB drawn anywhere on the liquid surface, the force of surface tension acts at right angles to this line on both of its sides and also along the tangent to the liquid surface. Fig. 10.46. The force acting per unit length of such a line gives the quantitative measure of surface tension.

If F is the total force acting on an imaginary line of length l , drawn on the liquid surface, the force of surface tension T , by definition, is

$$T = \frac{F}{l}$$

S.I. unit of T is Nm^{-1} .

Dimension of $T = [M^1 L^0 T^{-2}]$

181. What are cohesive and adhesive forces ? Give examples to illustrate these forces. Define molecular range, sphere of influence and surface film.

Ans :

The force of attraction between the molecules of the same substance is called the force of cohesion or the cohesive force. It is a short range molecular force and decreases very rapidly, when the distance between the molecules increases. Solids maintain a definite shape and volume due to strong force of cohesion. In a solid crystal various molecules are bound together due to force of cohesion. While tearing off a sheet of paper we apply force in order to overcome the cohesive force between paper molecules. Cohesive forces are the strongest in solids and the weakest in gases.

The force of attraction between the molecules of two different substances is called the force of adhesion or the adhesive force e.g., attraction between a glass and a water molecule, attraction between glue and paper molecules etc.

Sticking of one substance to another is due to force of adhesion between them. Some common examples of adhesive forces are sticking to powder and dirt to anybody, glue to wood, cement to brick, paint to walls, adhesive tape to the skin and ink to the paper while writing.

The maximum distance upto which a molecule exerts its influence on other molecules is called molecular range.

Sphere of influence : A sphere of radius equal to the molecular range is called sphere of influence.

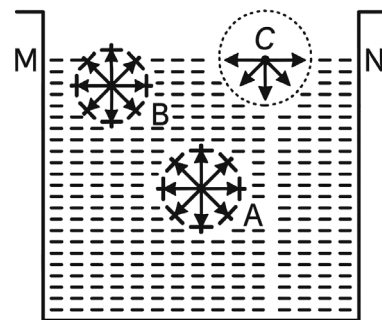
Surface film : It is the topmost layer of liquid at rest with thickness equal to molecular range.

182. Give molecular' theory to explain surface tension.

Ans :

Let $MNQP$ be the surface film where $MP = NQ$ = molecular range. Consider three molecules A, well within liquid; B within surface film and C, at the top of surface film. Draw sphere of influence of each molecule. The force of cohesion of all the neighbouring atoms to A mutually cancel and there is no net force of cohesion on molecule A.

Sphere of influence of molecule B lies partially outside the liquid, hence a net downward force acts on it. Sphere of influence of molecule C lies half outside the liquid, hence number of molecules in lower half of sphere of influence of C, acting downward is very large as compared to gas molecules above it. Hence there is a maximum resultant downward force acting on molecule C.

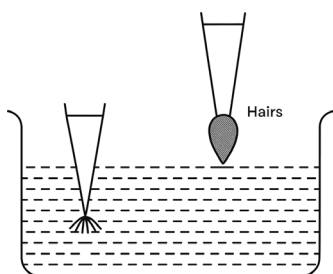


Thus, we conclude that no resultant force acts on a molecule below surface film. The downward force increases as we go up in the surface film. It is maximum at the top of the surface film. Therefore to take a molecule inside the surface film from anywhere below it, some work has to be done against downward force of cohesion. This work done appears as potential energy. Thus, surface film has more potential energy than the lower layers. But in state of equilibrium the surface should have least

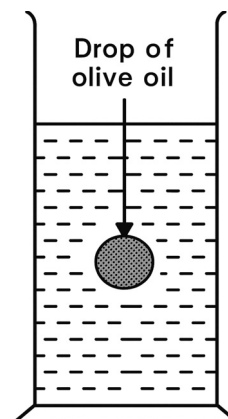


Shape of a drop, lying on a horizontal surface, is controlled, by the combined influence of surface tension and the force of gravity. For tiny drops, the force of gravity i.e. weight of drops is negligible. Therefore, under the action of surface tension drops become spherical. However, in case of larger drops force of gravity is appreciable and tries to flatten the drops so as to lower the position of its centre of gravity. Thus, under the combined action of surface tension and force of gravity drop acquires an oval (or elliptical) shape.

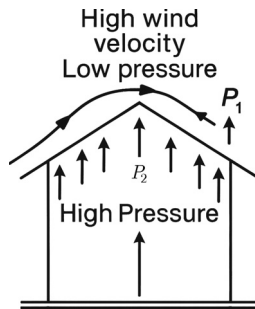
6. **Rain drops falling freely**, tiny dew drops and small water drops ejected from a pipette or burette are always spherical due to property of surface tension. However, a water drop, on coming in contact with some solid surface, flattens due to force of adhesion between drop and solid surface.
7. **Camel Brush** : If we dip a camel hair paint brush or a shaving brush into water, its hair keep apart. However on raising the brush out of water, the hair cling together. This is because the free surface of water film tends to contract.



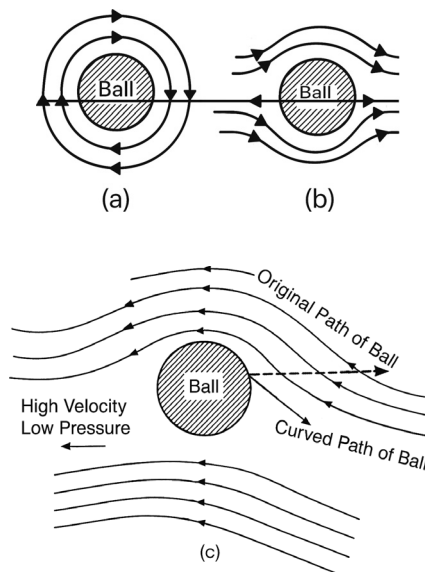
8. **Plateau's experiment** : Density of olive oil is less than that of water but more than alcohol. Prepare a solution of water and alcohol, so that the density of the mixture is equal to that of olive oil. Pour this solution in a jar and now drop a small quantity of olive oil by means of a pipette. It takes up a spherical shape due to force of surface tension. Here the effect of gravity is eliminated as the weight of olive oil drop is equal and opposite to the upward thrust due to weight of mixture displaced.



9. **Dancing camphor** : If a small irregular piece of camphor or gum is floated on the surface of pure water, it does not remain steady but dances about along the surface. The irregular-shaped camphor dissolves unequally thus, decreasing the surface tension of water unequally. Consequently, it experiences unequal pull from different directions and appears to be dancing irregularly.
10. **Antiseptic creams** : Antiseptic creams used for cuts and wounds are generally prepared in petroleum jelly or grease base. Due to its low surface tension even a small quantity of cream spreads on the whole wound.
11. **Oil spreads over the surface of water** : If we pour a drop of oil over water surface, it expands and acquires the shape of a thin film over the water surface. This is due to the fact that surface tension of oil is much less than that of water.
12. **Shrinking bubble** : If a soap bubble is blown at the end of a narrow glass tube and is made to stand for some time, bubble is seen to shrink slowly due to its tendency of contraction.
13. **Paper boat** : A paper boat can be made to propel itself on the surface of water by attaching a bit of soap at its back. The required power is supplied by the differential forces of surface tension of water and soap solution. The surface tension of water is greater than that of soap solution and it pulls the paper boat in the forward direction.
14. **Washing of clothes** : On adding soap or detergent powder into water its surface tension is lowered by a large percentage. Thus, this solution can more easily seep



- (iv) **Motion of two ships or two boats :** If two ships, small distance apart move parallel to each other at a very high velocity, the air starts moving with high velocity through the narrow space between them. Due to it the pressure is reduced in the space between the two ships. Now the outwards high pressure pushes the ships inwards. As a result of it, the ships come closer to each other and may even collide and be destroyed. A person standing near a fast moving train falls towards train can also be accounted as discussed above.
- (v) **Spinning of a ball :** When a spherical ball is rotated about an axis perpendicular to the plane of paper, the surrounding air is set into motion in stream lines of concentric circles in a plane parallel to the plane of paper and the direction of rotation of the ball (as shown in figure). The more rough the surface of the sphere, the thicker is the layer of air set in motion.



When the ball is given only translatory (linear) motion forward, the air in front is pushed aside. It flows backward on the sides and towards its rear end (as shown in figure). However, if the ball possesses

translational as well as rotatory motion, as in case of a spinning ball, then the two streams on the upper side are in opposite directions hence the resultant velocity of air is decreased and pressure increased. On its lower side, the two sets of streamlines are in the same direction and help each other. Thus, the velocity of air is increased and pressure decreased. Hence the ball experiences a net lateral force in downward direction and it is for this reason that the ball describes a curved path, which is convex on the side of greater pressure. This is called Magnus effect and is exhibited by a tennis or golf ball when given spin.

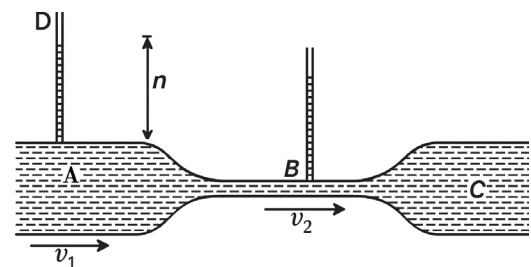
178. What is venturimeter ? Find expression for flow of liquid through a venturimeter.

Ans :

A venturimeter is a guage used for measuring the rate of flow of liquids through pipes.

Principle : When a liquid flows through a tube of varying cross-section, the velocity flow becomes more, where area of cross-section is small and vice versa. According to Bernoulli's principle, the hydrostatic pressure is less, where velocity of flow is more. Hence by measuring the pressure difference, the rate of flow of the liquid can be determined.

Construction : As shown in the figure a venturimeter consists of two identical co-axial tubes A and C connected by a narrow co-axial tube B. Two vertical tubes D and E are mounted on the tubes A and B to measure the pressure of the flowing liquid.



Let a_1, a_2 = areas of cross-section of tubes A and B respectively.
 v_1, v_2 = velocities of flow of liquid through A and B respectively,
 P_1, P_2 = pressures of liquid flow through A and B respectively,

ρ = density of the liquid.

V = rate of flow (i.e. volume/sec.) of the liquid through the pipe

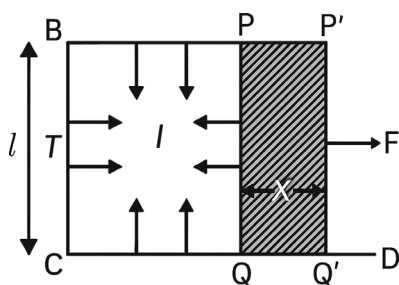
potential energy. For this mass of the surface film and hence the area of the film should be minimum. We know for a given volume, sphere has minimum area. In order to acquire minimum area, the surface tends to contract into spherical surface and behaves like a stretched membrane.

- 183.** Define surface energy. Find the relation between surface energy and surface tension.

Ans :

Surface energy is defined as the amount of work done against the force of surface tension, in forming the liquid surface of a given area at a constant temperature.

Consider a rectangular wire from ABCD carrying a wire PQ which can slide along the sides AB and DC as shown in the figure. On dipping the frame in soap solution, a thin soap film BCQP is formed. The force of surface tension acts (along the tangent) on both the free surfaces of the film as shown in the figure. All other sides being fixed, we can increase the surface area of the soap film by moving the sliding wire PQ.



Let T = force of surface tension of the soap solution.

l = length of the wire PQ .

As there are two free surfaces of the film and surface tension acts on both, therefore, total length upon which the force of surface tension acts is $2l$.

Total inward force on the wire PQ is $F = T \times 2l$... (1)

To increase the area of the soap film, we have to pull the sliding wire PQ outward with a force F .

Let the film be stretched by moving the wire PQ , through a distance x to the position $P'Q'$

Hence, Work done in stretching the film.

W = force applied $\times$ distance moved

$W = (T \times 2l) \times x$... (2)

$W = T \times \text{increase in area of the film on both sides.}$... (3)

(Since, $2l \times x$ = increase in area on one side of the film)

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

From (3), $T = \frac{W}{a}$. Hence surface tension may also be defined as the surface energy per unit area. Also, if increase in the area is unity i.e.,

$$a = 1,$$

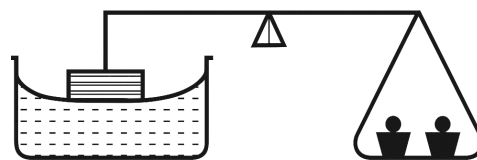
Then surface energy = T

Hence Surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension.

- 184.** How will you determine surface tension of a liquid experimentally ?

Ans :

Take some liquid in a vessel. Suspend a flat glass plate to one arm of balance, over the surface of liquid. On the other side of balance weights are added to balance the glass plate so that its horizontal edge is just above water. The vessel is raised slightly till the liquid just touches the glass plate and pull it down a little because of surface tension. Weights are added till the plate just clears water.



Let W = Additional weight added

Hence, Surface tension,

$$\sigma = \frac{W}{2l} = \frac{mg}{2l}$$

Where l = length of plate edge.

- 185.** (a) Discuss excess of pressure on curved surface of a liquid.
(b) Determine excess of pressure in-side (i) liquid drop (ii) air bubble (iii) soap solution bubble.

Ans :

- (a) We know that the similar molecules of a liquid attract each other by a force of cohesion and force of surface tension acts tangentially to the liquid surface at rest.

If a liquid surface is plane, the molecule O on the liquid surface is acted upon by the forces (T, T) of surface tension, tangentially to the liquid surface in opposite directions. Thus the resultant force on the molecule of a liquid surface would be zero. [as shown in the figure].

Hence, Increase in surface energy

$\Delta E = \text{surface tension} \times \text{increase in area}$

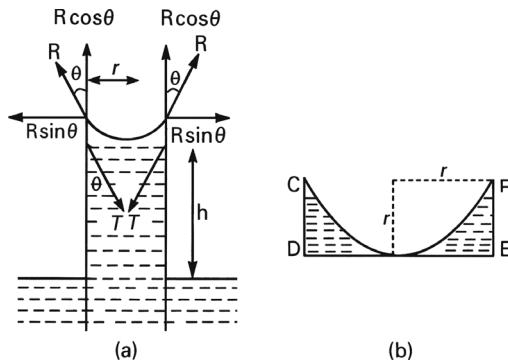
$$\Delta E = T \times \pi D^2 \left[n^{\frac{1}{3}} - 1 \right]$$

188. Derive an expression for the rise of liquid in a capillary tube and show that the height of the liquid column supported is directly proportional to the curvature of the tube.

Ans :

Let a capillary tube of radius r and density ρ stands upto a height h at an angle θ with the wall. T is the surface tension acting tangential to the point of contact and R is an equal and opposite reaction of the wall.

Reaction R can be resolved into two rectangular components.



- (i) $R \sin \theta$ perpendicular to the wall and
- (ii) $R \cos \theta$ acting upwards along the wall of the tube.

This is the component responsible for upward lift of the liquid. $R \sin \theta$ just pulls the wall outwards.

$R \cos \theta$ acts all along circumference of the circle of contact i.e. along $2\pi r$

Hence,

$$\begin{aligned} \text{Total upward force} &= T \cos \theta \times 2\pi r \\ &= T \cos \theta \times 2\pi r \quad \dots(1) \end{aligned}$$

Volume of liquid in cylindrical column of height h and radius $r = \pi r^2 h$

If tube is narrow, the radius of curvature of the meniscus is approximately equal to the radius r of the capillary tube.

Hence, Volume of liquid under meniscus = Volume of cylinder CDEF of height h and radius r minus volume of hemisphere EFGE of radius r

$$\begin{aligned} V &= \pi r^2 h - \frac{1}{2} \times \frac{4}{3} \pi r^3 \\ &= \pi r^2 h - \frac{2}{3} \pi r^3 \end{aligned}$$

$$V = \frac{1}{3} \pi r^3$$

$$\text{Total volume of liquid in the tube} = \pi r^2 h + \frac{1}{3} \pi r^3$$

$$= \pi r^2 \left(h + \frac{r}{3} \right)$$

Weight of the liquid in capillary tube acting downward

$$= (\text{volume}) (\text{density}) (g)$$

$$= \pi r^2 \left(h + \frac{r}{3} \right) \rho g \quad \dots(2)$$

In equilibrium

Total upward force = Total downward force

$$T \cos \theta \times 2\pi r = \pi r^2 \left(h + \frac{r}{3} \right) \rho g$$

$$\left(h + \frac{r}{3} \right) = \frac{2T \cos \theta}{r \rho g}$$

$$h = \frac{2T \cos \theta}{r \rho g} - \frac{r}{3}$$

which is ascent formula.

If $r \ll h$, then $\frac{r}{3}$ may be neglected.

$$\text{Hence,} \quad h = \frac{2T \cos \theta}{r \rho g}$$

$$h \propto \frac{1}{r}$$

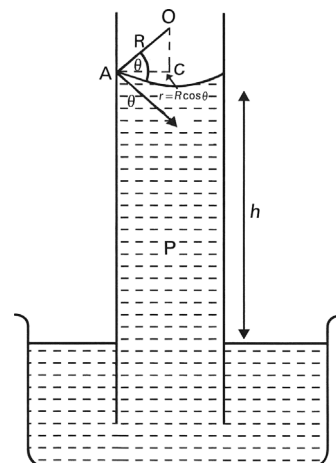
$$\text{and} \quad r \propto \frac{1}{\text{curvature}}$$

Combining above two, we have

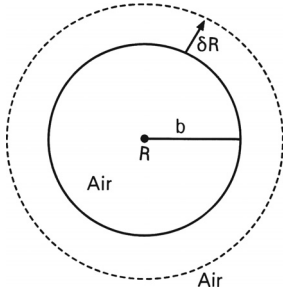
$$h \propto \text{curvature.}$$

For water the value of θ is very small and for all practical purpose, we may consider $r \cos \theta \rightarrow 1$, hence for water, we have

$$h = \frac{2T}{r \rho g}$$



Let p = Excess of pressure.
Hence, Work done by the excess pressure
= force $\times$ distance
= pressure $\times$ area $\times$ distance
= $p \times 4\pi R^2 \times \delta R$



The soap bubble has two free liquid surfaces, one outside and other inside the bubble.

Hence, Increase in area

$$= 2[4\pi(R + \delta R)^2 - 4\pi R^2]$$

$$= 2 \times 8\pi R \delta R = 16\pi R \delta R$$

Increase in surface = Increase in surface area
 $\times$ Surface tension
= $16\pi R \delta R \times T$

$$p \times 4\pi R^2 \times \delta R = 16\pi R \delta R T$$

$$p = \frac{4T}{R}$$

- 186.** When two (or more) mercury drops are brought into contact, they form one drop. Explain it.

Ans :

When a few drops of mercury are brought in contact, they form one big drop. Let us consider that n drops, each of a radius r , combine together to form a bigger drop of radius R .

Then volume of each small drop = $\frac{4}{3}\pi r^3$

Volume of n small drops = $\frac{4}{3}n\pi r^3$

When these drops unite to form a single drop, its volume $\frac{4}{3}\pi R^3$ must be equal to the original volume

$$\text{i.e. } \frac{4}{3}n\pi r^3 = \frac{4}{3}\pi R^3$$

$$R = (n)^{\frac{1}{3}}r \quad \dots(1)$$

Now total surface area of n small drops

$$= n4\pi r^2$$

and the final surface area of single bigger drop

$$= 4\pi R^2 = 4\pi(n)^{\frac{2}{3}}r^2$$

Decrease in surface area

$$= \text{Initial area} - \text{Final area}$$

$$= 4\pi n r^2 - 4\pi(n)^{\frac{2}{3}}r^2$$

$$= 4\pi r^2[n - (n)^{\frac{2}{3}}]$$

Hence, Potential energy released, E

$$= \text{surface tension} \times \text{decrease in area}$$

$$\text{Hence, } E = T \times 4\pi r^2[n - n^{\frac{2}{3}}]$$

- 187.** A liquid drop of Diameter 'D' breaks up into n tiny drops. Find the resulting change in energy.

Ans :

If we break a drop of liquid into a large number (say n) of smaller drops we shall have to do some work on the liquid drop in order to increase the surface area of n tiny drops. This work done is stored in the form of potential energy of the tiny drops. Thus, their surface energy increases.

Let diameter of bigger drop = D

Hence, Volume of bigger drop

$$= \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{\pi D^3}{6}$$

If all the n tiny drops are of equal size, each having a diameter d , then their total volume must be equal to the original volume of a single drop.

$$\text{Hence, } n\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi D^3}{6}$$

$$\frac{n\pi d^3}{6} = \frac{\pi D^3}{6}$$

$$d^3 = \frac{D^3}{n}$$

$$d = D(n)^{-\frac{1}{3}} \quad \dots(1)$$

Now initial surface area of a single drop

$$S_1 = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$$

Final surface area of n tiny drops taken together

$$S_2 = n \times 4\pi\left(\frac{d}{2}\right)^2 = n\pi d^2$$

$$= n\pi D^2(n)^{-\frac{2}{3}} = (n)^{\frac{1}{3}}D^2$$

Hence, Increase in surface area due to splitting of a drop

$$\Delta S = S_2 - S_1$$

$$= (n)^{\frac{1}{3}}\pi D^2 - \pi D^2$$

$$= \pi D^2[(n)^{\frac{1}{3}} - 1]$$

On differentiating both sides w.r.t. h , we get

$$2R \frac{dR}{dh} = 4H - 8h$$

$$\text{Hence, } \frac{dR}{dh} = \frac{4H - 8h}{2R} \quad \dots(3)$$

To find the condition of maximum range, let us put

$$\frac{dR}{dh} = \frac{4H - 8h}{2R} = 0$$

$$\text{Hence, } 4H = 8h$$

$$h = \frac{H}{2} \quad \dots(4)$$

Thus, for maximum horizontal range $h = \frac{H}{2}$ i.e. the hole O should be situated exactly mid-way between the bottom and free water surface.

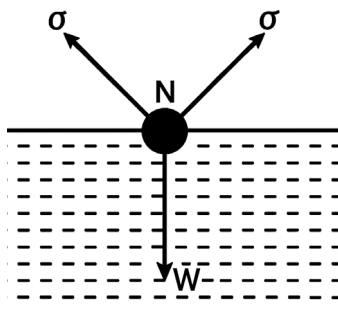
180. (a) Define surface tension. Illustrate with examples.
(b) How will you measure surface tension ?

Ans :

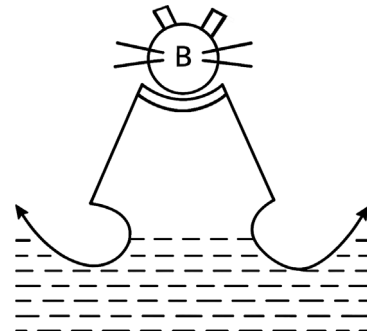
- (a) Surface tension is the property of a liquid due to which the free surface of the liquid at rest tries to have minimum area and as such it behaves like a stretched membrane.

Illustrations

- Needle supported on a free surface. If a greased needle of steel is gently placed on the surface of water, it floats, although it is about 8 times heavier than water. In fact the weight W of needle is balanced by the vertical component of the force of surface tension of water. However, if some detergent is added into water, surface tension of solution is lowered and is unable to balance weight of needle. As a result, the needle sinks.

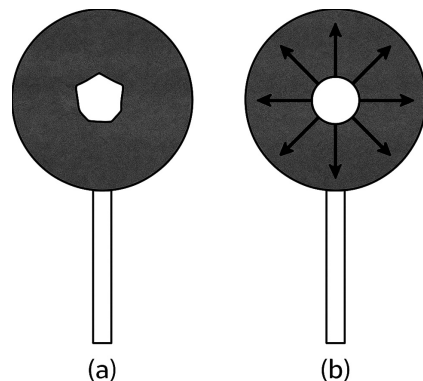


- Insects on surface 'skin' :** Fly, mosquitoes and water beetles creep freely on the water surface, as if



moving over thin leather sheet. The beetle B bends its legs on the surface of water such that its weight is balanced by the upward components of the surface tension forces acting tangentially to the deformed surface.

- Endless wet thread on a soap film :** If we take a circular frame of a stiff wire and dip it into a soap solution a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape. But when the film is pricked at the centre, the loop is stretched outwards and takes a symmetrical circular shape. This is because for a given length a circle has the maximum surface area and thus, the outer liquid film tries to occupy minimum possible area like a stretched elastic membrane.



- Formation of spherical balls :** When a molten metal is poured into water from a suitable height, the falling stream of molten metal breaks up and the detached portions acquire spherical shape. This technique is used for preparing lead shots and glass marbles etc.
- Mercury drops :** If mercury is spilled on a table top, the small drops are always spherical, but larger ones are somewhat flattened.

NUMERICAL QUESTIONS

- 189.** In a hydraulic press used for compressing wool, the area of the piston is 0.1 m^2 and the force exerted along the piston rod is 200 N . If the area of the larger cylinder is 0.8 m^2 , find the pressure produced in the cylinder and the total crushing force exerted on a bale of wool.

Ans :

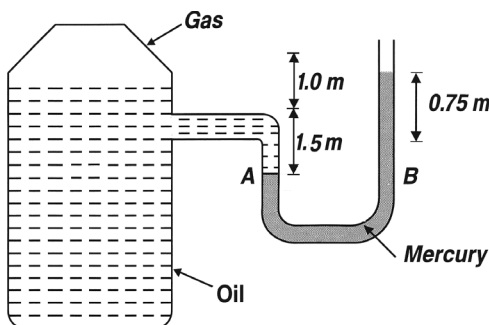
Pressure produced in the cylinder

$$= \frac{200 \text{ N}}{0.1 \text{ m}^2} = 2000 \text{ Nm}^{-2}$$

Total crushing force exerted on a bale of wool

$$\begin{aligned} &= \text{Pressure} \times \text{area of larger cylinder} \\ &= 2000 \text{ Nm}^{-2} \times 0.8 \text{ m}^2 \\ &= 1600 \text{ N} \end{aligned}$$

What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in the following figure ? Density of oil $= 820 \text{ kg m}^{-3}$, density of mercury $= 13.6 \times 10^3 \text{ kg m}^{-3}$, atmospheric pressure $= 1.01 \times 10^5 \text{ Pa}$



Ans :

Points A and B of the mercury column in the U-tube are at the same level and therefore, are at the same pressure. Now

$$P_A = P + (1.50 + 1.00) \times 8.20 \times 9.8$$

$$\text{and } P_B = P + (1.50 + 0.75) \times 13.6 \times 10^3 \times 7.8$$

$$\text{and } P_B = P + (1.50 + 0.75) \times 13.6 \times 10^3 \times 7.8$$

where P is the pressure of the gas in the tank (called absolute pressure), and P' is the atmospheric pressure. Since $P_A = P_B$, we have

$$\begin{aligned} P + (2.5 \times 8.20 \times 9.8) &= P' + (2.25 \times 13.6 \times 9.8 \times 10^3) \text{ that it,} \\ P &= P' + 2.80 \times 10^5 \end{aligned}$$

$$= 1.01 \times 10^5 + 2.80 \times 10^5$$

$$= 3.81 \times 10^5 \text{ Pa}$$

The gauge pressure P is the difference between absolute pressure and atmospheric pressure $P = P - P'$. The gauge pressure of the gas in the tank is,

$$P = 2.80 \times 10^5 \text{ Pa}$$

- 190.** A piece of wood of relative density 0.25 floats in a pail containing oil of relative density 0.81. What is the fraction of volume of the wood above the surface of the oil ?

Ans :

Let V be the volume of the piece of wood and ρ its density.

Let V' be the volume of the piece of wood submerged in the liquid of density ρ' .

V' equals the volume of liquid displaced by the wood. According to the Archimedes' principle, the upward thrust on the piece of wood is equal to the weight of the liquid displaced.

That is upthrust $= V' \rho' g$

Since the floating wood is in equilibrium, the weight of the wood is equal to the upthrust in magnitude (and opposite in direction). Thus

$$V' \rho' g = V \rho g$$

$$\frac{V'}{V} = \rho / \rho'$$

For $\rho = 0.25 \times 10^3 \text{ kg m}^{-3}$, $\rho' = 0.81 \times 10^3 \text{ kg m}^{-3}$
The fraction of volume above the surface of the oil is $1 - 0.31 = 0.69$.

- 191.** Calculate the amount of energy evolved when 8 droplets of water (Surface tension 0.072 Nm^{-1}) of radius $\frac{1}{2} \text{ mm}$ each combines into one.

Ans :

$$\text{Given, } \sigma = 0.072 \text{ Nm}^{-1}$$

$$r = 0.5 \text{ mm} = 0.05 \text{ cm}$$

$$= 0.0005 \text{ m}$$

Let R be the radius of big drop formed.

As, volume of big drop = volume of 8 small drops.

$$\text{Hence, } R = 2r = 2 \times 0.0005 = 0.001 \text{ m}$$

Surface area of big drop

$$= 4\pi R^2 = 4 \times \pi \times (0.001)^2$$

$$= 4\pi \times 10^{-6} \text{ m}^2$$

$$= -\frac{0.2 \times 0.465 \times 0.766}{13.6 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{ m}$$

Negative sign indicates that the amount of mercury dip down by $5.34 \times 10^{-3} \text{ m} = 5.34 \text{ mm}$.

- 205.** Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shape tube open at both levels. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is $7.3 \times 10^{-2} \text{ Nm}^{-1}$. Take the angle of contact to be zero and density of water to be $1.0 \times 10^3 \text{ kg m}^{-3}$ ($g = 9.8 \text{ ms}^{-2}$).

Ans :

For zero angle of contact, the radius meniscus equals radius of the bore. The concave side of the surface in each bore is at 1 atm.

For the first bore, pressure difference (between the concave and convex sides)

$$\text{Using } P = \frac{2T}{R}$$

$$= \frac{2 \times 7.3 \times 10^{-2}}{\frac{3}{2} \times 10^{-3}} = 97.4 \text{ Pa}$$

Similarly for the second bore pressure difference

$$= \frac{2 \times 7.3 \times 10^{-2}}{\frac{6}{2} \times 10^{-3}} = 48.7 \text{ Pa}$$

$$h\rho g = (97.4 - 48.7)$$

$$h \times 1 \times 10^3 \times 9.8 = 48.7$$

$$h = \frac{48.7}{10^3 \times 9.8} = 4.97 \text{ mm}$$

$$= 5.0 \text{ mm}$$

CASE BASED QUESTIONS

- 206.** A hydraulic lift in an automobile service station is used to raise a car of mass 2000 kg. The car rests on a platform attached to a large piston with a cross-sectional area A_2 . A smaller piston with a cross-sectional area $A_1 = 10 \text{ cm}^2$ is used to apply a force F_1 to lift the car. The oil used in the hydraulic system is incompressible.

Questions:

- What principle explains the working of a hydraulic lift?
- How is the pressure P_1 applied to the smaller piston related to the pressure P_2 experienced by the larger piston?

- If the ratio $A_2/A_1 = 100$, what force F_1 must be applied to the smaller piston to lift the 2000 kg car? (Use $g = 9.8 \text{ m/s}^2$).
- If the smaller piston is pushed down by a certain distance d_1 , how does the distance d_2 by which the larger piston (and the car) moves up relate to d_1 , A_1 , and A_2 ?

Ans :

- Pascal's law explains the working of a hydraulic lift.
- According to Pascal's law, the pressure is transmitted undiminished throughout the fluid. Therefore, $P_1 = P_2$.
- The weight of the car

$$F_2 = mg = 2000 \text{ kg} \times 9.8 \text{ m/s}^2 = 19600 \text{ N}.$$

Since $P_1 = P_2$, we have $F_1/A_1 = F_2/A_2$. So,

$$F_1 = F_2(A_1/A_2) = 19600 \text{ N} \times (1/100)$$

$$= 196 \text{ N}.$$

- Since the volume of fluid displaced is the same, $A_1 d_1 = A_2 d_2$. Therefore, $d_2 = d_1(A_1/A_2)$. The larger piston moves up a smaller distance.

- 207.** An experienced diver plans to explore a deep freshwater lake. The pressure at the surface of the lake is equal to the atmospheric pressure, $P_a = 1.01 \times 10^5 \text{ Pa}$. The density of freshwater is approximately 1000 kg/m^3 .

Questions:

- How does pressure in a fluid at rest vary with depth?
- What is the difference between absolute pressure and gauge pressure?
- Calculate the absolute pressure at a depth of 20 m in the lake. (Use $g = 9.8 \text{ m/s}^2$).
- If the diver inflates a flexible balloon at the surface and takes it down to 20 m, what will happen to the volume of the balloon, assuming the temperature of the air inside remains constant? Explain why.

Ans :

- The pressure in a fluid at rest increases with depth 'h' according to the relation $P = P_a + \rho gh$.
- Absolute pressure is the total pressure at a point. Gauge pressure is the difference between the absolute pressure and the atmospheric pressure ($P - P_a$).
- Absolute pressure

$$P = P_a + \rho gh = 1.01 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 20 \text{ m})$$

$$= 1.01 \times 10^5 \text{ Pa} + 1.96 \times 10^5 \text{ Pa}$$

$$= 2.97 \times 10^5 \text{ Pa}.$$

- 195.** With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 Nsm^{-2} and specific gravity 0.9 ? What is the terminal velocity of the same bubble in water ?

Ans :

Given, $v = ?$

$$r = \frac{0.8}{2} \text{ mm} = 0.4 \times 10^{-3} \text{ m}$$

$$\eta = 0.15 \text{ Nsm}^{-2}$$

Density of liquid, $\sigma = 0.9 \text{ g cm}^{-3} = 900 \text{ kg m}^{-3}$

Density of air, $\rho = 1.293 \text{ kg m}^{-3}$

Terminal velocity of the bubble in the liquid

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$v = \frac{2}{9} \times \frac{(0.4 \times 10^{-3})^2 \times (1.293 - 900) \times 9.8}{0.15}$$

$$= -2.09 \times 10^{-3} \text{ ms}^{-1}$$

Negative sign indicates that the bubble will rise upward.

Density of water, $\sigma = 1000 \text{ kg m}^{-3}$

Viscosity of water, $\eta = 10^{-3} \text{ Nsm}^{-2}$

Hence, Terminal velocity of the bubble in water

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2}{9} \times \frac{(0.4 \times 10^{-3})^2 \times (1.293 - 900) \times 9.8}{10^{-3}}$$

$$= -0.0348 \text{ ms}^{-1}$$

Again, negative sign indicates that the bubble will rise upwards.

- 196.** A glass plate of length 20 cm, breadth 4 cm and thickness 0.4 cm weighs 40 g in air. If it is held vertically with long side horizontal and the plate half immersed in water, what will be its apparent weight ? Surface tension of water = 70 dyne cm^{-1} .

Ans :

Given, $l = 20 \text{ m}$, $b = 4 \text{ cm}$

$$t = 0.4 \text{ cm}, S = 70 \text{ dyne/cm}.$$

Following three forces are acting on the plate :

- (i) Weight of the plate = 40 gf acting vertically downward.
- (ii) Force due to surface tension acting vertically downward.

Now, force due to surface tension, $F = S \times l$

Total length in contact with water = 2 (length + thickness)

$$= 2(20 + 0.4) = 40.8 \text{ cm}$$

Hence, Force due to surface tension,

$$F = 70 \times 40.8$$

$$= 2856 \text{ dynes}$$

$$= \frac{2856}{980} \text{ gf} = 2.9143 \text{ gf}$$

- (iii) Upward thrust = $V \times \rho \times g$

Volume of water displaced

$$= l \times \frac{b}{2} \times t$$

$$= 20 \times \frac{4}{2} \times 0.4$$

$$= 16 \text{ cm}^3$$

Hence, upward thrust = $16 \times 1 \times 980 \text{ dyne}$

$$= \frac{16 \times 980}{980} \text{ gf}$$

Net weight = $40 + 2.9143 - 6$

$$= 26.9143 \text{ gf}$$

- 197.** A body of density ρ floats with a volume V_1 of its total volume V immersed in one liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2 , where $\rho_1 > \rho_2$. Find the relative volumes immersed in two liquids.

Ans :

$$\text{Weight of the body} = V\rho g$$

$$\text{Weight of first liquid displaced} = V_1\rho_1 g$$

$$\text{Weight of second liquid displaced} = V_2\rho_2 g$$

When the body floats,

Weight of the body = Weight of two liquids displaced

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g$$

$$V\rho = V_1\rho_1 + V_2\rho_2 \quad \dots(1)$$

$$\text{Also} \quad V = V_1 + V_2 \quad \dots(2)$$

$$V\rho_1 = V_1\rho_1 + V_2\rho_1 \quad \dots(3)$$

Subtracting (3) from (2), we get

$$V(\rho - \rho_1) = V_2(\rho_2 - \rho_1)$$

$$V(\rho_1 - \rho) = V_2(\rho_1 - \rho_2)$$

$$V_2 = \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2} \right) V \quad \dots(4)$$

Substituting the value of V_2 in (2), we get

$$V = V_1 + \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2} \right) V$$

CHAPTER 10

Thermal Properties of Matter

SUMMARY

1. TEMPERATURE

Temperature is a physical property that indicates the thermal equilibrium state of a body. It helps determine the direction of heat flow. When two bodies are in thermal equilibrium, no heat flows between them, and they are said to have the same temperature.

2. HEAT

Heat is a form of energy that flows from one body to another solely due to a temperature difference. It always flows from a hotter body to a colder body until thermal equilibrium is achieved. Heat is measured in joules (J) in the SI system.

2.1 Calorie (or Kilo Calorie)

It is defined as the heat required to raise the temperature of one gram (or kilogram) of water by 1°C, from 14.5°C to 15.5°C.

3. THERMAL EXPANSION

Thermal expansion is the tendency of matter to change its shape, area, or volume when its temperature changes due to heat transfer. When a substance is heated, its particles gain energy and vibrate more, leading to an increase in dimensions.

3.1 Coefficient of Linear Expansion

It is defined as the increase in length per unit rise in temperature of the body.

$$\text{i.e., } \alpha = \frac{\Delta l}{l \Delta T}$$

3.2 Coefficient of Superficial Expansion

It is defined as the increase in area per unit rise in temperature of the body.

$$\text{i.e., } \beta = \frac{\Delta A}{A \Delta T}$$

3.3 Coefficient of Volume Expansion

It is defined as the increase in volume per unit volume of a body of 1 degree rise in temperature.

$$\gamma = \frac{\Delta V}{V \Delta T}$$

4. JOULES MECHANICAL EQUIVALENT OF HEAT

It is defined as the ratio of mechanical work done to the heat produced.

$$\text{i.e., } J = \frac{W}{Q}$$

5. SPECIFIC HEAT

The specific heat is defined as the amount of heat required to raise the temperature of 1 kilogram of substance through 1 kelvin. It is denoted by c . Its unit is $J \text{ kg}^{-1} \text{ K}^{-1}$.

$$C = \frac{\Delta Q}{m \Delta T}$$

5.1 Molar Specific Heat (C)

The amount of heat required to raise the temperature of 1 mole of a body through 1K (or 1°C) is called molar specific heat.

$$\text{and } C = Mc,$$

Where M is the molecular weight of the body.

6. THERMAL CAPACITY OR HEAT CAPACITY

The amount of heat required to raise the temperature of a given mass of a body through 1K (or 1°C) is called thermal capacity of the body.

7. WATER EQUIVALENT

It is defined as the mass of water which absorbs or emits the same amount of heat as is done by the given body for the same rise or fall in temperature.

$$W = mc$$

$$\begin{aligned}
 \text{Hence, } h &= \frac{P}{\rho g} \\
 &= \frac{2 \times 10^3}{1.06 \times 10^3 \times 9.8} = \frac{2}{10.4} = 0.19 \\
 &= 0.2 \text{ m}
 \end{aligned}$$

- 201.** (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m, if the flow must remain steady ?
 (b) What is the corresponding flow rate ?
 (Taking viscosity of blood to be 2.084×10^{-3} Pa s)

Ans :

- (a) For laminar flow, the Reynolds number R is given by,

$$\begin{aligned}
 R &= \frac{\rho v r}{\eta} \\
 \text{Hence, } v &= \frac{R \eta}{\rho r} = \frac{1000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 2 \times 10^{-3}} \\
 &= 0.98 \text{ ms}^{-1}
 \end{aligned}$$

- (b) Flow rate i.e. volume of liquid flowing per second

$$\begin{aligned}
 \frac{Q}{t} &= v \times \pi r^2 = 0.98 \times 3.14 \times 4 \times 10^{-6} \\
 &= 1.23 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}
 \end{aligned}$$

- 202.** A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2 . If the speed of air is 180 km h^{-1} over the lower wing and 234 km h^{-1} over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg m^{-3})

Ans :

Given,

$$\begin{aligned}
 v_u &= 180 \text{ km h}^{-1} = 180 \times \frac{5}{18} = 50 \text{ ms}^{-1} \\
 v_l &= 234 \text{ km h}^{-1} = 234 \times \frac{5}{18} = 65 \text{ ms}^{-1} \\
 A &= 25 \text{ m}^2
 \end{aligned}$$

From Bernoulli's equation and the equation of continuity, we have

$$P_u + \frac{1}{2} \rho v_u^2 = P_l + \frac{1}{2} \rho v_l^2$$

$$P_u - P_l = \frac{\rho}{2} (v_l^2 - v_u^2)$$

$$\Delta P = \frac{1}{2} (65^2 - 50^2) = 862.5 \text{ Nm}^{-2}$$

Hence, force acting on each wing,

$$\begin{aligned}
 F &= \Delta P \times A \\
 &= 862.5 \times 25 = 21562.5 \text{ N}
 \end{aligned}$$

$$\text{Force on two wings} = 21562.5 \times 2 = 43125 \text{ N}$$

$$\text{Weight of plane} = 43125 \text{ N}$$

$$\begin{aligned}
 \text{Mass of plane} &= \frac{43125}{g} = \frac{43125}{9.8} \\
 &= 4400 \text{ kg}
 \end{aligned}$$

- 203.** In Millikan's oil drop experiment, what is the terminal speed of a drop of radius, 2.0×10^{-5} m and density $1.2 \times 10^3 \text{ kg m}^{-3}$? Take the viscosity of air at the temperature of experiment to be 1.8×10^{-5} Pa-s. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.

Ans :

The terminal velocity v is given by

$$\begin{aligned}
 6\pi\eta r v &= \frac{4}{3}\pi\alpha^3\rho g = \frac{2\alpha^2\rho g}{9\eta} \\
 &= \frac{2 \times (2.0 \times 10^{-5})^2 \times (1.2 \times 10^3) \times 9.8}{9 \times 1.8 \times 10^{-5}} \\
 6\pi\eta r v &= \frac{2 \times 4 \times 10^{-10} \times 1.2 \times 10^3 \times 9.8}{9 \times 1.8 \times 10^{-5}} \\
 &= \frac{8 \times 12 \times 98 \times 10^{-3}}{9 \times 18 \times 10^{-5}} \\
 6\pi\eta r v &= \frac{8 \times 12 \times 98 \times 10^{-3}}{9 \times 18} = 5.8 \text{ cm s}^{-1}
 \end{aligned}$$

Viscous force,

$$\begin{aligned}
 F &= 6\pi\eta r v \\
 &= 6 \times \frac{22}{7} \times 2.0 \times 10^{-6} \times 1.8 \times 10^{-5} \times 5.8 \times 10^{-2} \\
 &= \frac{275616}{70000} \times 10^{-10} \text{ N} \\
 &= 3.94 \times 10^{-10} \text{ N}
 \end{aligned}$$

- 204.** Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of the glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment of 0.465 Nm^{-1} . Density of mercury = $13.6 \times 10^3 \text{ m}$.

Ans :

$$\text{Radius of the tube} = 1 \times 10^{-3} \text{ m}$$

$$\begin{aligned}
 h &= \frac{2T \cos \theta}{r \rho g} \\
 &= \frac{2 \times 0.465 \times \cos 140^\circ}{1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8} \\
 h &= \frac{2 \times 0.465 \times (-\sin 50^\circ)}{13.6 \times 9.8}
 \end{aligned}$$

$$V_1 = V \left(1 - \frac{\rho_1 - \rho}{\rho_1 - \rho_2} \right)$$

$$= \left(\frac{\rho - \rho_2}{\rho_1 - \rho_2} \right) V \quad \dots(5)$$

198. A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of partition has a small hinged door of area 20 cm^2 . The tank is filled with water in one compartment and an acid (of relative density 1.7) in the other, both to a height at 4.0. Compute the force necessary to keep the door closed.

Ans :

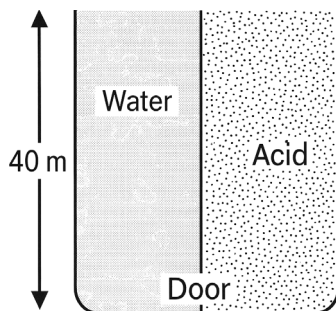
As we know that,

Force on the door = Pressure cliff. $\times$ area of door

Pressure exerted by the acid $p_1 = h\rho g$

$$P_1 = 4 \times 1.7 \times 10^3 \times 9.8$$

$$P_1 = 66.64 \times 10^3 \text{ Pa}$$



Pressure exerted by the water p_2

$$= 4 \times 1000 \times 9.8$$

$$= 39.2 \times 10^3 \text{ Pa}$$

Difference of pressure,

$$\Delta p = p_1 - p_2$$

$$= (66.64 - 39.2) \times 10^3$$

$$= 27.4 \times 10^3 \text{ Pa}$$

Area of door, $A = 20 \times 10^{-3} \text{ m}^2$

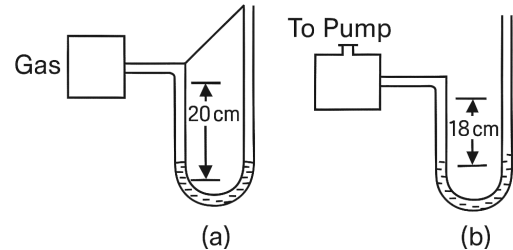
Force applied = $\Delta p \times A$

$$= 27.4 \times 10^3 \times 20 \times 10^{-3}$$

$$= 55 \text{ N}$$

199. A manometer reads the pressure of a gas in an enclosure as shown in figure (a). When some of the gas is removed by a pump, the manometer reads as in figure (b). The liquid used in manometers is mercury and the atmospheric pressure is 76 cm of mercury.

- (i) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.
- (ii) How would the levels change in case of figure. (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limbs of the manometer ? (Ignore the small change in volume of the gas).



Ans :

- (i) For (a)

Absolute pressure of the gas, $p = p_A + h$ of the gas

$$= 76 + 20 = 96 \text{ cm of Hg}$$

Gauge pressure of the gas

$$= p - p_A = 96 - 76$$

$$= 20 \text{ cm of Hg}$$

For (b)

Absolute pressure, $p = p_A - h$

$$= 76 - 18 = 58 \text{ cm of Hg}$$

Gauge pressure = $p - p_A$

$$= 58 - 76 = -18 \text{ cm}$$

- (ii) $h_1 \rho_1 \times g = h_2 \times 13.6 \times g$

$$h_2 = 1 \text{ cm}$$

Therefore, mercury would rise in the left limb such that the difference in its levels in the two limbs becomes

$$18 + 1 = 19 \text{ cm.}$$

200. During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ?

Ans :

Given, $P = 2000$,

$$P_a = 2 \times 10^3 \text{ Nm}^{-2}$$

[Density of blood, $\rho = 1.06 \times 10^3 \text{ kg m}^{-3}$ (known)]

Since, $P = h\rho g$

- The volume of the balloon will decrease. As the balloon is taken deeper, the external pressure increases. According to Boyle's Law (assuming constant temperature), for a fixed amount of gas, pressure is inversely proportional to volume ($PV = \text{constant}$). So, an increase in external pressure will cause the volume to decrease.

208. Water is flowing steadily through a horizontal pipe that gradually tapers in its cross-sectional area. At point A, the cross-sectional area is A_1 and the speed of water is v_1 . At point B, downstream from A, the cross-sectional area A_2 is smaller than A_1 .

Questions:

- What is the equation of continuity for incompressible fluid flow?
- Does the speed of the water change as it flows from point A to point B? If so, how?
- If the area at point B (A_2) is half the area at point A ($A_1/2$), what is the speed of water v_2 at point B in terms of v_1 ?
- According to Bernoulli's principle, how would the pressure at point B compare to the pressure at point A (assuming the pipe is horizontal)?

Ans :

- The equation of continuity for incompressible fluid flow is $A_1 v_1 = A_2 v_2$.
- Yes, the speed of the water increases as it flows from point A to point B because the cross-sectional area decreases.
- From the equation of continuity, $A_1 v_1 = A_2 v_2$. Given $A_2 = A_1/2$, we have $A_1 v_1 = (A_1/2) v_2$, which implies $v_2 = 2v_1$.
- According to Bernoulli's principle ($P + \frac{1}{2}\rho v^2 = \text{constant}$ for horizontal flow), where the speed is higher (point B), the pressure will be lower compared to where the speed is lower (point A).

209. A perfume atomizer (or spray bottle) works by forcing air at high speed across the top of a tube dipped into the liquid perfume. The high-speed airflow creates a region of low pressure above the tube.

Questions:

- State Bernoulli's principle relating pressure, speed, and height of a flowing fluid.
- Why does the pressure above the tube become lower when air is blown at high speed across it?
- What causes the perfume liquid to rise up the tube?
- If the air is blown less vigorously (lower speed), how would this affect the atomization process?

Ans :

- Bernoulli's principle states that for a streamline flow of an ideal fluid, the sum of pressure energy, kinetic energy per unit volume, and potential energy per unit volume remains constant ($P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$).
- According to Bernoulli's principle, where the speed of the fluid (air) is high, the pressure is low.
- The atmospheric pressure acting on the surface of the perfume liquid in the bottle is now greater than the reduced pressure above the tube. This pressure difference pushes the liquid up the tube.
- If the air is blown less vigorously, the speed of air across the top of the tube will be lower. This means the pressure drop above the tube will be less significant. Consequently, the pressure difference might not be sufficient to lift the perfume effectively, or the atomization (breaking the liquid into fine droplets) will be less efficient.

210. A small raindrop falls through the air. As its speed increases, the viscous drag force from the air also increases. Eventually, the drag force plus the buoyant force balance the gravitational force, and the raindrop reaches a constant speed called terminal velocity.

Questions:

- What is Stokes' Law, which describes the viscous drag force on a spherical object moving through a fluid?
- What are the forces acting on the raindrop when it achieves terminal velocity? What is the net force?
- If the terminal velocity v_t of a spherical raindrop of radius ' r ' and density ' ρ ' falling through air of density ' σ ' and viscosity ' η ' is given by

$$v_t = \frac{2r^2 g(\rho - \sigma)}{9\eta},$$

What happens to the terminal velocity if the radius of the raindrop doubles (assuming other factors remain constant)?

- Why does a tiny mist particle fall much slower than a large raindrop?

Ans :

- Stokes' Law states that the viscous drag force F on a sphere of radius ' r ' moving with velocity ' v ' through a fluid of viscosity ' η ' is $F = 6\pi\eta rv$.
- When terminal velocity is achieved, the forces acting are: gravity (downwards), buoyant force (upwards), and viscous drag force (upwards). The net force is zero.

Surface area of 8 small drops

$$\begin{aligned} &= 8 \times 4\pi r^2 \\ &= 8 \times 4\pi \times (.0005)^2 \\ &= 8\pi \times 10^{-6} \text{m}^2 \end{aligned}$$

Decrease in surface area

$$\begin{aligned} &= 8\pi \times 10^{-6} - 4\pi \times 10^{-6} \\ &= 4\pi \times 10^{-6} \text{m} \end{aligned}$$

As energy evolved

$$= \text{Surface tension} \times \text{decrease in area}$$

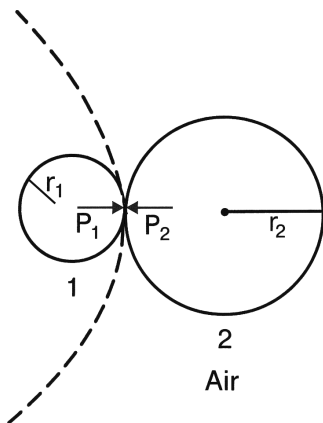
Hence, Energy evolved

$$\begin{aligned} &= .072 \times 4\pi \times 10^{-6} \\ &= 9.05 \times 10^{-7} \text{J} \end{aligned}$$

- 192.** Two separate air bubbles (radii 0.002 m and 0.004m) formed of the same liquid (surface tension 0.70 Nm^{-1}) came together to form a double bubble. Find the radius and sense of curvature of the internal film surface common to both the bubbles.

Ans :

Let r_1 and r_2 be the radii of curvature of two air bubbles and r be radius of curvature of big bubble.



If p_1 and p_2 be pressure in bubbles 1 and 2 respectively, then

$$\begin{aligned} p_1 - p_2 &= \frac{4T}{r} \\ \frac{4T}{r_1} - \frac{4T}{r_2} &= \frac{4T}{r} \\ \frac{1}{r_1} - \frac{1}{r_2} &= \frac{1}{r} \\ r &= \frac{r_1 r_2}{r_2 - r_1} = \frac{0.002 \times 0.004}{0.004 - 0.002} \\ &= 0.004 \text{ m} \end{aligned}$$

The concave surface is concave towards the centre of smaller bubble (dotted curve) because excess pressure is more towards the concave side and inversely proportional to the radius.

- 193.** The density of air in atmosphere decreases with height and can be expressed by the relation $\rho = \rho_0 e^{-ah}$ where ρ_0 is the density at sea level, a is a constant and h is the height. Calculate the atmospheric pressure at sea level. Assume g to be constant. The numerical values of constants are :

$$g = 9.8 \text{ ms}^{-2}, \rho_0 = 1.3 \text{ kg m}^{-3}, a = 1.2 \times 10^{-4} \text{ m}^{-1}$$

Ans :

$$\text{Given, } \rho = \rho_0 e^{-ah}$$

Now, the pressure due to small air column of length dh at height h is

$$\begin{aligned} dp &= dh\rho g \\ &= \rho g dh = \rho_0 e^{-ah} g dh \end{aligned}$$

Total atmospheric pressure is given by

$$\begin{aligned} p &= \int dp = \rho_0 g \int_0^\infty e^{-ah} dh \\ p &= \frac{\rho_0 g}{a} (e^{-\infty} - e^0) = -\frac{\rho_0 g}{a} (0 - 1) \\ p &= \frac{\rho g}{a} = \frac{1.3 \times 9.8}{1.2 \times 10^{-4}} \\ p &= 1.06 \times 10^4 \text{ Nm}^{-2} \end{aligned}$$

- 194.** A tank containing water upto the height 15 m from the bottom of the tank. Find the speed and horizontal range of the water jet coming out through an orifice which is at 5 m above the bottom of the water tank. (Use $g = 10 \text{ ms}^{-2}$)

Ans :

$$\text{Given, } H = 15 \text{ m, } h' = 5 \text{ m}$$

$$\text{So } h = H - h' = 10 \text{ m}$$

$$\begin{aligned} \text{Speed, } v &= \sqrt{2gh} \\ &= \sqrt{2(10 \text{ ms}^{-2})(10 \text{ m})} \\ &= 10\sqrt{2} \text{ ms}^{-1} \end{aligned}$$

$$\text{Hence, } v = 14 \text{ ms}^{-1}$$

$$\begin{aligned} \text{Horizontal range, } R &= 2\sqrt{hh'} = 2\sqrt{(10 \text{ m})(5 \text{ m})} \\ &= 10\sqrt{2} \text{ m} \end{aligned}$$

$$\text{Hence, } R = 14 \text{ m}$$

3. The higher pressure on the lower surface and lower pressure on the upper surface create a net upward force (pressure difference multiplied by wing area) on the wing, which is the dynamic lift.
 4. The angle of attack, which is the angle the wing makes with the direction of air flow, also influences the lift. A small positive angle of attack can increase lift.
- 214.** Blood flows through arteries. If an artery becomes constricted due to plaque buildup (arteriosclerosis), the speed of blood flow must increase in the constricted region to maintain the same flow rate (volume per second).

Questions:

1. What principle governs the relationship between the speed of blood flow and the cross-sectional area of the artery for a constant flow rate?
2. According to Bernoulli's principle, how does the pressure in the constricted region of the artery compare to the pressure in the unconstricted regions?
3. What is the consequence of this pressure drop in the constricted region on the artery walls?
4. How can this phenomenon potentially lead to a more severe blockage or even a collapse of the artery, as explained in the context of a heart attack?

Ans :

1. The equation of continuity ($Av = \text{constant}$) governs this relationship.
2. In the constricted region where the speed of blood flow is higher, the pressure is lower according to Bernoulli's principle.
3. The external pressure (from surrounding tissues) on the artery walls may now be greater than the internal blood pressure in the constricted region.
4. This pressure difference can cause the artery to collapse or squeeze in further in the constricted region, leading to a more severe blockage. If this happens in a coronary artery supplying the heart, it can lead to a heart attack.

- 215.** A Venturi-meter is a device used to measure the flow speed of an incompressible fluid. It consists of a tube with a broad entry (area A_1) and a narrow constriction (throat, area A_2). A manometer connected to these two sections measures the pressure difference ($P_1 - P_2$).

Questions:

1. Why is the fluid speed (v_2) in the throat of the Venturi-meter greater than the fluid speed (v_1) in the broad entry section?

2. If the height difference in the manometer liquid (density ρ_m) is 'h', what is the pressure difference $P_1 - P_2$ in terms of ρ_m , g, and h?
3. Using Bernoulli's equation and the equation of continuity, how can the speed of flow v_1 in the broad entry section be determined from the pressure difference and the areas A_1 and A_2 ? (Provide the conceptual steps or the final formula structure if recalled).
4. What is one practical limitation of Bernoulli's equation mentioned in the text that might affect the accuracy of a Venturi-meter?

Ans :

1. Due to the equation of continuity ($A_1 v_1 = A_2 v_2$), since $A_2 < A_1$, it must be that $v_2 > v_1$.
2. The pressure difference $P_1 - P_2 = \rho_m gh$.
3. From Bernoulli's equation for a horizontal tube:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

Using continuity $v_2 = (A_1/A_2) v_1$. Substitute v_2 into Bernoulli's equation and $P_1 - P_2$ from the manometer reading to solve for v_1 . The formula is

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho((A_1/A_2)^2 - 1)}}.$$

4. One practical limitation is that Bernoulli's equation assumes no viscous (frictional) losses, but real fluids are viscous and experience some energy loss, which can affect accuracy.

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3. From the formula, $v_t \propto r^2$. If the radius 'r' doubles, the terminal velocity v_t will increase by a factor of $2^2 = 4$.
4. A tiny mist particle has a much smaller radius 'r' than a large raindrop. Since terminal velocity is proportional to r^2 , the mist particle will have a significantly lower terminal velocity, causing it to fall much slower.

211. A soap bubble has two liquid surfaces in contact with air, one inside and one outside. Due to surface tension, the pressure inside the soap bubble is greater than the pressure outside.

Questions:

1. What is surface tension of a liquid?
2. Why is there an excess pressure inside a liquid drop or a bubble?
3. If P_i is the pressure inside a soap bubble of radius R and P_o is the pressure outside, and S is the surface tension of the soap solution, what is the formula for the excess pressure ($P_i - P_o$)?
4. How does the excess pressure inside a soap bubble change if its radius is doubled, assuming the surface tension remains the same?

Ans :

1. Surface tension is a property of liquids by virtue of which their free surface behaves like a stretched elastic membrane, tending to contract so as to occupy the minimum possible surface area. It is measured as force per unit length.
2. There is an excess pressure inside a liquid drop or bubble because the surface tension forces try to minimize the surface area, compressing the fluid inside.
3. For a soap bubble (which has two surfaces), the excess pressure is

$$P_i - P_o = \frac{4S}{R}.$$
4. Since the excess pressure is inversely proportional to the radius R ($P_i - P_o \propto 1/R$), if the radius R is doubled, the excess pressure will be halved.

212. Water rises up narrow tubes (xylem vessels) in tall plants due to a phenomenon called capillary action. This action depends on the surface tension of water, the angle of contact between water and the tube material, and the radius of the tube.

Questions:

1. Define the angle of contact.
2. For water to rise in a capillary tube, should the angle of contact be acute or obtuse? What does an acute angle of contact imply about the adhesive forces compared to cohesive forces?

3. The height 'h' to which a liquid rises in a capillary tube of radius 'r' is given by

$$h = \frac{2S \cos \theta}{\rho g r},$$

where S is surface tension, θ is angle of contact, ρ is density, and g is acceleration due to gravity. If the radius 'r' of the xylem vessel is halved, how would the height of capillary rise change?

4. Why is capillary action more effective in very narrow tubes for transporting water to significant heights in plants?

Ans :

1. The angle of contact between a liquid and a solid surface is the angle enclosed between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.
2. For water to rise, the angle of contact should be acute ($\theta < 90^\circ$). An acute angle of contact implies that the adhesive forces (between water and tube material) are stronger than the cohesive forces (between water molecules).
3. From the formula, $h \propto 1/r$. If the radius 'r' is halved, the height 'h' of capillary rise will be doubled.
4. Capillary action is more effective in very narrow tubes because the height of rise 'h' is inversely proportional to the radius 'r' of the tube. Smaller 'r' leads to a larger 'h', allowing water to be transported to greater heights against gravity.

213. The wing of an airplane (airfoil) is designed such that the air flows faster over its curved upper surface than its flatter lower surface. This difference in airspeeds creates a pressure difference, resulting in an upward force called dynamic lift.

Questions:

1. According to Bernoulli's principle, how does the pressure on the upper surface of the wing compare to the pressure on the lower surface?
2. Which surface of the wing (upper or lower) experiences lower air pressure?
3. How does this pressure difference contribute to the dynamic lift on the airplane wing?
4. Besides the curved shape, what other factor related to the wing's orientation with respect to airflow (angle of attack) can influence the lift?

Ans :

1. According to Bernoulli's principle, where the speed of air is higher (upper surface), the pressure is lower. Thus, the pressure on the upper surface is lower than the pressure on the lower surface.
2. The upper surface of the wing experiences lower air pressure.

where m is the mass of the body and c is the specific heat of the body.

7.1 Two Specific Heats of Gases

Gases can be heated either at constant pressure or at constant volume, so a gas has two specific heats

- (i) specific heat at constant volume c_v and
- (ii) specific heat at constant pressure c_p .

8. LATENT HEAT (L)

It is defined as the quantity of heat required to change the unit mass of a substance completely from one state into another at constant temperature.

8.1 Latent Heat of Fusion

It is the amount of heat required to change the unit mass of a solid into its liquid state at its melting point at constant temperature.

e.g. Latent heat of fusion of ice = 80 kcal kg^{-1}

8.2 Latent Heat of Vaporisation

It is the amount of heat required to change the unit mass of a liquid to its vapour state at its boiling point at constant temperature.

e.g. latent heat of vaporisation of H_2O
 $= 540 \text{ kcal kg}^{-1}$

9. TRANSFER OF HEAT

Transfer of heat is the movement of Thermal energy from one body to another of different temperature. These objects could be two solids, a solid and a liquid or gas or even within a liquid or gas.

Transfer of heat is normally from high temperature object to low temperature object. There are three methods of transfer of heat namely conduction, convection and radiation.

9.1 Conduction

It is that process of transmission of heat in which heat travels from particle to particle in the direction of fall of temperature, the particles themselves remaining at rest in their relative positions.

Solids are generally heated by this process. Those substances which allow heat to pass through them more rapidly are called good conductors. Metals as a rule are good conductors. Silver is the best conductor of heat and next in order to it are copper and aluminium. Liquids and gases are, in general, poor conductors, the only exception being mercury.

9.2 Convection

It is the process in which heat travels from one part of a body to another by the actual motion of the heated particles.

Conduction as well as convection is a slow process in which heat is transmitted by material particles and along any path straight or curve.

Radiations in autumn also work on same principle of convection currents. The heated water flows through the pipes, where it is cooled and passes down a lower pipe and then enters the cylinder again at the bottom.

9.3 Radiation

It is that process in which heat is transmitted without heating the intervening medium.

Radiation is a form of wave-motion similar to light and travels through empty space with the same velocity. It does not require any material medium for its transmission. Some media allow heat radiation to pass through them freely. They are called Diathermanous. Those media which absorb the heat radiations and are themselves heated are called athermanous. We get heat from the sun by radiations.

Temperature gradient is the rate of change of temperature with distance in the direction of flow of heat.

9.4 Coefficient of Thermal Conductivity

Coefficient of thermal conductivity (K) of a material is the quantity of heat energy that flows in unit time between the opposite faces of a unit cube whose opposite faces are maintained at temperature difference of one degree.

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10. NEWTON'S LAW OF COOLING

Newton's law of cooling states that the rate of loss of heat from a body is directly proportional to the difference of temperature between the body and the surroundings provided the temperature difference is small. i.e.

$$E \propto T - T_0$$

20. Why are burns from steam generally more serious than those from boiling water at the same temperature?

- (A) Steam is at a higher temperature than boiling water.
- (B) Steam has a higher specific heat capacity than water.
- (C) Steam releases a large amount of latent heat of vaporisation upon condensation.
- (D) Steam penetrates the skin more easily than water.

Ans :

(C) Steam releases a large amount of latent heat of vaporisation upon condensation.

Steam at 100°C contains significantly more energy per unit mass than water at 100°C due to its latent heat of vaporisation. This extra energy is released upon contact with skin, causing more severe burns.

21. The mechanism of heat transfer between two adjacent parts of a body because of their temperature difference, without actual motion of matter, is called:

- (A) Conduction
- (B) Convection
- (C) Radiation
- (D) Sublimation

Ans :

(A) Conduction

Conduction is the transfer of thermal energy through direct contact between particles of a substance, without any net movement of the substance itself.

22. What is the SI unit of thermal conductivity (K)?

- (A) $\text{Jsm}^{-1}\text{K}^{-1}$
- (B) WmK^{-1}
- (C) $\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$
- (D) $\text{Wm}^{-2}\text{K}^{-1}$

Ans :

(C) $\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$

The SI unit for thermal conductivity is Joules per second per meter per Kelvin ($\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$), which is equivalent to Watts per meter per Kelvin ($\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$).

23. Which of the following materials generally have the highest thermal conductivities?

- (A) Gases
- (B) Liquids
- (C) Non-metals
- (D) Metals

Ans :

(D) Metals

Metals are generally good conductors of heat due to the presence of free electrons that can efficiently transfer thermal energy.

24. The mode of heat transfer that involves the actual motion of matter is:

- (A) Conduction
- (B) Convection
- (C) Radiation
- (D) Emission

Ans :

(B) Convection

Convection is heat transfer by the movement of a fluid (liquid or gas) due to differences in density caused by temperature variations.

25. Which mode of heat transfer does not require a material medium?

- (A) Conduction
- (B) Convection
- (C) Radiation
- (D) Advection

Ans :

(C) Radiation

Radiation is the transfer of heat through electromagnetic waves, which can travel through a vacuum, unlike conduction or convection that require a medium.

26. The electromagnetic radiation emitted by a body by virtue of its temperature is called:

- (A) Visible light
- (B) Thermal radiation
- (C) X-rays
- (D) Gamma rays

Ans :

(B) Thermal radiation

Thermal radiation is electromagnetic radiation generated by the thermal motion of particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation.

27. According to Wien's Displacement Law, as the temperature of a blackbody increases, the wavelength (λ_m) at which the energy radiated is maximum:

- (A) Increases
- (B) Decreases
- (C) Remains constant
- (D) Becomes zero

Ans :

- (a) Triple point has a unique temperature. Fusion point and boiling point temperatures depend on pressure.
- (b) The other fixed point is the absolute zero i.e. (-273.16°C).
- (c) Triple point is 0.01°C , not 0°C .
- (d) Temperature of triple point of water on absolute scale is given by

$$T = 273.15 \times \frac{9}{5}$$

$$= 491.69 \text{ K}$$

5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made :

Temperature	Pressure Thermometer A	Pressure Thermometer B
Triple point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting-point of sulphur	$1.757 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

- (a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?
- (b) What do you think to the reason for slightly different answers from A to B ? (The thermometers are not faulty).

Ans :

- (a) (i) For pressure thermometer A,

$$T_{\text{tr}} = 273 \text{ K},$$

$$P_{\text{tr}} = 1.250 \times 10^5 \text{ Pa}$$

$$p = 1.797 \times 10^5 \text{ Pa}$$

Normal freezing point of sulphur,

$$T = \frac{p}{p_{\text{tr}}} \times T_{\text{tr}}$$

$$= \frac{1.797 \times 10^5 \times 273}{1.250 \times 10^5}$$

$$= 392.028 \text{ K}$$

- (ii) For pressure thermometer B,

$$T_{\text{tr}} = 273 \text{ K}$$

$$P_{\text{tr}} = 0.200 \times 10^5 \text{ Pa}$$

$$p = 0.287 \times 10^5 \text{ Pa}$$

$$T = \frac{0.287 \times 10^5 \times 273}{0.200 \times 10^5}$$

$$= 391.75 \text{ K}$$

- (b) The slight difference is due to the fact that, oxygen and hydrogen do not behave strictly as ideal gases.

6. A steel tape 1 metre long is correctly calibrated for temperature of 27.0°C . The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45°C . What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear expansion of steel $= 1.20 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$:

Ans :

Actual length of the rod at 45°C

$$= l_0(1 + \alpha t) = l_0 + l_0 \alpha t$$

$$= 63.0 + 63 \times 1.20 \times 10^{-5} \times 18$$

$$= 63.0 + 0.0136$$

$$= 63.0136 \text{ cm}$$

Length of the same rod at $27^\circ\text{C} = 63.0 \text{ cm}$

7. A large steel wheel is to be fitted on a shaft of the same material. At 27°C , the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft ? Assume coefficient of linear expansion of the steel is to be constant over the required temperature range, $\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.

Ans :

Given, $T_1 = 27^\circ\text{C} = 273 + 27 = 300 \text{ K}$

Length at temperature,

$$T_1 = l_1 = 8.70 \text{ cm}$$

Length at temperature,

$$T_2 = l_2 = 8.70 \text{ cm}$$

$$\text{Change in length} = \Delta l = l_2 - l_1 = l_1 \times \alpha \times \Delta T$$

$$8.69 - 8.7 = 8.7 \times 1.2 \times 10^{-5} (T_2 - 300)$$

$$T_2 - 300 = \frac{-0.01}{8.7 \times 1.2 \times 10^{-5}} = -95.8$$

$$T_2 = 300 - 95.8$$

$$= 204.2 \text{ K} = -68.8^\circ\text{C}$$

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the molar specific heat of diatomic gases is nearly $\frac{5}{2}R$, which agrees with the observations for all the gases listed in the table except chlorine. The higher value of molar specific heat of chlorine indicates that besides rotational modes, vibrational modes are also present in chlorine at room temperature.

16. A child running a temperature of 101°F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min., what is the average rate of extra evaporation caused by the drug ? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water and latent heat of evaporation of water at that temperature is about 580 cal g^{-1} .

Ans :

According to the question,

$$\text{Fall of temperature} = 101 - 98 = 3^\circ\text{F}$$

$$= 3 \times \frac{5}{9} = \frac{5}{3}^\circ\text{C}$$

$$\text{Heat lost by child} = 30 \times 1000 \times 1 \times \frac{5}{3}$$

$$= 50,000 \text{ cal}$$

$$\text{Latent heat of evaporation of water} = 580 \text{ cal g}^{-1}$$

$$\text{Water evaporated in 20 minutes} = \frac{50,000}{580}$$

$$\text{Water evaporated in 1 minute} = \frac{50,000}{580 \times 20}$$

$$= \frac{250}{58}$$

$$= 4.3 \text{ g min}^{-1}$$

17. A thermocole box is a cheap and efficient method for storing small quantity of cooked food in summer in particular. A cubical box of side 30 cm has a thickness of 5 cm. If 4 kg of ice are put in a box. Estimate the amount of ice remaining after 6 h. The outside temperature 45°C , and coefficient $K = 0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ [Heat of fusion of water $= 335 \times 10^3 \text{ J kg}^{-1}$].

Ans :

Let m = mass of ice melted in 6 h

Hence, Amount of heat required to melt m gram of ice

$$Q = m \times 335 \times 10^3 \text{ J kg}^{-1}$$

$$= 335 \times 10^3 m \text{ J kg}^{-1}$$

$$\text{Since } Q = \frac{KA(\theta_1 - \theta_2)t}{d}$$

$$\text{Hence, } = 335 \times 10^3 m$$

$$= \frac{.01 \times 6 \times 30 \times 30 \times 10^{-4} \times 45 \times 6 \times 60 \times 60}{5 \times 10^{-2}}$$

$$= \frac{.01 \times 6 \times 30 \times 30 \times 10^{-4} \times 45 \times 6 \times 60 \times 60}{335 \times 10^3 \times 5 \times 10^{-2}}$$

$$m = \frac{3 \times 3 \times 45 \times 6 \times 6 \times 6 \times 6}{335 \times 5 \times 1000}$$

$$= 0.30 \text{ kg}$$

Amount of ice left

$$= 4.00 - 0.30 = 3.7 \text{ kg}$$

18. A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min. When placed on a gas stove. Estimate the temperature of the part of the flame in contact with boiler. Thermal conductivity of brass $= 109 \text{ J s}^{-1} \text{ m}^{-1} \text{ }^\circ\text{C}^{-1}$.

Ans :

As we know that,

$$Q = mL = \frac{KA(\theta_1 - \theta_2)t}{d}$$

$$\frac{6000}{60} \times 2256 = \frac{109 \times 0.015(\theta - 100) \times 1}{0.01}$$

$$\theta - 100 = \frac{6000 \times 0.01 \times 2256}{60 \times 109 \times 0.15}$$

$$\theta - 100 = 138$$

$$\theta = 238^\circ\text{C}$$

19. Explain why :

- A body with large reflecting is a poor emitter.
- A brass tumbler feels much colder than a wooden tray on a chilly day.
- An optical pyrometer (for measuring high temperature) calibrated for an ideal black body radiation gives too low value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.
- The earth without its atmosphere would be in hospitably cold.
- Heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Ans :

- Good reflectors are poor absorbers and poor emitters.
- Because brass tumbler is good conductor of heat, on touching, heat quickly flows from our

12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = $0.91 \text{ J g}^{-1} \text{ }^{\circ}\text{C}^{-1}$.

Ans :

$$\text{Total energy} = 10^4 \times 2.5 \times 60 \text{ J}$$

$$\begin{aligned} \text{And } C &= 0.91 \text{ J g}^{-1} \text{ K}^{-1} \\ &= 0.91 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \end{aligned}$$

As 50% Energy used in raising the temperature

Hence, Thermal energy available

$$\begin{aligned} \Delta Q &= 10^4 \times 2.5 \times 60 \times \frac{50}{100} \\ &= 10^4 \times 2.5 \times 30 \text{ J} \end{aligned}$$

$$M \times C \times \Delta T = 10^4 \times 2.5 \times 30$$

$$8 \times 0.91 \times 10^3 \times \Delta T = 10^4 \times 2.5 \times 30$$

$$\begin{aligned} T &= \frac{10^4 \times 2.5 \times 30}{8 \times 0.91 \times 10^3} \\ &= 103^{\circ}\text{C} \end{aligned}$$

13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt ? Specific heat of copper = $0.39 \text{ J g}^{-1} \text{ }^{\circ}\text{C}^{-1}$, heat of fusion of water = 335 J g^{-1} .

Ans :

Given, Mass of copper block

$$= 2.5 \text{ kg} = 2500 \text{ g}$$

Temperature of copper block

$$= 500^{\circ}\text{C}$$

Specific heat of copper

$$\begin{aligned} &= 0.39 \text{ J g}^{-1} \text{ }^{\circ}\text{C}^{-1} \\ &= 0.39 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \end{aligned}$$

Heat content of copper

$$= 2500 \times 0.39 \times 10^3 \times 500 \text{ J}$$

Heat of fusion of water

$$= 335 \text{ J g}^{-1}$$

$$\begin{aligned} \text{Mass of ice melted} &= \frac{2500 \times 0.39 \times 10^3 \times 500}{335 \times 10^3} \\ &= 1.5 \text{ kg} \end{aligned}$$

14. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025

kg) containing 150 cc of water at 27°C . The final temperature is 40°C . Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal ?

Ans :

Let c be the specific heat of the metal

Heat lost by the metal = $200 \times c \times (150 - 40) \text{ cal}$

Heat gained by water and calorimeter

$$\begin{aligned} &= 150 \times 1 \times (40 - 27) + 25 \times (40 - 27) \\ &= 150 \times 13 + 25 \times 13 \\ &= 1950 + 325 = 2275 \text{ cal} \end{aligned}$$

Heat lost = Heat gained

$$\begin{aligned} c &= \frac{2275}{22000} = 0.1 \text{ cal}^{-1} \text{ g}^{\circ}\text{C}^{-1} \\ &= 0.1 \times 4.2 = 0.42 \text{ J g}^{-1} \text{ }^{\circ}\text{C}^{-1} \end{aligned}$$

The answer is smaller than the actual value for specific heat of the metal.

Given below are observations on molar specific heats at room temperature of some common gases :

Gas	Molar specific heat (C) (cal/mol ⁻¹ K ⁻¹)
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases.

(Typically, molar specific heat of a mono-atomic gas is 2.92 cal/mol K). Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

Ans :

The gases are diatomic, and have other degrees of freedom (that is have other modes of motion) possible besides the translational degrees of freedom. To raise the temperature of the gas by a certain amount, heat is to be supplied to increase the average energy of all the modes. Consequently, molar specific heat of diatomic gases is more than that of monoatomic gases. It can be shown that if only rotational modes of motion are considered,

NCERT TEXT-BOOK QUESTIONS

1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Ans :

As we know that,

$$\frac{C}{100} = \frac{F - 32}{180}$$

$$F = \frac{9}{5}C + 32$$

Neon : $T_C = T - 273.15 = 24.57 - 273.15$
 $= -248.58^\circ\text{C} = -248.6^\circ\text{C}$

$$T_F = \frac{9}{5}T_C + 32$$

$$T_F = \frac{9}{5}(-248.6) + 32$$

$$= -415.44^\circ\text{F}$$

Carbon dioxide :

$$T_C = T - 273.15$$

$$= 216.55 - 273.15 = -56.6^\circ\text{C}$$

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-56.6) + 32$$

$$= -69.88^\circ\text{F}$$

2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

Ans :

$$\text{Triple point of water} = 200 \text{ A} = 350 \text{ B}$$

$$= 273.16 \text{ K}$$

$$1 \text{ A} = \frac{273.16}{200} \text{ K}$$

$$1 \text{ B} = \frac{273.16}{350} \text{ K}$$

If T_A and T_B represent the triple point of water on scales A and B,

$$\frac{273.16}{200} T_A = \frac{273.16}{350} T_B$$

$$\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$$

Hence, $T_A = \frac{4}{7} T_B$

3. The electrical resistance in ohms of any specimen of material varies with temperature according to the approximate law.

$$R = R_0[1 + \alpha(T - T_0)]$$

The resistance is 101.6Ω at triple point of water and 165.5Ω at normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4Ω ?

Ans :

Given, $R = 101.6 \Omega$, $T = 273 \text{ K}$

$$101.6 = R_0[1 + \alpha(273 - T_0)] \quad \dots(1)$$

Again, $R = 165.5$, $T = 600.5 \text{ K}$

$$165.5 = R_0[1 + \alpha(600.5 - T_0)] \quad \dots(2)$$

Dividing (1) and (2),

$$\frac{101.6}{165.5} = \frac{1 + \alpha(273 - T_0)}{1 + \alpha(600.5 - T_0)}$$

Hence, $T_0 = -49.3 \text{ K}$

Substituting in (1),

$$R_0 = \frac{101.6}{1 + \alpha(600.5 - T_0)}$$

$$= 38.9 \Omega$$

When $R = 123.4 \Omega$

$$123.4 = 38.9[1 + \alpha(T + 49.3)]$$

$$1 + \alpha(T + 49.3) = \frac{123.4}{38.9}$$

$$5 \times 10^{-3}(T + 49.3) = \frac{84.5}{38.9}$$

$$T + 49.3 = \frac{84.5}{38.9 \times 5 \times 10^{-3}}$$

$$T = 384.83 \text{ K}$$

4. Answer the following :

- The triple-point of water is a standard fixed point in modern thermometry, why ? What is wrong in taking the melting point of ice and the boiling point of water as standard point 1 as was originally done in the celsius scale ?
- There were two fixed points in the original celsius scale as mentioned above which were assigned the number 0°C and 100°C respectively. On the absolute scale, one of the fixed points is the triple point of water which on the kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (kelvin) scale ?
- The absolute temperature (kelvin scale) T is related to the temperature T_C on the celsius scale by $T_C = T - 273.15$. Why do we have 273.15 in this relation and not 273.16 ?
- What is the temperature of the triple point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?

body to the tumbler and it appears colder. On the other hand, wood is a bad conductor of heat so on touching heat does not flow from our body to the wood and it appears comparatively hotter.

- (c) Let T be temperature of hot body.

When body is in open having temperature of surroundings T_0 ,

Heat radiated per second per unit area

$$E_1 = \sigma(T^4 - T_0^4) \quad \dots(1)$$

When body is in the furnace having temperature T , then

Heat radiated per second per unit area

$$E_2 = \sigma T^4 \quad \dots(2)$$

From (1) and (2),

$E_1 < E_2$. So the optical pyrometer gives low value of the temperature of the body placed in open.

- (d) The temperature of earth is maintained by convection currents set in air which will not be possible with atmosphere.
(e) Because steam contains more heat than boiling water.

20. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C . The temperature of surrounding is 20°C .

Ans :

According to Newton's law of cooling,

Rate of loss of heat

$$= k(\text{mean temperature} - \text{atmospheric temperature})$$

$$\text{i.e., } \frac{\theta_1 - \theta_2}{t} = k \left[\frac{\theta_1 + \theta_2}{2} - \theta \right]$$

Where, θ_1 = Initial temperature

θ_2 = Final temperature

θ = Atmospheric temperature.

When body cools from 80°C to 50°C in 5 minutes.

We have

$$\frac{80 - 50}{5} = k \left[\frac{80 + 50}{2} - 20 \right]$$

$$\frac{30}{5} = k[65 - 20]$$

$$k = \frac{2}{15}$$

When a body cools from 60°C to 30°C in time t , then we have

$$\frac{60 - 30}{t} = \frac{2}{15} \left[\frac{60 + 30}{2} - 20 \right]$$

$$\frac{30}{t} = \frac{2}{15} [45 - 20]$$

$$\frac{30}{t} = \frac{50}{15}$$

$$t = \frac{50}{15} \times 30 = 9 \text{ minutes.}$$

OBJECTIVE QUESTIONS

1. What is defined as the relative measure or indication of hotness or coldness?

- (A) Heat
(B) Temperature
(C) Energy
(D) Power

Ans :

- (B) Temperature

Temperature is a measure that indicates the degree of hotness or coldness of a body. It helps in comparing the thermal states of different objects.

2. What is the SI unit of heat energy transferred?

- (A) Kelvin (K)
(B) Celsius ($^\circ\text{C}$)
(C) Joule (J)
(D) Watt (W)

Ans :

- (C) Joule (J)

Heat is a form of energy, and its SI unit is the Joule (J). This unit is used to quantify the amount of heat transferred between systems.

3. At what temperature do the Celsius and Fahrenheit scales show the same numerical value?

- (A) 0°
(B) -40°
(C) 100°
(D) 32°

Ans :

- (B) -40°

The relationship between Fahrenheit (t_F) and Celsius (t_C) is. If $t_F = t_C = x$, then $x = (9/5)x + 32$, which solves to $x = -40$.

4. Which law states that for a given quantity of gas, when the temperature is held constant, the pressure and volume are related as $PV = \text{constant}$?

- (A) Charles's Law
(B) Boyle's Law
(C) Ideal Gas Law
(D) Newton's Law

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Ans :

(B) Boyle's Law

Boyle's Law describes the inverse relationship between the pressure and volume of a gas at constant temperature. It was discovered by Robert Boyle.

5. What is the value of the universal gas constant (R)?

- (A) $8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$
- (B) $9.81 \text{ Jmol}^{-1} \text{ K}^{-1}$
- (C) $6.67 \text{ Jmol}^{-1} \text{ K}^{-1}$
- (D) $1.38 \text{ Jmol}^{-1} \text{ K}^{-1}$

Ans :

(A) $8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$

The universal gas constant (R) is a fundamental physical constant used in the ideal gas law, and its value is approximately $8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$.

6. The increase in the dimensions of a body due to the increase in its temperature is called:

- (A) Thermal contraction
- (B) Thermal expansion
- (C) Thermal equilibrium
- (D) Thermal conductivity

Ans :

(B) Thermal expansion

Thermal expansion refers to the tendency of matter to change its dimensions (length, area, or volume) in response to a change in temperature.

7. The coefficient of linear expansion (α_l) is characteristic of the:

- (A) Shape of the rod
- (B) Temperature of the rod
- (C) Material of the rod
- (D) Length of the rod

Ans :

(C) Material of the rod

The coefficient of linear expansion is an intrinsic property of a material that describes how much it expands per degree Celsius (or Kelvin) change in temperature.

8. Which of the following statements regarding thermal expansion is incorrect?

- (A) Most substances expand on heating and contract on cooling.
- (B) The expansion in area is called area expansion.
- (C) Water always expands on heating.
- (D) The coefficient of volume expansion for gases is generally larger than for solids.

Ans :

(C) Water always expands on heating.

Water exhibits anomalous expansion; it contracts on heating between 0°C and 4°C . Above 4°C , it expands on heating like most other substances.

9. Which of the following correctly describes the relationship between the coefficient of volume expansion (α_V) and the coefficient of linear expansion (α_l) for an isotropic solid?

- (A) $\alpha_V = \alpha_l$
- (B) $\alpha_V = 2\alpha_l$
- (C) $\alpha_V = 3\alpha_l$
- (D) $\alpha_V = (\alpha_l)^3$

Ans :

(C) $\alpha_V = 3\alpha_l$

For an isotropic solid, the coefficient of volume expansion is approximately three times the coefficient of linear expansion because expansion occurs in three dimensions.

10. Why does water at the bottom of a lake remain at 4°C even when the surface freezes?

- (A) Water has a low specific heat capacity.
- (B) Water has maximum density at 4°C .
- (C) Ice is denser than water.
- (D) The ground keeps the bottom warm.

Ans :

(B) Water has maximum density at 4°C .

Water has its maximum density at 4°C . As surface water cools to this temperature, it sinks, and warmer, less dense water rises. Below 4°C , colder water becomes less dense and stays at the surface, where it freezes.

11. The quantity of heat required to change the temperature of a unit mass of a substance by one unit is called:

- (A) Heat capacity
- (B) Specific heat capacity
- (C) Latent heat
- (D) Thermal conductivity

Ans :

(B) Specific heat capacity

Specific heat capacity is an intrinsic property of a substance that defines the amount of heat needed to raise the temperature of one unit of mass of that substance by one degree Celsius (or Kelvin).

12. What is the SI unit of specific heat capacity?

- (A) JkgK^{-1}
- (B) $\text{Jmol}^{-1} \text{ K}^{-1}$

- (C) Jkg^{-1}
(D) JK^{-1}

Ans :

- (A) JkgK^{-1}

The SI unit for specific heat capacity is joules per kilogram per Kelvin ($\text{Jkg}^{-1}\text{K}^{-1}$), representing the energy required per unit mass per unit temperature change.

13. Which of the following substances listed has the highest specific heat capacity?

- (A) Aluminium
(B) Copper
(C) Water
(D) Iron

Ans :

- (C) Water

Water has a very high specific heat capacity compared to many other common substances, meaning it requires a significant amount of heat to change its temperature.

14. What is a device used for heat measurement called?

- (A) Thermometer
(B) Barometer
(C) Calorimeter
(D) Manometer

Ans :

- (C) Calorimeter

A calorimeter is a device specifically designed for measuring the amount of heat involved in a chemical or physical process.

15. The change of state from solid to liquid is called:

- (A) Vaporisation
(B) Sublimation
(C) Melting
(D) Condensation

Ans :

- (C) Melting

Melting, also known as fusion, is the process by which a substance transitions from a solid phase to a liquid phase, typically by absorbing heat.

16. During a change of state, the temperature of the substance:

- (A) Increases
(B) Decreases
(C) Remains constant
(D) Fluctuates

Ans :

- (C) Remains constant

When a substance undergoes a phase change (e.g., melting or boiling), the heat added or removed is used to change the state, and the temperature remains constant until the transition is complete.

17. What is the term for the phenomenon of refreezing of water that has melted under pressure?

- (A) Sublimation
(B) Regelation
(C) Condensation
(D) Vaporisation

Ans :

- (B) Regelation

Regelation is the phenomenon where ice melts under pressure and refreezes when the pressure is removed. This allows a wire with weights to pass through an ice block.

18. Which statement about boiling point is incorrect?

- (A) Boiling point is the temperature at which liquid and vapour states coexist in thermal equilibrium.
(B) Boiling point increases with an increase in pressure.
(C) Boiling point decreases with a decrease in pressure.
(D) All substances pass through a liquid state before boiling.

Ans :

- (D) All substances pass through a liquid state before boiling.

Some substances, like dry ice (solid CO_2) and iodine, undergo sublimation, changing directly from solid to vapor without passing through a liquid state.

19. The heat required per unit mass for a substance to change from solid to liquid state at its melting point is known as:

- (A) Latent heat of vaporisation
(B) Specific heat capacity
(C) Latent heat of fusion
(D) Thermal capacity

Ans :

- (C) Latent heat of fusion

Latent heat of fusion is the amount of heat energy absorbed or released by a substance during a phase change from solid to liquid (melting) or liquid to solid (freezing) at constant temperature and pressure.

8. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27°C. What is the change in the diameter of the hole when the sheet is heated to 227°C ? Coefficient of linear expansion of copper = $1.70 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$.

Ans :

As we know that,

$$\begin{aligned}\Delta l &= l\alpha \Delta T \\ &= 4.24 \times 1.70 \times 10^{-5} (227 - 27) \\ &= 4.24 \times 1.70 \times 10^{-5} \times 200 \\ &= 1.44 \times 10^{-2} \text{ cm}\end{aligned}$$

9. A brass wire 1.8 m long at 27°C is held when taut with little tension between two rigid supports. If the wire is cooled to a temperature -39°C , what is the tension developed in the wire, if its diameter is 2.0 mm ? Coefficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$, Young's modulus of brass = $0.91 \times 10^{11} \text{ Pa}$.

Ans :

$$\begin{aligned}\Delta T &= -39 - (27) = -66^\circ\text{C} \\ A &= \pi r^2 = 3.14 \times (1 \times 10^{-3})^2 \\ \alpha &= 2.0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1} \\ Y &= 0.91 \times 10^{11}\end{aligned}$$

Tension developed,

$$\begin{aligned}F &= -AY\alpha \Delta T = -\pi r^2 Y\alpha \Delta T \\ &= -3.14 \times (1 \times 10^{-3}) \times 0.91 \times 10^{11} \\ &\quad \times (2.0 \times 10^{-5}) \times (-60) \\ &= -3.14 \times 0.91 \times 2 \times 66 \text{ N} \\ &= -3.8 \times 10^2 \text{ N}\end{aligned}$$

Here the negative sign indicates that the force is inwards leading to contraction of the wire.

10. A brass rod of length 50 cm and diameter 3.00 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are at 40.0°C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand coefficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$, steel = $1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$.

Ans :

Since the ends of the combined rod are not clamped, each rod expands freely

Using, $\Delta l_{\text{brass}} = l\alpha \Delta t$

$$\begin{aligned}&= 50 \times 2.0 \times 10^{-5} \times (250 - 40) \\ &= 50 \times 2.0 \times 10^{-5} \times 210 \\ &= 0.21 \text{ cm}\end{aligned}$$

Using $\Delta l_{\text{steel}} = l\alpha \Delta t$

$$\begin{aligned}&= 50 \times 1.2 \times 10^{-5} \times (250 - 40) \\ &= 50 \times 1.2 \times 10^{-5} \times 210 \\ &= 0.126 \text{ cm} = 0.13 \text{ cm}\end{aligned}$$

Total change in length = $0.21 + 0.13 = 0.34 \text{ cm}$

No 'thermal stress' is developed at the junction since the rods freely expand.

11. The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$. What is the fractional change in its density for 30°C rise in temperature ?

Ans :

The mass of a substance remains unchanged on heating,

$$\begin{aligned}\text{Mass} &= \text{Volume} \times \text{Density} \\ &\quad (\text{or } Vd = \text{constant})\end{aligned}$$

On differentiating,

$$V\Delta d + d\Delta V = 0$$

Fractional change in density,

$$\frac{\Delta V}{V} = -\frac{\Delta d}{d}$$

On heating the volume increases, so density decreases.

Fractional change in density,

$$\frac{\Delta d}{d} = -\frac{\Delta V}{V}$$

But, $\Delta V = V\gamma \Delta T$

Hence, $\frac{\Delta d}{d} = -\frac{V\gamma \Delta T}{V} = -\gamma \Delta T$

Given, $\gamma = 49 \times 10^{-5} \text{ K}^{-1}$

$$\Delta T = 30 \text{ K}$$

Hence, $\frac{\Delta d}{d} = -49 \times 10^{-5} \times 30$

$$= -1.47 \times 10^{-2}$$

$$= -1.5 \times 10^{-2}$$

(upto proper significant figures)

Negative sign shows that density decreases.

Ans :

(B) Decreases

Wien's Displacement Law states that the peak wavelength of emitted radiation by a blackbody is inversely proportional to its absolute temperature. Thus, as temperature increases, λ_m decreases.

28. Which of the following statements about blackbody radiation is incorrect?

- (A) A perfect blackbody absorbs all incident electromagnetic radiation.
- (B) The radiation emitted by a blackbody depends only on its temperature.
- (C) Blackbody radiation curves show that energy is radiated at a single wavelength.
- (D) Wien's Displacement Law applies to blackbody radiation.

Ans :

(C) Blackbody radiation curves show that energy is radiated at a single wavelength.

Blackbody radiation has a continuous spectrum, meaning it emits energy over a range of wavelengths, not just a single one. The intensity varies with wavelength.

29. The Stefan-Boltzmann law states that the total energy radiated per unit time by a perfect blackbody is proportional to:

- (A) t
- (B) T^2
- (C) T^3
- (D) T^4

Ans :

(D) T^4

The Stefan-Boltzmann law describes that the total radiant heat energy emitted from a surface is proportional to the fourth power of its absolute temperature.

30. Why are the bottoms of cooking utensils often blackened?

- (A) To make them look aesthetically pleasing.
- (B) To increase their thermal conductivity.
- (C) To make them absorb maximum heat from the source.
- (D) To prevent food from sticking.

Ans :

(C) To make them absorb maximum heat from the source.

Black surfaces are good absorbers of radiant heat. Blackening the bottom of cooking utensils helps

them to absorb more heat from the stove, leading to efficient cooking.

31. Which of the following modes of heat transfer is LEAST significant in the heating of a room by a radiator placed on one side?

- (A) Conduction through air
- (B) Convection of air
- (C) Radiation from the radiator
- (D) All are equally significant

Ans :

(A) Conduction through air

Air is a poor thermal conductor. While some heat is conducted directly to the air in contact with the radiator, convection (movement of heated air) and radiation are more significant modes of heat transfer for heating the entire room.

32. How does the thermal conductivity of gases generally compare to that of metals?

- (A) Gases have much higher thermal conductivity.
- (B) Gases have slightly higher thermal conductivity.
- (C) Gases have much lower thermal conductivity.
- (D) Gases and metals have similar thermal conductivity.

Ans :

(C) Gases have much lower thermal conductivity.

Metals are excellent thermal conductors due to free electrons, while gases are poor thermal conductors because their molecules are far apart and collisions are less frequent.

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33. Assertion: Water has a higher specific heat capacity than land.

Reason: Water warms up and cools down faster than land.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
- (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
- (C) Assertion is true, but Reason is false.
- (D) Assertion is false, but Reason is true.

Ans :

(C) Assertion is true, but Reason is false.

Water indeed has a high specific heat capacity. This means it requires more heat to change its temperature, causing it to warm up and cool down slower than land, not faster.

- 34.** Assertion: Burns from steam are usually more serious than those from boiling water at the same temperature.

Reason: Steam possesses additional energy in the form of latent heat of fusion.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(C) Assertion is true, but Reason is false.
 Burns from steam are more severe because steam at 100°C contains extra energy as latent heat of vaporisation, not fusion, which is released upon condensation on the skin.

- 35.** Assertion: A body with large reflectivity is a poor emitter of thermal radiation.

Reason: According to Kirchhoff's law of thermal radiation, good absorbers are good emitters.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 Good reflectors are poor absorbers. Since good absorbers are good emitters (Kirchhoff's law), it follows that good reflectors (poor absorbers) are poor emitters.

- 36.** Consider the following statements about thermal expansion:

1. All materials expand at the same rate when heated.
2. The coefficient of volume expansion is generally three times the coefficient of linear expansion for isotropic solids.
3. Water shows anomalous expansion between 0°C and 4°C .
4. Thermal stress develops in a rod if its expansion is prevented.

Which of the statements are correct?

- (A) 1, 2, and 3
 (B) 2, 3, and 4
 (C) 1, 3, and 4
 (D) 1, 2, and 4

Ans :

- (B) 2, 3, and 4

Statement 1 is incorrect as different materials have different coefficients of expansion. Statements 2, 3, and 4 accurately describe phenomena related to thermal expansion.

- 37.** Regarding the change of state:

1. During melting, the temperature of a substance continuously rises.
2. Latent heat of vaporisation is the heat required to change a unit mass of liquid to vapour at its boiling point.
3. Sublimation is the process where a solid directly changes to a liquid.
4. The boiling point of a liquid increases with an increase in external pressure.

Which of the above statements are correct?

- (A) 1 and 3
 (B) 2 and 4
 (C) 1 and 2
 (D) 3 and 4

Ans :

- (B) 2 and 4

Statement 1 is incorrect; temperature remains constant during melting. Statement 3 is incorrect; sublimation is solid to vapor. Statements 2 and 4 are correct descriptions of phase changes.

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- 38.** Identify the correct statements regarding heat transfer:

1. Conduction can occur in vacuum.
2. Convection involves the bulk movement of fluid particles.
3. Radiation is the fastest mode of heat transfer.
4. Black bodies are poor absorbers and poor emitters of radiation.

Which statements are correct?

- (A) 1 and 4
 (B) 2 and 3
 (C) 1, 2, and 3
 (D) 2, 3, and 4

Ans :

- (B) 2 and 3

Statement 1 is incorrect; conduction requires a medium. Statement 4 is incorrect; black bodies are good absorbers and good emitters. Statements 2 and 3 are correct.

- 39.** Match the physical quantity in Column I with its SI unit in Column II.

	Column I		Column II
1.	Heat Capacity	P.	$\text{Jkg}^{-1}\text{K}^{-1}$
2.	Specific Heat Capacity	Q.	JK^{-1}
3.	Latent Heat	R.	Jkg^{-1}
4.	Thermal Conductivity	S.	$\text{Wm}^{-1}\text{K}^{-1}$

- (A) 1-Q, 2-P, 3-R, 4-S
 (B) 1-P, 2-Q, 3-R, 4-S
 (C) 1-Q, 2-R, 3-P, 4-S
 (D) 1-S, 2-P, 3-Q, 4-R

Ans :

- (A) 1-Q, 2-P, 3-R, 4-S
 1 Heat Capacity (S) unit is JK^{-1} .
 2 Specific Heat Capacity (s) unit is $\text{Jkg}^{-1}\text{K}^{-1}$.
 3 Latent Heat (L) unit is Jkg^{-1} .
 4 Thermal Conductivity (K) unit is $\text{Wm}^{-1}\text{K}^{-1}$ or $\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$.

- 40.** Match the process in Column I with the primary mode of heat transfer involved in Column II.

	Column I		Column II
1.	Heating a metal rod at one end	P.	Radiation
2.	Land breeze and sea breeze	Q.	Conduction
3.	Heat from the Sun to Earth	R.	Convection
4.	Warming hands near a bonfire	S.	Both Radiation and Convection

- (A) 1-Q, 2-R, 3-P, 4-S
 (B) 1-R, 2-Q, 3-P, 4-S
 (C) 1-Q, 2-P, 3-R, 4-S
 (D) 1-P, 2-R, 3-Q, 4-S

Ans :

- (A) 1-Q, 2-R, 3-P, 4-S
 1 Heating a metal rod primarily involves conduction.
 2 Land and sea breezes are due to convection currents.
 3 Heat from the Sun reaches Earth through radiation.
 4 Warming hands near a bonfire involves both radiation (direct heat waves) and convection (heated air rising).

- 41.** Match the law/principle in Column I with its description in Column II.

	Column I		Column II
1.	Boyle's Law	P.	Rate of cooling is proportional to temperature difference
2.	Charles's Law	Q.	At constant temperature, $PV = \text{constant}$
3.	Stefan-Boltzmann Law	R.	At constant pressure, $V/T = \text{constant}$
4.	Newton's Law of Cooling	S.	Total energy radiated by a blackbody is proportional to T^4

- (A) 1-Q, 2-R, 3-S, 4-P
 (B) 1-R, 2-Q, 3-S, 4-P
 (C) 1-Q, 2-S, 3-R, 4-P
 (D) 1-P, 2-R, 3-Q, 4-S

Ans :

- (A) 1-Q, 2-R, 3-S, 4-P
 1 Boyle's Law: $PV = \text{constant}$ at constant T.
 2 Charles's Law: $V/T = \text{constant}$ at constant P.
 3 Stefan-Boltzmann Law: $H = A\sigma T^4$ for a blackbody.
 4 Newton's Law of Cooling: $-dQ/dt = k(T_2 - T_1)$

- 42.** Which of the following pairs correctly matches a substance with its approximate coefficient of linear expansion (α_l) in 10^{-5}K^{-1} ?

- (A) Aluminium : 1.7
 (B) Copper : 2.5
 (C) Steel (Iron) : 1.2
 (D) Glass (pyrex) : 1.8

Ans :

- (C) Steel (Iron) : 1.2
 From Table 10.1, the coefficient of linear expansion for Iron (similar to steel) is $1.2 \times 10^{-5}\text{K}^{-1}$. Aluminium is 2.5, Copper is 1.7, and Glass (pyrex) is 0.32, all in 10^{-5}K^{-1} .

- 43.** Identify the correct pairing of a substance with its state change temperature at 1 atm and the associated latent heat.

- (A) Water : Melting Point 0°C , $L_f = 22.6 \times 10^5 \text{Jkg}^{-1}$
 (B) Ethanol : Boiling Point 78°C , $L_v = 8.5 \times 10^5 \text{Jkg}^{-1}$
 (C) Nitrogen : Melting Point -196°C , $L_f = 2.0 \times 10^5 \text{Jkg}^{-1}$
 (D) Lead : Boiling Point 328°C , $L_v = 0.25 \times 10^5 \text{Jkg}^{-1}$

(D) Convection currents inside the car.

Ans :

(B) The greenhouse effect, where glass traps infrared radiation.

Sunlight (short wavelengths) passes through the glass and heats the interior. The interior then radiates infrared energy (longer wavelengths), which is largely trapped by the glass, leading to a temperature increase.

59. Why are gaps left between railway tracks?

- (A) To reduce the noise of the train.
- (B) To allow for thermal expansion of the rails in summer.
- (C) To provide better grip for the train wheels.
- (D) To make the tracks lighter.

Ans :

(B) To allow for thermal expansion of the rails in summer.

Railway tracks are made of steel, which expands when heated in the summer. Gaps are left between sections of rail to accommodate this thermal expansion and prevent the tracks from buckling.

60. The amount of heat per unit mass transferred during a change of state of a substance at constant temperature and pressure is called:

- (A) Specific heat capacity
- (B) Heat capacity
- (C) Latent heat
- (D) Thermal energy

Ans :

(C) Latent heat

Latent heat is the energy absorbed or released by a substance during a change in its physical state (phase) that occurs without changing its temperature.

61. Which of the following has the highest specific heat?

- (a) Copper
- (b) Water
- (c) Hydrogen
- (d) Silver

Ans :

(c) Hydrogen

62. Boiling water is changing into steam. The specific heat of boiling water is:

- (a) Zero
- (b) One
- (c) Infinity
- (d) Less than one.

Ans :

(c) Infinity

$$c = \frac{\Delta Q}{m(\Delta T)} = \frac{\Delta Q}{m \times 0} = \infty.$$

63. One gram of ice at 0°C is added to 5 gram of water at 10°C . Final temperature of the mixture is :

- (a) -5°C
- (b) 5°C
- (c) 0°C
- (d) None of these

Ans :

(a) -5°C

Heat gained = Heat lost

$$m_1 + mc(\Delta T') = (m + m_1)C \cdot \Delta T$$

$$1 \times (-80) + 5 \times 1 \times 10 = (1 + 5) \times 1 \times \Delta T$$

$$\Delta T = -5^\circ\text{C}$$

64. An iron ball is heated. The percentage increase will be largest in :

- (a) Diameter
- (b) Volume
- (c) Surface area
- (d) Density.

Ans :

(b) Volume

Surface volume of ball $= \frac{4}{3}\pi r^3$ involves maximum value of r hence percentage increase in volume will be maximum.

65. Coefficient of cubical expansion of water is minimum at :

- (a) 0°C
- (b) 4°C
- (c) 15.5°C
- (d) 100°C .

Ans :

(b) 4°C

The coefficient of cubical expansion of water is minimum at 4°C because the density of water is minimum at 4°C .

66. 540 g of ice at 0°C is mixed with 540 g of water at 80°C . The final temperature of the mixture in $^\circ\text{C}$ will be :

- (a) 40°C
- (b) 79.9°
- (c) 0°C
- (d) 80°C .

Ans :

(c) 0°C

Heat gain by ice to melt at 0°C

$$= mL = 540 \times 80 \text{ cal.}$$

Heat lost by water in cooling from 80°C to $^\circ\text{C}$

$$= mc\Delta T = 540 \times 1 \times (80 - 0)$$

As heat gained = heat lost

Hence final temperature of the mixture is 0°C .

The Celsius scale defines the freezing point of water at standard atmospheric pressure as 0°C . This is one of the fixed reference points for the scale.

51. The absolute zero temperature, 0 K, is equivalent to:

(A) 0°C
 (B) -100°C
 (C) -273.15°C
 (D) 273.15°C

Ans :

(C) -273.15°C

Absolute zero (0 Kelvin) is the lowest possible temperature, at which particles have minimal vibrational motion. It corresponds to -273.15°C on the Celsius scale.

52. The expansion in area of a substance due to an increase in temperature is called:

(A) Linear expansion
 (B) Superficial expansion
 (C) Volume expansion
 (D) Cubical expansion

Ans :

(B) Superficial expansion

Area expansion, also known as superficial expansion, refers to the increase in the surface area of a substance when its temperature is raised.

53. The SI unit of the coefficient of volume expansion (α_V) is:

(A) m^3K^{-1}
 (B) K^{-1}
 (C) Km^{-3}
 (D) Dimensionless

Ans :

(B) K^{-1}

The coefficient of volume expansion is defined as the fractional change in volume per unit change in temperature. Its unit is per Kelvin (K^{-1}).

54. The normal boiling point of water at standard atmospheric pressure is:

(A) 0°C
 (B) 212°F
 (C) 373.15K
 (D) All of the above

Ans :

(D) All of the above

The normal boiling point of water is 100°C , which is equivalent to 212°F and 373.15K . These are all representations of the same physical temperature.

55. The change of state from liquid to solid is called:

(A) Melting
 (B) Boiling
 (C) Freezing
 (D) Sublimation

Ans :

(C) Freezing

Freezing, or solidification, is the phase transition in which a liquid turns into a solid when its temperature is lowered below its freezing point.

56. Which of the following statements about heat transfer is incorrect?

(A) Conduction is the primary mode of heat transfer in solids.
 (B) Convection relies on the movement of fluid.
 (C) Radiation requires a medium for propagation.
 (D) Metals are generally good conductors of heat.

Ans :

(C) Radiation requires a medium for propagation. Radiation can transfer heat through a vacuum (e.g., heat from the sun to Earth) and does not require a material medium.

57. Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the:

(A) Absolute temperature of the body.
 (B) Absolute temperature of the surroundings.
 (C) Difference in temperature between the body and its surroundings.
 (D) Specific heat capacity of the body.

Ans :

(C) Difference in temperature between the body and its surroundings.

Newton's law of cooling applies when the temperature difference between an object and its surroundings is not too large, stating that the rate of heat loss is proportional to this difference.

58. On a hot day, a car is left in direct sunlight with its windows closed. The inside of the car becomes significantly hotter than the outside air. This is primarily due to:

(A) High thermal conductivity of the car body.
 (B) The greenhouse effect, where glass traps infrared radiation.
 (C) Heat generated by the car's engine.

107. What is S.I. unit of specific heat ?

Ans :

$\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$.

108. What is the value of specific heat of water in S.I. units ?

Ans :

$4.18 \times 10^3 \text{ J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$

109. What is the principle of calorimetry ?

Ans :

According to the principle of calorimetry.

Heat gained by cold body = Heat lost by hot body.

110. The temperature of a gas is increased by 12°C . What is the corresponding change on kelvin scale ?

Ans :

12 K

111. Of metal and alloy, which has greater value of temperature coefficient ?

Ans :

Metals

112. What is the relation between α , β and γ ?

Ans :

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

113. What is thermal conductivity of a perfect heat conductor and perfect heat insulator ?

Ans :

$K = \infty$ for perfect conductors and $K = 0$ for perfect insulators.

114. Which metal is best conductor of heat ?

Ans :

Silver metal is best conductor of heat.

115. Which is the only way of heat transfer through a solid?

Ans :

Conduction

116. In which method of heat transfer, gravity does not play any role ?

Ans :

During conduction and radiation, gravity does not play any role.

117. There is a hole in a metal disc. What happens to the size of the hole if metal disc is heated ?

Ans :

The size of the hole will increase.

118. At what temperature will wood and iron appears equally hot or equally cold ?

Ans :

At the temperature of human body.

119. How does the heat energy from the sun reaches earth?

Ans :

By radiation

120. Why thick glass tumbler cracks when boiling liquid is put in it ?

Ans :

The inner and outer surfaces of thick tumbler undergo uneven expansion due to poor conductivity of glass, hence it cracks.

121. Why a gas is cooled when expanded ?

Ans :

Due to decrease in internal energy.

122. Why people in desert wear heavy clothes ?

Ans :

Because the heavy clothes protect them from both heat and cold.

123. Which mode of transfer of heat is quickest ?

Ans :

Radiation is the quickest mode of transfer or heat.

124. What is the basic condition for Newton's law of cooling to be obeyed ?

Ans :

The basic condition is that the temperature difference between the body and the surroundings should be small (not more than 40°C).

125. Stainless steel cooking pans are preferred with extra copper bottom. Why ?

Ans :

Because the thermal conductivity of copper is more than that of steel.

- 67.** A body takes 10 minutes to cool from 60°C to 50°C . If the temperature of surroundings is 25°C , then temperature of body after next 10 minutes will be :

- (a) 48°C (b) 46°C
(c) 49°C (d) 42.85°C

Ans :

- (d) 42.85°C

From Newton's law, we have

$$\frac{T_1 - T_2}{t} = k \left[\frac{T_1 + T_2}{2} - T_0 \right]$$

$$\text{Hence, } \frac{60 - 50}{10} = k \left[\frac{60 + 50}{2} - 25 \right] \quad \dots(1)$$

If T is the temperature after another 10 minute, then

$$\frac{50 - T}{10} = k \left[\frac{T + 50}{2} - 25 \right] \quad \dots(2)$$

Dividing (1) by (2), we get

$$\begin{aligned} \frac{10}{50 - T} &= \frac{30 \times 2}{T} \\ 10T &= 60(50 - T) \\ T &= 42.85^{\circ}\text{C} \end{aligned}$$

- 68.** Dimensional formula of specific heat capacity is :

- (a) $[ML^2 T^{-2} K]$ (b) $[ML^2 T^{-2} K^{-1}]$
(c) $[ML T^{-2} K^{-1}]$ (d) $[M^0 L^2 T^{-2} K^{-1}]$

Ans :

- (d) $[M^0 L^2 T^{-2} K^{-1}]$

Specific heat capacity

$$\begin{aligned} &= \frac{\text{Heat capacity}}{M} = \frac{[ML^2 T^{-2}]}{M} \\ &= [M^0 L^2 T^{-2} K^{-1}] \end{aligned}$$

- 69.** Boiling point of water:

- (a) increases with the decrease in pressure
(b) increases with the increase in pressure
(c) decreases with the increase in pressure
(d) remains the same with increase in pressure.

Ans :

- (b) increases with the increase in pressure

As the pressure is increased, the boiling point of water also increases.

- 70.** Two spheres are made of same metal and have same mass. One is solid and the other is hollow. When heated to the same temperature, percentage increase in diameter will be:

- (a) more for hollow sphere

- (b) less for hollow sphere
(c) same for both
(d) cannot say

Ans :

- (c) same for both

$$\text{Since, } \frac{\Delta l}{l} = \alpha \Delta T,$$

here α and ΔT is same for both the sphere, so the percentage increase in diameter will also be same for both spheres.

The specific heat of an ideal gas varies with temperature T as:

- (a) T^1 (b) T^2
(c) T^{-1} (d) T^0

Ans :

- (d) T^0

Specific heat on an ideal gas is independent of temperature.

- 72.** In a room containing air, heat, can go from one place to another by :

- (a) Conduction (b) Convection
(c) Radiation (d) All these

Ans :

- (d) All these

Heat can be lost by all the three modes.

- 73.** It is hotter at some distance over the fire than in front of it, because:

- (a) heat is radiated upwards only
(b) convection of heat occurs downwards only
(c) air conducts heat upwards only
(d) convection of heat occurs upwards only.

Ans :

- (d) Convection of heat occurs upwards only.

- 74.** The dimensional formula of coefficient of thermal conductivity is :

- (a) $[M^1 L^1 T^{-3} K^{-1}]$ (b) $[M^1 L^2 T^{-2} K^{-1}]$
(c) $[M^1 L^1 T^{-3} K^{-4}]$ (d) $[M^1 L^0 T^{-2} K^{-1}]$

Ans :

- (a) $[M^1 L^1 T^{-3} K^{-1}]$

$$\begin{aligned} k &= \frac{\Delta Q}{\Delta t} = \frac{\Delta l}{\Delta T \times A} \\ &= \frac{[ML^2 T^{-2}]}{[T]} = \frac{L}{kL^2} = [M^1 L^1 T^{-3} K^{-1}] \end{aligned}$$

Ans :

(B) Ethanol : Boiling Point 78°C , $L_v = 8.5 \times 105 \text{ J kg}^{-1}$

1. Water: Melting Point 0°C , $L_f = 3.33 \times 105 \text{ J kg}^{-1}$.
2. Ethanol: Boiling Point 78°C , $L_v = 8.5 \times 105 \text{ J kg}^{-1}$.
3. Nitrogen: Melting Point -210°C , $L_f = 0.26 \times 105 \text{ J kg}^{-1}$.
4. Lead: Boiling Point 1744°C , $L_v = 8.67 \times 105 \text{ J kg}^{-1}$.

44. Which of the following pairs INCORRECTLY matches a gas with its molar specific heat capacity at constant volume (C_v) in $\text{J mol}^{-1} \text{K}^{-1}$?

- (A) Helium (He) : 12.5
- (B) Nitrogen (N_2) : 20.8
- (C) Oxygen (O_2) : 25.1
- (D) Carbon Dioxide (CO_2) : 28.5

Ans :

(C) Oxygen (O_2) : 25.1

From Table 10.4, the C_v for Oxygen (O_2) is $21.1 \text{ J mol}^{-1} \text{K}^{-1}$, not 25.1. The values for He, N_2 , and CO_2 are correctly listed.

45. Identify the incorrect pairing of a material and its thermal conductivity (K) in $\text{Js}^{-1} \text{m}^{-1} \text{K}^{-1}$.

- (A) Silver : 406
- (B) Glass : 0.8
- (C) Wood : 1.2
- (D) Air : 0.024

Ans :

(C) Wood : 1.2

From Table 10.6, the thermal conductivity for Wood is $0.12 \text{ Js}^{-1} \text{m}^{-1} \text{K}^{-1}$, not 1.2. The values for Silver, Glass, and Air are correctly listed.

46. Arrange the following materials in increasing order of their coefficient of linear expansion: Copper, Glass (pyrex), Aluminium, Steel.

- (A) Glass (pyrex), Steel, Copper, Aluminium
- (B) Aluminium, Copper, Steel, Glass (pyrex)
- (C) Steel, Glass (pyrex), Aluminium, Copper
- (D) Glass (pyrex), Copper, Steel, Aluminium

Ans :

(A) Glass (pyrex), Steel, Copper, Aluminium

The coefficients are (in 10^{-5}K^{-1}): Glass (pyrex) 0.32, Steel (Iron) 1.2, Copper 1.7, Aluminium 2.5. Thus, the increasing order is Glass (pyrex), Steel, Copper, Aluminium.

47. A quantity of ice at -10°C is heated until it becomes steam at 100°C . Arrange the following stages in the correct sequence of occurrence:

1. Temperature of water rises from 0°C to 100°C .

2. Ice melts at 0°C .
3. Temperature of ice rises from -10°C to 0°C .
4. Water vaporises at 100°C .

(A) 3, 2, 1, 4

(B) 2, 3, 1, 4

(C) 3, 1, 2, 4

(D) 2, 1, 3, 4

Ans :

(A) 3, 2, 1, 4

The sequence is: heating ice from -10°C to 0°C (solid phase heating); ice melting at 0°C (phase change solid to liquid); heating water from 0°C to 100°C (liquid phase heating); water vaporising at 100°C (phase change liquid to gas).

48. A blacksmith heats an iron ring before fitting it onto the rim of a wooden wheel. This is an application of:

- (A) Thermal contraction
- (B) Specific heat capacity
- (C) Thermal expansion
- (D) Latent heat of fusion

Ans :

(C) Thermal expansion

Heating the iron ring causes it to expand (thermal expansion). This allows the slightly larger ring to fit over the wooden wheel. Upon cooling, the ring contracts and fits tightly.

49. Heat is a form of energy transferred between two systems by virtue of:

- (A) Mass difference
- (B) Volume difference
- (C) Temperature difference
- (D) Pressure difference

Ans :

(C) Temperature difference

Heat transfer occurs between systems or a system and its surroundings when there is a difference in their temperatures. Energy flows from the hotter to the colder body.

50. The freezing point of pure water at standard pressure on the Celsius scale is:

- (A) 32°C
- (B) 0°C
- (C) 100°C
- (D) -273.15°C

Ans :

(B) 0°C

89. Define latent heat.

Ans :

Latent heat of a substance is the amount of heat energy required to change the state of unit mass of the substance from solid to liquid to vapour form without any rise in temperature.

90. What is the relation between α , β , γ ?

Ans :

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

91. What is the value of J ?

Ans :

$$4.18 \text{ J cal}^{-1}$$

92. Which is greater C_p or C_v ?

Ans :

$$C_p$$

93. What is heat ?

Ans :

Heat is a kind of energy that gives us a sensation of hotness or coldness.

94. What is temperature ?

Ans :

Temperature is the thermal state of a body which gives the direction of flow of heat when two bodies are placed in thermal contact with each other.

95. When an object is heated, what physical changes are observed ?

Ans :

When an object is heated, there is expansion, change of state, change of colour and change in resistance and electric conductivity take place.

96. What is the name of temperature measuring device?

Ans :

Thermometer

97. What is the principle of a Thermometer ?

Ans :

The variation of any physical property of the substance with temperature forms the principle of a Thermometer.

98. Can a substance contract on heating ?

Ans :

Yes, Ice contracts on heating.

99. Are coefficients of thermal expansion constant for a given solid ?

Ans :

No, the coefficients of thermal expansion of a solid change with temperature.

100. If the temperature of a body falls through 1°C . What will be change on kelvin scale ?

Ans :

1 K

101. What is lowest possible temperature ?

Ans :

0 K (or -273°C)

102. How many joules are there in 1 calorie ?

Ans :

$$1 \text{ cal} = 4.2 \text{ joules}$$

103. What is the highest value of density of water ?

Ans :

$$1000 \text{ kg m}^{-3} \text{ or } 1 \text{ g cm}^{-3}$$

104. Do water and ice have the same specific heats ?

Ans :

No

$$\text{specific heat of water} = 1 \text{ cal}^{-1} \text{ g}^\circ\text{C}^{-1}$$

$$\text{specific heat of ice} = 0.5 \text{ cal}^{-1} \text{ g}^\circ\text{C}^{-1}$$

105. What should be the properties of a thermometric substance ?

Ans :

The length, volume, resistance etc. of a thermometric substance should change uniformly with temperature.

106. At what temperature, the density of water is maximum?

Ans :

The density of water is maximum at 4°C and its value is 1000 kg m^{-3} (1 g cm^{-3}).

75. Steam at 100°C is more dangerous than the same mass of water at 100°C because :
- steam contains more internal energy
 - steam moves faster than water
 - steam is less dense than water
 - steam has a higher specific heat capacity than water

Ans :

- (a) Steam contains more internal energy.

76. Which is the fastest mode of transfer of heat ?

- Conduction
- Convection
- Radiation
- (a) and (b)

Ans :

- (c) Radiation

Radiation is fastest mode of transfer of heat.

77. Solids are heated by the mode of :

- Conduction
- Convection
- Radiation
- None of these

Ans :

- (a) Conduction

The transfer of heat from one part to another part in a solid takes place through conduction.

78. If m is the mass, θ is temperature and c is specific heat, then thermal capacity k is given by:

- $k = mc\theta$
- $k = m\theta$
- $k = \frac{mc}{\theta}$
- $k = mc$

Ans :

- (d) $k = mc$

79. The presence of gravitational field is required for the heat transfer by:

- Stirring off liquid
- Conduction
- Natural convection
- Radiation

Ans :

- (c) Natural convection

Natural convection requires the presence of gravitational field.

80. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is difference of temperature of the body and the surroundings, and n is equal to :

- 1
- 2
- 3
- 4

Ans :

- (a) 1

According to Newton's law of cooling, Rate of cooling $\propto \Delta\theta$

so $n = 1$

ONE MARK QUESTIONS

81. What is heat ?

Ans :

Heat is a kind of energy which produces in us the sensation of warmth.

82. What is S.I. unit of heat ?

Ans :

joule (J).

83. What is the name of temperature measuring device?

Ans :

Thermometer.

84. Define specific heat.

Ans :

Specific heat is the amount of heat required to raise the temperature of unit mass of substance through unit temperature.

It is measured in $\text{cal g}^{-1}\text{K}^{-1}$ or $\text{J kg}^{-1}\text{K}^{-1}$.

85. What is absolute zero ?

Ans :

The lowest possible temperature attainable by an object is -273.15°C . This temperature is called absolute temperature.

86. Are coefficient of thermal expansion constant for a given solid ?

Ans :

No, the coefficient of thermal expansion of a solid changes with temperature.

87. What is the principle of calorimetry ?

Ans :

According to the principle of calorimetry

Heat gained by one body = Heat lost by another body.

88. Do all bodies expand on heating ?

Ans :

No, e.g. water contracts when heated from 0°C to 4°C .

- 126.** Why a wooden table fixed with iron nails become loose after sometime ?

Ans :

Due to uneven expansion of wood and nail.

TWO MARKS QUESTIONS

- 127.** What is the difference between rate of cooling and rate of loss of heat ?

Ans :

Rate of cooling of a body at a temperature is the fall in temperature per second at that temperature. Rate of loss of heat is the quantity of heat lost per second from a body at a given temperature.

- 128.** Is it possible that there is no increase in the temperature of a body despite being heated ?

Ans :

Yes. During boiling of water (or at melting of ice), the heat added to the system is used to change the phase i.e. from liquid to vapour (or from solid to liquid) at constant temperature.

- 129.** What is thermal capacity ? Give its SI unit.

Ans :

Thermal or heat capacity is the amount of heat required to raise the temperature of a body through 1K and is measured in J.

- 130.** What is water equivalent ?

Ans :

Water equivalent of a body is defined as the amount of water which will absorb or evolve the same amount of heat as is done by the body in raising or falling through the same range of temperature.

It is equal to product of mass of substance and its specific heat. Its unit is that of mass i.e., kg.

- 131.** Distinguish clearly between temperature and heat.

Ans :

Temperature is that thermal property which determines the condition of thermal equilibrium between the given thermal system. Moreover, temperature governs the direction of flow of heat energy. On the other hand, heat is a form of energy and is possessed by a system due to motion of the

system molecules. Heat energy always flows from a body at higher temperature to a body at lower temperature.

- 132.** Why do we pack ice in gunny bags ?

Ans :

Gunny bags have a number of fine pores, which contain air in them. Air with $K = 0.00005$ is a bad conductor of heat. Therefore, it does not allow the external heat to go in and melt the ice.

- 133.** Why do Eskimos make double walled houses ?

Ans :

This is done to avoid the conduction of heat from inside the house. Outside conduction is very low through the air gap in two walls. Thus, the Eskimos can keep their houses warm in winter. The outside temperature is very low.

- 134.** On a winter night, you feel warmer when clouds cover the sky than when the sky is clear. Explain.

Ans :

The clouds being bad conductor of heat do not conduct heat radiated from the earth to the outer surface. That is why a cloudy night is warmer than a clear night.

- 135.** Why do we wear woollen clothes in winter ?

Ans :

We prefer to wear woollen clothes in winter because wool ($K = 0.00001$) does not conduct the heat from our body to the external atmosphere and hence we are kept warm. It does not conduct heat because its value of coefficient of thermal conductivity is very low. We have $Q = \frac{KA(\theta_1 - \theta_2)t}{x}$. If K is less, the amount of heat conducted will be less.

- 136.** The lid of a tea pot is provided with a hole, why ?

Ans :

If there is no hole in the lid, then it will be air tight. As hot tea inside the pot produces vapours which get condensed, so a vacuum is produced inside. Thus, it becomes difficult to pour tea out of the pot. If there is a hole in the lid, air can enter through the hole producing no vacuum and so it becomes easier to pour tea out of the pot.

B at a lower temperature T_0 , then the amount of radiant energy lost by the body A per unit area per unit time is given by

$$\begin{aligned} E &= \sigma T^4 - \sigma T_0^4 \\ &= \sigma (T^4 - T_0^4) \end{aligned} \quad \dots(2)$$

This is known as Stefan Boltzmann law.

153. State and prove Newton's law of cooling.

Ans :

If we consider a hot body such that the temperature difference between the body and its surroundings is small (not more than 30°C), there is a simplified law for rate of loss of heat energy. This simplified law was given by Newton. According to Newton's law of cooling, rate of loss of heat from a body is directly proportional to the difference of temperature between the body and the surroundings. Mathematically,

$$E \propto (T - T_0)$$

where E = heat energy lost per second,

T = temperature of body

and T_0 = temperature of surroundings.

Consider a hot body having temperature T , specific heat c and placed in a surrounding having temperature T_0 ($T > T_0$). The small amount of heat dQ is lost in time dt . Then from Newton's law of cooling, we have

$$\begin{aligned} -\frac{dQ}{dt} &\propto T - T_0 \\ \frac{dQ}{dt} &= -k(T - T_0) \end{aligned} \quad \dots(1)$$

where k is constant of proportionality and its value depends upon the area and nature of the surface of the body. If the temperature of the body decreases by dT in time dt , then the rate of loss of heat is given by

$$\frac{dQ}{dt} = -mc \frac{dT}{dt} \quad \dots(2)$$

From equations (1) and (2), we get

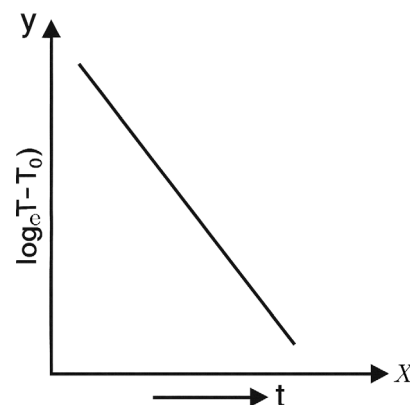
$$\begin{aligned} mc \frac{dT}{dt} &= -k(T - T_0) \\ \frac{dT}{T - T_0} &= \frac{k}{mc} dt \\ \frac{dT}{T - T_0} &= -K dt \end{aligned} \quad \dots(3)$$

where $K = \frac{k}{mc}$ is another constant. And negative sign indicates that the temperature of the body decreases as the time elapses.

Integrating both sides of Eq. (iii), we get

$$\int \frac{dT}{T - T_0} = -K \int dt$$

$$\log_e (T - T_0) = -Kt + K_1 \quad (4)$$



Where K_1 , is constant of integration.

If we plot a graph between $\log_e (T - T_0)$ and t as shown in figure. The equation (iv) represents a straight line having slope k and making an intercept K_1 on the y axis, hence Newton's law of cooling is verified.

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154. Answer the following questions based on the P-T phase diagram of carbon dioxide :

- At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?
- What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?
- What are the critical temperature and pressure for CO_2 ? What is their significance ?
- In CO_2 solid, liquid or gas (a) -70°C under 1 atm. (b) -60°C under 10 atm. (c) 15°C under 56 atm.

Ans :

- At the triple point temperature $= -56.6^\circ\text{C}$ and pressure $= 5.11$ atm.
- Both the boiling point and freezing point of CO_2 decrease if pressure decreases.
- The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm respectively. Above this temperature, CO_2 will not liquefy even if compressed to high pressures.
- (a) Vapour (b) Solid (c) Liquid.

The rate of flow of heat.

$$\text{For rod X} \quad \frac{Q_1}{t_1} = \frac{k_1 A_1 (\theta_1 - \theta_2)}{d}$$

$$\text{For rod Y} \quad \frac{Q_2}{t_2} = \frac{k_2 A_2 (\theta_1 - \theta_2)}{d}$$

Hence, Condition for equal rate of flow of heat through rods X and Y is

$$\begin{aligned} \frac{Q_1}{t_1} &= \frac{Q_2}{t_2} \\ \frac{k_1 A_1 (\theta_1 - \theta_2)}{d} &= \frac{k_2 A_2 (\theta_1 - \theta_2)}{d} \\ k_1 A_1 &= k_2 A_2 \end{aligned}$$

FIVE MARKS QUESTIONS

143. Distinguish between heat and work and justify the statement that heat and work are interrelated.

Ans :

Heat is defined as that form of energy which flows from a body at a higher temperature to a body at lower temperature.

Work is said to be done when under the action of a force acting, a body moves along the direction of action of force through a certain displacement. Work done is measured by the product of the displacement and the component of the force in the direction of the displacement.

Heat and Work are interrelated. At first glance it appears that heat and work represent two separated concepts entirely independent of each other, but actually heat and work are interrelated. Truly speaking both heat and work are forms of energy and can be transformed from one form into another. It was Joule, who first of all concluded that a definite relation exists between the work done and the heat gained.

Mathematically, he expressed the relation as $W = JH$,

where W is the work done in joules, H is the heat gained in calories and J is a constant. The constant J is called the mechanical equivalent of Heat and from his experiment Joule determined its value as 4.18 joules per calorie.

144. Explain different scales of temperature and hence explain the concept of absolute zero from absolute zero from absolute scale of temperature.

Ans :

The branch of physics which deals with the measurement of temperature is called thermometry. Following are different scales of temperature.

- (i) **Celsius Scale** : On this scale, lower fixed point is melting point of ice i.e. 0°C and upper fixed point is boiling point of water (at normal temperature) i.e. 100°C . The distance between these two fixed points is divided into 100 equal parts. Each part represents 1°C .
- (ii) **Fahrenheit Scale** : On this scale, lower fixed point is melting point of ice 32°F and upper fixed point is boiling point of water 212°F . The distance between these two points is divided into 180 equal parts. Each part represent 1°F .
- (iii) **Reaumer Scale** : On this scale, lower fixed point is melting point of ice 0°R and upper fixed point is boiling point of water 80°R . The distance between these two points is divided into 80 equal parts. Each part represents 1°R .
- (iv) **Kelvin Scale** : On this scale, lower fixed point is melting point of ice 273.15 K and upper fixed point is boiling point of water 373.15 K . The distance between these two points is divided into 100 equal parts. Each part represent 1 K . For one mole of an ideal gas,

$$PV = RT$$

$$\text{where,} \quad R = 8.31\text{ J mol}^{-1}\text{K}^{-1}$$

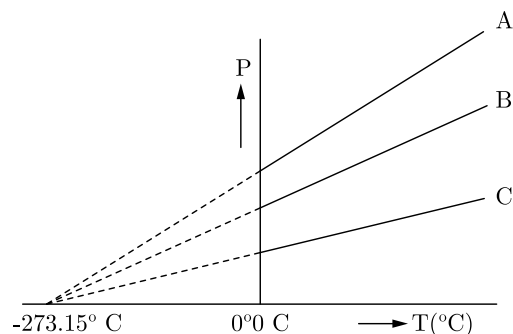
$$PV \propto T$$

So for a given volume V , of one mole of a gas

$$P \propto T$$

Let us plot the variation of P with T ($^\circ\text{C}$) for three different gases say A, B and C.

The graph is as shown in the figure.



From the graph, we find that for each gas, the graph is a straight line and when extrapolated backward, all the three lines meet at -273.15°C called absolute zero denoted by 0 K or 0°A . It appears that if a gas is cooled to -273.15°C , the

- 159.** The opposite faces of a cubical block, of iron and cross-section 4 sq. cm are kept in contact with steam of melting ice. Calculate the amount of ice melted at the end of 10 minutes if $K = 0.2 \text{ cal cm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{C}^{-1}$ for iron.

Ans :

Given, $A = 4 \text{ cm}^2$, $d = 2 \text{ cm}$

$$(\theta_2 - \theta_1) = 100 - 0 = 100 \text{ }^{\circ}\text{C}$$

Time, $t = 10 \text{ min} = 600 \text{ s}$, K
 $= 0.2 \text{ cal cm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{C}^{-1}$

Now $Q = \frac{KA(\theta_2 - \theta_1)t}{d}$
 $= \frac{0.2 \times 4 \times 100 \times 600}{2}$
 $= 24000 \text{ cal}$

If m is the mass of ice melted, then

$$Q = m \times 80 = 24000$$

$$m = 300 \text{ g}$$

- 160.** The heat of combustion of ethane gas is 373 K Cal per mole. Assuming that 50% of heat is lost, how many litres of ethane measured at S.T.P. must be burnt to convert 50 kg of water at $10 \text{ }^{\circ}\text{C}$ to steam at $100 \text{ }^{\circ}\text{C}$? One mole of gas occupies 22.4 litres at S.T.P. Latent heat (L) of steam $= 2.25 \times 10^6 \text{ J K}^{-1}$.

Ans :

Given, $L = 2.25 \times 10^6 \text{ J K}^{-1}$

$$= \frac{2.25}{4.2} \times 10^6 \text{ Cal } ^{\circ}\text{C}^{-1}$$

$$Q = \text{Heat of combustion}$$

$$= 373 \times 10^3 \text{ Cal/mole}$$

$$C = 10^3 \text{ J kg}^{-1} \text{ }^{\circ}\text{C}^{-1}, m = 50 \text{ kg.}$$

$$\Delta \theta = 100 - 10 = 90 \text{ }^{\circ}\text{C},$$

$$V = 22.4 \text{ litres.}$$

If Q_1 = Total heat energy required to convert 50 kg of water at $10 \text{ }^{\circ}\text{C}$ to steam at $100 \text{ }^{\circ}\text{C}$

Then, $Q_1 = mc\Delta\theta + mL$

$$= 5.0 \times 1000 \times 90 + 50 \times \frac{2.25 \times 10^6}{4.2}$$

$$= 4.5 \times 10^6 + 26.79 \times 10^6$$

$$= 31.29 \times 10^6 \text{ Cal}$$

As 50% of heat is lost,

$$\text{Hence, total heat produced} = \frac{100}{50} \times 31.29 \times 10^6$$

Let n = no. of moles of ethane to be burnt, then

$$n = \frac{2 \times 31.29 \times 10^6}{373 \times 10^3} \text{ mole}$$

Hence, Volume of ethane $= nV$

$$= \frac{2 \times 31.29 \times 10^6}{373 \times 10^3} \times 22.4 \text{ litres}$$

$$= 3758.2 \text{ litres}$$

- 161.** Calculate the heat of combustion of coal when 10 g of coal on burning raises the temperature of 2 litres of water from $20 \text{ }^{\circ}\text{C}$ to $55 \text{ }^{\circ}\text{C}$.

Ans :

Given, $C = 1 \text{ Cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}$

$$m = 2 \times 10^3 \text{ gm}$$

$$\Delta \theta = 55 - 20 = 35 \text{ }^{\circ}\text{C}$$

$$M = 10 \text{ g}$$

If Q be the heat produced, then

$$Q = mc\Delta\theta$$

$$= 2 \times 10^3 \times 1 \times 35 = 70 \times 10^3 \text{ cal.}$$

Hence, If Q_1 be the heat of combustion, then

$$Q_1 = \frac{Q}{M} = \frac{70 \times 10^3}{10} = 7000 \text{ Cal g}^{-1}$$

- 162.** The coefficient of cubical expansion of mercury is 0.00018 and that of brass 0.00006 per deg. C. If a barometer having a brass-scale were to read 74.5 cm at $30 \text{ }^{\circ}\text{C}$, find the true barometric height at $0 \text{ }^{\circ}\text{C}$. The scale is supposed to be correct at $15 \text{ }^{\circ}\text{C}$.

Ans :

The brass-scale is true at $15 \text{ }^{\circ}\text{C}$, and it reads 74.5 cm at $30 \text{ }^{\circ}\text{C}$ therefore if it is cooled to $15 \text{ }^{\circ}\text{C}$, it contracts and shows actual height to be l_{30} given by

$$l_{30} = l_{15}[1 + \alpha(30 - 15)]$$

Given, $\alpha_{\text{brass}} = \frac{\gamma r}{3} = \frac{0.00006}{3} = 0.00002$

and $l_{15} = 74.5 \text{ cm}$

Hence, $l_{30} = 74.5[1 + 0.00002 \times 15]$
 $= 74.522 \text{ cm}$

Assuming area of cross-section to be constant. we have

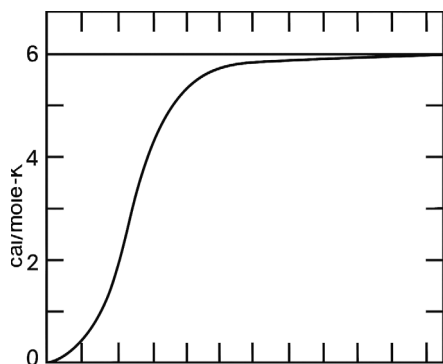
$$V_0\rho_0 = V_{30}\rho_{30} \text{ or } \alpha h_0\rho_0 = \alpha h_{30}\rho_{30}$$

Hence, True height at $0 \text{ }^{\circ}\text{C}$,

$$h_0 = h_{30} \frac{\rho_{30}}{\rho_0} = \frac{h_{30}}{(1 + \gamma_{Hg} t)}$$

$$= \frac{74.522}{1 - 0.00018 \times 30} = \frac{74.522}{1.0054}$$

$$= 74.122 \text{ cm.}$$



From figure we find that at $T = 0 \text{ K}$, $C_v \rightarrow 0$. As T increases, C_v also increases and becomes constant ($= 3R$) at some temperature depending upon the solid. Classical theory could not explain this behaviour. Debye explained it on the basis of quantum mechanics.

- 147.** Define calorimetry. What is its principle ? How will you determine the specific heat by using a calorimeter?

Ans :

The branch of physics which deals with the measurement of heat is called calorimetry.

Principle of Calorimetry :

When two bodies at two different temperatures are mixed, then heat gained by cold body is equal to heat lost by hot body provided no heat is allowed to escape to the surroundings.

Measurement of unknown specific heat of substance

Let m_1 = mass of water in the calorimeter

T_1 = temperature of water

W = water equivalent (mass $\times$ specific heat) of calorimeter at temperature T_1

Let m_2 = mass of substance added into water contained in calorimeter

T_2 = temperature of substance ($T_2 > T_1$).

Let T_3 = temperature of mixture.

$$\begin{aligned} \text{Heat gained by water} &= mC\theta \\ &= m_1 \times 1 (T_3 - T_1) \quad (\text{Since, } C = 1) \end{aligned}$$

$$\begin{aligned} \text{Heat gained by calorimeter} &= mC\theta \\ &= w(T_3 - T_1) \quad (w = mC) \end{aligned}$$

Total heat gained by water and calorimeter

$$\begin{aligned} &= m_1(T_3 - T_1) + w(T_3 - T_1) \\ &= (m_1 + w)(T_3 - T_1) \end{aligned}$$

Heat lost by substance

$$= m_2 \times C \times (T_2 - T_3)$$

Since, heat lost = heat gained

$$\begin{aligned} m_2 C (T_2 - T_3) &= (m_1 + w)(T_3 - T_1) \\ C &= \frac{(m_1 + w)(T_3 - T_1)}{m_2(T_2 - T_3)} \end{aligned}$$

Hence, C can be determined.

- 148.** (a) What do you mean by phases of a substance ?
(b) Describe briefly the study of different phases of a substance from isothermals.

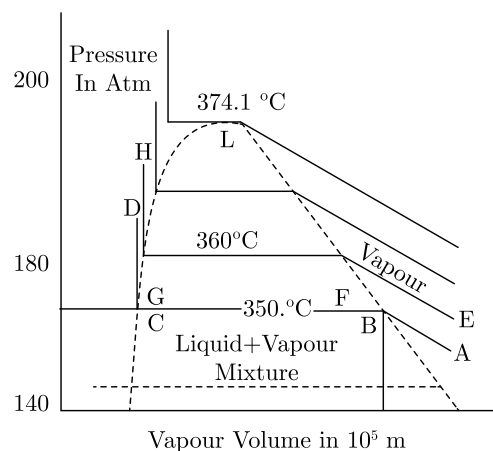
Ans :

- (a) If a substance can exist in different forms like solid, liquid and gas which are homogeneous, physically distinct but mechanically separable from one another are called different phases of a substance.

- (b) Figure shows the isothermal curves for one mole of water at different temperatures. Let us discuss these isothermal curves in detail.

At 350°C corresponding to point A, the water is in unsaturated vapour. Boyle's law is obeyed. At point B, a small increase in pressure causes water vapour to liquefy, volume decreases, vapour phase and liquid phase are in $\equiv m$ along BC.

As pressure is further increased, the portion CD of the isothermal is obtained. The approximate vertical portion CD indicates that even large increase in pressure causes a very little decrease in volume of the liquids. It shows that liquids are practically incompressible.



At 360°C , The isothermal EFGH at 360°C shows the same series of changes but with the following differences :

- (i) As the portion EF of the isothermal at 360°C is longer than the portion AB of

the isothermal at 350°C , it follows that at higher temperature, water vapour obeys Boyle's law over greater ranges of pressure and volume.

- (ii) The line FG of the isothermal at 360°C is smaller, it means that at higher temperature, the range over which the vapour and liquid phase co-exist reduces.
- (iii) Also, the decrease in horizontal portion indicates that at higher temperature, the difference in specific volume of saturated vapour and the liquid decreases.

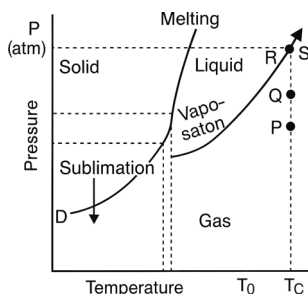
At 374.1°C , The horizontal portion of the isothermal reduces to a point L indicating that there is no physical difference between the two phases. The temperature of 374.1°C is called critical temperature of water. The isothermal at 374.1°C for water is known as critical isothermal.

The temperature 374.1°C is called critical temperature for water for the reason that it is the maximum temperature at which water can exist in liquid phase. It will be in gaseous phase above 374.1°C . Below 374.1°C , water in vapour state can be changed to liquid phase by applying pressure alone.

- 149.** Discuss pressure-temperature phase diagram and define Triple point.

Ans :

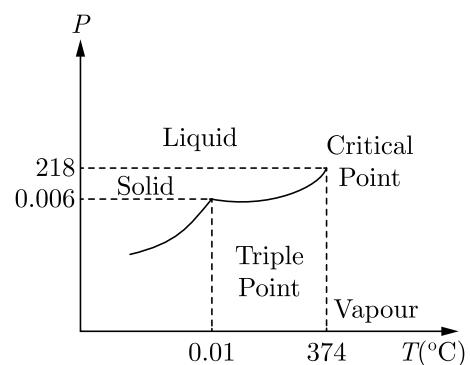
In the pressure-temperature plane, each phase corresponds to a region of area. The boundary between two areas is a curve in the p — T plane on which the two phases coexist. This is shown in the figure where the solid (S), liquid (L) and gas (G) regions are indicated.



The isotherm PQRS of figure appears as a vertical constant temperature line PS. The two phase co-existence region QR is shrunk into a point on the boundary T. In the p — T diagram, the volume does not appear explicitly. At QR, the volume lies between the liquid and gas values. Thus the lines in

a p — T phase diagram mark discontinuous change in some physical properties of a substance. The liquid gas coexistence line T ends in a critical point, where gas line do not. Thus the liquid gas transition can be continuous while solid-liquid and solid-gas transitions are always discontinuous.

If three phases are possible, namely, solid, liquid and gas, their boundaries meet at only one point. At this pressure and temperature, called the triple point (shown by T in figure), the three phases are equally stable. This is thus a unique condition and can be used to define say absolute temperature. For example, the triple point of water is at a pressure of 4.6 mm Hg and temperature 272.16 K.



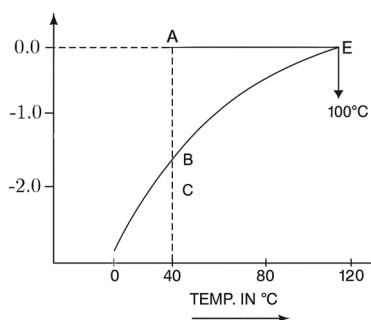
- 150.** (a) Discuss Equilibrium processes and Phase changes.
(b) Explain Evaporation and Boiling.

Ans :

- (a) When a substance is in thermodynamic equilibrium, its state is described entirely by pressure, volume and temperature, i.e. by a point in the p — T plane. Heating it or pushing it changes this state. If this change takes place such that at each step of the process, the substance is in thermodynamic equilibrium, the process can be described by a curve in the p — T plane. For example, heating of a solid at constant pressure p . The isothermal expansion of a liquid at temperature T_c is shown by the vertical line PQRS. This process leads to a change i.e. conversion of liquid to vapour or gas.
- (b) At ordinary temperature water is in liquid form and water vapour in atmosphere.

Consider figure which shows the liquid vapour of the p — T phase diagram. At atmospheric pressure and room-temperature (20°C) one is at the point A of the phase diagram, water is liquid. However at the same temperature, provided the pressure p is lower than that for

the point B, vapour is stable. The amount of water vapour is say a unit volume of air or equivalently its density is such that the pressure due to it is less than P_B . Depending on conditions, the vapour of water (i.e. partial pressure due to water vapour) may be close too but less than P_B much less than P_B . On a rainy day, the first is likely as on a dry summer day, the second.



Suppose on a particular day the partial pressure of water vapour is P_C while the maximum pressure at that temperature is P_B . Then the relative humidity is (P_C/P_B) .

The maximum pressure P_B at which a vapour can coexist with its liquid is called the saturated vapour pressure (SVP). It depends strongly on temperature and increases with it.

Suppose water is heated at atmospheric pressure. This process is described by the line ADE. Below 100°C , liquid water is more stable and above 100°C , water vapour is more stable. Consider a vessel containing water and open to the atmosphere being heated. The pressure everywhere is the same. The water, its immediate surroundings (e.g. the region just above water) and the air around are all at one atmospheric pressure. As the water temperature increases, the pressure of the water-vapour just above the water surface goes up, that is a large and larger friction of the gas, there is water-vapour. More and more water evaporates. At 100°C , the SVP of water is one atmosphere. Thus, the gas just above water surface is all water vapour. Further, water-vapour begins to form inside the liquid water, and bubble of this rise up. Clearly now, conversion of liquid into vapour is very rapid. This is called boiling.

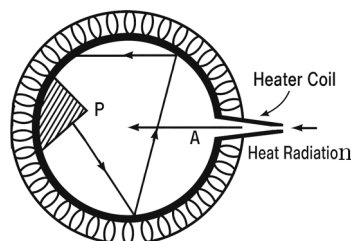
- 151.** Define a black body and explain how it can be realised in practice.

Ans :

A perfect black body is that which absorbs all the heat radiations, of whatever wavelength, incident

upon it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of incident radiation. A black body is a very good source of heat radiation in which intensity of radiation depends only on the temperature of black body.

To realise a black body in practice. A simple black body, designed by Fery, consists of a hollow copper sphere having a fine small hole A in the surface. The sphere is completely blackened from inside. When heat radiations enter the hole, they suffer multiple reflections at the inner wall of the sphere until it is completely absorbed. To minimise reflection of radiation entering normally a central projection P is made in front of hole A. We fit a heating element in between the walls of copper sphere.



When a current is passed through the element, heat is produced and temperature of sphere rises. When the temperature of the enclosure wall is increased sufficiently, the hole A will become self luminous and heat radiation starts coming out of the hole. Thus, the hole A acts as black body radiator. By changing the value of current passing through the heating element we can adjust the temperature of the black body and thus, control the intensity of heat radiation emitted by the hole.

- 152.** State and explain Stefan Boltzmann law for the black body radiation.

Ans :

According to Stefan's law the total radiant energy emitted per unit time from unit surface area of a perfect black body is directly proportional to the fourth power of the temperature 'T' of the radiating surface. Symbolically it states that

$$E \propto T^4$$

$$E = \sigma T^4 \quad \dots(1)$$

where σ is a constant known as Stefan's constant. Its value has been experimentally found to be $5.672 \times 10^{-8} \text{ watt } m^{-2} s^{-1} K^{-4}$.

If a perfect black body A maintained at a higher temperature T is surrounded by another black body

THREE MARKS QUESTIONS

137. Define temperature ?

Ans :

Temperature is the degree of hotness of a body. Temperature can also be defined in two other ways.

- (i) Temperature is thermal condition which determines the direction of flow of heat when two bodies touch each other. Heat always flows from a body at a higher temperature to a body at lower temperature.
- (ii) Temperature is that quantity which remains constant when two bodies are in thermal equilibrium with each other.

138. Why mercury is used in a thermometer ?

Ans :

A mercury thermometer has following advantages :

1. Mercury is bright and opaque.
2. The coefficient of expansion is uniform over a wide range of temperature.
3. Mercury is a good conductor of heat.
4. Its specific heat is low and hence takes negligible amount of heat.
5. It does not stick to tube.
6. The range of mercury thermometer is very large i.e. -39°C to 357°C .

139. Why C_p and C_v are called the principal specific heats of a gas ?

Ans :

Specific heats of a gas under different conditions is different. The specific heat of a gas under isothermal condition is

$$Q = mc\theta$$

$$c = \frac{Q}{m\theta}$$

If $\theta = 0$ (Isothermal conditions)

$$c = \frac{Q}{m \cdot 0} = \infty$$

i.e. c is infinite

If $Q = 0$ (adiabatic conditions)

$$\text{Then } c = \frac{Q}{m\theta} = 0$$

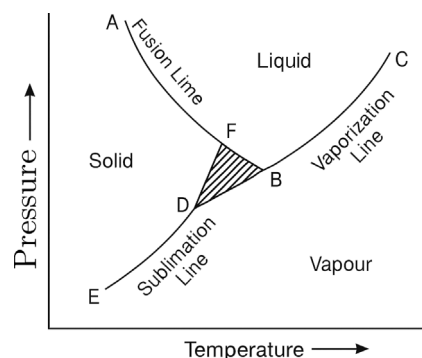
i.e. c is zero.

Thus, we find that specific heats of gases vary from zero to infinity. Thus, we define two specific heats, one at constant volume and other at constant pressure and call them principal specific heats.

140. Prove that there is only one triple point.

Ans :

To prove that there is only one triple point (or all the three curves meet in a single point). Let us suppose that the fusion, vaporisation and the sublimation curves do not meet in a single point but intersect as shown in figure. Let us consider the state of the substance within the shaded area BDF. As this area is below the fusion line, it represents only the solid state. As it is above the vaporisation curve, it represents only the liquid state and as it is to the right of the sublimation line, it represents only vapour state. But these three lines are contradictory conclusions. Hence the area BDF cannot exist which means that the three curves must meet in a point.



141. There are two spheres of same material and same radius at same temperature but one being solid while the other hollow. Which sphere will expand more if (a) they are heated to the same temperature (b) same amount of heat is given to each of them ?

Ans :

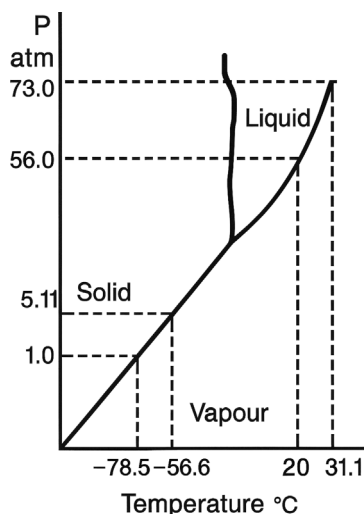
- (a) The expansion of a cavity is same as if it were a solid body of the same material because $\Delta V = \gamma V \Delta T$. So the expansion for both spheres will be same.
- (b) When same amount of heat is given, the hollow sphere will be more hot due to lesser mass because $\Delta T = \frac{Q}{mc}$. So the expansion ($\Delta V = \gamma V \Delta T$) will be more for hollow sphere.

142. Two rods X and Y are of equal lengths. Each rod has its ends at temperature T_1 and T_2 . What is the condition that will ensure equal rate of flow of heat through the rods X and Y ?

Ans :

Let d be the length of each rod. A_1, A_2 be area of cross-section and K_1, K_2 the coefficient of thermal conductivity of the rods X and Y respectively.

155. Answer the following questions based on the P-T phase diagram of CO_2 as shown in the figure.



- CO_2 at 1 atm pressure and temperature -60°C is compressed isothermally. Does it go through a liquid phase?
- What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?
- Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature -65°C as it is heated upto room temperature at constant pressure.
- CO_2 is heated to a temperature 70°C and compressed isothermally. What changes in its properties do we expect to observe?

Ans :

- No, vapour condenses to solid directly.
- It condenses to solid directly without passing through the liquid phase.
- It turns to liquid phase and then to vapour phase. The fusion and boiling points are when the horizontal line on P-T diagram at constant pressure of 10 atm intersects the fusion and vaporization curves.
- It will not exhibit any clear phase transition to liquid phase, but it will depart more and more from ideal gas behaviour as its pressure increases.

NUMERICAL QUESTIONS

156. What space must be left between two rails 60 m long so as to allow for an expansion upto 40°C assuming that the rails are laid down at 15°C . $\alpha = 0.000012$?

Ans :

Given, Initial length at 15°C ,

$$l_{15} = 60 \text{ m}$$

Rise of temperature = $40 - 15 = 25$

Final length at $40^\circ\text{C} = 60(1 + 0.000012 \times 25)$

$$= 60.081 \text{ m}$$

Space to be left = $60.081 - 60$

$$= 0.018 \text{ m}$$

157. An iron ring which is 1 mm in diameter is to be shrunk on a pulley which is 1.005 m in diameter. If the temperature of the ring is 10°C , find the temperature to which it must be raised so as that it will just slip on the circumference on the pulley. α for iron = 0.00012.

Ans :

Given,

Original temperature = 10°C

Circumference of pulley = $\pi \times 1.005 \text{ m}$

Circumference of ring = $\pi \times 1 \text{ m}$

In order that ring may slip over the pulley it should be heated till its circumference equals that of the pulley. Let t be the temperature to which it must be heated, then

$$\pi \times 1.005 = \pi \times 1 + 0.00012(t - 10)$$

$$t = \frac{0.005}{0.00012} + 10$$

$$= 426.8^\circ\text{C}$$

158. A metal plate 5 millimetre thick and 10 centimetre square has a temperature difference of 35°C between its surfaces. If 1820 calories of heat are transmitted every second, what is the co-efficient of thermal conductivity of the metal?

Ans :

Given, $A = 10 \times 10 = 100 \text{ sq. cm}$; $d = 0.5 \text{ cm}$.

$$(\theta_2 - \theta_1) = 35^\circ\text{C},$$

$$Q = 1820 \text{ cal/sec.}$$

$$\text{Now, } K = \frac{Qd}{A(\theta_2 - \theta_1)t} = \frac{1820 \times 0.5}{100 \times 35} = 0.26$$

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pressure and volume of the gas will be zero and at lower temperature a negative volume, which is impossible. Thus -273.15°C is the lowest possible temperature and is called absolute zero, i.e. $0\text{ K} = -273.15^{\circ}\text{C}$, the temperature scale which begins at absolute zero is called kelvin scale or ideal gas scale or absolute scale. The size of the unit for Kelvin temperature is the same celsius degree, so temperature on these scales are related by

$$T = t_c + 273.15$$

145. (a) Define heat and calorie. What is SI unit of heat?
(b) Define Joule's Mechanical equivalent of heat.

Ans :

- (a) Heat is defined as total kinetic energy due to random motion of the molecules.
Heat is associated with disorderly total energy of the molecules.

It is a scalar quantity and is a form of energy which gives up sensation of warmth.

Calorie : It is a unit of heat in cgs system. It is defined as the heat required to raise the temperature of one gram of water by 1°C from 14.5°C to 15.5°C .

Another unit of heat is kilo calorie.

1 kilo calorie = 1000 calories.

In S.I. unit, heat is expressed in joule.

- (b) **Joule's mechanical equivalent of heat :** Joule established that a definite amount of work has to be done to produce a definite amount of heat or when a certain amount of heat is spent, a definite amount of mechanical energy appears.
Mechanical work $\propto$ Heat produced

$$W \propto H$$

$$W = JH$$

where J is called Joule's mechanical equivalent of heat

$$\frac{W}{H} = J$$

If $H = 1$, then $J = W$

Hence, Joule's mechanical equivalent of heat is equal to the amount of mechanical work done to produce one calorie of heat.

$$J = 4.2\text{ J cal}^{-1}$$

$$= 4.2 \times 10^3\text{ J kcal}^{-1}$$

In S.I. units, $J = 1$

It is called conversion factor because it converts heat into work and work into heat.

146. (a) Define heat capacity and specific heat.
(b) How does specific heat vary with temperature ?
or

Discuss Specific heat capacity of solids (Dulong and Petit law).

Ans :

- (a) **Heat Capacity :** Heat capacity C of a substance may be defined as the amount of heat required to raise the temperature of a body by certain amount.

If ΔQ = Amount of heat absorbed

ΔT = Rise in temperature from T to $T + \Delta T$

$$\text{Hence capacity, } C = \frac{\Delta Q}{\Delta T}$$

Heat capacity depends upon

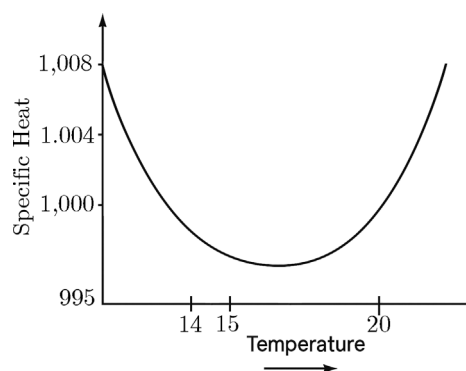
- (i) Amount of substance and
(ii) Amount of temperature

Specific heat : Specific heat of a substance is defined as the amount of heat required to raise the temperature of 1 g of substance through 1 kelvin (1°C). We sometimes use the term molar specific heat.

Molar specific heat : It is defined as the amount of heat required to raise the temperature of 1 mole of substance through 1 K

Units. $\text{joule mole}^{-1}\text{K}^{-1}$

- (b) **Variation of specific heat with temperature:**
The variation of specific heat of water in the temperature range from 0 to 100 is as shown in the figure.



The specific heat first decreases and then increases with temperature. In, general at lower temperature specific heat increases with increase in temperature and becomes constant at certain high temperature.

According to Dulong and Petit, the heat capacity of a solid is $3R$ or $6\text{ cal mol}^{-1}\text{K}^{-1}$ and is as shown in the figure.

- 163.** A brass wire 1.8 m long at 27°C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of 39°C , what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass is $2.0 \times 10^{-5}^\circ\text{C}^{-1}$, Young's modulus of brass is 0.91×10^{11} Pa.

Ans :

Original length of the wire, $l = 1.8$ m

Change in temperature, $\Delta T = 27 - (-39)$
 $= 66^\circ\text{C}$

Diameter of the wire, $d = 2.0$ mm

Hence, radius of the wire, $r = 1.0$ mm

Coefficient of linear expansion,

$$\alpha = 2.0 \times 10^{-5}^\circ\text{C}^{-1}$$

Young's modulus of elasticity,

$$Y = 0.91 \times 10^{11} \text{ Pa}$$

Decrease in length, $\Delta l = \alpha l \Delta T$

Strain produced, $\frac{\Delta l}{l} = \frac{\alpha l \Delta T}{l} = \alpha \Delta T$

Also, $Y = \frac{\text{stress}}{\text{strain}}$ or stress = $Y \times \text{strain}$

$$\frac{F}{A} = Y \times \alpha \Delta T$$

$$F = YA \times \alpha \Delta T = \pi r^2 Y \alpha \Delta T$$

$$F = \frac{22}{7} \times 0.1^2 \times 10^4 \times 0.91 \times 10^{11} \times 2.0 \times 10^{-5} \times 66$$

$$= \frac{22}{7} \times 0.91 \times 2 \times 66 \times 10^2 \times 10^2 \text{ N}$$

$$= 3.37 \times 10^2 \text{ N}$$

- 164.** A cylinder of diameter exactly 1 cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30°C . To what temperature must the plate be heated? For steel $\alpha = 1.1 \times 10^{-5}^\circ\text{C}^{-1}$.

Ans :

The hole will expand in the same way as a circle of steel fitting it would expand. The diameter of the hole needs to be changed by $\Delta l = 1 - 0.99970 = 0.00030$ cm

But, $\Delta l = \alpha l \Delta T$

$$\Delta T = \frac{\Delta l}{\alpha l} = \frac{0.00030}{1.1 \times 10^{-5} \times 0.99970}$$

$$= 27.3^\circ\text{C}$$

Hence, the plate must be raised to a temperature of
 $30 + 27.3 = 57.3^\circ\text{C}$

- 165.** A metal rod A of 25 cm length expands by 0.050 cm when its temperature is raised from 0°C to 100°C . Another rod B of a different metal of length 40 cm expands by 0.040 cm for the same rise in temperature. A third rod C of 50 cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C . Find the lengths of each portion of the composite rod C.

Ans :

$$\text{For the rod A, } \sigma_A = \frac{\Delta L}{L \times \Delta T} = \frac{0.050}{25 \times 100}$$

$$= 2 \times 10^{-5} \text{ per } ^\circ\text{C}$$

$$\text{For the rod B, } \alpha_B = \frac{\Delta L'}{L' \times \Delta T} = \frac{0.040}{40 \times 100}$$

$$= 1 \times 10^{-5} \text{ per } ^\circ\text{C}$$

Let the composite rod C is a made of portions L_A of A and L_B of rod B. Then

$$L_A + L_B = 50 \quad \dots(1)$$

After expansions, let the lengths of the two rods be L'_A and L'_B respectively. Then

$$L'_A + L'_B = 50.03 \quad \dots(2)$$

$$\text{But, } L_A = L'_A(1 + 2 \times 10^{-5} \times 50)$$

$$\text{and } L'_B = L_B(1 + 10^{-5} \times 50)$$

Hence,

$$L_A(1 + 2 \times 10^{-5} \times 50) + L_B(1 + 1 \times 10^{-5} \times 50) = 50.03 \quad \dots(3)$$

From equation (1), we have

$$L_A = 50 - L_B$$

Substituting this value in equation (3), we get

$$(50 - L)(1.001) + L_B(1.0005) = 50.3$$

$$\text{Solving we get, } L_B = 40 \text{ cm.}$$

Hence, $L_B = 40$ cm and $L_A = 10$ cm.

CASE BASED QUESTIONS

- 166.** A blacksmith needs to fit a new iron ring onto the rim of a wooden wheel of a horse cart. At room temperature (27°C), the diameter of the iron ring is 5.231 m, and the diameter of the wooden rim is 5.243 m. To make the ring fit, the blacksmith heats the iron ring. The coefficient of linear expansion of iron (α_i) is $1.20 \times 10^{-5} \text{ K}^{-1}$.

Questions:

- What is thermal expansion? What are the three types of thermal expansion?
- Why does the blacksmith heat the iron ring before attempting to fit it onto the wooden wheel?

Thermodynamics fundamentally studies the processes by which work can be transformed into heat and, conversely, how heat can be utilized to perform work. This inter-conversion is a central theme in the laws and principles of thermodynamics.

3. According to the historical caloric theory, how did heat flow between a hot and a cold body?
- Heat flowed from the colder body to the hotter body.
 - Heat flowed from the hotter body to the colder body.
 - Heat flowed until the pressure equalized.
 - Heat did not flow, but work was done by the fluid.

Ans :

(B) Heat flowed from the hotter body to the colder body.

The caloric theory, now discarded, posited that heat was a fluid called caloric. This fluid was believed to flow from a hotter body to a colder body until their “caloric levels” or temperatures equalized.

4. Who performed a significant experiment in 1798 demonstrating that heat is a form of energy by observing the boring of a brass cannon?
- James Prescott Joule
 - Benjamin Thomson (Count Rumford)
 - Sadi Carnot
 - Rudolf Clausius

Ans :

(B) Benjamin Thomson (Count Rumford)

Benjamin Thomson, also known as Count Rumford, conducted experiments in 1798 involving the boring of brass cannons. He observed that a significant amount of heat was generated, leading to the conclusion that heat is a form of energy.

5. Thermodynamics is primarily concerned with which type of systems?
- Microscopic systems involving individual molecules.
 - Bulk systems at the macroscopic level.
 - Systems involving only mechanical motion.
 - Systems at absolute zero temperature.

Ans :

(B) Bulk systems at the macroscopic level. Thermodynamics deals with bulk systems and their macroscopic properties like pressure, volume, and temperature. It does not delve into the detailed molecular constitution or behavior of individual particles within the system.

6. Which of the following is NOT typically considered a macroscopic variable in thermodynamics?
- Pressure
 - Volume
 - Temperature
 - Velocity of individual molecules

Ans :

(D) Velocity of individual molecules

Macroscopic variables in thermodynamics, such as pressure, volume, and temperature, describe the bulk properties of a system and are usually measurable directly. The velocity of individual molecules is a microscopic property dealt with in kinetic theory.

7. What is an adiabatic wall?
- A conducting wall that allows heat to flow.
 - An insulating wall that does not allow heat to flow.
 - A wall that allows work to be done.
 - A wall that maintains constant pressure.

Ans :

(B) An insulating wall that does not allow heat to flow.

An adiabatic wall is an insulating wall that prevents the flow of heat (energy) between a system and its surroundings or between two systems. This means no heat transfer occurs across an adiabatic wall.

8. Which of the following statements about thermal equilibrium is incorrect?
- In thermal equilibrium, the temperatures of two systems are equal.
 - Macroscopic variables of systems in thermal equilibrium do not change over time.
 - Energy flow continues indefinitely between two systems in thermal equilibrium.
 - A diathermic wall allows systems to reach thermal equilibrium.

Ans :

(C) Energy flow continues indefinitely between two systems in thermal equilibrium.

When two systems are in thermal equilibrium, their temperatures are equal, and there is no net flow of energy (heat) between them. Energy flow ceases once thermal equilibrium is attained.

9. Which statement correctly describes the Zeroth Law of Thermodynamics?
- Energy cannot be created or destroyed.
 - The entropy of an isolated system always increases.

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4. Heat required to melt ice

$$= m_{ice} L_f = (0.15 \text{ kg}) L_f.$$

Heat required to raise temperature of melted ice water from 0°C to

$$\begin{aligned} 6.7^\circ\text{C} &= m_{ice} s_w (6.7^\circ\text{C} - 0^\circ\text{C}) \\ &= (0.15 \text{ kg}) (4186 \text{ J kg}^{-1} \text{ K}^{-1}) (6.7^\circ\text{C}) \\ &= 4206.93 \text{ J}. \end{aligned}$$

Heat lost = Heat gained:

$$54376.14 \text{ J} = (0.15 \text{ kg}) L_f + 4206.93 \text{ J}.$$

$$L_f = (54376.14 - 4206.93) / 0.15$$

$$= 50169.21 / 0.15 \approx 3.34 \times 10^5 \text{ J/kg}.$$

- 169.** An iron bar ($L_1 = 0.1 \text{ m}$, $A_1 = 0.02 \text{ m}^2$, $K_1 = 79 \text{ W m}^{-1} \text{ K}^{-1}$) and a brass bar ($L_2 = 0.1 \text{ m}$, $A_2 = 0.02 \text{ m}^2$, $K_2 = 109 \text{ W m}^{-1} \text{ K}^{-1}$) are soldered end to end. The free end of the iron bar is maintained at 373 K , and the free end of the brass bar is maintained at 273 K . The system is insulated from the surroundings.

Questions:

- What is thermal conductivity (K) of a material?
- In the steady state, what can be said about the rate of heat flow (H) through any cross-section of the iron bar compared to the brass bar?
- Let T_0 be the temperature of the junction. Write the expressions for the rate of heat flow through the iron bar (H_1) and through the brass bar (H_2) in terms of T_0 .
- Using the condition for steady state, derive an expression for the junction temperature T_0 in terms of $K_1, K_2, T_1, T_2, L_1, L_2, A_1, A_2$. (Simplify for $A_1 = A_2 = A$ and $L_1 = L_2 = L$ as in the example).

Ans :

- The thermal conductivity (K) is the constant of proportionality in the equation for the rate of heat flow by conduction ($H = KA \frac{T_C - T_D}{L}$).
It indicates how rapidly a material conducts heat.
- In the steady state, the rate of heat flowing across a cross-section of the rod is the same at every point along the length of the combined rod. Thus, heat current (H_1) through the iron bar is equal to the heat current (H_2) through the brass bar.

$$3. \quad H_1 = \frac{K_1 A_1 (T_1 - T_0)}{L_1}$$

$$\text{and} \quad H_2 = \frac{K_2 A_2 (T_0 - T_2)}{L_2}.$$

4. Since $H_1 = H_2$, and given $A_1 = A_2 = A$ and $L_1 = L_2 = L$:

$$\frac{K_1 A (T_1 - T_0)}{L} = \frac{K_2 A (T_0 - T_2)}{L}$$

$$K_1 (T_1 - T_0) = K_2 (T_0 - T_2)$$

$$K_1 T_1 - K_1 T_0 = K_2 T_0 - K_2 T_2$$

$$K_1 T_1 + K_2 T_2 = T_0 (K_1 + K_2)$$

$$T_0 = \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}.$$

- 170.** Coastal regions often experience a phenomenon where the wind blows from the sea towards the land during the day (sea breeze) and from the land towards the sea during the night (land breeze). This is due to the different rates at which land and water heat up and cool down. Water has a high specific heat capacity.

Questions:

- What is convection as a mode of heat transfer?
- Why does land heat up more quickly than large bodies of water during the day?
- Explain how the difference in heating leads to a sea breeze during the day.
- How and why does this convection cycle reverse at night to form a land breeze?

Ans :

- Convection is a mode of heat transfer by actual motion of matter; it is possible only in fluids.
- Land heats up more quickly than large bodies of water because water has a greater specific heat capacity and also because mixing currents disperse the absorbed heat throughout the great volume of water.
- During the day, the ground heats up more quickly than water. The air in contact with the warm ground is heated by conduction, expands, and becomes less dense. This warm air rises, and cooler air from over the sea moves in to fill the space, creating a sea breeze.
- At night, the ground loses its heat more quickly, and the water surface is warmer than the land. The air above the warmer water rises, and cooler air from the land moves in to take its place, thus reversing the cycle and forming a land breeze.

- 171.** A piece of iron is heated in a furnace until it becomes red hot. When taken out, it emits thermal radiation. The wavelength (λ_m) at which the emitted energy is maximum depends on its temperature.

Questions:

- What is Wien's Displacement Law?
- What kind of spectrum does thermal radiation have at any temperature?

NCERT TEXT-BOOK QUESTIONS

1. A geyser heats water flowing at the rate of 3.0 litres per minute from 27°C to 77°C . If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is $40 \times 10^4 \text{ J/g}$?

Ans :

Heat gained by water

$$\begin{aligned} &= 3 \times 1000 \times 1 \times (77 - 27) \\ &= 3000 \times 50 \text{ cal} \\ &= 3000 \times 50 \times 4.2 \end{aligned}$$

$$\text{Heat of combustion} = 4.0 \times 10^4 \text{ J/g}$$

Rate of consumption of fuel

$$\begin{aligned} &= \frac{3000 \times 50 \times 4.2}{4.0 \times 10^4} \\ &= 16 \text{ g min}^{-1} \end{aligned}$$

2. What amount of heat must be supplied to $2 \times 10^{-2} \text{ kg}$ of nitrogen (at room temperature) to raise its temperature by 45° , at constant pressure (Molecular mass of $\text{N}_2 = 28$ $R = 8.3 \text{ J.mol}^{-1}$).

Ans :

$$\text{Mass of gas, } m = 2 \times 10^{-2} \text{ kg} = 20 \text{ g}$$

$$\text{Increase in temperature, } \Delta T = 45^{\circ}\text{C}$$

$$\text{Molecular mass, } M = 28$$

$$\text{No. of moles, } n = \frac{m}{M} = \frac{20}{28} = 0.714$$

Molar specific heat at constant pressure

$$C_P = \frac{7}{2}R \text{ [Nitrogen is diatomic gas]}$$

$$C_P = \frac{7}{2} \times 8.3 \text{ J mol}^{-1}\text{K}^{-1}$$

$$\text{So } \Delta Q = nC_P \Delta T$$

$$= 0.714 \times \frac{7}{2} \times 8.3 \times 45 \text{ J}$$

$$= 933.4 \text{ J}$$

3. Explain why

- Two bodies at different temperatures T_1 and T_2 , if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$.
- The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat ?
- Air pressure in a car tyre increases during driving.

Ans :

- The temperature of both the bodies will not be mean. It depends upon the conductivity and specific heat of the body.
- If the specific heat is large, it can absorb large amount of heat, before it starts boiling.
- Due to increase in temperature pressure increases.

4. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulate and the piston is insulated by having a pile of sand on it. By what factor does pressure of gas increase if the gas is compressive to half its original volume ?

Ans :

Let

$$V_1 = x$$

$$V_2 = \frac{x}{2}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{x}{\frac{x}{2}}\right)^{1.4}$$

$$P_2 = P_1(2)^{1.4}$$

$$P_2 = 2.64P_1$$

5. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas taken from state A to B via a process in which net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in latter case. (take $1 \text{ cal} = 4.19 \text{ J}$)

Ans :

According to first law of thermodynamics,

$$\Delta W = \Delta Q - \Delta U$$

$$\Delta W = 9.35 \times 4.19 - 22.3 = 16.87 \text{ J}$$

6. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened.

Answer the following :

- What is the final pressure of the gas in A and B?
- What is the change in internal energy of the gas?
- What is the change in the temperature of the gas?

4. Why do measurements on real gases deviate from the ideal gas law at low temperatures, and how is absolute zero inferred graphically?

Ans :

- For an ideal gas at constant volume, pressure (P) is directly proportional to the absolute temperature (T), i.e., $P \propto T$.
- A thermometer that uses a gas gives the same readings regardless of which gas is used (at low densities), unlike liquid-in-glass thermometers which show different readings due to differing expansion properties.
- Since $P/T = \text{constant}$ for a constant volume gas thermometer, $P_{tr}/T_{tr} = P/T$. Thus,

$$T = T_{tr}(P/P_{tr}) = 273.16K \times (P/P_{tr}).$$

- Measurements on real gases deviate from the ideal gas law at low temperatures due to intermolecular forces and finite molecular size becoming significant. Absolute zero is inferred by extrapolating the linear part of the pressure versus temperature graph (at constant volume) for low-density gases to the point where the pressure would be zero.

174. Water has a notably high specific heat capacity ($4186 \text{ J kg}^{-1} \text{ K}^{-1}$) compared to many other common substances. This property has significant implications in daily life and nature.

Questions:

- How is the specific heat capacity (s) of a substance defined?
- What does a high specific heat capacity imply about the amount of heat needed to change a substance's temperature?
- Give two examples where the high specific heat capacity of water is utilized or has a noticeable effect.
- If equal amounts of heat are added to equal masses of water and copper (specific heat capacity $386.4 \text{ J kg}^{-1} \text{ K}^{-1}$), which substance will experience a larger temperature increase? Explain.

Ans :

- Specific heat capacity (s) is defined as the amount of heat per unit mass absorbed or given off by the substance to change its temperature by one unit.
- A high specific heat capacity implies that a relatively large amount of heat is needed to cause a significant change in the substance's temperature.
- Two examples are: water is used as a coolant in automobile radiators, and as a heater in hot

water bags. Also, due to its high specific heat, water warms up more slowly than land, leading to cooling sea breezes in summer.

- Copper will experience a larger temperature increase. Since $\Delta Q = ms\Delta T$, for the same ΔQ and m, $\Delta T = \Delta Q/(ms)$. A smaller specific heat capacity 's' (like that of copper) will result in a larger ΔT .

175. A Dewar flask, commonly known as a thermos bottle, is designed to minimize heat transfer between its contents and the outside environment, allowing it to keep hot liquids hot or cold liquids cold for extended periods. It typically consists of a double-walled glass vessel.

Questions:

- What are the three modes of heat transfer?
- How does the silver coating on the inner and outer walls of the double-walled glass vessel help in minimizing heat transfer?
- Why is the space between the walls of the vessel evacuated?
- If the cork supporting the flask were made of a good thermal conductor instead of an insulator, how would it affect the performance of the Dewar flask?

Ans :

- The three distinct modes of heat transfer are conduction, convection, and radiation.
- The silver coating reflects radiation from the inner wall back to the contents and reflects any incoming radiation from the outer wall, minimizing heat transfer by radiation.
- The space between the walls is evacuated to reduce heat loss through conduction and convection, as these modes require a material medium.
- If the cork were a good thermal conductor, it would allow more heat to be conducted between the contents of the flask and the surroundings through the support point, thus reducing the flask's effectiveness in maintaining the temperature of its contents.

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CHAPTER 11

Thermodynamics

SUMMARY

1. INTERNAL ENERGY,

Internal energy (U) includes only the (disordered) energy (i.e. sum of kinetic and potential energies) associated with the random motion of molecules of the system. Its value depends only on the given state of the system, not on the path taken to arrive at that state.

2. HEAT AND WORK

Heat is the energy transfer due to temperature difference between the systems and the surroundings.

Work is the energy transfer brought about by means that do not involve such a temperature difference.

Heat and work are two different modes of altering the state of a thermodynamic system that results in change in its internal energy.

3. ZEROth LAW OF THERMODYNAMICS

It states that if two bodies are in thermal equilibrium with a third body individually, then the first two bodies will also be in thermal equilibrium with each other.

4. FIRST LAW OF THERMODYNAMICS

If the heat is supplied to a system which is capable of doing work then the quantity of heat observed by the system will be equal to the sum of external work done by the system and the increase in its internal energy.

Mathematically, $dQ = dU + dW$ i.e. $dW < dQ$

It means that it is impossible to construct a machine that would do more work than the energy supplied.

5. SPECIFIC HEAT CAPACITY

Specific heat at constant volume (C_v) is the amount of heat required to raise the temperature of 1 g of a gas through 1°C at constant volume.

Specific heat at constant pressure (C_p) is the amount of heat required to raise the temperature of 1 g of a gas through 1°C at constant pressure.

6. THERMODYNAMIC PROCESSES

6.1 Adiabatic Process

A process in which no heat enters or leaves a system is called an adiabatic process. For an adiabatic process, $Q = \text{constant}$ or $dQ = 0$

6.2 Isochoric Process

When a substance undergoes a process in which the volume remains unchanged ($\Delta V = 0$), the process is called isochoric.

Hence, $dW = P dV = 0$

6.3 Isothermal Process

A process which takes place at constant temperature is said to be isothermal. In general, none of the quantities Q , W or $(V_2 - V_1)$ is zero.

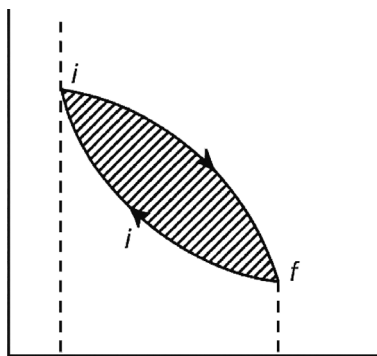
6.4 Isobaric Process

A process which takes place at constant pressure is called an isobaric process.

6.5 Cyclic Process

A cyclic process is one in which system returns to its initial stage after undergoing a series of changes.

In cyclic process, the work done per cycle is numerically equal to the area enclosed by the loop representing the loop, and if loop is traced clockwise, the work done is positive and if it is traced anticlockwise, the work done is negative.



More work is required in adiabatic compression than an isothermal compression. This is due to the reason that in an adiabatic compression, the internal energy of the system also increases whereas in an isothermal process, it remains the same.

The slope of adiabatic is γ times that of the isothermal. As the value of $\gamma > 1$, an adiabatic is steeper than an isothermal curve.

7. WORK DONE IN DIFFERENT PROCESS

Work done during isothermal process. It is given by

$$W_{\text{iso}} = RT \log_e \frac{V_2}{V_1} = RT 2.3026 \log_{10} \frac{V_2}{V_1}$$

Work done during adiabatic process. It is given by

$$W_{\text{adi}} = \frac{R}{\gamma - 1} [T_1 - T_2]$$

Change in internal energy is independent of the path and depends only on the initial and final states of a system.

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8. SECOND LAW OF THERMODYNAMICS

It is based on experimental observations and is stated in different forms out of which following two forms are important :

- (i) **Kelvin statement** : It is impossible to obtain continuous supply of work by cooling a body to a temperature below that of coldest of its surroundings.
- (ii) **Clausius statement** : It is impossible to construct a self-acting machine operating in a cycle unaided by any external agency to transfer heat from colder to a hotter body.

9. REVERSIBLE AND IRREVERSIBLE PROCESS

9.1 Reversible Process

The process which can be retraced to the initial state by reversing the controlling factors which occurred in the direct process is called a reversible process.

9.2 Irreversible Process

The process which cannot be retraced to their original state by reversing the controlling factors which occurred in the direct process is called irreversible process.

10. HEAT ENGINE

A device which converts heat energy into mechanical work is called a heat engine.

10.1 Efficiency of a Heat Engine (η)

It is defined as the ratio of exeter work done to the amount of energy absorbed from the heat source Mathematically

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

11. REFRIGERATOR OR HEAT PUMP

A refrigerator is a heat engine working in reverse direction.

The coefficient of performance (β) of a refrigerator is defined as the ratio of the quantity of heat removed per cycle from the contents of the refrigerator to the amount of work done by the external agency to remove it.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

12. CARNOT ENGINE

Carnot engine is an ideal heat engine which is perfectly reversible operating between two temperatures.

It consists of four processes (Isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression.)

The efficiency of Carnot heat engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

A perfect heat engine is one which converts all heat into work and have 100% efficiency.

3. If the iron piece first becomes dull red, then reddish-yellow, and finally white-hot as its temperature increases, what does Wien's law tell us about the shift in λ_m ?
4. The surface of the Sun has a temperature of about 6060 K, and its radiation has a maximum at $\lambda_m = 4753\text{\AA}$. If a distant star has a surface temperature of 3030 K, estimate the wavelength at which its radiation energy is maximum.

Ans :

1. Wien's Displacement Law states that the wavelength (λ_m) for which energy is maximum decreases with increasing temperature, and the relation is $\lambda_m T = \text{constant}$.
2. Thermal radiation at any temperature has a continuous spectrum from small to long wavelengths.
3. As the temperature increases, Wien's law implies that λ_m (the wavelength of maximum emission) decreases, shifting from red (longer wavelength) towards yellow and then white (a mixture including shorter wavelengths).
4. From Wien's law, $\lambda_{m1} T_1 = \lambda_{m2} T_2$.

$$(4753\text{\AA})(6060\text{K}) = \lambda_{m2}(3030\text{K}).$$

$$\begin{aligned}\lambda_{m2} &= (4753\text{\AA}) \times \frac{6060}{3030} \\ &= 4753\text{\AA} \times 2 = 9506\text{\AA}.\end{aligned}$$

- 172.** A cup of hot coffee at 94°C is placed in a room where the temperature is 20°C . The coffee begins to cool down. Initially, it cools from 94°C to 86°C in 2 minutes.

Questions:

1. State Newton's law of cooling.
2. Does Newton's law of cooling hold good for large temperature differences or small ones?
3. For the initial cooling from 94°C to 86°C in 2 minutes, what is the average excess temperature of the coffee over the surroundings?
4. If the coffee later cools from 71°C to 69°C , will it take more time, less time, or the same 2 minutes as the initial 8°C drop? Explain using Newton's law of cooling. (Based on Example 10.8, it will take less time for a smaller temperature drop when the excess temperature is also smaller, but here we are comparing different stages of cooling). The question is about a 2-degree drop at a lower average temperature.

Ans :

1. Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference of temperature of the body and the surroundings.

2. The law holds good only for small differences in temperature. However, it is often used as an approximation.
3. The average temperature during the first phase is $(94 + 86)/2 = 90^\circ\text{C}$. The excess temperature is $90^\circ\text{C} - 20^\circ\text{C} = 70^\circ\text{C}$.
4. When cooling from 71°C to 69°C , the average temperature is $(71 + 69)/2 = 70^\circ\text{C}$. The excess temperature is $70^\circ\text{C} - 20^\circ\text{C} = 50^\circ\text{C}$.
The rate of cooling is proportional to the excess temperature.

For the first stage:

$$\text{Rate} = \frac{8^\circ\text{C}}{2\text{min}} = 4^\circ\text{C/min}.$$

This is proportional to 70°C . So $4 = K(70)$.

For the second stage:

$$\text{Rate} = \frac{2^\circ\text{C}}{\text{Time}}.$$

This is proportional to 50°C . So

$$\frac{2}{\text{Time}} = K(50).$$

Dividing the equations:

$$\frac{4\text{min}^{-1}}{2/\text{Time}} = \frac{70}{50}$$

$$\frac{4 \times \text{Time}}{2} = \frac{7}{5}$$

$$2 \times \text{Time} = \frac{7}{5}$$

$$\text{Time} = \frac{7}{10} = 0.7 \text{ minutes or } 42 \text{ sec}.$$

This is for a 2°C drop. The rate of cooling is slower because the temperature difference is smaller (50°C vs 70°C).

- 173.** A constant-volume gas thermometer uses a low-density gas. When the thermometer is calibrated, it is found that the pressure of the gas is P_1 at the triple point of water (273.16 K). When the thermometer is placed in contact with an object whose temperature is to be measured, the pressure of the gas changes to P_2 , while the volume is kept constant.

Questions:

1. For an ideal gas at constant volume, how is pressure (P) related to absolute temperature (T)?
2. Why is a gas thermometer advantageous over a liquid-in-glass thermometer for accurate temperature measurements across different gases?
3. If the pressure at the triple point of water ($T_{tr} = 273.16\text{K}$) is P_{tr} , and the pressure at an unknown temperature T is P, what is the relation for T?

- (d) Do the intermediate states of the system (before settling to the final equilibrium rate) on its P-V-T surface ?

Ans :

- (a) 0.5 atm,

Since, $P_2 V_2 = P_1 V_1$

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{1 \times V}{2V} = 0.5 \text{ atm}$$

- (b) Zero,

- (c) Zero (assuming the gas to be ideal).

- (d) No, since the process (called free expansion) is rapid and cannot be controlled. The intermediate states are non-equilibrium states and do not satisfy the gas equation. In due course, the gas does return to an equilibrium state, which lies on its P-V-T surface.

7. An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate of 75 joule per second. At what rate is the internal energy increasing ?

Ans :

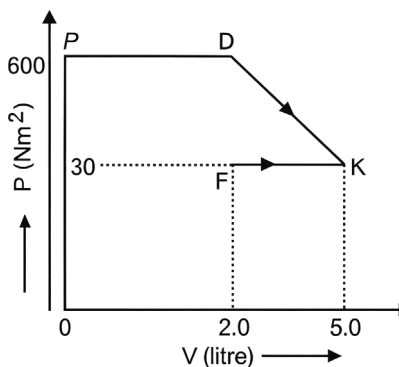
Heat supplied, $Q = 100 \text{ W}$

Work done per second, $dW = 75 \text{ J/s} = 75 \text{ W}$

Hence, Increase in internal energy,

$$\begin{aligned} dU &= Q - dW \\ &= 100 - 75 = 25 \text{ W} \end{aligned}$$

8. A thermodynamic system is taken from an original state to an intermediate state by the linear process as shown in figure.



Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the work done by the gas from D to E to F

Ans :

Work done by the gas from P to E (expansion)

$$W_{DE} = \text{Area of trapezium DEKLD}$$

$$W_{DE} = \frac{1}{2}(DL + EK) \times LK$$

Here, $DL = 600 \text{ Nm}^{-2}$,

$$EK = 300 \text{ Nm}^{-2}$$

$$\begin{aligned} LK &= 5.0 - 2.0 = 3.0 \text{ litre} \\ &= 3 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$\text{So } W_{DE} = \frac{1}{2}(600 + 300) \times 3 \times 10^{-3}$$

$$W_{DE} = 1350 \times 10^{-3} \text{ J} = 1.35 \text{ J}$$

Work done during the process from E to F (compression)

$$\begin{aligned} W_{EF} &= -\text{Area EFLK} = -KL \times EK \\ &= -3 \times 10^{-3} \times 0.300 = -0.9 \text{ J} \end{aligned}$$

Net Work done,

$$\begin{aligned} W &= W_{DE} + W_{EF} \\ &= 1.35 - 0.9 = 0.45 \text{ J} \end{aligned}$$

OBJECTIVE QUESTIONS

- Which of the following best defines thermodynamics?
 - The study of heat as a form of invisible fluid.
 - The branch of physics dealing with heat, temperature, and their inter-conversion.
 - The study of molecular motion in bulk systems.
 - The science of converting work entirely into heat.

Ans :

- (B) The branch of physics dealing with heat, temperature, and their inter-conversion.

Thermodynamics is the branch of physics that focuses on the concepts of heat and temperature, and how heat can be converted into other forms of energy, such as work, and vice versa. It examines the macroscopic behaviour of systems.

- What is the primary concern of thermodynamics regarding energy conversion?
 - Conversion of kinetic energy to potential energy.
 - Conversion of nuclear energy to electrical energy.
 - Processes where work is converted into heat and vice versa.
 - Conversion of chemical energy into light energy.

Ans :

- (C) Processes where work is converted into heat and vice versa.

- To what temperature should the iron ring be heated so that its diameter becomes 5.243 m, allowing it to fit the rim of the wheel?
- If the blacksmith overheats the ring significantly beyond the required temperature and then fits it, what might happen to the wooden wheel as the iron ring cools down?

Ans :

- The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion. The three types are linear expansion, area expansion, and volume expansion.
- The blacksmith heats the iron ring so that it expands, increasing its diameter, which allows it to easily slip onto the wooden wheel. Upon cooling, the ring will contract and fit tightly.
- Let $L_{T_1} = 5.231\text{m}$ at $T_1 = 27^\circ\text{C}$ (300 K), and we need $L_{T_2} = 5.243\text{m}$ at temperature T_2 .

Using

$$L_{T_2} = L_{T_1}[1 + \alpha_l(T_2 - T_1)].$$

$$5.243 = 5.231[1 + 1.20 \times 10^{-5}(T_2 - 27)].$$

$$\frac{5.243}{5.231} - 1 = 1.20 \times 10^{-5}(T_2 - 27).$$

$$0.002294 - 1 \approx 0.002294 = 1.20 \times 10^{-5}(T_2 - 27)$$

$$T_2 - 27 = \frac{0.002294}{1.20 \times 10^{-5}} \approx 191.17$$

$$T_2 \approx 191.17 + 27 = 218.17^\circ\text{C}. \text{ (Example 10.2 gives } 218^\circ\text{C).}$$

- If the ring is significantly overheated, it will expand more. Upon fitting and cooling, it will contract more significantly. This excessive contraction could exert a very large compressive force (thermal stress) on the wooden wheel, potentially damaging or crushing the wood.

- 167.** A freshwater lake is located in a region where the winter temperatures drop well below 0°C . As the lake cools, the water near the surface loses energy to the atmosphere. The density of water changes unusually between 0°C and 4°C .

Questions:

- Describe the anomalous behavior of water's volume (and density) when it is cooled from room temperature down to 0°C .
- At what temperature does water have its maximum density?
- Explain the process of how a lake freezes from the top down, referencing the density changes of water.
- What would be the ecological impact if water did not have this anomalous expansion property and instead behaved like most other substances

(i.e., density continuously increasing with cooling)?

Ans :

- As water is cooled from room temperature, its volume decreases until its temperature reaches 4°C . Below 4°C , the volume increases, and therefore, the density decreases.
- Water has its maximum density at 4°C .
- As a lake cools towards 4°C , water near the surface loses energy, becomes denser, and sinks. The warmer, less dense water from below rises. However, once the colder water on top reaches temperatures below 4°C , it becomes less dense and remains at the surface, where it freezes.
- If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy much of their animal and plant life.

- 168.** 0.15 kg of ice at 0°C is mixed with 0.30 kg of water at 50°C in an insulated container. After some time, the mixture reaches a steady final temperature of 6.7°C . The specific heat capacity of water (s_w) is $4186\text{Jkg}^{-1}\text{K}^{-1}$.

Questions:

- What is latent heat of fusion?
- What is the principle of calorimetry used when mixing substances at different temperatures in an isolated system?
- Calculate the heat lost by the water initially at 50°C .
- Using the heat lost by the water and the heat gained by the ice (to melt and then to warm up to 6.7°C), calculate the heat of fusion of ice (L_f).

Ans :

- The latent heat for a solid-liquid state change is called the latent heat of fusion (L_f). It is the amount of heat per unit mass transferred during the change of state from solid to liquid at a constant temperature.
- The principle of calorimetry states that when a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings.
- Heat lost by water

$$= m_w s_w (\theta_i - \theta_f)$$

$$= (0.30\text{kg})(4186\text{Jkg}^{-1}\text{K}^{-1})(50.0^\circ\text{C} - 6.7^\circ\text{C})$$

$$= 54376.14\text{J}.$$

(C) Two systems in thermal equilibrium with a third system are in thermal equilibrium with each other.

(D) It is impossible to reach absolute zero in a finite number of steps.

Ans :

(C) Two systems in thermal equilibrium with a third system are in thermal equilibrium with each other.

The Zeroth Law of Thermodynamics states that if system A is in thermal equilibrium with system C, and system B is also in thermal equilibrium with system C, then systems A and B are in thermal equilibrium with each other. This law defines temperature.

10. What is the significance of the Zeroth Law of Thermodynamics?

- (A) It establishes the concept of internal energy.
- (B) It provides the formal definition of temperature.
- (C) It defines the direction of heat flow.
- (D) It explains the efficiency of heat engines.

Ans :

(B) It provides the formal definition of temperature. The Zeroth Law of Thermodynamics is crucial because it leads to the concept of temperature. It implies that there is a scalar quantity, temperature, which is the same for all systems in thermal equilibrium.

11. All of the following are considered modes of energy transfer that can change a system's internal energy EXCEPT:

- (A) Heat supplied to the system.
- (B) Work done by the system.
- (C) The temperature of the system.
- (D) Work done on the system.

Ans :

(C) The temperature of the system.

Heat and work are the two modes of energy transfer that can alter a system's internal energy. Temperature is a property of the system itself, indicating its 'hotness' or 'coldness', not a mode of energy transfer.

12. How does internal energy differ from heat?

- (A) Internal energy is energy in transit, while heat is stored energy.
- (B) Internal energy is a state variable, while heat is energy in transit.
- (C) Internal energy only includes kinetic energy, while heat includes potential energy.

(D) Internal energy depends on the path taken, while heat does not.

Ans :

(B) Internal energy is a state variable, while heat is energy in transit.

Internal energy (U) is a state variable, meaning it depends only on the current state of the system. Heat (Q) is not a state variable; it represents energy transferred due to a temperature difference, i.e., energy in transit.

13. What does ΔU represent in the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$?

- (A) Heat supplied to the system.
- (B) Work done by the system.
- (C) Change in internal energy of the system.
- (D) Change in the enthalpy of the system.

Ans :

(C) Change in internal energy of the system.

In the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$, ΔU represents the change in the internal energy of the system. This law is essentially the principle of conservation of energy applied to thermodynamic systems.

14. If heat supplied to a system is used up entirely by the system in doing work on the environment, what is the change in internal energy?

- (A) $\Delta U = \Delta Q$
- (B) $\Delta U = \Delta W$
- (C) $\Delta U = 0$
- (D) $\Delta U = -\Delta W$

Ans :

(C) $\Delta U = 0$

If the heat supplied (ΔQ) is entirely used for work done by the system (ΔW), then according to the first law ($\Delta Q = \Delta U + \Delta W$), if $\Delta Q = \Delta W$, then ΔU must be zero. This often occurs in an isothermal expansion of an ideal gas.

15. What is the approximate latent heat of vaporization for water at atmospheric pressure?

- (A) 169.2 J/g
- (B) 2086.8 J/g
- (C) 2256 J/g
- (D) 1.013×10^5 J/g

Ans :

(C) 2256 J/g

The measured latent heat of water, which is the heat required to change 1 gram of water from liquid to vapor phase at constant temperature and pressure, is approximately 2256 J/g.

- (A) 1 and 2 only
 (B) 1, 2, and 3 only
 (C) 2, 3, and 4 only
 (D) 1, 2, 3, and 4

Ans :

- (D) 1, 2, 3, and 4

All four statements are correct definitions used in thermodynamics. An open system allows both mass and energy transfer. A closed system allows energy transfer but not mass transfer. An isolated system allows neither. A diathermic wall permits heat flow, whereas an adiabatic wall prevents it.

41. Regarding the First Law of Thermodynamics, which of the following statements are true?

1. It is a restatement of the law of conservation of energy.
2. It implies that heat supplied to a system is always equal to the work done by the system.
3. The change in internal energy of a system depends only on the initial and final states.
4. Work done by the system is always positive.

- (A) 1 and 3 only
 (B) 2 and 4 only
 (C) 1, 2, and 3 only
 (D) 1, 3, and 4 only

Ans :

- (A) 1 and 3 only

Statement 1 is true as the First Law is the conservation of energy principle. Statement 3 is true because internal energy is a state function. Statement 2 is only true if $\Delta U = 0$ (e.g., isothermal ideal gas). Statement 4 is incorrect; work done on the system is negative.

42. Which of the following statements about specific heat capacity are correct?

1. It is an intensive property.
2. It is the amount of heat required to raise the temperature of a unit mass of a substance by one degree Celsius.
3. Its value is the same for all substances.
4. For gases, specific heat at constant pressure (C_p) is generally greater than specific heat at constant volume (C_v).

- (A) 1, 2, and 4
 (B) 1 and 3 only
 (C) 2 and 4 only
 (D) All are correct

Ans :

- (A) 1, 2, and 4

Statement 1 is correct as specific heat capacity does not depend on the amount of substance. Statement 2 is the definition of specific heat capacity. Statement 3 is incorrect as different substances have different specific heat capacities. Statement 4 is correct for gases.

43. Match the thermodynamic process in Column I with its defining characteristic in Column II.

	Column I		Column II
(a)	Isothermal	(i)	$\Delta Q = 0$
(b)	Adiabatic	(ii)	Volume constant
(c)	Isobaric	(iii)	Temperature constant
(d)	Isochoric	(iv)	Pressure constant

- (A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
 (B) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)
 (C) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
 (D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Ans :

- (A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Isothermal processes occur at constant temperature. Adiabatic processes involve no heat exchange. Isobaric processes occur at constant pressure. Isochoric processes occur at constant volume. These matches align correctly with their definitions.

44. Match the term in Column I with its appropriate description in Column II.

	Column I		Column II
(a)	Internal Energy	(i)	Energy in transit due to temperature difference
(b)	Heat	(ii)	Measure of disorderness in the system
(c)	Entropy	(iii)	Sum of molecular kinetic and potential energies
(d)	Enthalpy	(iv)	Measure of total heat content of the system

- (A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
 (B) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
 (C) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
 (D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Ans :

- (A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

Internal energy is the sum of molecular kinetic and potential energies. Heat is energy transfer due to temperature difference. Entropy is a measure of disorder. Enthalpy represents the total heat content. These pairings accurately reflect their definitions.

Intensive variables, like temperature and pressure, do not depend on the size or amount of the system. If a system is divided, the intensive variables remain unchanged for each part. Extensive variables, like volume and mass, scale with the system's size.

- 23.** All of the following are thermodynamic processes where a specific variable is kept constant EXCEPT:
- (A) Isothermal process (temperature constant)
 - (B) Isobaric process (pressure constant)
 - (C) Quasi-static process (entropy constant)
 - (D) Isochoric process (volume constant)

Ans :

- (C) Quasi-static process (entropy constant)

A quasi-static process is one that occurs infinitely slowly, allowing the system to remain in equilibrium at every stage. While an adiabatic quasi-static process involves constant entropy (isentropic), quasi-static itself doesn't inherently mean constant entropy. The other options correctly define processes by their constant variable.

- 24.** What is the main difference between a quasi-static process and a non-equilibrium process?
- (A) Quasi-static processes are rapid, while non-equilibrium processes are slow.
 - (B) Quasi-static processes maintain equilibrium throughout, while non-equilibrium processes do not.
 - (C) Quasi-static processes involve heat transfer, while non-equilibrium processes are adiabatic.
 - (D) Quasi-static processes only occur in ideal gases, non-equilibrium in real gases.

Ans :

(B) Quasi-static processes maintain equilibrium throughout, while non-equilibrium processes do not. A quasi-static process is an idealized, infinitely slow process where the system remains in thermal and mechanical equilibrium with its surroundings at every stage. In contrast, during a non-equilibrium process, the system passes through states that are not in equilibrium.

- 25.** What is an isothermal process?
- (A) A process where the pressure of the system is kept fixed.
 - (B) A process where the volume of the system is kept fixed.
 - (C) A process where the temperature of the system is kept fixed.
 - (D) A process where no heat flows between the system and surroundings.

Ans :

- (C) A process where the temperature of the system is kept fixed.

An isothermal process is a thermodynamic process in which the temperature of the system remains constant throughout. Any heat exchange with the surroundings occurs in such a way that the temperature does not change.

- 26.** In an isothermal expansion of an ideal gas, what is the relationship between heat supplied (Q) and work done (W) by the gas?
- (A) $Q = -W$
 - (B) $Q = W$
 - (C) $Q = \Delta U$
 - (D) $W = \Delta U$

Ans :

- (B) $Q = W$

For an ideal gas undergoing an isothermal process, the internal energy (U) depends only on temperature. Since the temperature is constant, the change in internal energy (ΔU) is zero. According to the first law ($\Delta Q = \Delta U + \Delta W$), this means $Q = W$.

- 27.** For an adiabatic process of an ideal gas, which of the following relations is true, where $\gamma = C_p/C_v$?
- (A) $PV = \text{constant}$
 - (B) $PV^\gamma = \text{constant}$
 - (C) $P^\gamma V = \text{constant}$
 - (D) $P/V = \text{constant}$

Ans :

- (B) $PV^\gamma = \text{constant}$

In an adiabatic process for an ideal gas, where no heat is exchanged with the surroundings ($\Delta Q = 0$), the relationship between pressure (P) and volume (V) is given by $PV^\gamma = \text{constant}$, where γ is the ratio of specific heats.

- 28.** What is the work done by an ideal gas during an isochoric process?
- (A) $P\Delta V$
 - (B) $\mu R\Delta T$
 - (C) Zero
 - (D) ΔU

Ans :

- (C) Zero

An isochoric process is one in which the volume of the system remains constant ($\Delta V = 0$). Since work done by a gas is given by $W = P\Delta V$ (for constant pressure, or $\int P dV$ in general), if $\Delta V = 0$, then the work done is zero.

Ans :

(c) no change in temperature whether heat is taken in or given out.

64. Air in a cylinder is suddenly compressed by a piston, which is then maintained at the same position. With passage of time :

- (a) The pressure increases.
- (b) The pressure may increase or decrease.
- (c) The pressure remains the same.
- (d) The pressure decreases.

Ans :

(d) The pressure decreases.

When cylinder is suddenly compressed, there is increase in temperature. With passage of time, the temperature falls and hence the pressure decreases.

65. The specific heat of a gas in an isothermal process is :

- (a) Infinite
- (b) Zero
- (c) Negative
- (d) Remains constant

Ans :

(a) Infinite

In isothermal process $\Delta T = 0$

and $c = \frac{\Delta Q}{m \Delta T} = \infty$

66. Which of the following is not true about the processes?

- (a) For isothermal process $dT = 0$
- (b) For isobaric process $dP = 0$
- (c) For isochoric process $dE = 0$
- (d) For adiabatic process $dQ = 0$

Ans :

(c) For isochoric process $dE = 0$.

67. Which of the following is not a state function?

- (a) Temperature
- (b) Entropy
- (c) Pressure
- (d) Work

Ans :

(d) Work

Work is not a function to describe the state of a system.

68. Which of the following parameters does not characterize the thermodynamic state of matter ?

- (a) Temperature
- (b) Pressure
- (c) Work
- (d) Volume

Ans :

(c) Work

Thermodynamic state of matter are characterized by P , V and T .

69. Which of the following is incorrect regarding the first law of thermodynamics ?

- (a) It introduces the concept of the internal energy.
- (b) It introduces the concept of the entropy.
- (c) It is not applicable to any cyclic process.
- (d) It is a restatement of the principle of conservation of energy.

Ans :

(b) It introduces the concept of the entropy

First law is unable to introduce to concept of entropy.

70. "Heat cannot itself flow from a body at lower temperature to a body at higher temperature" is the statement or consequence of:

- (a) Second law of thermodynamics.
- (b) Conservation of momentum.
- (c) Conservation of mass.
- (d) First law of thermodynamics.

Ans :

(a) Second law of thermodynamics

71. Which one of the following statements is true in respect of usual quantities represented by ΔQ , ΔU and ΔW :

- (a) ΔU and ΔW are path dependent.
- (b) ΔQ and ΔU are path dependent.
- (c) ΔU does not depend on path.
- (d) ΔQ does not depend upon path.

Ans :

(c) ΔU does not depend on path.

ΔU depends on initial and final states and independent of the path followed.

72. Which law of the thermodynamics leads us to conclude that it is not possible to convert whole of heat into work continuously?

- (a) Third law
- (b) Second law
- (c) First law
- (d) Zeroth law

Ans :

(c) First law

73. What is the temperature of the source in carnot cycle of 10% efficiency when heat exhausts at 270 K

- (a) 400 K
- (b) 500 K
- (c) 300 K
- (d) 600 K

- 35.** The Carnot cycle consists of all the following processes EXCEPT:

(A) Isothermal expansion
(B) Adiabatic expansion
(C) Isochoric compression
(D) Isothermal compression

Ans :

(C) Isochoric compression

The Carnot cycle, which describes the operation of a Carnot engine, consists of two isothermal processes (expansion and compression) and two adiabatic processes (expansion and compression). It does not include any isochoric (constant volume) processes.

- 36.** How does the efficiency of a practical (irreversible) engine compare to that of a Carnot engine operating between the same two temperature reservoirs?

(A) The practical engine has higher efficiency.
(B) The practical engine has lower efficiency.
(C) Both engines have the same efficiency.
(D) The comparison depends on the working substance.

Ans :

(B) The practical engine has lower efficiency.

According to Carnot's theorem, no engine operating between two given temperatures can be more efficient than a reversible Carnot engine operating between the same two temperatures. Therefore, any practical (irreversible) engine will have lower efficiency.

- 37.** Assertion (A): The internal energy of an ideal gas depends only on its temperature.

Reason (R): For an ideal gas, intermolecular forces are negligible.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The internal energy of an ideal gas consists only of the kinetic energy of its molecules. Since intermolecular forces are negligible in an ideal gas, there is no potential energy component related to these forces. Kinetic energy is directly proportional to temperature, hence internal energy depends only on temperature.

- 38.** Assertion (A): Work done in an adiabatic process is path independent.

Reason (R): In an adiabatic process, the change in internal energy is equal to the negative of the work done.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(D) A is false but R is true.

Work done, in general, is path-dependent, even in an adiabatic process. However, for an adiabatic process, $\Delta Q = 0$. According to the first law, $\Delta Q = \Delta U + \Delta W$, so $0 = \Delta U + \Delta W$, which means $\Delta W = -\Delta U$. The change in internal energy (ΔU) is path-independent as it's a state function. Thus, Reason (R) is true, but Assertion (A) is false.

- 39.** Assertion (A): The efficiency of a Carnot engine is 100% if the sink temperature is 0 K.

Reason (R): The efficiency of a Carnot engine is given by $\eta = 1 - (T_2/T_1)$.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The efficiency of a Carnot engine is indeed given by $\eta = 1 - (T_2/T_1)$, where T_2 is the sink temperature and T_1 is the source temperature. If $T_2 = 0\text{K}$ (absolute zero), then $\eta = 1 - 0 = 1$, which corresponds to 100% efficiency. Thus, both assertion and reason are true, and the reason correctly explains the assertion.

- 40.** Consider the following statements regarding thermodynamic systems:

1. An open system can exchange both energy and matter with its surroundings.
2. A closed system can exchange only energy but not matter with its surroundings.
3. An isolated system cannot exchange either energy or matter with its surroundings.
4. A diathermic wall allows heat transfer, while an adiabatic wall does not.

Which of the statements are correct?

29. In a cyclic process, what is the change in internal energy (ΔU)?
- (A) Equal to heat absorbed.
 (B) Equal to work done.
 (C) Zero.
 (D) Always positive.

Ans :

(C) Zero.

A cyclic process is one in which the system returns to its initial state after a series of changes. Since internal energy is a state variable, its value depends only on the state. If the initial and final states are the same, the change in internal energy (ΔU) is zero.

30. Which law of thermodynamics states that the efficiency of a heat engine can never be unity?
- (A) Zeroth Law
 (B) First Law
 (C) Second Law
 (D) Third Law

Ans :

(C) Second Law

The Second Law of Thermodynamics, in one of its formulations (Kelvin-Planck statement), implies that it is impossible for any device operating in a cycle to absorb heat from a single reservoir and convert it entirely into work. This means the efficiency of a heat engine can never be 100% (or unity).

31. What is the Clausius statement of the Second Law of Thermodynamics?
- (A) No process can convert heat entirely into work.
 (B) Entropy of the universe tends to a maximum.
 (C) No process can transfer heat from a colder to a hotter object without external work.
 (D) Absolute zero temperature is unattainable in finite steps.

Ans :

(C) No process can transfer heat from a colder to a hotter object without external work.

The Clausius statement of the Second Law of Thermodynamics asserts that it is impossible for any process, working in a cycle, to have as its sole result the transfer of heat from a colder body to a hotter body. This implies refrigerators need external work.

32. Which of the following is an incorrect statement about irreversible processes?

- (A) Most spontaneous processes in nature are irreversible.
 (B) They can be reversed to bring both system and surroundings to their initial states.
 (C) Friction and viscosity contribute to irreversibility.
 (D) Free expansion of a gas is an example of an irreversible process.

Ans :

(B) They can be reversed to bring both system and surroundings to their initial states.

An irreversible process is one that cannot be reversed such that both the system and its surroundings return to their exact initial states without any other changes elsewhere. Most natural processes are irreversible.

33. Which statement correctly describes a reversible process?
- (A) It is a rapid process that occurs spontaneously.
 (B) It always involves an increase in the entropy of the universe.
 (C) It is quasi-static and free from dissipative effects.
 (D) It can only occur in isolated systems.

Ans :

(C) It is quasi-static and free from dissipative effects.

A reversible process is an idealized process that is carried out quasi-statically (infinitely slowly, remaining in equilibrium) and involves no dissipative effects like friction or viscosity. It can be reversed without leaving any net change in the system or surroundings.

34. What is the significance of the Carnot engine?

- (A) It is the most practical engine design.
 (B) It has an efficiency of 100%.
 (C) It provides the maximum possible efficiency for an engine operating between two temperatures.
 (D) It operates using only adiabatic processes.

Ans :

(C) It provides the maximum possible efficiency for an engine operating between two temperatures.

The Carnot engine is a theoretical, ideal reversible heat engine. Its significance lies in establishing the maximum possible efficiency that any heat engine can achieve when operating between two given temperature reservoirs.

45. Match the law of thermodynamics in Column I with its consequence or statement in Column II.

	Column I		Column II
(a)	Zeroth Law	(i)	$\Delta Q = \Delta U + \Delta W$
(b)	First Law	(ii)	No engine can be 100% efficient
(c)	Second Law (Kelvin-Planck)	(iii)	Concept of temperature
(d)	Second Law (Clausius)	(iv)	Heat cannot flow from cold to hot without external work

- (A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
 (B) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
 (C) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
 (D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Ans :

- (A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

The Zeroth Law establishes the concept of temperature. The First Law is the conservation of energy, $\Delta Q = \Delta U + \Delta W$. The Kelvin-Planck statement of the Second Law implies limits on engine efficiency. The Clausius statement of the Second Law concerns the direction of heat flow.

46. Which of the following pairs correctly matches a thermodynamic variable with its characteristic?

- (A) Heat – State variable
 (B) Work – Path independent
 (C) Internal Energy – Energy in transit
 (D) Temperature – Marker of hotness

Ans :

- (D) Temperature – Marker of hotness

Temperature is a physical property that quantitatively expresses the common notions of hot and cold. Heat is energy in transit, not a state variable. Work is generally path-dependent. Internal energy is stored energy, a state variable, not energy in transit.

47. Identify the correct pair regarding thermodynamic processes and their work done expression for an ideal gas.

- (A) Isochoric process – $W = P(V_2 - V_1)$
 (B) Isothermal process – $W = \mu R T \ln(V_2/V_1)$
 (C) Adiabatic process – $W = 0$
 (D) Isobaric process – $W = \mu R(T_2 - T_1)/(\gamma - 1)$

Ans :

- (B) Isothermal process – $W = \mu R T \ln(V_2/V_1)$

The work done during an isothermal expansion or compression of an ideal gas is correctly given by $W = \mu R T \ln(V_2/V_1)$. For an isochoric process, $W = 0$. For an adiabatic process, $W = \mu R(T_2 - T_1)/(\gamma - 1)$. For an isobaric process, $W = P(V_2 - V_1) = \mu R(T_2 - T_1)$.

48. Which of the following pairs incorrectly matches a thermodynamic concept with its description?

- (A) Diathermic wall – Allows heat flow
 (B) Thermodynamic equilibrium – Macroscopic variables change with time
 (C) Quasi-static process – Infinitely slow process
 (D) Carnot engine – Reversible heat engine

Ans :

- (B) Thermodynamic equilibrium – Macroscopic variables change with time

In a state of thermodynamic equilibrium, the macroscopic variables (like pressure, volume, temperature) that characterize the system do not change with time. All other pairs are correctly matched.

49. Identify the incorrect pair regarding variables and their classification.

- (A) Pressure – Intensive variable
 (B) Volume – Extensive variable
 (C) Density – Extensive variable
 (D) Mass – Extensive variable

Ans :

- (C) Density – Extensive variable

Density (mass per unit volume) is an intensive variable because if you divide a system into two equal parts, the density of each part remains the same. Pressure is intensive, while volume and mass are extensive.

50. Arrange the following steps of a Carnot cycle in the correct sequence, starting from isothermal expansion:

1. Adiabatic compression
2. Isothermal expansion
3. Adiabatic expansion
4. Isothermal compression

- (A) 2, 3, 4, 1
 (B) 2, 4, 3, 1
 (C) 1, 2, 3, 4
 (D) 3, 2, 1, 4

Ans :

- (A) 2, 3, 4, 1

The correct sequence of operations in a Carnot cycle is: Isothermal expansion (heat absorbed

from source), followed by Adiabatic expansion (temperature drops), then Isothermal compression (heat rejected to sink), and finally Adiabatic compression (temperature rises back to initial state).

51. Consider the energy transformations when a bullet is fired from a gun and then pierces a wooden block and stops. Arrange the dominant energy forms of the bullet in chronological order.

1. Internal energy (due to increased temperature)
2. Kinetic energy (due to motion)
3. Chemical potential energy (in the gunpowder before firing, considering the entire system)

- (A) 3, 2, 1
(B) 2, 3, 1
(C) 1, 2, 3
(D) 2, 1, 3

Ans :

- (A) 3, 2, 1

Initially, the system (gunpowder + bullet) possesses chemical potential energy. Upon firing, this is converted into the kinetic energy of the bullet. When the bullet pierces the wood and stops, its kinetic energy is converted into heat, increasing the internal energy (temperature) of the bullet and wood.

52. When you rub your palms together in winter, they feel warmer. This is an example of:

- (A) Heat being converted into work.
(B) Work done being converted into heat.
(C) An increase in internal energy due to heat transfer from surroundings.
(D) A decrease in the entropy of the palms.

Ans :

- (B) Work done being converted into heat.

Rubbing your palms together involves doing mechanical work against frictional forces. This work done is converted into thermal energy, which manifests as an increase in the temperature of your palms, hence they feel warmer.

53. What is meant by 'thermal equilibrium'?

- (A) The net external force on a system is zero.
(B) The macroscopic variables of a system do not change with time.
(C) The system is completely isolated from its surroundings.
(D) The system is undergoing a rapid change in temperature.

Ans :

- (B) The macroscopic variables of a system do not change with time.

A system is in thermal equilibrium if its macroscopic variables, such as pressure, volume, and temperature, remain constant over time. This implies no net flow of heat within the system or with its surroundings if it is in contact with them.

54. If two systems A and B are separated by a diathermic wall, what happens over time?

- (A) They remain in their initial states indefinitely.
(B) They exchange work but not heat until their pressures equalize.
(C) They exchange heat until their temperatures equalize and reach thermal equilibrium.
(D) They exchange heat until their volumes equalize and reach mechanical equilibrium.

Ans :

- (C) They exchange heat until their temperatures equalize and reach thermal equilibrium.

A diathermic wall is a conducting wall that allows heat to flow between systems. If systems A and B are at different temperatures and separated by a diathermic wall, heat will flow between them until their temperatures become equal, at which point they reach thermal equilibrium.

55. What fundamental thermodynamic variable has the same value for two systems in thermal equilibrium?

- (A) Pressure
(B) Volume
(C) Internal Energy
(D) Temperature

Ans :

- (D) Temperature

The Zeroth Law of Thermodynamics implies that when two systems are in thermal equilibrium, there is a physical quantity that has the same value for both. This thermodynamic variable is defined as temperature.

56. The First Law of Thermodynamics is essentially a statement of:

- (A) The law of increasing entropy.
(B) The impossibility of perpetual motion machines of the first kind.
(C) The general law of conservation of energy.
(D) The unattainability of absolute zero temperature.

Ans :

- (C) The general law of conservation of energy.

The First Law of Thermodynamics ($\Delta Q = \Delta U + \Delta W$) is a specific application of the general law of conservation of energy to thermodynamic systems, accounting for energy transfer through heat and work, and changes in internal energy.

57. In the equation $\Delta W = P\Delta V$, what does ΔV represent?

(A) Change in pressure of the gas.
 (B) Change in temperature of the gas.
 (C) Change in volume of the gas.
 (D) Change in internal energy of the gas.

Ans :

(C) Change in volume of the gas.

In the expression for work done by a system against a constant pressure P , $\Delta W = P\Delta V$, ΔV represents the change in the volume of the gas. This formula applies when the pressure is constant during the volume change.

58. What are extensive thermodynamic variables?

(A) Variables that are independent of the size of the system.
 (B) Variables that indicate the 'size' or amount of the system.
 (C) Variables that describe the path of a thermodynamic process.
 (D) Variables that remain constant during an adiabatic process.

Ans :

(B) Variables that indicate the 'size' or amount of the system.

Extensive thermodynamic variables, such as volume, mass, and internal energy, depend on the size or amount of the system. If the system is divided, the value of an extensive variable for each part is proportionally smaller.

59. Why is it meaningless to say "a gas in a given state has a certain amount of heat"?

(A) Because heat is a form of potential energy.
 (B) Because heat is not a state variable but energy in transit.
 (C) Because heat can only be measured during a phase change.
 (D) Because heat is always equal to the internal energy.

Ans :

(B) Because heat is not a state variable but energy in transit.

Heat is a mode of energy transfer between a system and its surroundings due to a temperature difference. It is not a property that a system "possesses" in a given state. Internal energy, on the other hand, is a state variable.

60. A steam engine uses the 'heat' of steam to do useful work in moving pistons. This is an application related to which fundamental concept in thermodynamics?

(A) The Zeroth Law and temperature measurement.
 (B) The conversion of heat into work, central to the First and Second Laws.
 (C) The concept of specific heat capacity of steam.
 (D) The process of achieving thermal equilibrium.

Ans :

(B) The conversion of heat into work, central to the First and Second Laws.

A steam engine operates by converting the thermal energy (heat) of steam into mechanical work. This process of energy conversion is a core concept studied in thermodynamics, particularly in the context of the First Law (conservation of energy) and the Second Law (limitations on efficiency).

61. First law of thermodynamics can be explained on the basis of :

(a) Boyle's law (b) Maxwell's law
 (c) Charle's law (d) Joule's law

Ans :

(d) Maxwell's law

First law of thermodynamics is $dQ = dU + dW$ and this is special form of Joule's law $W = JQ$.

62. A given system undergoes a change in which work done by the system equals decrease in its internal energy. The system must have undergone an :

(a) Isothermal change (b) Isobaric change
 (c) Adiabatic change (d) Isochoric change

Ans :

(c) Adiabatic change

Given, $dW = -dU$

So $dW + dU = 0$ or $dQ = 0$

i.e. adiabatic change.

63. Heat capacity of a substance is infinite. It means :

(a) Heat is given out
 (b) Heat is taken in
 (c) no change in temperature whether heat is taken in or given out.
 (d) All of these

16. How is the heat capacity (S) of a substance defined?

- (A) $S = \Delta Q \cdot \Delta T$
 (B) $S = \frac{\Delta T}{\Delta Q}$
 (C) $S = \frac{\Delta Q}{\Delta T}$
 (D) $S = m \cdot \Delta Q \cdot \Delta T$

Ans :

(C) $S = \frac{\Delta Q}{\Delta T}$

The heat capacity (S) of a substance is defined as the amount of heat (ΔQ) supplied to the substance divided by the corresponding change in its temperature (Δt). It represents the heat required for a unit rise in temperature.

17. What is the SI unit of specific heat capacity?

- (A) $\text{J} \cdot \text{kg} \cdot \text{K}$
 (B) $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
 (C) $\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$
 (D) $\text{cal} \cdot \text{g}^{-1} \cdot ^\circ\text{C}^{-1}$

Ans :

(B) $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$

The SI unit for specific heat capacity (s) is joules per kilogram per kelvin ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$). This unit reflects that specific heat capacity is the heat required per unit mass per unit change in temperature.

18. For a mole of a solid at ordinary temperatures, the molar specific heat capacity (C) at constant pressure is approximately:

- (A) R
 (B) $2R$
 (C) $3R$
 (D) $(3/2)R$

Ans :

(C) $3R$

According to the law of equipartition of energy, for a mole of a solid, where atoms vibrate in three dimensions, the molar specific heat capacity at constant volume (and approximately at constant pressure for solids) is $3R$, where R is the universal gas constant.

19. One calorie is defined as the amount of heat required to raise the temperature of 1g of water from:

- (A) 0°C to 1°C
 (B) 14.5°C to 15.5°C
 (C) 99°C to 100°C
 (D) 4.5°C to 5.5°C

Ans :

(B) 14.5°C to 15.5°C

For a precise definition, one calorie is the amount of heat needed to raise the temperature of 1 gram of water from 14.5°C to 15.5°C . This specification is necessary because the specific heat of water varies slightly with temperature.

20. Which of the following statements regarding C_p and C_v for an ideal gas is incorrect?

- (A) C_p is the molar specific heat capacity at constant pressure.
 (B) C_v is the molar specific heat capacity at constant volume.
 (C) For an ideal gas, $C_p - C_v = R$
 (D) For an ideal gas, C_p is always less than C_v .

Ans :

(D) For an ideal gas, C_p is always less than C_v .

For an ideal gas, C_p (molar specific heat at constant pressure) is always greater than C_v (molar specific heat at constant volume). This is because at constant pressure, some heat supplied is used to do work during expansion, in addition to increasing internal energy.

21. Which statement correctly describes thermodynamic state variables?

- (A) They depend on the path taken to reach a particular state.
 (B) Their values depend only on the particular state of the system.
 (C) Heat and work are examples of state variables.
 (D) They describe non-equilibrium states of systems primarily.

Ans :

(B) Their values depend only on the particular state of the system.

Thermodynamic state variables, such as pressure, volume, temperature, and internal energy, describe the equilibrium state of a system. Their values depend solely on the current state, not on the process or path taken to reach that state.

22. Which of the following is an example of an intensive variable?

- (A) Internal energy (U)
 (B) Volume (V)
 (C) Total mass (M)
 (D) Temperature (T)

Ans :

(D) Temperature (T)

Ans :

(c) 300 K

$$\text{Since, } \eta = 1 - \frac{T_2}{T_1}$$

$$\text{So, } \frac{T_2}{T_1} = 1 - \eta = 1 - \frac{10}{100} = \frac{90}{100}$$

$$\text{So } \frac{T_1}{T_2} = \frac{100}{90}$$

$$T_1 = \frac{10}{9} \times T_2 = \frac{10}{9} \times 270 = 300 \text{ K}$$

74. For 100% efficiency of Carnot engine, temperature of the sink should be :

- (a) 0°C (b) 0 K
(c) 273 K (d) 0° F

Ans :

(b) 0 K

$$\text{Since, } \eta = 1 - \frac{T_2}{T_1}$$

$$\text{Now, } \eta = 1 \text{ (or 100\%)} \text{ if } T_2 = 0$$

75. Four engines are working between :

- (a) 100 K, 80 K (b) 40 K, 20 K
(c) 60 K, 40 K (d) 120 K, 100 K

Which one has maximum efficiency ?

Ans :

(b) 40 K, 20 K

$$\text{Since, } \eta = 1 - \frac{T_2}{T_1}$$

$$\text{In all four cases } T_1 - T_2 = 20 \text{ K}$$

So, η will be maximum if T_1 is minimum (i.e. $T_1 = 20 \text{ K}$)

76. A Carnot engine works between 600 K and 300 K. In each cycle of operation, the engine draws 1000 J of heat energy from source. The efficiency of the engine is :

- (a) 50% (b) 70%
(c) 20% (d) 80%

Which one has maximum efficiency ?

Ans :

(a) 50%

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{600 - 300}{600} = \frac{1}{2} = 50\%$$

77. A carnot engine works between constant temps. T_1 and T_2 of source and sink resp. For efficiency to be greatest :

- (a) Both T_1 and T_2 should be low.
(b) Both T_1 and T_2 should be high.

(c) T_1 should be high and T_2 should be low.(d) T_1 should be high and T_2 should be high.

Ans :

(c) T_1 should be high and T_2 should be low.

$$\text{Since, } \eta = 1 - \frac{T_2}{T_1}$$

So η will be maximum if T_1 is high and T_2 is low.

78. In a given process on an ideal gas $dW = 0$ and $dQ < 0$. Then for the gas :

- (a) The temperature will decrease.
(b) The volume will increase.
(c) The pressure will remain constant.
(d) The temperature will increase.

Ans :

(c) The pressure will remain constant.

$$\text{Since, } dQ = dU + dW$$

$$\text{If } dW = 0, dQ = dU$$

If $dQ < 0$, then $dU < 0$

i.e. internal energy decreases, hence the temperature will decrease.

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79. The efficiency of a carnot engine working between the steam point and ice point, is :

- (a) 28.8% (b) 27.8%
(c) 26.8% (d) 23.8%

Ans :

(c) 26.8%

$$\begin{aligned} \eta &= \frac{T_1 - T_2}{T_1} \\ &= \frac{(273 + 100) - (273 + 0)}{273 + 100} \\ &= \frac{100}{373} = 0.268 = 26.8\% \end{aligned}$$

80. When the door of a refrigerator in a room is kept open, the temperature of the room :

- (a) Decreases (b) Increases
(c) Remains constant (d) Cannot say

Ans :

(b) Increases

Because the work done by the compressor of the refrigerator reject heat to the air in the room and hence the temperature of the room will increase.

ONE MARK QUESTIONS

81. What do you mean by “internal energy” ?

Ans :

The energy possessed by a body within itself by virtue of molecular motion and intermolecular attraction is called internal energy of that body.

82. What is the value of specific heat of water in SI units?

Ans :

$$4.18 \times 10^3 \text{ J kg}^{-1} \text{ C}^{-1}.$$

83. Which is greater C_p or C_v ?

Ans :

C_p is more than C_v .

84. What is adiabatic gas equation ?

Ans :

$$PV^\gamma = \text{Constant}$$

85. Does the internal energy of an ideal gas change in (a) an isothermal process (b) an adiabatic process ?

Ans :

- (a) No
- (b) Yes

86. Can the temperature of an isolated system change ?

Ans :

Yes, in an adiabatic process, the temperature of an isolated system changes. It increases during adiabatic compression and decreases during adiabatic expansion.

87. Why does a gas gets heated on compression ?

Ans :

Because during compression, work is done on the system which increases the internal energy of the gas.

88. Can two isothermal curves intersect each other ? Why?

Ans :

No, If they intersect, then at the point of intersection the volume and pressure of the gas will be same at two different temperatures, which is impossible.

89. What is heat engine ?

Ans :

An heat engine is a device used for converting heat energy into mechanical work.

90. What is heat pump ?

Ans :

A heat pump is a device which uses mechanical work to remove heat.

91. How is heat engine differ from refrigerator ?

Ans :

A refrigerator is a heat engine working in reverse direction.

92. Under what conditions can the efficiency of a Carnot engine be 100% ?

Ans :

The efficiency of a Carnot engine be 100% if either temperature of sink is 0 K or temperature of source is infinite.

93. Is it possible to convert internal energy into work ?

Ans :

Yes, during the explosion of a bomb, chemical energy (internal energy) is converted into work.

94. Can whole of work be converted into heat ? Is the reverse true ?

Ans :

Yes, we can convert whole work into heat through friction. But the reverse is not true because of second law of thermodynamics, the whole of heat cannot be converted into work.

95. Is rusting of iron a reversible process ?

Ans :

No, it is irreversible process.

96. Is Carnot's engine a practical engine ?

Ans :

No, it is an ideal engine and cannot be realised in practice.

97. How is the efficiency of a Carnot engine affected by the nature of the working substance ?

Ans :

Carnot engine is independent of nature of working substance.

- 98.** Can the temperature of a gas be increased by keeping its pressure and volume constant ?

Ans :

No

- 99.** Do the internal energy of ideal gas depend upon volume ?

Ans :

No, it depends upon temperature only for an ideal gas.

- 100.** What is zeroth law of thermodynamics ?

Ans :

It states that when two bodies are in thermal equilibrium with a third body, then they are in thermal equilibrium with each other also.

- 101.** How does the internal energy of an ideal gas differ from that of real gas ?

Ans :

The internal energy of an ideal gas depends only on the kinetic energy of the particles of the gas and internal energy of real gas depends on both the kinetic as well as potential energies.

- 102.** A match stick can be lighted by rubbing it against a rough surface. Why ?

Ans :

Because when a match stick is rubbed against a rough surface, work is done against friction which appears as heat and lights up the match stick.

- 103.** An ideal gas is compressed at a constant temperature. Will its internal energy increase or decrease ?

Ans :

The internal energy of an ideal gas depends only on temperature and independent of its volume. Hence when temperature is constant, then its internal energy will not change.

- 104.** Write adiabatic gas equation.

Ans :

$PV^\gamma = \text{constant.}$

- 105.** What thermodynamical variable is defined by (a) Zeroth law (b) First law ?

Ans :

Zeroth law defines temperature and first law defines internal energy.

- 106.** If an inflated tyre bursts, the air escaping out is cooled. Why ?

Ans :

Because when tyre burst, an adiabatic expansion takes place and during expansion work is done, so internal energy decreases hence temperature falls.

- 107.** Can temperature of a gas be increased by keeping its pressure and volume constant ?

Ans :

No

- 108.** In a Carnot engine, the temperature of sink is increased. What will happen to its efficiency ?

Ans :

Since efficiency of a Carnot's engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

If T_2 is increased, $\frac{T_2}{T_1}$ will increase and η will decrease.

- 109.** An engine working under isothermal conditions can produce no useful work. Explain.

Ans :

Because, the efficiency of an engine working between the same temperatures is zero.

- 110.** What is Carnot's theorem ?

Ans :

It states that no heat engine working between two given temperatures of sources and sink can be more efficient than a perfectly reversible engine working between the same two temperatures.

- 111.** Can specific heat of a gas be infinity ?

Ans :

Yes

- 112.** What is the relation between C_p and C_v ?

Ans :

$$C_p - C_v = \frac{R}{J}$$

- 113.** Can a practical engine have 100% efficiency ?

Ans :

No

- Momentum conservation:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}.$$

- Kinetic energy conservation:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2.$$

(v_{1f} , v_{2f} are final velocities).

- 135.** In a 1D elastic collision with m_2 initially at rest, what happens if $m_1 = m_2$? Describe final velocities.

Ans :

If two masses are equal ($m_1 = m_2$) and the collision is elastic with m_2 initially at rest, the first mass (m_1) comes to rest ($v_{1f} = 0$), and the second mass (m_2) moves off with the initial velocity of the first mass ($v_{2f} = v_{1i}$).

- 136.** In a 1D elastic collision, what happens if a very light mass m_1 hits a very heavy mass m_2 ($m_2 \gg m_1$) initially at rest?

Ans :

If $m_2 \gg m_1$ and m_2 is initially at rest, the heavier mass m_2 remains almost undisturbed ($v_{2f} \approx 0$). The lighter mass m_1 reverses its velocity with nearly the same speed ($v_{1f} \approx -v_{1i}$).

- 137.** Why is a moderator like deuterium or carbon used in a nuclear reactor for slowing down neutrons?

Ans :

Moderators containing light nuclei (like deuterium or carbon) are used to slow down fast neutrons produced in nuclear fission. Neutrons lose a significant portion of their kinetic energy in elastic collisions with these light nuclei, increasing the probability of further fission with isotopes like Uranium-235.

- 138.** A neutron (m_1) collides elastically and head-on with a stationary deuterium nucleus ($m_2 = 2m_1$). What fraction of its kinetic energy is transferred to deuterium?

Ans :

The fractional kinetic energy retained by the neutron is

$$\begin{aligned} f_1 &= \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 = \left(\frac{m_1 - 2m_1}{m_1 + 2m_1} \right)^2 \\ &= \left(\frac{-1}{3} \right)^2 = \frac{1}{9}. \end{aligned}$$

The fraction transferred to deuterium is

$$f_2 = 1 - f_1 = 1 - \frac{1}{9} = \frac{8}{9} \text{ (approx 90\%).}$$

- 139.** What defines a one-dimensional or head-on collision for small spherical bodies?

Ans :

A collision is one-dimensional or head-on if the initial and final velocities of both bodies lie along the same straight line. For small spherical bodies, this occurs if the direction of motion of the incident body passes through the centre of the target body which is initially at rest.

- 140.** If a collision is two-dimensional, where do the initial and final velocities generally lie?

Ans :

In a general two-dimensional collision, the initial velocities and the final velocities of the colliding particles all lie in a single plane.

- 141.** For an elastic glancing collision between two billiard balls of equal mass, if one is initially at rest, what is the angle between their final velocities?

Ans :

When two equal masses undergo a glancing elastic collision with one of them initially at rest, they will move off at right angles ($\theta_1 + \theta_2 = 90^\circ$) to each other after the collision.

- 142.** What is scattering, and how does it generally differ from collisions like those in the game of marbles?

Ans :

Scattering is an event where particles interact via forces that can act at a distance, like a comet near the sun or an alpha particle near a nucleus. Collisions in marbles involve direct physical contact. In scattering, the interacting particles may not “touch” in the conventional sense.

- 143.** Briefly state the work-energy theorem.

Ans :

The work-energy theorem states that the change in the kinetic energy of a body is equal to the work done on it by the net force ($K_f - K_i = W_{net}$).

- 144.** List two defining characteristics of a conservative force.

Ans :

A force is conservative if:

- The work done by it on an object moving between two points is independent of the path taken.
- The work done by it on an object moving in an arbitrary closed path, returning to its initial position, is zero.

- 124.** What are the SI unit and dimensions of power? Name another common unit for power.

Ans :

The SI unit of power is the watt (W), defined as one joule per second (1 J s^{-1}). The dimensions of power are $[ML^2 T^{-3}]$. Another common unit for power is horsepower (hp), where $1 \text{ hp} = 746 \text{ W}$.

- 125.** How is instantaneous power expressed in terms of force F and velocity v ?

Ans :

Instantaneous power (P) is the scalar product of the force vector (F) and the instantaneous velocity vector (v) of the point of application of the force. It is given by the equation $P = F \cdot v$.

- 126.** A 100-watt bulb is on for 10 hours. Calculate the energy consumed in kilowatt-hours and joules.

Ans :

Energy consumed = Power $\times$ Time.

Energy = $100 \text{ W} \times 10 \text{ h} = 1000 \text{ Wh} = 1 \text{ kWh}$.

Since $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$, the energy consumed is $3.6 \times 10^6 \text{ J}$.

- 127.** An elevator motor must balance a total downward force of 22000 N while moving up at 2 ms^{-1} . Determine the minimum power delivered.

Ans :

The minimum power (P) delivered by the motor to move the elevator at a constant speed (v) against a force (F) is given by $P = F \cdot v$.

$$P = 22000 \text{ N} \times 2 \text{ ms}^{-1} = 44000 \text{ W}.$$

This is equivalent to 59 hp.

- 128.** What fundamental physical quantity is always conserved in all collisions? Explain briefly.

Ans :

The total linear momentum of an isolated system is always conserved in all types of collisions. This is because the impulsive forces exerted by the colliding bodies on each other are internal to the system and, by Newton's third law, are equal and opposite, resulting in zero net change in system momentum.

- 129.** Is the total kinetic energy of a system conserved in all collisions? Explain.

Ans :

No, the total kinetic energy is not conserved in all collisions. It is conserved only in perfectly elastic collisions. In inelastic collisions, some kinetic energy

is transformed into other forms like heat, sound, or deformation energy.

- 130.** Distinguish between an elastic collision and a completely inelastic collision.

Ans :

In an elastic collision, both the total linear momentum and the total kinetic energy of the system are conserved. In a completely inelastic collision, the colliding particles stick together and move as a single body after the collision; total linear momentum is conserved, but kinetic energy is not.

- 131.** What is an inelastic collision (intermediate case)?

Ans :

An inelastic collision is one where the deformation during collision is partly relieved, and some of the initial kinetic energy is lost, typically to heat or sound. While total linear momentum is conserved, the total kinetic energy of the system decreases.

- 132.** For a completely inelastic collision in one dimension where mass m_1 (velocity v_{1i}) hits stationary m_2 , find the final velocity v_f .

Ans :

By conservation of linear momentum for a completely inelastic collision:

$$m_1 v_{1i} = (m_1 + m_2) v_f.$$

Thus, the final velocity of the combined mass is

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}.$$

- 133.** Derive the expression for the loss in kinetic energy (ΔK) in a one-dimensional completely inelastic collision, where m_2 is initially at rest.

Ans :

The loss in kinetic energy

$$\begin{aligned} \Delta K &= K_{\text{initial}} - K_{\text{final}} \\ &= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} (m_1 + m_2) v_f^2. \end{aligned}$$

Substituting $v_f = \frac{m_1 v_{1i}}{m_1 + m_2}$, we get

$$\Delta K = \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) v_{1i}^2.$$

This value is always positive.

- 134.** Write the momentum and kinetic energy conservation equations for a one-dimensional elastic collision where m_1 (velocity v_{1i}) hits stationary m_2 .

Ans :

For a one-dimensional elastic collision:

169. Two reversible engines are working between same source and sink. Whether the two will be equally efficient or have different efficiencies, discuss.

Ans :

If two reversible engines are operating between same source and sink, their efficiency will be equal because efficiency of a reversible engine depends only on the temperature of the source and the sink and is entirely independent of the nature and properties of the working substances.

170. Is the efficiency of a heat engine more in hilly areas than in the plains ?

Ans :

Yes. Since the efficiency of a heat engine working between two temperatures T_1 and T_2 ($T_1 > T_2$) is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

In hilly areas, the temperature of surroundings is lower than that in the plains, so $\frac{T_2}{T_1}$ is less in hilly areas than in plains.

Hence, efficiency η is more in hilly areas than in plains.

171. What is thermal equilibrium in thermodynamics? How do adiabatic and diathermic walls affect it between two systems?

Ans :

Thermal equilibrium in thermodynamics refers to a state where the macroscopic variables of a system, such as pressure, volume, and temperature, do not change with time. The nature of the wall separating two systems significantly affects their ability to reach thermal equilibrium:

- An **adiabatic wall** is an insulating boundary that prevents any flow of heat (energy) between the two systems it separates. Consequently, if systems A and B are separated by an adiabatic wall, they will maintain their individual states irrespective of each other, and any initial difference in their macroscopic variables will persist.
- A **diathermic wall**, on the other hand, is a conducting boundary that allows heat to flow freely between the systems. When systems A and B are separated by a diathermic wall, energy will transfer from the hotter system to the colder one until their temperatures become equal, and they reach a state of mutual thermal equilibrium.

172. State and explain the Zeroth Law of Thermodynamics. What fundamental concept does this law introduce?

Ans :

The Zeroth Law of Thermodynamics provides a fundamental basis for the concept of temperature. It states: "Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other."

- **Explanation:** If system A is in thermal equilibrium with system C (meaning they are at the same temperature, $T_A = T_C$), and system B is also separately in thermal equilibrium with system C ($T_B = T_C$), then it necessarily follows that system A and system B will be in thermal equilibrium with each other ($T_A = T_B$).
- **Concept Introduced:** This law formally introduces **temperature** as a fundamental thermodynamic property. It implies the existence of a scalar quantity, temperature, which is the same for all systems that are in thermal equilibrium.

173. Define the internal energy of a system. On what factors does the internal energy of an ideal gas depend?

Ans :

The internal energy (U) of a thermodynamic system is defined as the sum of the kinetic energies and potential energies of its molecular constituents, as viewed from a frame of reference in which the center of mass of the system is at rest.

- It encompasses energies due to the random translational, rotational, and vibrational motions of molecules, as well as potential energies from intermolecular forces.
- For an **ideal gas**, intermolecular forces are considered negligible. Consequently, its internal energy consists only of the sum of the kinetic energies of its molecules and depends solely on its absolute temperature, not on its pressure or volume. Internal energy is a state variable.

174. Distinguish between heat and internal energy in thermodynamics. Why is this distinction critically important?

Ans :

Heat and internal energy are fundamentally different concepts in thermodynamics, though both relate to energy.

- **Internal Energy (U):** This is the energy possessed by a system due to the kinetic and potential energies of its constituent molecules. It is a thermodynamic state variable, meaning

- 145.** How is a potential energy function $V(x)$ related to a one-dimensional conservative force $F(x)$?

Ans :

For a one-dimensional conservative force $F(x)$, the potential energy function $V(x)$ is defined such that the force is the negative derivative of the potential energy with respect to position:

$$F(x) = -\frac{dV(x)}{dx}.$$

- 146.** What is the principle of conservation of mechanical energy?

Ans :

The principle of conservation of mechanical energy states that if only conservative forces are doing work on a body, its total mechanical energy (the sum of its kinetic and potential energies) remains constant.

- 147.** Write the expression for the gravitational potential energy $V(x)$ of a particle of mass m at a height x above Earth's surface, ignoring g 's variation.

Ans :

The gravitational potential energy $V(x)$ of a particle of mass m at a height x above the Earth's surface is given by $V(x) = mgx$, where g is the acceleration due to gravity.

- 148.** What is the elastic potential energy $V(x)$ stored in a spring of force constant k when it is extended by an amount x ?

Ans :

The elastic potential energy $V(x)$ stored in a spring with force constant k and extension x is given by $V(x) = \frac{1}{2}kx^2$.

- 149.** How is the scalar product of two vectors A and B interpreted geometrically?

Ans :

The scalar product $A \cdot B$ can be interpreted as the product of the magnitude of vector A and the component of vector B along A , or alternatively, the product of the magnitude of B and the component of A along B .

- 150.** What are the values of $\hat{i} \cdot \hat{i}$ and $\hat{i} \cdot \hat{k}$ for orthogonal unit vectors?

Ans :

For orthogonal unit vectors, $\hat{i} \cdot \hat{i} = 1$ (scalar product of a unit vector with itself) and $\hat{i} \cdot \hat{k} = 0$ (scalar product of two perpendicular unit vectors).

- 151.** What are the dimensions of kinetic energy, and what is its SI unit?

Ans :

The dimensions of kinetic energy are $[ML^2 T^{-2}]$. Its SI unit is the joule (J).

- 152.** What are the dimensions of power, and what is its SI unit?

Ans :

The dimensions of power are $[ML^2 T^{-3}]$. Its SI unit is the watt (W).

- 153.** How is power P related to work W and time t ? How is it related to force F and velocity v ?

Ans :

Power P is the rate at which work is done, $P = dW/dt$. It can also be expressed as the scalar product of force F and velocity v , $P = F \cdot v$.

- 154.** Why is the phrase 'calculate the work done' often incomplete? What specific information is needed?

Ans :

The phrase is incomplete because work is always done by a specific force (or a set of forces) on a particular body over a certain displacement. This context – the force, the body, and the displacement – must be clearly stated or implied.

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- 155.** Can work done be negative? Provide an example of negative work.

Ans :

Yes, work done is a scalar quantity that can be positive or negative. For instance, the work done by the force of friction on a moving body is negative because the frictional force opposes the displacement.

- 156.** If the sum of mutual forces between two bodies is zero by Newton's Third Law, does the sum of work done by these forces necessarily cancel?

Ans :

No, the sum of the work done by the two mutual forces ($W_{12} + W_{21}$) need not always be zero, even though the forces are equal and opposite ($F_{12} + F_{21} = 0$). This is because the displacements of the points of application of these forces might differ.

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THREE MARKS QUESTIONS

157. Give two examples of (i) isothermal changes (ii) adiabatic changes.

Ans :

Isothermal change :

- (i) A gas contained in a metallic cylinder is compressed or allowed to expand slowly.
- (ii) Boyle's law.

Adiabatic Change :

- (i) A gas contained in a non-conducting cylinder is compressed or allowed to expand quickly.
- (ii) Air escaping from the cycle tube on removing the valve undergoes adiabatic expansion.

158. How is the heat absorbed used by a system in a cyclic process ?

Ans :

From first law of thermodynamics, we have

$$dQ = dU + dW$$

In cyclic process, after undergoing a series of changes, the system returns to its initial state.

Hence, $dU = 0$

So $dQ = dW$

So in a cyclic process, heat absorbed by the system is equal to net work done by the system.

159. Explain why it is impossible to design a heat engine with 100% efficiency ?

Ans :

The efficiency of a heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Now, $\eta = 1$ (or 100%) if either

$$T_2 = 0 \text{ K}$$

$$T_1 = \infty$$

which is not possible, so it is impossible to design a heat engine with 100% efficiency.

160. Explain the reason for cooling when a gas suffers adiabatic expansion.

Ans :

In an adiabatic change no heat is supplied to the gas. Thus, according to the first law of thermodynamics, we have

$$dU + dW = 0$$

As during the expansion some external work is to be done by the gas given by $\int PdV$, it is clear that there will be a corresponding decrease in internal energy and hence in temperature of the gas. Thus, cooling takes place.

161. Explain what do you understand by the term "Internal energy"

Ans :

Internal energy of a thermodynamic system. The energy possessed by a body within, itself by virtue of its molecular motion and inter-molecular attraction is called the internal energy of that body. In other words, energy possessed by a body due to its thermal state is called its internal energy. Internal energy of a body is generally denoted by the symbol U . The internal energy of a body is partly kinetic and partly potential. If temperature of a body increases, molecular motion inside the body and hence the internal energy of the body increases. Therefore, in general, the internal energy of a thermodynamic system is measured by the amount of heat energy present in it. Thus, if temperature of a body of mass m and specific heat C increases from θ_1 to θ_2 , increase in internal energy of the body will be $U_2 - U_1$ given by the relation

$$U_2 - U_1 = mC(\theta_2 - \theta_1)$$

In case of gases the same result may be written as

$$U_2 - U_1 = nC_v(\theta_2 - \theta_1)$$

where quantity of the gas is n gram-mole and C_v is the specific heat of the gas under constant volume condition.

162. What is a heat engine ? What are its types ?

Ans :

A heat engine is a device which converts heat energy into mechanical energy. Steam engine, petrol engine, diesel engine are some of the examples of heat engine.

Types of heat engines. There are two types of heat engines in actual use :

1. External combustion engine. In this type of heat engine, the fuel is burnt in a separate unit (called the boiler) which is situated outside the engine. This heat is then led to the engine by means of a suitable working substance. Steam engine is a familiar example of an external combustion engine. ($\eta = 8\%$ to 18%).
2. Internal combustion engine. In this engine, fuel is burnt in the cylinder of the engine itself

and the products of combustion are themselves the working substances. The petrol engine and the diesel engine are familiar examples of the internal combustion engine.

Efficiency for petrol engine is 25% and that for diesel engine is 40%.

163. What are cyclic and non-cyclic processes ?

Ans :

A cyclic process is the process in which, after a series of changes of pressure and volume, the working substance finally returns to its initial state i.e., the final temperature, pressure and volume after the changes are again same as in the beginning. Indicator diagram of a cyclic process is always a closed path.

A non-cyclic process is a process in which final condition of the working substance is different from that at the start. Indicator diagram for a non-cyclic change is a line either straight or curved.

164. Is the heat supplied to a system always equal to the increase in its internal energy ?

Ans :

According to first law of thermodynamics,

$$dQ = dU + PdV$$

If heat is supplied in such a manner that volume does not change ($dV = 0$ i.e. isochoric change), then whole of the heat energy supplied to the system will increase internal energy only. Normally, a part is used up by the system in performing the external work.

165. How do you justify that first law of thermodynamics is the law of conservation of energy ?

Ans :

If dQ be the heat supplied to a system, dU be the increase in internal energy of the system and dW be the amount of external work done, then according to first law of thermodynamics,

$$dQ = dU + dW$$

Heat is a form of energy so it means that when one form of energy disappears another equivalent form of energy appears, but the sum total of energy remains the same. It is nothing but the law of conservation of energy.

166. What are the limitations of first law of thermodynamics ?

Ans :

Limitations of the first law of thermodynamics. The first law of thermodynamics establishes the equivalence between heat and mechanical energy and that one can be converted into the other. But it fails to specify :

- (i) Whether heat can be converted wholly or only partially into mechanical work i.e. the extent to which the change is permissible is not defined by first law.
- (ii) The direction in which the change occurs. For example, when we rub our hands against each other, some heat is produced. But if the same heat is given to the hands, they do not start rubbing themselves. Thus, mechanical energy is converted into heat, but heat in this case is not converted into work. First law of thermodynamics says nothing about the direction of the change.

The above two points are said to form the insufficiency or the limitations of first law of thermodynamics.

167. Temperature of the source and sink of a Carnot's engine are increased by the same amount, how will the efficiency be affected ?

Ans :

$$\eta = \frac{T_1 - T_2}{T_1}$$

Let each temperature be increased by x , then new efficiency is

$$\eta' = \frac{(T_1 + x) - (T_2 + x)}{(T_1 + x)} = \frac{T_1 - T_2}{T_1 + x}$$

$$\eta' = \frac{T_1 - T_2}{T_1 + x} < 1$$

$$\eta' < \eta$$

i.e., the efficiency will decrease when temperature of source and sink are increased by the same amount.

168. What is meant by a reversible engine ? Explain as to why the efficiency of a reversible engine is maximum.

Ans :

A reversible engine is that in which the process can be reversed at any stage of its operation and there is no wastage of energy by any means. As in a reversible engine there is no wastage of energy due to conduction, convection or radiation or work done against friction, etc., hence it is self-evident that efficiency of a reversible engine will be maximum.

- 114.** Is the internal energy of an ideal gas a function of its pressure ?

Ans :

No

- 115.** Is Carnot's cycle a cyclic process ?

Ans :

Yes

TWO MARKS QUESTIONS

- 116.** What is the first law of thermodynamics ?

Ans :

When some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system dQ is equal to the sum of increase in internal energy of the system dU and the external work done dW .

$$dQ = dU + dW$$

- 117.** State the first law of thermodynamics.

Ans :

It states that the energy can neither be created nor it can be destroyed but can change itself from one form to another (i.e. law of conservation of energy)

or

It state that, if an amount of heat dQ is added in a system, a part of it is used to increase the internal energy of the system (dU) and the remaining part is used to do external work (dW).

i.e. $dQ = dU + dW$

- 118.** Why cooling takes place when a gas suffer adiabatic expansion ?

Ans :

In adiabatic process

$$dQ = 0$$

From first law of thermodynamics

$$0 = dU + dW$$

On expansion, the gas does external work for its expansion on the cost of internal energy due to which temperature of the gas falls and hence cooling takes place.

- 119.** Explain why the internal energy of a compressed gas is less than that of a rarefied gas at the same temperature.

Ans :

On compressing a real gas, its molecules come closer and mutual interaction become stronger. So there is an increase in potential energy. Since potential energy is negative, so the total energy of the gas decreases but the kinetic energy remains the same so the temperature of the gas remains constant.

- 120.** Explain why friction is considered a non-conservative force.

Ans :

Friction is a non-conservative force because the work done by friction depends on the path taken between two points. Furthermore, work done by friction over a closed path is not zero, and this energy is typically dissipated as heat, meaning mechanical energy is not conserved when friction is present.

- 121.** What is the significance of the arbitrary nature of the zero of potential energy?

Ans :

The zero of potential energy can be chosen at any convenient reference point. This choice is arbitrary because physical phenomena depend on differences in potential energy, not its absolute value. For example, gravitational potential energy is often zero at ground level, while for a spring, it's at the equilibrium position.

- 122.** How is the principle of conservation of mechanical energy modified when non-conservative forces are present?

Ans :

When non-conservative forces like friction act, the total mechanical energy ($E = K + V$) of a system is not conserved. The change in total mechanical energy ($\Delta E = E_f - E_i$) is equal to the work done (W_{nc}) by these non-conservative forces on the system.

- 123.** Define average power and instantaneous power. How are they related?

Ans :

Average power (P_{av}) is the total work (W) done divided by the total time (t) taken: $P_{av} = W/t$. Instantaneous power (P) is the limiting value of the average power as the time interval approaches zero, or the rate at which work is done: $P = dW/dt$.

its value depends only on the current state of the system. A system has a certain amount of internal energy.

- **Heat (Q):** This is the energy transferred between a system and its surroundings solely due to a temperature difference between them. Heat is energy in transit; it is not stored in a system and is not a state variable.

The distinction is critical: internal energy describes the energy within a system, while heat describes one method of energy transfer.

- 175.** State the First Law of Thermodynamics using its standard equation and explain each term involved in the equation.

Ans :

The First Law of Thermodynamics is essentially the principle of conservation of energy applied to thermodynamic systems, accounting for energy transfer through heat and work. It is mathematically stated as:

$$\Delta Q = \Delta U + \Delta W$$

The terms are:

- ΔQ : This represents the amount of **heat supplied to the system** by its surroundings. If heat is released by the system, ΔQ is negative.
- ΔU : This represents the **change in the internal energy** of the system. It depends only on the initial and final states of the system.
- ΔW : This represents the **work done by the system** on its surroundings. If work is done on the system, ΔW is negative.

- 176.** A gas contained in a cylinder with a movable piston expands against a constant external pressure P . Derive an expression for the work done by the gas.

Ans :

When a gas expands against a constant external pressure P , it performs work on its surroundings. Consider the gas in a cylinder fitted with a movable piston of cross-sectional area A .

- The constant force (F) exerted by the gas on the piston is given by $F = P \times A$.
- If the piston moves outwards by a small distance Δx , the change in volume of the gas is $\Delta V = A \times \Delta x$.
- The work done by the gas (ΔW) during this expansion is the product of the force and displacement: $\Delta W = F \Delta x$.
- Substituting $F = PA$, we get

$$\Delta W = (PA) \Delta x = P(A \Delta x).$$
- Therefore, the work done by the gas is

$$\Delta W = P \Delta V.$$

- 177.** Explain the concepts of specific heat capacity (s) and molar specific heat capacity (C) of a substance. State their respective SI units.

Ans :

Specific heat capacity and molar specific heat capacity quantify how a substance's temperature changes upon heat exchange.

Specific Heat Capacity (s): This is defined as the amount of heat (ΔQ) required to change the temperature (ΔT) of a unit mass (m) of a substance by one unit. It is given by

$$s = \left(\frac{1}{m}\right) \frac{\Delta Q}{\Delta T}.$$

The SI unit for specific heat capacity is joules per kilogram per kelvin ($\text{J kg}^{-1} \text{K}^{-1}$).

Molar Specific Heat Capacity (C): This is defined as the amount of heat (ΔQ) required to change the temperature (ΔT) of one mole (μ) of a substance by one unit. It is given by

$$C = \left(\frac{1}{\mu}\right) \frac{\Delta Q}{\Delta T}.$$

The SI unit for molar specific heat capacity is joules per mole per kelvin ($\text{J mol}^{-1} \text{K}^{-1}$). Both depend on the substance and its temperature.

- 178.** Using the law of equipartition of energy, determine the molar specific heat capacity at constant volume (C_v) for an ideal monatomic gas.

Ans :

The law of equipartition of energy states that in thermal equilibrium, the total energy of a system is equally distributed among all its possible energy modes, with each mode having an average energy of $\frac{1}{2} k_B T$, where k_B is Boltzmann's constant and T is the absolute temperature.

- A molecule of a monatomic ideal gas has 3 translational degrees of freedom (along x, y, and z axes).
- Therefore, the average energy per molecule is

$$3 \times \left(\frac{1}{2} k_B T\right) = \frac{3}{2} k_B T.$$

- For one mole of the gas (containing N_A molecules, where N_A is Avogadro's number), the total internal energy is

$$U = N_A \times \left(\frac{3}{2} k_B T\right) = \frac{3}{2} RT,$$

since $R = N_A k_B$.

- The molar specific heat at constant volume is

$$C_v = \left(\frac{dU}{dT}\right)_v = \frac{d}{dT} \left(\frac{3}{2} RT\right) = \frac{3}{2} R.$$

- 179.** For an ideal gas, derive the relationship $C_p - C_v = R$, where C_p and C_v are the molar specific heat capacities at constant pressure and volume, respectively.

Ans :

The relationship $C_p - C_v = R$ can be derived using the First Law of Thermodynamics ($\Delta Q = \Delta U + P\Delta V$ for one mole) and the definitions of molar specific heats.

- At constant volume ($\Delta V = 0$), $\Delta Q_v = \Delta U$. Thus,

$$C_v = \left(\frac{\Delta Q}{\Delta T}\right)_v = \left(\frac{\Delta U}{\Delta T}\right)_v.$$

Since for an ideal gas, U depends only on T , we can write $C_v = \frac{dU}{dT}$.

- At constant pressure, $\Delta Q_p = \Delta U + P\Delta V$. Thus,

$$C_p = \left(\frac{\Delta Q}{\Delta T}\right)_p = \left(\frac{\Delta U}{\Delta T}\right)_p + P\left(\frac{\Delta V}{\Delta T}\right)_p.$$

- Using $C_v = \frac{dU}{dT}$ and for one mole of an ideal gas $PV = RT$, so

$$P\left(\frac{\Delta V}{\Delta T}\right)_p = R.$$

- Substituting these into the equation for C_p gives $C_p = C_v + R$, or $C_p - C_v = R$.

- 180.** Distinguish between extensive and intensive thermodynamic state variables, providing two examples for each category.

Ans :

Thermodynamic state variables describe the equilibrium state of a system and are classified as either extensive or intensive.

- Extensive variables** are those whose values depend on the size or amount of the system. If a system is divided into two equal parts, the value of an extensive variable for each part is halved. Examples include:

- Volume (V)
- Internal energy (U)
- Total mass (M)

- Intensive variables** are those whose values are independent of the size or amount of the system. If a system is divided, the value of an intensive variable remains unchanged for each part. Examples include:

- Pressure (P)
- Temperature (T)
- Density (ρ)

- 181.** What is meant by a quasi-static process in thermodynamics? Explain why it is considered an idealised concept.

Ans :

A quasi-static process is a thermodynamic process that is conducted so slowly that the system remains infinitesimally close to an equilibrium state at all times, both internally and with its surroundings.

- This means that at every stage of the process, the difference in pressure between the system and the external pressure, and the difference in temperature between the system and its surroundings, are infinitesimally small.

It is an idealised concept because real-world processes occur at finite rates and typically involve the system passing through non-equilibrium states where variables like pressure or temperature might not be uniform throughout.

- 182.** State the Clausius statement of the Second Law of Thermodynamics. What does this imply about the operation of a refrigerator?

Ans :

The Clausius statement of the Second Law of Thermodynamics declares: "No process is possible whose sole result is the transfer of heat from a colder object to a hotter object."

- This means that heat cannot spontaneously flow from a body at a lower temperature to a body at a higher temperature without external work being performed on the system.
- For a refrigerator, which transfers heat from a cold space to a warmer environment, this implies that it requires an input of work to operate. The coefficient of performance of a refrigerator can therefore never be infinite.

- 183.** What defines a reversible process in thermodynamics? Mention the two main conditions that must be met for a process to be ideally reversible.

Ans :

A thermodynamic process is defined as reversible if the process can be reversed in such a way that both the system and its surroundings are returned to their original states, with no other net changes occurring anywhere else in the universe.

The two main conditions for a process to be ideally reversible are:

- Quasi-static nature:** The process must occur infinitely slowly (quasi-statically), ensuring the system is always in an equilibrium state with its surroundings at every stage.
- Absence of dissipative effects:** There should be no dissipative forces such as friction, viscosity, or electrical resistance, which would convert useful energy into internal energy and prevent perfect reversal.

- 184.** Identify and briefly explain the two primary causes that lead to the irreversibility of most processes observed in nature.

Ans :

Most processes observed in nature are irreversible. This irreversibility primarily stems from two main causes:

- **Involvement of Non-Equilibrium States:** Many spontaneous processes, like the free expansion of a gas into a vacuum or an explosive chemical reaction, involve the system passing through non-equilibrium states. In such states, thermodynamic variables like pressure and temperature are not well-defined throughout the system, making a precise reversal impossible.
- **Presence of Dissipative Effects:** Most real-world processes involve dissipative effects such as friction (e.g., a block sliding to rest), viscosity (e.g., stirring a liquid), or electrical resistance. These effects convert organized energy (like mechanical work) into disorganized internal energy (heat), a transformation that cannot be completely undone to restore the original state without other changes.

- 185.** What is a Carnot engine? Describe the four reversible processes that constitute a Carnot cycle using an ideal gas as the working substance.

Ans :

A Carnot engine is a theoretical, reversible heat engine that operates in a cycle between two heat reservoirs, one at a high temperature T_1 (source) and the other at a low temperature T_2 (sink). The Carnot cycle, when an ideal gas is the working substance, consists of the following four reversible steps:

- **Isothermal Expansion (at T_1):** The gas expands isothermally, absorbing heat Q_1 from the hot reservoir.
- **Adiabatic Expansion:** The gas expands adiabatically, doing work and cooling down from T_1 to T_2 .
- **Isothermal Compression (at T_2):** The gas is compressed isothermally, releasing heat Q_2 to the cold reservoir.
- **Adiabatic Compression:** The gas is compressed adiabatically, with work done on it, heating it back up from T_2 to T_1 .

- 186.** Provide the mathematical expression for the efficiency (η) of a Carnot engine, first in terms of the heat absorbed (Q_1) and heat released (Q_2), and then in terms of the reservoir temperatures (T_1, T_2).

Ans :

The efficiency (η) of a Carnot engine, like any heat engine, is the ratio of the net work done (W) by the engine to the heat absorbed (Q_1) from the hot reservoir.

- Since $W = Q_1 - Q_2$ (where Q_2 is the heat released to the cold reservoir), the efficiency in terms of heat quantities is:

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

- For a Carnot engine (which is reversible), this efficiency can also be expressed solely in terms of the absolute temperatures of the hot reservoir (T_1) and the cold reservoir (T_2):

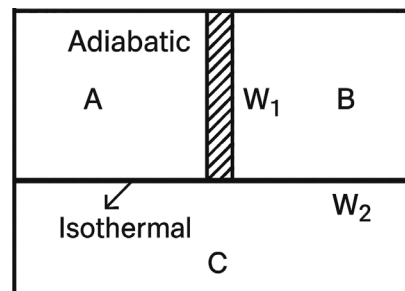
$$\eta = 1 - \frac{T_2}{T_1}$$

FIVE MARKS QUESTIONS

- 187.** What do you mean by “thermal equilibrium” and “Zeroth law of thermodynamics” ? Hence give the concept of temperature.

Ans :

A thermodynamic system is said to be in thermal equilibrium if all parts of it are at the same temperature and this temperature is same as that of surroundings too.



Zeroth Law of Thermodynamics states that if two systems are in thermal equilibrium with a third system, then they must be in thermal equilibrium with each other as well i.e. their temperatures must be the same.

Figure shows bodies A and B separated by wall W_1 which does not allow the heat flow between A and B. Both are placed on another body C. The wall W_2 separating them, allows heat exchange between A and C or B and C.

Let A and B be separately in thermal equilibrium with C. Now remove the wall W_1 and replace it by one like W_2 , which allows heat exchange. It will be

201. What are the main essentials of Carnot's reversible heat engine ? Explain its working and efficiency of the engine.

Ans :

Carnot's device is an ideal cycle of operation of heat engine.

Construction : It consists of

- Source of heat of infinite thermal capacity maintained at $T_1 K$.
- Working substance is a perfect gas contained in a cylinder with perfectly insulated walls and fitted with a frictionless piston.
- Sink of heat is having infinite thermal capacity maintained at $T_2 K$ so that $T_2 < T_1$.

Working : Consider a mole of perfect gas.

- Isothermal Expansion :** Let P_1, V_1, T_1 be the initial pressure, volume and temperature represented respectively by A. Place the cylinder on source and move out the piston slowly so that it absorbs heat Q_1 isothermally.

Work done by the gas in expanding isothermally A ($P_1 V_1$) to B ($P_2 V_2$).

$$Q_1 = W_1 = \int_{V_1}^{V_2} P dV$$

$$= R T_1 \log_e \frac{V_2}{V_1} = \text{area } ADda$$

Adiabatic Expansion : Cylinder is now removed and placed on insulating pad and gas is allowed to expand from B (P_2, V_2) to C (P_3, V_3). In this adiabatic process, temperature falls from T_1 to T_2 . Work done adiabatically,

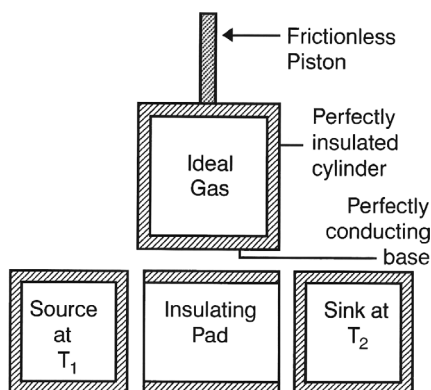


Figure: (a)

Isothermal compression : Cylinder is now placed on sink. The piston is moved slowly and compressed to pressure P_4 and volume V_4 keeping the temperature constant. Work done

on the gas is equivalent to heat given to sink i.e. Q_2 .

$$Q_2 = W_3 = - \int_{V_3}^{V_4} P dV$$

$$= \int_{V_3}^{V_4} P dV = R T_2 \log_e \frac{V_3}{V_4}$$

$$= \text{area } CDdc$$

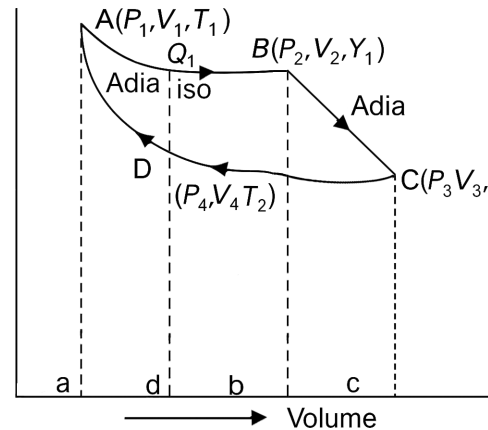


Figure: (b)

Adiabatic compression : Cylinder is now placed on insulating pad. Piston is moved downward that gas is again compressed from V_4 to V_1 adiabatically.

$$W_4 = - \int_{V_4}^{V_1} P dV = \frac{R(T_2 - T_1)}{\gamma - 1}$$

$$= \text{area } DdaA$$

Net work done, $W = W_1 + W_2 - (W_3 + W_4)$

But, $W_2 = W_4$

Hence, $W = W_1 - W_3 = Q_1 - Q_2$

$$= \text{area } ABCDA$$

Now Efficiency, $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{R T_2 \log_e \frac{V_3}{V_4}}{R T_1 \log_e \frac{V_2}{V_1}}$$

A and D lie on the same adiabatic curve

$$\text{Hence, } T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

Again B and C lie on the same adiabatic curve

$$\text{Hence, } T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

From (2) and (3), we have

$$\left(\frac{V_1}{V_2} \right)^{\gamma-1} = \left(\frac{V_4}{V_3} \right)^{\gamma-1}$$

so as to ensure free exchange of heat with the surroundings in order to maintain temperature of the system constant.

Equation of Isothermal process : According to gas equation for one gram mole of a gas,

$$PV = RT$$

But in an isothermal change, temperature T is constant, hence we say that

$$PV = \text{constant} \quad \dots(1)$$

If the pressure changes from P_1 and P_2 , then volume changes from V_1 to V_2 s.t.

$$P_1 V_1 = P_2 V_2 \quad \dots(2)$$

We may say $PV = \text{constant}$.

According to first law of thermodynamics,

$$dQ = dU + dW$$

In isothermal change

$$dU = 0,$$

Since, $dQ = dW$

i.e. during isothermal process heat added (or subtracted)

= work done by (or on) the gas.

- 191.** What do you mean by reversible and irreversible processes ? Give examples.

or

Distinguish between reversible and irreversible process.

Ans :

The process which can be retraced to the initial state by reversing the controlling factors which occurred in the direct process is called a reversible process.

For a process to be reversible, following conditions must be satisfied :

- Process should be extremely slow, so that the system is in mechanical, thermal and chemical equilibrium.
- The system should be free from friction, viscosity, inelasticity and electrical resistance.

Since these conditions cannot be obtained practically so no process can be called completely reversible process.

Following are the examples of some processes which can be called reversible to some extent but are not practical processes.

- Working substance in a Carnot's engine.
- An ideal gas allowed to expand slowly.

- All isothermal and adiabatic changes are reversible provided there is no friction and no loss of heat by conduction, convection and radiation.

Irreversible process

The process which cannot be retraced to the initial state by reversing the controlling factors which occurred in the direct process is called irreversible process.

Every process in nature is an irreversible process.

Examples :

- Work done against friction.
- Rusting of iron.
- Diffusion of gases.
- Falling of water from certain height.

- 192.** Give two statements of second law of thermodynamics.

or

Give the statements of second law of thermodynamics. Show that they are equivalent.

Ans :

- Clausius statement of second law of thermodynamics is as follows :
Heat cannot flow from a colder body to a hotter body without some work being done by an external agency.
- Kelvin and Planck states second law of thermodynamics as follows :
It is impossible to get a continuous supply of work by cooling a body to a temperature lower than that of the coldest of its surroundings.
Equivalence of two statements
These two statements are equivalent to each other and if one statement fails, other automatically fails. Let us see how.

Consider a heat engine which performs work when cooled below the temperature of its surroundings. This is against Kelvin statement. Let the work done by this engine be used to derive dynamo which produces current and this current is used to heat the surroundings by using a heater coil. Hence we get a machine in which heat flows from lower to higher temperature without any external agent. This is against Clausius statement. Hence both the statements are equivalent.

- 193.** Show that adiabatics are steeper than isotherms.

Ans :

Slopes of isothermal and adiabatic curves. The slope of an isothermal or adiabatic curve is given by $\frac{dP}{dV}$.

For an isothermal change,

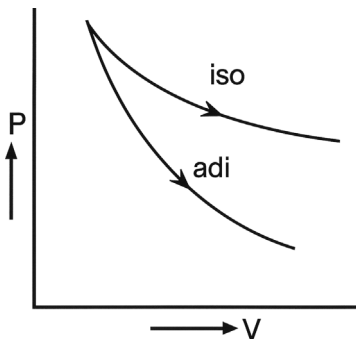
$$PV = K$$

Differentiating both sides, we get

$$PdV + VdP = 0$$

$$V.dP = -P.dV$$

$$\left(\frac{dP}{dV}\right)_{iso} = -\frac{P}{V} \quad \dots(1)$$



For an adiabatic change,

$$PV^\gamma = K$$

Differentiating both sides, we get

$$P\gamma V^{\gamma-1}.dV + V^\gamma.dP = 0$$

$$V^\gamma dP = -P\gamma V^{\gamma-1} dV$$

$$\left(\frac{dP}{dV}\right)_{adi} = -\frac{P\gamma V^{\gamma-1}}{V^\gamma} = -\frac{\gamma P}{V} \quad \dots(2)$$

$$\left(\frac{dP}{dV}\right)_{adi} = \gamma\left(-\frac{P}{V}\right)$$

Using equation (1), we get

$$\left(\frac{dP}{dV}\right)_{adi} = \gamma\left(-\frac{dP}{dV}\right)_{iso}$$

Thus, we find that slope of an adiabatic is γ times the slope of an isothermal. As $\gamma > 1$, therefore, adiabatic at any point is steeper than the isothermal at that point.

- 194.** State first law of thermodynamics. Apply this to derive an expression for the change in internal energy during boiling process.

Ans :

According to the first law of thermodynamics, when some heat dQ is supplied to a system capable of doing external work, then quantity of heat dQ absorbed by the system is equal to the sum of the increase in the internal energy of the system dU and external work done dW i.e.,

$$dQ = dU + dW$$

First law applied to boiling process : We can obtain the increase in internal energy of a liquid in the process of boiling. A liquid changes into vapour state at constant pressure and constant temperature.

If m is the mass and the volume of liquid at its boiling point T and P be the constant pressure. Let V_2 be the volume of vapours and L be the latent heat of vaporisation of liquid, then

$$dQ = mL \text{ and } dW = P(V_2 - V_1)$$

$$\text{Hence, } mL = dU + P(V_2 - V_1)$$

$$dU = mL - P(V_2 - V_1)$$

- 195.** Give two statements of second law of thermodynamics.

Ans :

- (i) Clausius statement of second law of thermodynamics is as follows :

Heat cannot flow from a colder body to a hotter body without some work being done by an external agency.

For example, in a refrigerator, heat flows from cold bodies to hotter surroundings. But this flow of heat is not by itself. Electrical energy is spent on an outside agency (the compressor) to drive this heat out.

- (ii) Kelvin and Planck states second law of thermodynamics as follows :

It is impossible to get a continuous supply of work by cooling a body to a temperature lower than that of the coldest of its surroundings.

For example, a heat engine absorbs heat from a hot body, converts it partially into work and rejects the rest of heat to a body at lower temperature. When temperatures of both bodies become equal, flow of heat stops and the engine stops working. Hence, it is impossible to obtain any work from the engine, if the source cools down to a temperature lower than that of the coldest body in the surroundings.

Obviously, both the forms of the second law of thermodynamics are identical. Though there is no direct proof of the validity of this law, yet it is a universally true experimental law.

- 196.** Derive the relation between C_p and C_v .

Ans :

Consider one mole of an ideal gas. Heat the gas at constant volume so that its temperature increases by dT .

$$\text{Heat supplied} = 1 \times C_v \times dT = C_v dT$$

As the gas is heated at constant volume, it will not perform external work and in accordance with the first law of thermodynamics, the heat supplied will be just equal to increase in internal energy of the gas.

Hence,
$$dU = C_v dT \quad \dots(1)$$

Now, let the gas be heated at constant pressure to again increase its temperature by dT . If dQ is the amount of heat supplied, then

$$\begin{aligned} dQ &= 1 \times C_p dT \\ dQ &= C_p \times dT \quad \dots(2) \end{aligned}$$

The heat supplied at constant pressure increases the temperature of the gas by dT i.e. increases its internal energy by dU and as well as enables the gas to perform work dW . If dV is increase in volume, then work performed by the gas

$$dW = PdV \quad \dots(3)$$

According to the first law of thermodynamics

$$dQ = dU + dW$$

Using equations (1), (2) and (3), we get

$$C_p dT = C_v dT + PdV \quad \dots(4)$$

For one mole of a perfect gas,

$$PV = RT$$

Now, heat is supplied at constant pressure.

Therefore, treating P as constant, differentiate both sides w.r.t. T and we obtain,

$$\begin{aligned} \frac{d}{dT}(PV) &= \frac{d}{dT}(RT) \\ P \frac{dV}{dT} &= R \quad \dots(5) \end{aligned}$$

$$PdV = RdT$$

From equations (4) and (5), we have

$$C_p dT = C_v dT + R dT \quad \dots(6)$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

The above equation is a relation between two principal specific heats of the gas, when C_p , C_v and R are measured in the units of either heat or of work. In case, C_p and C_v are measured in heat units and R in units of work, then

$$C_p = C_v + \frac{R}{J}$$

For one gram of a gas, the relation will be

$$C_p = C_v + \frac{r}{J}$$

where C_p and C_v are specific heats and r is gas constant for 1 gram of the gas.

- 197.** (a) Define thermal equilibrium, thermodynamical state, thermodynamical variables, equation of state.
(b) Define isothermal process, adiabatic process, isochoric (isovolumetric) process, isobaric process and isotherms.

Ans :

- (a) (i) Thermal equilibrium : A system is said to be in a state of thermal equilibrium if pressure, temperature and volume remain unchanged in that state.
(ii) Thermodynamical state : A thermodynamical state is the state which is described by pressure, volume and temperature of a system.
(iii) Thermodynamical variables : Parameters like pressure, temperature and volume which determine the state of body are called thermodynamical variables.
(iv) Equation of state : The relation between P , V and T describing the state of thermodynamical system is called equation of state.

e.g. $pV = \mu RT$ is an equation of state.

- (b) Isothermal Process : A thermodynamical process in which pressure and volume change keeping the temperature constant is called isothermal process.

In this process, $PV = \text{constant}$.

Adiabatic Process. A thermodynamical process in which internal energy of the system remains the same is called adiabatic process.

In this process, $PV^\gamma = \text{constant}$

Isochoric or Isovolumetric process. A thermodynamical process in which pressure and temperature change, keeping volume constant is called Isochoric process.

Isobaric process : A thermodynamical process in which volume and temperature change, keeping pressure constant is called Isobaric process.

Isotherms : These are graphs between pressure and volume at constant temperature.

- 198.** Define adiabatic process. Give examples. Derive adiabatic equations.

Ans :

A change in pressure and volume of a gas in which temperature also changes is called an adiabatic change. In such a change, no heat is allowed to enter into or escape from the gas (i.e., there is no exchange of heat between the gas and its surroundings).

A few examples of nearly adiabatic changes are :

- (i) the propagation of sound waves through air
- (ii) the sudden bursting of a cycle tube
- (iii) the compression of the mixture of petrol vapour and air that takes place during compression stroke of a petrol engine (temperature rises here).

For an adiabatic change, the exchange of heat between the gas and the surroundings must be prevented.

Hence the two essential conditions for a perfect adiabatic change are :

- (i) the walls of the container must be perfectly non-conducting in order to prevent any exchange of heat between the gas and its surroundings, and
- (ii) the process of compression or expansion should be rapid so that there is no time for the exchange of heat.

Both these conditions are difficult to be realised strictly in practice. Therefore, perfect adiabatic changes are practically impossible.

Adiabatic operation is that in which no heat is allowed to enter or escape from the gas i.e. there is no exchange of heat between the gas and the surroundings.

Derivation of Equation of Adiabatic changes. According to first law of thermodynamics, we have

$$dQ = dU + dW \quad \dots(1)$$

During adiabatic operation, $dQ = 0$;

$$dU = 1 C_v dT \text{ and } dW = PdV$$

[considering mole of gas]

Substituting in equation (1), we have

$$0 = C_v dT + PdV \quad \dots(2)$$

Now for a mole of gas, $PV = RT$

Differentiating, we have

$$PdV + VdP = RdT$$

$$dT = \frac{PdV + VdP}{R}$$

Substituting in equation (2), we have

$$C_v \left(\frac{PdV + VdP}{R} \right) + PdV = 0$$

$$\frac{C_v PdV + C_v VdP + RPdV}{R} = 0$$

$$PdV(C_v + R) + C_v VdP = 0$$

[Since, $C_p - C_v = R$ Hence, $C_v + R = C_p$]

Dividing throughout by $C_v PV$, we have

$$\frac{PdV C_p}{C_v PV} + \frac{C_v VdP}{C_v PV} = 0 \left[\text{But } \frac{C_p}{C_v} = \gamma \right]$$

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

$$\gamma \int \frac{1}{V} dV + \int \frac{1}{P} dP = 0$$

$$\gamma \log_e V + \log_e P = \text{constant}$$

$$\log_e V^\gamma + \log_e P = \text{constant}$$

$$\log_e PV^\gamma = \text{constant}$$

$$PV^\gamma = \text{constant} \quad \dots(3)$$

This is an adiabatic equation between P and V .

Adiabatic Equation between V and T . We know that

$$PV = RT$$

$$P = \frac{RT}{V}$$

Substituting in equation (3), we have

$$\frac{RT}{V} V^\gamma = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

Adiabatic Equation between P and T .

We know that,

$$PV = RT$$

$$V = \frac{RT}{P}$$

Substitution in (3), we have

$$P \left(\frac{RT}{P} \right)^\gamma = \text{constant}$$

$$P^{1-\gamma} T^\gamma = \text{constant}$$

- 199.** Calculate the external work done when an ideal gas is expanded (i) isothermally (ii) adiabatically.

Ans :

Consider one mole of an ideal gas in a cylinder having tight and frictionless piston. Let the piston be moved very slowly through a distance dx such that small change in volume dV takes place.

Hence, Small work done,

$$W = \int PdV \quad \dots(1)$$

- (i) Work done in isothermal expansion

If the expansion of a gas is isothermal, temperature of the gas remains constant.

Hence,

we have

$$PV = RT$$

$$P = \frac{RT}{V} \quad \dots(2)$$

Putting equation (2) in (1), we get

$$W = \int \frac{RT}{V} dV$$

If the volume changes from V_1 to V_2 then

$$\begin{aligned} W &= RT \int_{V_1}^{V_2} \frac{1}{V} dV \\ &= RT [\log_e V]_{V_1}^{V_2} \\ &= RT [\log_e V_2 - \log_e V_1] \\ W &= RT \log_e \frac{V_2}{V_1} \\ &= 2.3026 RT \log_{10} \frac{V_2}{V_1} \end{aligned}$$

(ii) Work done in adiabatic expansion

If the change is adiabatic, then we have

$$\begin{aligned} PV^\gamma &= \text{constant (K)} \\ P &= V^{-\gamma} K \quad \dots(3) \end{aligned}$$

Putting equation (3) in (1), we get

$$W = \int V^{-\gamma} K dV$$

If the volume changes from V_1 to V_2 then

$$\begin{aligned} W &= K \int_{V_1}^{V_2} V^{-\gamma} dV = K \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2} \\ &= \frac{1}{-\gamma+1} [KV_2^{-\gamma+1} - KV_1^{-\gamma+1}] \end{aligned}$$

But,

$$\begin{aligned} K &= P_1 V_1^\gamma = P_2 V_2^\gamma \\ W &= \frac{1}{-\gamma+1} [P_2 V_2^\gamma \cdot V_2^{-\gamma+1} - P_1 V_1^\gamma V_1^{-\gamma+1}] \\ &= \frac{1}{-\gamma+1} [P_2 V_2 - P_1 V_1] \\ &= \frac{1}{-\gamma+1} [RT_2 - RT_1] \\ &= \frac{R}{-\gamma+1} [T_2 - T_1] \\ W &= \frac{1}{\gamma-1} [T_1 - T_2] \end{aligned}$$

Here T_1 and T_2 are initial and final temperatures of the gas.

200. What is a heat engine ? How is its efficiency defined?

Ans :

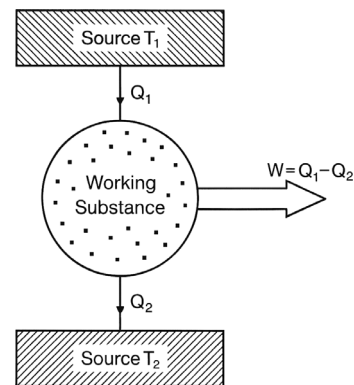
It is a device, which converts continuously heat energy into mechanical energy in a cyclic process. In general, a heat engine must have three main parts :

- (i) Heat source : Source of heat is a hot body maintained at a high temperature (say T_1), from which engine may draw heat.
- (ii) Heat sink : Sink of heat is a cold body maintained at a low temperature (say T_2 , where $T_2 < T_1$), to which heat rejected by the engine may be given.

(iii) Working substance : Working substance is any material (may be gas, liquid or solid), which performs mechanical work when heat energy is supplied to it.

(iv) Mechanical parts : Mechanical parts are needed for containing the working substance and for transfer of mechanical energy to the appropriate parts of the machine.

Working. A heat engine has been symbolically shown in figure. The working substance absorbs a definite amount of heat from the source and transforms a part of this heat energy into mechanical work. The remaining amount of heat is rejected to the sink. The same cycle is repeated again and again and we continuously obtain mechanical work from the engine.



Efficiency (η)

Let Q_1 = Heat absorbed by the working substance from the source.

Q_2 = Heat rejected by the working substance to the sink.

So, net mechanical work done by the heat engine

$$W = Q_1 - Q_2$$

Efficiency of a heat engine is defined as the ratio of the net work done by the engine in one cycle (i.e. output) to the amount of heat absorbed by the working substance from the source (i.e. input)

$$\text{i.e., } \eta = \frac{\text{Work done by engine (output)}}{\text{Heat absorbed from the source (input)}}$$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

As heat is always rejected to the sink

$$\text{So } Q_2 \neq 0$$

$$\text{Hence, } \eta < 1 \quad (\text{practically})$$

Theoretically, the maximum value of $\eta = 1$ when $Q_2 = 0$.

found that no exchange of heat will occur between A and B. That is, A and B will also be in thermal equilibrium with each other.

Concept of Temperature : According to Zeroth law of thermodynamics all thermal systems in equilibrium possess a common property and this property is temperature. Thus, the temperature of a system is that property of the system, which determines whether or not the system is in thermal equilibrium with the neighbouring systems.

It is now very much clear that two systems in thermal equilibrium must be at the same temperature. On the other hand, if two systems are not in thermal equilibrium, they must be at different temperatures.

188. State and explain first law of thermodynamics.

Ans :

First law of thermodynamics : According to this law, when heat energy is converted into any other form of energy, the energy appearing in one form is exactly equal to the amount of energy disappearing in the other form, whatever may be the mode of conversion.

Let us consider a system whose total internal energy is U_i to start with. Let it undergo a thermodynamical operation in which it absorbs Q units of heat energy and does W units of useful work. If U_f is the total energy of the system at the end, then

Net amount of energy absorbed by the system

$$= Q - W$$

Increase in internal energy of the system

$$= U_f - U_i$$

According to First law of thermodynamics,

$$U_f - U_i = Q - W \quad \dots(1)$$

Note that in eqn. (1), Q and W both are in the same units of energy. The eqn. is true whatever path of changes the system may follow between the given initial and final state.

If dU is a small increase in internal energy of the system, dQ is a small amount of heat energy absorbed and dW is a small amount of useful work done by the system, then eqn. (1) can be written as :

$$dU = dQ - dW$$

$$\text{or } dU + dW = dQ$$

This is general mathematical form of the 1st law of thermodynamics.

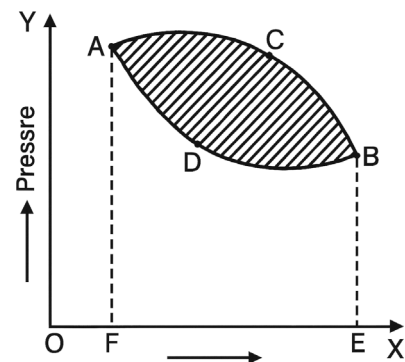
189. From the indicator diagram find an expression for work done in a cyclic process.

Ans :

Work done in a cyclic process. Let us consider a cyclic change being represented by an indicator diagram ABCDA. The cycle is completed in two parts ACB and then BDA.

According to the property of indicator diagram, work done by the system W_1 in going from A to B along the curve ACB is given by

$$W_1 = \text{area ACBEF}$$



and the work done on the system W_2 during the change from B to A along the curve BDA is given by

$$W_2 = \text{area BDAFE}$$

The net amount of work done in one complete cyclic change along ACBDA path is

$$\begin{aligned} W &= W_1 - W_2 \\ &= \text{area ACBEF} - \text{area BDAFE} \\ &= \text{area ACBDA} \end{aligned}$$

Here work W_2 is taken negative, because this work is being done upon the system.

Thus, in a cyclic process the work done is numerically equal to the area enclosed by the indicator diagram of cyclic process shown in figure.

190. What is an isothermal change ? Apply first law of thermodynamics to an isothermal change.

Ans :

Isothermal change : A change is said to be isothermal change if a system suffers physical change under the condition that its temperature remains constant during the change.

The essential condition for an isothermal change is that the system must be contained in a conducting chamber and the change should be slow enough

$$\frac{V_1}{V_2} = \frac{V_4}{V_3} \text{ or } \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Substituting above value in (1), we have

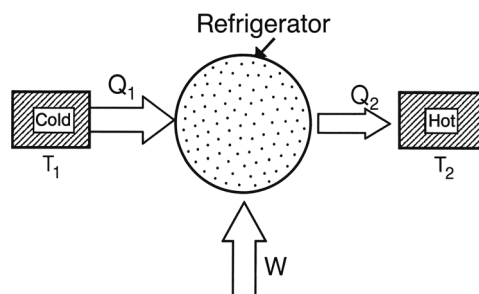
$$\eta = 1 - \frac{T_2}{T_1}$$

202. (a) What is a refrigerator ?

Calculate its coefficient of performance : (b) Why is defrosting necessary ?

Ans :

- (a) Refrigerator : A refrigerator may be considered as a heat engine operated in reverse. For its operation, it takes heat Q_2 from a cold reservoir, has mechanical work W done upon it by the compressor (external agency) and rejects a larger amount of heat Q_1 to the hot reservoir. In a household refrigerator, the food and the ice cubes comprise the cold reservoir, an electric motor acts as an external agency to supply the input work and the air in the kitchen acts as a hot reservoir.



Working : The working substance in a tube is suddenly expanded by a process called Throttling. The gas escapes suddenly from out of this tube into a region of low pressure. This cools the fluid and the fluid converts into a mixture of liquid and vapour. This cold fluid circulates in the region which is to be cooled and absorbs heat. The expanded fluid, now completely vapour, is compressed by a compressor. Work is done on the vapour and the pressure increases. The heat of this vapour is given out to the surrounding through a set of tubes called condenser.

The coefficient of performance (generally denoted by β) of a refrigerator is determined by the ratio of the heat Q_2 extracted from the cold reservoir to the mechanical work W done upon it. Thus

$$\beta = \frac{\text{heat extracted}}{\text{input work}} = \frac{Q_2}{W} \quad \dots(1)$$

If Q_1 is the heat rejected to the hot reservoir, we have

$$Q_1 = Q_2 + W$$

$$W = Q_1 - Q_2$$

$$\text{Hence,} \quad \beta = \frac{Q_2}{Q_1 - Q_2} \quad \dots(2)$$

The smaller the amount of mechanical work (Q) spent in removing the heat Q_2 from the hot reservoir, the greater is the coefficient of performance. Dividing numerator and denominator of Equation (2) by Q_2 , we have

$$\begin{aligned} \beta &= \frac{1}{\frac{Q_1}{Q_2} - 1} \\ &= \frac{1}{\frac{T_1}{T_2} - 1} \quad \left(\text{Since } \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right) \end{aligned}$$

- (b) When refrigerator has been worked for sufficient time and a good amount of ice has been formed in freezer, it is desirable to defrost the refrigerator. Formation of too much ice decreases T_2 and increases $(T_1 - T_2)$. This results in decrease in value of β (the coefficient of performance) and correspondingly decreases the efficiency of refrigerator.

Defrosting results in increase of T_2 and decrease in value of $(T_1 - T_2)$. These two factors increase the value of coefficient of performance and hence its efficiency.

NUMERICAL QUESTIONS

- 203.** Calculate the difference between the two specific heats of Helium gas per gram. Given that molecular weight of He = 4, $J = 4.18$ joule/cal and $R = 8.31 \text{ J mole}^{-1} \text{ K}^{-1}$.

Ans :

$$\text{Given,} \quad C_p - C_v = ?$$

$$M = 4$$

$$R = 8.31 \text{ joule/mole/K}$$

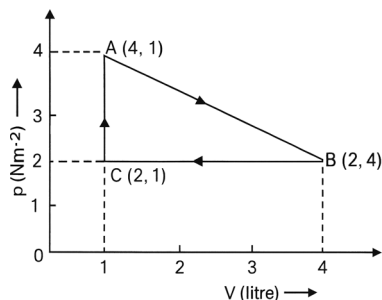
$$J = 4.18 \text{ joule/cal}$$

$$\text{As} \quad C_p - C_v = \frac{r}{J} = \frac{R}{M \cdot J}$$

$$\text{Hence,} \quad C_p - C_v = \frac{8.31}{4 \times 4.18} = 0.497 \text{ cal kg}^{-1} \text{ K}^{-1}$$

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211. The p-V diagram for a cyclic process is a triangle ABC drawn in order (fig. 12.23) alongside. The coordinates A, B, C are (4, 1) (2, 4) and (2, 1). The coordinates are in the order (p-V). pressure is in Nm^{-2} and volume is in litre. Calculate the work done during the process from A to B; B to C and C to A. Also calculate the work done in the complete cycle.



Ans :

The cyclic process is a triangle ABC represented by A (4, 1); B (2, 4); C (2, 1) on the p-V diagram.

Work done during the process from A to B :

W_{AB} is positive, as the system expands along AB

Hence, W_{AB} = area of trapezium ABDEA

$$= \frac{AE + BD}{2} \times DE$$

Now, $DE = 4 - 1 = 3 \text{ litres} = 3 \times 10^{-3} \text{ m}^3$

$$AE = 4 \text{ Nm}^{-2}$$

and $BD = 2 \text{ Nm}^{-2}$

$$W_{AB} = \frac{4+2}{2} \times 3 \times 10^{-3} = 9 \times 10^{-3} \text{ J}$$

Work done during the process from B to C :

Now, W_{BC} is negative, as the system is compressed along BC.

$$W_{BC} = + \text{ area of rectangle BCED}$$

$$= - DE \times CE$$

Now, $DE = 3 \times 10^{-3} \text{ m}^3$

and $CE = 2 \text{ Nm}^{-2}$

$$W_{BC} = 3 \times 10^{-3} \times 2 = 6 \times 10^{-3} \text{ J}$$

Work done during the process from C to A :

Since in going from C to A, there is no change in volume;

$$W_{CA} = 0$$

Work done in the complete cycle,

$$= 9 \times 10^{-3} + (-6 \times 10^{-3}) + 0$$

$$= 3 \times 10^{-3} \text{ J}$$

CASE BASED QUESTIONS

212. A cylinder with a movable piston contains 3 moles of hydrogen gas at standard temperature and pressure (STP). The walls of the cylinder are made of a heat insulator (adiabatic walls), and the piston is also insulated. The gas is then compressed to half its original volume.

Questions:

1. What is an adiabatic process?
2. In an adiabatic compression, what happens to the internal energy of the gas? What about its temperature?
3. For an adiabatic process of an ideal gas, what is the relationship between pressure (P) and volume (V)?
4. By what factor does the pressure of the hydrogen gas increase if it is compressed adiabatically to half its original volume? (Assume hydrogen behaves as an ideal diatomic gas with $\gamma = C_p/C_v = 7/5$).

Ans :

1. In an adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero ($\Delta Q = 0$).
2. In an adiabatic compression, work is done on the gas (ΔW is negative). Since $\Delta Q = 0$, by the First Law ($\Delta U = \Delta Q - \Delta W = -\Delta W$), the internal energy (ΔU) of the gas increases. For an ideal gas, an increase in internal energy leads to an increase in its temperature.
3. For an adiabatic process of an ideal gas,

$$PV^\gamma = \text{constant}$$

4. Given $P_1 V_1^\gamma = P_2 V_2^\gamma$. If $V_2 = V_1/2$, then

$$P_1 V_1^\gamma = P_2 (V_1/2)^\gamma$$

$$P_2 = P_1 (V_1/V_2)^\gamma$$

$$= P_1 (V_1/(V_1/2))^\gamma = P_1 (2)^\gamma$$

For a diatomic gas like hydrogen, $\gamma = 7/5 = 1.4$
So, $P_2 = P_1 (2)^{1.4}$. $2^{1.4} \approx 2.64$.

The pressure increases by a factor of approximately 2.64.

213. A certain amount of hot water at temperature T_1 is mixed with an equal amount of cold water at temperature T_2 in an insulated container. After some time, the mixture settles to a final common temperature T_f . Assume no heat is lost to the surroundings or the container.

Questions:

1. What is the general principle of conservation of energy as stated by the First Law of Thermodynamics?

$$V_1 = V$$

$$P_2 = P_1$$

$$V_2 = 2V$$

Hence, $PV = P_1 2V$

$$\frac{P}{P_1} = 2 \quad \dots(1)$$

(ii) For adiabatic expansion, $P_1 V_1^\gamma = P_2 V_2^\gamma$

Here $P_1 = P$

$$V_2 = V$$

$$P_2 = P_a$$

$$V_2 = 2V$$

Hence, $PV^\gamma = P_a (2V)^\gamma$

$$\frac{P}{P_a} = 2^\gamma \quad \dots(2)$$

Dividing equation (1) by (2), we get

$$\begin{aligned} \frac{P_a}{P} \frac{2}{2^\gamma} &= 2^{1-\gamma} \\ &= 2^{1-1.67} = 2^{-0.67} = 0.6285 \end{aligned}$$

- 208.** A litre of hydrogen at 27°C and 10^5 Nm^{-2} pressure expands isothermally until its volume is doubled and then adiabatically until its volume is redoubled. Find the final pressure of the gas. ($\gamma = 1.4$)

Ans :

Initial pressure, $P_1 = 10^5 \text{ Nm}^{-2}$

Initial temperature, $T_1 = 300 \text{ K}$

Let initial volume, $V_1 = V$

Hence, Volume after isothermal expansion, $V_2 = 2V$

Volume after adiabatic expansion, $V_3 = 4V$

Final pressure after adiabatic expansion, $P_3 = ?$

For isothermal change, $P_1 V_1 = P_2 V_2$

Hence, $\frac{P_2}{P_1} = \frac{V_1}{V_2} = \frac{1}{2}$

$$P_2 = 0.5 \times 10^5 \text{ Nm}^{-2}$$

For adiabatic change, $PV^\gamma = \text{Constant}$

$$P_2 V_2^\gamma = P_3 V_3^\gamma$$

$$\frac{P_3}{P_2} = \left(\frac{V_2}{V_3}\right)^\gamma = \left(\frac{1}{2}\right)^\gamma = (0.5)^\gamma$$

$$P_3 = P_2 \times (0.5)^\gamma$$

$$\begin{aligned} P_3 &= 0.5 \times 10^5 \times (0.5)^{1.4} \\ &= 1.893 \times 10^4 \text{ Nm}^{-2} \end{aligned}$$

- 209.** A volume of gas at atmospheric pressure is compressed adiabatically to half its original volume. Calculate the resulting pressure ($\gamma = 1.4$).

Ans :

Given that, $P_1 = 76 \text{ cm of Hg}$

$$\frac{V_2}{V_1} = \frac{1}{2}$$

Now we know that for adiabatic change,

$$PV^\gamma = \text{constant}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma$$

Putting the values, $\frac{76}{P_2} = \left(\frac{1}{2}\right)^{1.4}$

$$\begin{aligned} P_2 &= 76 \times (2)^{1.4} \\ &= 200.5 \text{ cm of Hg} \end{aligned}$$

- 210.** The efficiency of an ideal engine is $\frac{1}{8}$. By lowering the temperature of the sink by 100 K , it increases to $\frac{1}{4}$. Find initial and final temperature of sink.

Ans :

Given, $\eta_1 = \frac{1}{8}$ in 1st case

$$\eta_2 = \frac{1}{4} \text{ in 2nd case}$$

Let $T_1 = \text{Temperature of source}$

$$T_2 = \text{Initial temperature of sink}$$

and $T_2 = \text{Final temperature of sink}$
 $= T_2 - 100$

As $\eta = 1 - \frac{T_2}{T_1}$, we get

Case I : $\frac{1}{8} = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = 1 - \frac{1}{8} = \frac{7}{8} \quad \dots(1)$

Case II : $\frac{1}{4} = 1 - \frac{T_2 - 100}{T_1}$

$$\frac{T_2 - 100}{T_1} = 1 - \frac{1}{4} = \frac{3}{4} \quad \dots(2)$$

$$\frac{7}{8} - \frac{100}{T_1} = \frac{3}{4}$$

$$\frac{100}{T_1} = \frac{7}{8} - \frac{3}{4} = \frac{1}{8}$$

Hence, $T_1 = 800 \text{ K} \quad \dots(3)$

From (1) and (2), we get,

$$T_2 = \frac{7}{8} \times 800 = 700 \text{ K}$$

$$T_2 = 600 \text{ K}$$

- 204.** The specific heats of a gas at a constant pressure and at constant volume are 0.20 and 0.15 cal/g/K respectively. Calculate its density at N.T.P. Given $J = 4.2 \times 10^7$ ergs/cal.

Ans :

$$\begin{aligned}\text{Given, } C_p &= 0.20 \text{ cal/g/}^\circ\text{K} \\ C_v &= 0.15 \text{ cal/g/}^\circ\text{K} \\ J &= 4.2 \times 10^7 \text{ ergs/cal}\end{aligned}$$

$$\text{Density, } \rho = ?$$

$$\text{As } C_p - C_v = \frac{r}{J}$$

$$\text{Hence, } r = (C_p - C_v)J$$

$$\text{Now, } r = \frac{PV}{T} = 0.05 \times 4.2 \times 10^7$$

This is the volume occupied by 1 g of gas.

$$\text{As density} = \frac{\text{mass}}{\text{volume}}$$

$$\begin{aligned}\text{Hence, } \rho &= \frac{m}{V} = \frac{76 \times 13.6 \times 980}{21 \times 10^5 \times 273} \\ &= 1.766 \times 10^{-3} \text{ g/c.c.} \\ \rho &= 1.766 \text{ g/litre}\end{aligned}$$

- 205.** Find the quantity of heat required to increase the volume of 1 litre (dm^3) of air at N.T.P. to double its volume at constant pressure. Density of air at N.T.P. = $0.001293 \text{ g cm}^{-3}$, Specific heat of air at constant volume = $0.168 \text{ cal g}^{-1}\text{C}^{-1}$.

Ans :

As the pressure remains constant

$$\text{Hence, } \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 , T_1 are the initial and V_2 , T_2 the final volume and absolute temperatures respectively.

$$\text{Now, } V_1 = 1,000 \text{ cm}^3, T_1 = 273 \text{ K}$$

$$V_2 = 2,000 \text{ cm}^3, T_2 = ?$$

$$\begin{aligned}T_2 &= \frac{V_2}{V_1} \cdot T_1 = \frac{2,000}{1,000} \times 273 \\ &= 546 \text{ K}\end{aligned}$$

Hence, Rise of temperature,

$$\theta = 546 - 273 = 273 \text{ K}$$

Mass of the gas, $m = \text{Volume} \times \text{density}$

$$= 1,000 \times 0.001293 = 1.293 \text{ g}$$

Specific heat at constant pressure.

$$C_p = C_v + \frac{r}{J}$$

where r is the principle gas constant of air

$$\text{Now, } r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 1}{273 \times 0.001293}$$

$$\begin{aligned}\frac{r}{J} &= \frac{76 \times 13.6 \times 980}{273 \times 0.001293 \times 4.2 \times 10^7} \\ &= 0.0631\end{aligned}$$

$$\begin{aligned}\text{Hence, } C_p &= 0.168 + 0.0631 \\ &= 0.2311\end{aligned}$$

Hence amount of heat required,

$$\begin{aligned}Q &= m \cdot c_p \cdot \theta \\ &= 1.293 \times 0.2311 \times 273 \\ &= 81.58 \text{ cal}\end{aligned}$$

- 206.** A gas occupying 1 dm^3 at 80 cm pressure expands adiabatically to $1,190 \text{ cm}^3$. If the pressure falls to 60 cm in the process, determine the value of γ .

Ans :

$$\text{Given, } V_1 = 1 \text{ dm}^3 = 1,000 \text{ cm}^3;$$

$$V_2 = 1,190 \text{ cm}^3$$

$$P_1 = 80 \text{ cm}, P_2 = 60 \text{ cm}$$

Now for an adiabatic expansion,

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$$

$$\left(\frac{1,000}{1,190}\right)^\gamma = \frac{60}{80}$$

$$\left(\frac{100}{119}\right)^\gamma = \frac{3}{4}$$

$$\gamma(\log 100 - \log 119) = \log 3 - \log 4$$

$$\gamma(21.0000 - 2.0755) = 0.4771 - 0.6021$$

$$-\gamma(0.0755) = -(0.1250)$$

$$\gamma = \frac{0.1250}{0.0755} = 1.65$$

- 207.** An ideal gas with pressure P , volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i . If the same gas is expanded adiabatically to a volume $2V$ and a final pressure P_a , find the value of ratio $\frac{P_a}{P_i}$. The ratio of the specific heats for the gas is 1.67.

Ans :

$$\text{Given, } \gamma = 1.67$$

$$(i) \text{ For isothermal expansion, } P_1 V_1 = P_2 V_2$$

$$\text{Here, } P_1 = P$$

- When the hot and cold water are mixed in an insulated container, how is heat transferred between them? What is the condition for thermal equilibrium?
- If the system (hot water + cold water) is isolated, what can be said about the net heat exchanged with the surroundings (ΔQ_{system})?
- If two bodies at different temperatures T_1 and T_2 are brought into thermal contact, they do not necessarily settle to the mean temperature $(T_1 + T_2)/2$. Why might this be the case, considering specific heat capacities? (Refer to exercise 11.3a which points to this).

Ans :

- The First Law of Thermodynamics states that energy (ΔQ) supplied to the system goes in partly to increase the internal energy of the system (ΔU) and the rest in work on the environment (ΔW); $\Delta Q = \Delta U + \Delta W$.
- Heat is transferred from the hot water to the cold water due to the temperature difference. Thermal equilibrium is reached when both parts of the system attain the same temperature.
- If the system is isolated, no heat is exchanged with the surroundings, so ΔQ_{system} (net heat from surroundings) is zero.
- The final temperature would be the mean $(T_1 + T_2)/2$ only if the masses and specific heat capacities of the two bodies of water were identical (which they are, in this specific case of "equal amount" if it means equal mass). However, if the substances were different, or if "amount" referred to volume and densities were different, or if heat capacities of containers were involved, the final temperature would generally not be the simple mean. The specific heat capacity determines how much heat is needed to change the temperature of a substance, so different substances will have different temperature changes for the same amount of heat exchanged.

- 214.** A heat engine operates by taking heat Q_1 from a high-temperature source (hot reservoir at T_1), converting part of it into work W , and releasing the remaining heat Q_2 to a lower-temperature sink (cold reservoir at T_2). The efficiency (η) of such an engine is a critical parameter.

Questions:

- How is the efficiency (η) of a heat engine defined?
- What does the Kelvin-Planck statement of the Second Law of Thermodynamics imply about the possibility of an engine with 100% efficiency?

- If an engine absorbs 1000 J of heat from the source and performs 300 J of work in one cycle, how much heat is released to the sink? What is its efficiency?
- Can a Carnot engine, operating between the same two temperatures T_1 and T_2 , have an efficiency greater than any other engine operating between these temperatures? What does its efficiency depend on?

Ans :

- The efficiency (η) of a heat engine is defined as the ratio of the net work done W per cycle by the engine to the total amount of heat absorbed Q_1 per cycle by the working substance from the source: $\eta = W/Q_1$.
- The Kelvin-Planck statement says: "No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work." This implies that an engine cannot have 100% efficiency (Q_2 cannot be zero if Q_1 is not zero and W is done).
- From the First Law for a cycle, $W = Q_1 - Q_2$
So,

$$Q_2 = Q_1 - W \\ = 1000\text{J} - 300\text{J} = 700\text{J}.$$

Efficiency

$$\eta = W/Q_1 \\ = 300\text{J}/1000\text{J} = 0.3 \text{ or } 30\%.$$

- No engine operating between two given temperatures can have efficiency more than that of the Carnot engine. The efficiency of the Carnot engine ($\eta = 1 - T_2/T_1$) depends only on the temperatures of the hot and cold reservoirs.

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- 215.** One mole of an ideal gas is allowed to expand isothermally at a temperature T from an initial volume V_1 to a final volume V_2 . During this process, the gas remains in thermal contact with a large heat reservoir at temperature T .

Questions:

- What defines an isothermal process?
- What happens to the internal energy (U) of an ideal gas during an isothermal process?
- What is the work done (W) by the gas during this isothermal expansion in terms of μ , R , T , V_1 , and V_2 ?
- According to the First Law of Thermodynamics, how much heat (Q) is absorbed or released by the gas during this isothermal expansion?

- 221.** The Zeroth Law of Thermodynamics can be stated as: “Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.” This law implies the existence of a physical quantity called temperature, which is equal for two systems in thermal equilibrium.

Questions:

1. What does it mean for two systems to be in thermal equilibrium?
2. How does the Zeroth Law lead to the concept of temperature?
3. System A is in thermal equilibrium with system C. System B is also in thermal equilibrium with system C. According to the Zeroth Law, what can be said about systems A and B?
4. If the Zeroth Law were not true, could we reliably use a thermometer (system C) to compare the “hotness” or “coldness” of two different objects (systems A and B)? Explain.

Ans :

1. When two systems are in thermal equilibrium, their macroscopic variables do not change with time, and there is no net flow of heat between them. Their temperatures are equal.
2. The Zeroth Law suggests that when two systems are in thermal equilibrium, there must be a physical quantity that has the same value for both. This thermodynamic variable is called temperature.
3. According to the Zeroth Law, systems A and B are in thermal equilibrium with each other.
4. If the Zeroth Law were not true, then even if A is in equilibrium with C (thermometer) and B is in equilibrium with C, A and B might not be in equilibrium with each other. This would mean that the thermometer reading (temperature) would not be a consistent or reliable indicator of whether A and B would exchange heat if brought into contact. The very basis of temperature measurement and comparison would be flawed.

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- Does the scientist's claim violate the Second Law of Thermodynamics? Explain.
- How do practical refrigerators work to transfer heat from a colder to a hotter body, in light of the Second Law?

Ans :

- The Clausius statement of the Second Law is: "No process is possible whose sole result is the transfer of heat from a colder object to a hotter object."
- The claim does not necessarily violate the First Law. Energy could still be conserved if the amount of heat removed from the cold body equals the amount of heat transferred to the hot body. The First Law does not forbid this direction of spontaneous flow.
- Yes, the scientist's claim violates the Clausius statement of the Second Law of Thermodynamics because it describes a process whose sole result is the transfer of heat from a colder to a hotter object without any external work input.
- Practical refrigerators use an external energy source (e.g., electricity to run a compressor) to perform work. This work allows them to extract heat from a cold reservoir and transfer it to a hotter reservoir, thus not violating the Second Law, as the process is not solely heat transfer from cold to hot.

219. An engineer is designing a heat engine that will operate between a high-temperature reservoir at $T_1 = 800\text{K}$ and a low-temperature reservoir (sink) at $T_2 = 400\text{K}$. She hopes to achieve the maximum possible efficiency for these operating temperatures. Questions:

- What type of engine achieves the maximum possible efficiency operating between two given temperatures?
- What are the four processes that constitute a Carnot cycle?
- Calculate the maximum possible efficiency (Carnot efficiency) for an engine operating between $T_1 = 800\text{K}$ and $T_2 = 400\text{K}$.
- If a real engine operating between these temperatures has an efficiency of 30%, is this possible? Why will its efficiency always be less than the Carnot efficiency?

Ans :

- A heat engine based on idealised reversible processes, known as a Carnot engine, achieves the highest efficiency possible.
- The Carnot cycle consists of:
Step 1: Isothermal expansion at T_1 (heat absorbed).

Step 2: Adiabatic expansion (temperature drops from T_1 to T_2).

Step 3: Isothermal compression at T_2 (heat released).

Step 4: Adiabatic compression (temperature rises from T_2 to T_1).

- The Carnot efficiency is

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400\text{K}}{800\text{K}} \\ = 1 - 0.5 = 0.5 \text{ or } 50\%.$$

- Yes, an efficiency of 30% is possible because it is less than the Carnot efficiency of 50%. The efficiency of real engines will always be less than the Carnot efficiency because real processes involve irreversibilities like friction and heat loss to the surroundings, which are not present in the ideal Carnot cycle.

220. A sample of an ideal monatomic gas is contained in a rigid vessel. Heat is slowly added to the gas. Its pressure, volume, and temperature are macroscopic variables that describe its state. The internal energy (U) of this gas is also a key thermodynamic variable. Questions:

- How is the internal energy (U) of a system defined at a microscopic level?
- Is internal energy a state variable or a path-dependent quantity? Explain.
- For an ideal monatomic gas, the internal energy is purely the sum of the kinetic energies of its molecules. How does the internal energy of one mole of such a gas relate to its absolute temperature T ?
- If heat is added to the gas in the rigid vessel (isochoric process), where does this energy go according to the First Law of Thermodynamics?

Ans :

- Internal energy is simply the sum of the kinetic energies and potential energies of the molecules of the system.
- Internal energy U of a system is a thermodynamic state variable; its value depends only on the given state of the system, not on the path taken to arrive at that state.
- For one mole of an ideal monatomic gas, the internal energy $U = (3/2)RT$. (Based on $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$ per molecule, so for N_A molecules,

$$U = N_A \frac{3}{2} k_B T = \frac{3}{2} RT).$$

- In an isochoric process (constant volume), the work done $\Delta W = P\Delta V = 0$. According to the First Law, $\Delta Q = \Delta U + \Delta W$. Since $\Delta W = 0$, all the heat added (ΔQ) goes into increasing the internal energy (ΔU) of the gas.

Ans :

1. An isothermal process is one in which the temperature of the system is kept fixed throughout.
2. For an ideal gas, internal energy depends only on temperature. Thus, there is no change in the internal energy of an ideal gas in an isothermal process ($\Delta U = 0$).
3. The work done by the gas during an isothermal expansion is $W = \mu RT \ln(V_2/V_1)$.
4. From the First Law, $\Delta Q = \Delta U + \Delta W$. Since $\Delta U = 0$ for an isothermal process of an ideal gas, $\Delta Q = \Delta W$. Thus, the heat supplied to the gas equals the work done by the gas. Since $V_2 > V_1$ (expansion), W is positive, so Q is positive, meaning heat is absorbed by the gas.

- 216.** A thermally insulated container is divided into two compartments by a partition. One compartment contains a gas, and the other is evacuated (vacuum). The partition is suddenly removed, allowing the gas to expand freely into the vacuum.

Questions:

1. What is a quasi-static process? Is free expansion a quasi-static process?
2. How much work is done by the gas during free expansion against a vacuum?
3. If the container is thermally insulated, what is the heat exchanged (ΔQ) with the surroundings?
4. According to the First Law of Thermodynamics, what is the change in the internal energy (ΔU) of the gas during this free expansion? If the gas is ideal, what does this imply about its temperature change?

Ans :

1. A quasi-static process is an infinitely slow process such that the system remains in thermal and mechanical equilibrium with the surroundings throughout. Free expansion is not a quasi-static process; it is a rapid expansion where the gas passes through non-equilibrium states.
2. Work is done when the force acts against some external pressure causing displacement. In free expansion against a vacuum, the external pressure is zero, so the work done by the gas ($\Delta W = P_{ext} \Delta V$) is zero.
3. Since the container is thermally insulated, no heat is exchanged with the surroundings, so $\Delta Q = 0$.
4. From the First Law, $\Delta U = \Delta Q - \Delta W$. Since $\Delta Q = 0$ and $\Delta W = 0$, the change in internal energy $\Delta U = 0$. If the gas is ideal, its internal

energy depends only on temperature. Therefore, if $\Delta U = 0$, its temperature does not change.

- 217.** Aluminium has a molar mass of approximately 27 g/mol. Its experimentally measured molar specific heat capacity at constant pressure (C_p) at room temperature is around $24.4 \text{ J mol}^{-1} \text{ K}^{-1}$. According to the law of equipartition of energy for a solid, the molar specific heat capacity at constant volume (C_v) is predicted to be $3R$.

Questions:

1. How is molar specific heat capacity (C) defined?
2. For solids, why is the change in volume (ΔV) considered negligible when heat is added at constant pressure, allowing the approximation $\Delta Q \approx \Delta U$?
3. Using $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, calculate the predicted value of molar specific heat capacity (C_v) for a solid based on the equipartition theorem.
4. For solids, C_p and C_v are not very different. How does the predicted value of $3R$ compare with the experimental C_p for aluminium ($24.4 \text{ J mol}^{-1} \text{ K}^{-1}$)? Is it a good agreement?

Ans :

1. Molar specific heat capacity C is defined as

$$C = \frac{1}{\mu} \frac{\Delta Q}{\Delta T},$$

where μ is the number of moles.

2. For solids, the thermal expansion (change in volume) is generally small, so the work done ($P \Delta V$) is negligible compared to the change in internal energy (ΔU). Thus, $\Delta Q \approx \Delta U$.
3. The predicted value of C_v for a solid from the equipartition theorem is

$$3R = 3 \times 8.31 \text{ J mol}^{-1} \text{ K}^{-1} \\ = 24.93 \text{ J mol}^{-1} \text{ K}^{-1}.$$

4. The predicted value of $3R \approx 24.9 \text{ J mol}^{-1} \text{ K}^{-1}$ is very close to the experimental C_p for aluminium ($24.4 \text{ J mol}^{-1} \text{ K}^{-1}$). This is generally good agreement for many solids at ordinary temperatures.

- 218.** A scientist claims to have built a refrigerator that, without any external work input, continuously transfers heat from a cold body (e.g., ice at 0°C) to a hotter body (e.g., a room at 25°C), making the cold body even colder and the hot body even hotter.

Questions:

1. State the Clausius statement of the Second Law of Thermodynamics.
2. Does the scientist's claim violate the First Law of Thermodynamics (conservation of energy)? Explain.

CHAPTER 12

Kinetic Theory

SUMMARY

1. IDEAL GAS

A gas which satisfies all the assumptions of kinetic theory and obeys the gas laws, is called an ideal gas.

Perfect gas equation : For 1 mole of an ideal gas; pressure, volume and absolute temperature of the gas are related as

$$PV = RT$$

2. BOYLE'S LAW

It states that temperature remaining constant, volume of a given mass of a gas is inversely proportional to the pressure applied.

i.e., $P \propto \frac{1}{V}$ (at constant temperature)

$$PV = \text{constant}$$

3. CHARLES' LAW

It states that the pressure remaining constant, volume of given mass of gas is directly proportional to absolute temperature.

i.e., $V \propto T$ (at constant pressure)

$$\frac{V}{T} = \text{constant}$$

4. GAY LUSSAC'S LAW

It states that the volume remaining constant, the pressure exerted by a gas is directly proportional to its temperature.

$$P \propto T \text{ (at constant volume)}$$

$$\frac{P}{T} = \text{constant}$$

5. AVOGADRO'S HYPOTHESIS

It states that equal volume of all the gases under similar conditions of temperature and pressure contain equal number of molecules.

6. PRESSURE EXERTED BY A GAS

On the basis of kinetic theory of gases, pressure exerted by a gas is given by

$$P = \frac{1}{3} \rho C^2$$

$$P = \frac{M}{3V} C^2$$

$$P = \frac{2}{3} E$$

DALTON'S LAW OF PARTIAL PRESSURE

It states that the resultant pressure exerted by a mixture of non-interacting gases is equal to the sum of their individual pressures.

8. AVOGADRO'S LAW

It states that equal volume of all the gases under similar conditions of temperature and pressure contains equal number of molecules.

9. DEGREES OF FREEDOM (f)

The total number of independent quantities or coordinates required to describe the configuration of a dynamic system is called its degrees of freedom.

The number of degrees of freedom of a dynamic system can be obtained by subtracting the number of independent relations from the total number of coordinates required to specify the position of constituent particle of the system. If a dynamic system consists of N particles and k independent relations between them, then

$$f = 3N - k$$

10. LAW OF EQUIPARTITION OF ENERGY

It states that for any system in thermal equilibrium, the total energy is equally distributed among all the degrees of freedom and energy associated with each degree of freedom is $\frac{1}{2}kT$.

For monoatomic gas, $f = 3$
 $U = \frac{3}{2}kT$

$$C_v = \frac{3}{2}R, C_p = \frac{5}{2}R, \gamma = 1.67$$

For diatomic gas, $f = 5$

$$U = \frac{5}{2}kT$$

So $C_v = \frac{3}{2}R, C_p = \frac{7}{2}R, \gamma = 1.4$.

Mean free path is the average distance travelled by a molecule between two successive collisions with each other and denoted by λ .

NCERT TEXT-BOOK QUESTIONS

1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the size of oxygen molecule to be roughly $3\text{\AA}$.

Ans :

$$\begin{aligned}\text{Molecular volume} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times 3.14 \times \left(\frac{3}{2} \times 10^{-8}\right)^3 \\ &= 0.14 \times 10^{-22} \text{cm}^3\end{aligned}$$

Actual volume of oxygen at STP

$$\begin{aligned}&= \frac{\text{Molecular volume}}{\text{Avogadro number}} \\ &= \frac{22.4 \times 10^3}{6.023 \times 10^{23}} \\ &= 3.72 \times 10^{-20} \text{cm}^3\end{aligned}$$

Hence, Ratio $= \frac{\text{molecular volume}}{\text{actual volume}}$

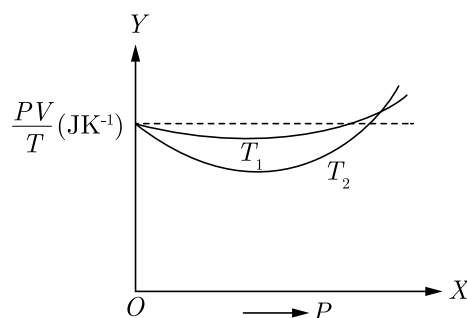
$$\begin{aligned}&= \frac{0.14 \times 10^{-22}}{3.72 \times 10^{-20}} = 3.79 \times 10^{-4} \\ &= 4 \times 10^{-4}\end{aligned}$$

2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (S.T.P.) 1 atmospheric pressure, (0°C). Show that it is 22.4 litres.

Ans :

$$\begin{aligned}P &= 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \\ \mu &= 1 \text{ mole}, R = 8.31 \text{ J mol}^{-1} \text{K}^{-1} \\ T &= 273 \text{ K} \\ PV &= \mu RT \\ V &= \frac{\mu RT}{P} = \frac{1 \times 8.31 \times 273}{1.013 \times 10^5} \\ &= 22.4 \text{ litres}\end{aligned}$$

3. Figure shows plots of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.
- What does the dotted plot signify ?
 - Which is true : $T_1 > T_2$ or $T_1 < T_2$?
 - What is the value of PV/T , where the curves meet on the y -axis
 - If we obtain similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y -axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure-high temperature region of the plot) (Molecular mass of $\text{H}_2 = 2.02$ of Oxygen = 32.0μ , $R = 8.31 \text{ J mol}^{-1} \text{K}^{-1}$) ?



Ans :

- The dotted plot corresponds to ideal gas behaviour,
- $T_1 > T_2$ (Since $\frac{PV}{T}$ curve is steeper)
- For $M = 32 \times 10^{-3}$ kg the value $\frac{PV}{T} (> R)$ is 8.31 J/mole/K
 Hence, For the value of $\frac{PV}{T} (> K) = \frac{8.31}{32 \times 10^{-3}} = 0.26 \text{ J/K}$
- The value of R , 831 J/K is true for $2.02 \times 10^{-3} \text{ kg}$ of H_2
 0.26 J/K is true for $\frac{2.02 \times 10^{-3} \times 8.31}{0.26}$
 $= 6.32 \times 10^{-5} \text{ kg}$

4. An oxygen cylinder of volume 30 litres has initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder the gauge pressure drop to 11 atm. and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder. ($R = 8.3 \text{ Jmol}^{-1}\text{K}^{-1}$) Molecular mass of $\text{O}_2 = 32$.

Ans :

$$P_1 = 15 \text{ atm.} = 15 \times 1.01 \times 10^5 \text{ Pa}$$

$$T_1 = 273 + 27 = 300 \text{ K}$$

$$V_2 = 30 \text{ litre}$$

$$P_2 = 11 \text{ atm.}$$

$$T_2 = 273 + 17 = 290 \text{ K}$$

$$\frac{P_1 V_1}{\mu_1 T_1} = \frac{P_2 V_2}{\mu_2 T_2}$$

$$\frac{\mu_2}{\mu_1} = \frac{11 \times 300}{15 \times 290} = 0.76$$

$$P_1 V = \mu_1 R T_1$$

$$\mu_1 = \frac{15 \times 1.01 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300}$$

$$\mu_1 = 18.23$$

Hence, $\mu_2 = 18.23 \times 0.76 = 13.85$

Since $\mu = \frac{m}{M}$

$$m_1 = \mu_1 m = 18.23 \times 32 \times 10^{-3} \\ = 0.58 \text{ kg}$$

$$m_2 = \mu_2 m = 13.85 \times 32 \times 10^{-3} \\ = 0.44 \text{ kg}$$

$$\text{Mass deduced} = m_1 - m_2 = 0.58 - 0.44 \\ = 0.14 \text{ kg}$$

5. An air bubble of volume of 1.0 cm³ rises from the bottom of lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface which is at a temperature of 35°C ?

Ans :

$$V_1 = 1 \text{ c.c.}, P_1 = P + 40 \times 1000 \times 9.8$$

$$= 0.76 \times 13600 \times 9.8 + 40 \times 1000 \times 98$$

$$T_1 = 273 + 12 = 285 \text{ K}$$

$$V_2 = ?, P_2 = 0.76 \times 13.6 \times 9.8 \times 1000 \text{ Nm}^{-2}$$

$$T_2 = 273 + 35 = 308 \text{ K}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \frac{P_1 V_1}{T_1} \times \frac{T_2}{P_2} \\ = \frac{493292.3 \times 10^{-3} \times 308}{101292.8 \times 285} \\ = 5.26 \times 10^{-6} \text{ m}^3$$

6. Estimate the total number of air molecules (inclusive oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m³ at a temperature of 27°C and 1 atm. pressure. ($K_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$)

Ans :

$$N = \frac{pV}{K_B T} \\ = \frac{(1.013 \times 10^5 \text{ Pa})(25 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ JK}^{-1})(300 \text{ K})} \\ = \frac{1.013 \times 25 \times 10^{24}}{1.38 \times 3} \\ = 6.10 \times 10^{26}$$

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7.

Estimate the average thermal energy of a helium atom (i) room temperature (27°C), (ii) the temperature on the surface of the star (6000°K), (iii) the temperature of 10 million degree kelvin (the typical core temperature in the case of a star). $K_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$

Ans :

$$(i) \quad \frac{1}{2} m v^2 = \frac{3}{2} K_B T \\ = \left(\frac{3}{2}\right)(1.28 \times 10^{-23} \text{ JK}^{-1})(300 \text{ K}) \\ = 6.2 \times 10^{-21} \text{ J}$$

$$(ii) \quad \frac{1}{2} m v^2 = \frac{3}{2} K_B T \\ = \left(\frac{3}{2}\right)(1.38 \times 10^{-23} \text{ JK}^{-1})(6000 \text{ K}) \\ = 1.24 \times 10^{-19} \text{ J}$$

$$(iii) \quad \frac{1}{2} m v^2 = \frac{3}{2} K_B T \\ = \left(\frac{3}{2}\right)(1.38 \times 10^{-23} \text{ JK}^{-1}) 10^6 \text{ K} \\ = 2.1 \times 10^{-16} \text{ J}$$

8. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules ? Is the root mean squared speed of molecules the same in the three cases ? If not in which case is v_{rms} the largest ?

Ans :

According to Avogadro's law "equal volumes of all gases contain equal number of molecules at the same temperature and pressure. Hence three vessels contain equal number of respective molecules.

No, The root mean square speed of molecules is not the same in the three cases.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$v_{\text{rms}} \propto \frac{1}{\sqrt{m}}$$

Therefore v_{rms} is largest for the lightest of the three gases i.e. Neon.

9. At what temperature is the root mean square speed of an atom in an argon gas equal to the rms speed of a helium gas atom at -20°C ? (Atomic mass of Ar = 39.9 μ of He = 4.0)

Ans :

$$\frac{M_1 \overline{C_1^2}}{T_1} = \frac{M_2 \overline{C_2^2}}{T_2} = \text{constant}$$

Where M_1 and M_2 are the molecular masses of two gases, $\overline{C_1^2}$ and $\overline{C_2^2}$ are the mean square velocities for the two gases and T_1 and T_2 are the absolute temperatures of the gases.

It is given that rms velocities of the two gases are equal, that is

$$\sqrt{\overline{C_1^2}} = \sqrt{\overline{C_2^2}}$$

$$\overline{C_1^2} = \overline{C_2^2}$$

Thus $\frac{M_1}{T_1} = \frac{M_2}{T_2}$

$$M_1 = 40, T_1 = ?, M_2 = 4,$$

$$T_2 = 273 - 20 = 253 \text{ K}$$

$$\frac{40}{T_1} = \frac{4}{253}$$

$$4T_1 = 253 \times 40$$

$$T_1 = \frac{253 \times 40}{4} = 2.53 \times 10^3 \text{ K}$$

10. Estimate the mean free path and collision frequency of N_2 molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C . Take the radius of nitrogen molecules to be $1.01 \text{ \AA}$. Compare the collision time with the time the molecules move freely between two successive collisions.

Ans :

$$P = 2 \text{ atm} = 2 \times 1.01 \times 10^5 \text{ Pa}$$

$$T = 290 \text{ K}$$

$$\text{Diameter} = 2 \times 10^{-10} \text{ m}$$

$$V = \frac{RT}{P} = \frac{8.31 \times 290}{2 \times 1.01 \times 10^5} = 11.93 \times 10^{-3} \text{ m}$$

Hence, No. of molecules in 1 m^3 of N_2

$$= \frac{6.023 \times 10^{23}}{11.93 \times 10^{-3}}$$

$$n = 5.05 \times 10^{25} \text{ m}^{-3} = 5.1 \times 10^{25} \text{ m}^{-3}$$

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v^2 = \frac{3 \times 1.38 \times 10^{-23} \times 290 \times 6.023 \times 10^{23}}{28 \times 10^{-3}}$$

$$v = 5.08 \text{ m s}^{-1} = 5.1 \times 10^2 \text{ m s}^{-1}$$

Mean free path,

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

$$= \frac{1}{\sqrt{2} \times 3.142 \times (2 \times 10^{-10})^2 \times 5.1 \times 10^{25}} = 10^{-7} \text{ m}$$

Collision frequency

$$= \frac{v}{l} = \frac{5.1 \times 10^2}{10^{-7}} = 5.1 \times 10^9 \text{ s}^{-1}$$

Time for collision

$$= \frac{d}{v} = \frac{2 \times 10^{-10}}{5.1 \times 10^2} = 4 \times 10^{-13} \text{ sec}$$

Time between successive collisions

$$\frac{l}{v} = 2 \times 10^{-10} \text{ sec}$$

Thus the time taken between two collisions is

$\left(\frac{2 \times 10^{-10}}{4 \times 10^{-13}}\right) = 500$ times more than time taken for a collision. Thus a mole cube is essentially free.

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OBJECTIVE QUESTIONS

- Who among the following was credited with discovering the law stating that for a fixed mass of gas at constant temperature, pressure is inversely proportional to volume?
(A) Isaac Newton
(B) James Clerk Maxwell
(C) Robert Boyle
(D) Ludwig Boltzmann

Ans :

(C) Robert Boyle

Robert Boyle, in 1661, discovered the relationship between the pressure and volume of a gas at constant temperature, famously known as Boyle's Law. This law was an early step in understanding gas behavior and laid groundwork for the development of kinetic theory by later scientists and researchers.

- The kinetic theory of gases explains the behavior of gases based on the idea that a gas consists of rapidly moving:
(A) Phonons and photons
(B) Atoms or molecules
(C) Electrons and protons
(D) Plasma particles

Ans :

(B) Atoms or molecules

Kinetic theory postulates that gases are composed of a large number of atoms or molecules that are in constant, random motion. This motion is responsible for the macroscopic properties of gases such as pressure and temperature, and the theory largely neglects inter-atomic forces for ideal gases.

- According to Richard Feynman, which hypothesis is considered highly significant for the next generation if all scientific knowledge were destroyed?
(A) The Law of Universal Gravitation
(B) The Atomic Hypothesis
(C) The Theory of Relativity
(D) The Laws of Thermodynamics

Ans :

(B) The Atomic Hypothesis

Richard Feynman emphasized the profound importance of the Atomic Hypothesis, which states that all things are made of atoms. He believed this single concept holds immense explanatory power for understanding the natural world, suggesting its crucial role for any civilization to rebuild scientific understanding in future.

- In ancient India, the Vaiseshika school of thought, founded by Kanada, proposed that atoms were:
(A) Divisible, temporary, and macroscopic parts
(B) Eternal, indivisible, and ultimate parts
(C) Mutable, continuous, and energetic parts
(D) Created, transformable, and interactive parts

Ans :

(B) Eternal, indivisible, and ultimate parts

The Vaiseshika school, attributed to Kanada, developed an atomic picture where 'Paramanu' (atoms) were considered eternal, indivisible, infinitesimal, and the ultimate constituents of matter. This ancient philosophical framework provided an early conceptualization of the particulate nature of matter, preceding modern atomic theories by centuries in detail.

- How did Democritus's view of atoms of water differ from atoms of earth in ancient Greece?
(A) Water atoms were thorny; earth atoms were smooth
(B) Water atoms were rough; earth atoms were jagged
(C) Water atoms were smooth; earth atoms were rough
(D) Water atoms were light; earth atoms were heavy

Ans :

(C) Water atoms were smooth; earth atoms were rough

Democritus proposed that atoms differed physically, influencing material properties. He imagined water atoms as smooth and round, allowing them to flow easily. In contrast, he thought earth atoms were rough and jagged, enabling them to hold together and form hard substances, explaining their differing states.

- John Dalton's atomic theory was proposed primarily to explain which laws?
(A) Laws of motion and gravitation
(B) Laws of thermodynamics
(C) Laws of definite and multiple proportions
(D) Laws of electromagnetism

Ans :

(C) Laws of definite and multiple proportions

John Dalton formulated his atomic theory to provide a scientific explanation for the empirically observed laws of definite and multiple proportions. These laws describe how elements combine to form chemical compounds, specifically concerning the

$$U = 3 \times (1/2) k_B T \times N_A = (3/2) RT.$$

Thus, $C_v = dU/dT = (3/2) R.$

- 32.** The relation $C_p - C_v = R$ is true for:

- (A) Only monatomic ideal gases
- (B) Only diatomic ideal gases
- (C) Only polyatomic ideal gases
- (D) Any ideal gas

Ans :

(D) Any ideal gas

The relation $C_p - C_v = R$, known as Mayer's relation, is a fundamental thermodynamic relationship that holds true for any ideal gas, irrespective of whether it is monatomic, diatomic, or polyatomic. It connects the molar specific heats at constant pressure and constant volume.

- 33.** For a rigid diatomic gas, the ratio of specific heats ($\gamma = C_p/C_v$) is:

- (A) $3/2$
- (B) $5/3$
- (C) $7/5$
- (D) $9/7$

Ans :

(C) $7/5$

A rigid diatomic molecule has 3 translational and 2 rotational degrees of freedom (total 5).

So, $C_v = (5/2) R$ and $C_p = C_v + R = (7/2) R.$

Therefore, $\gamma = C_p/C_v = ((7/2) R) / ((5/2) R)$
 $= 7/5 = 1.40.$

- 34.** Which of the following gases is likely to show the largest deviation from the predicted specific heat capacities (ignoring vibrational modes) at ordinary temperatures?

- (A) Helium (He)
- (B) Neon (Ne)
- (C) Chlorine (Cl_2)
- (D) Argon (Ar)

Ans :

(C) Chlorine (Cl_2)

Monatomic gases like He, Ne, and Ar generally show good agreement with theoretical specific heat values based only on translational modes. Diatomic gases like Cl_2 can have significant vibrational contributions even at ordinary temperatures, leading to experimental values greater than those predicted ignoring vibrations.

- 35.** What is the predicted molar specific heat at constant volume (C_v) for a polyatomic gas molecule having 3 translational, 3 rotational degrees of freedom, and 'f' vibrational modes?

- (A) $(3/2 + f) R$
- (B) $(3 + f) R$
- (C) $(6 + 2f) R$
- (D) $(3/2 + 3/2 + f/2) R$

Ans :

(B) $(3 + f) R$

The molecule has 3 translational degrees (each $(1/2) RT$ per mole), 3 rotational degrees (each $(1/2) RT$ per mole), and 'f' vibrational modes (each RT per mole).

So, $U = (3/2) RT + (3/2) RT + fRT$
 $= (3 + f) RT$

Thus, $C_v = dU/dT = (3 + f) R.$

- 36.** According to the law of equipartition of energy, the molar specific heat capacity (c) of a solid at ordinary temperatures is predicted to be approximately:

- (A) R
- (B) $(3/2) R$
- (C) $(5/2) R$
- (D) $3R$

Ans :

(D) $3R$

For a solid, atoms vibrate about their mean positions. Each atom can be considered as an oscillator in three dimensions. Each dimension contributes $k_B T$ (for kinetic and potential energy). For one mole (N_A atoms), $U = 3N_A k_B T = 3RT$. So, $C = dU/dT \approx 3R.$

- 37.** Which of the following pairs of gas type and its primary degrees of freedom at moderate temperatures is incorrect?

- (A) Monatomic gas : Only translational
- (B) Rigid diatomic gas : Translational and rotational
- (C) Non-rigid diatomic gas : Translational, rotational, and vibrational
- (D) Linear triatomic gas : Only translational and vibrational

Ans :

(D) Linear triatomic gas : Only translational and vibrational

A linear triatomic gas, like CO_2 , will have 3 translational degrees of freedom and 2 rotational degrees of freedom (similar to a diatomic molecule, as rotation about the linear axis is negligible). It will

Ans :

(B) Molecular size and intermolecular forces

Ideal gas laws assume molecules are point masses with no volume and that they do not exert forces on one another except during collisions. Real gases deviate from this because molecules do have finite sizes and experience intermolecular attractive and repulsive forces affecting behavior.

14. Charles's Law states that for a fixed pressure, the volume of a gas is proportional to its:

- (A) Absolute temperature (T)
- (B) Celsius temperature (t)
- (C) Square of absolute temperature (T^2)
- (D) Inverse of absolute temperature ($1/T$)

Ans :

(A) Absolute temperature (T)

Charles's Law describes the relationship where, at constant pressure, the volume occupied by a fixed amount of gas is directly proportional to its absolute temperature (measured in Kelvin). This means that as the absolute temperature increases, the volume of the gas also increases proportionately and consistently.

15. What is Dalton's law of partial pressures?

- (A) The pressure of a gas mixture is the average of individual pressures.
- (B) The total pressure of a mixture of ideal gases is the sum of partial pressures.
- (C) Each gas in a mixture exerts a pressure equal to the total pressure always.
- (D) The partial pressure of a gas is inversely proportional to its mole fraction value.

Ans :

(B) The total pressure of a mixture of ideal gases is the sum of partial pressures.

Dalton's law of partial pressures states that for a mixture of non-reacting ideal gases, the total pressure exerted is equal to the sum of the partial pressures that each individual gas would exert if it alone occupied the entire volume at the same given temperature.

16. If a vessel contains neon (monatomic) and oxygen (diatomic) with partial pressures in the ratio 3:2, what is the ratio of their number of molecules (N_{Ne}/N_{O_2})?

- (A) 2:3
- (B) 3:2
- (C) 9:4
- (D) 4:9

Ans :

(B) 3:2

According to the ideal gas law, $PV = Nk_B T$. Since volume (V), temperature (T), and Boltzmann constant (k_B) are the same for both gases in the mixture, the pressure P is directly proportional to the number of molecules N . Thus, the ratio of partial pressures equals the ratio of their molecules.

17. Which of the following is an assumption of the kinetic theory of an ideal gas?

- (A) Molecules are always at rest
- (B) Collisions between molecules are inelastic
- (C) The volume of molecules is negligible
- (D) Molecules exert strong attractive forces

Ans :

(C) The volume of molecules is negligible

One of the fundamental assumptions of the kinetic theory for an ideal gas is that the volume occupied by the molecules themselves is negligible compared to the total volume of the container. This simplification allows for easier mathematical treatment of gas behavior, especially at lower pressures.

18. In the kinetic theory of gases, collisions between molecules and the walls of the container are assumed to be:

- (A) Perfectly inelastic
- (B) Partially elastic
- (C) Perfectly elastic
- (D) Absorptive

Ans :

(C) Perfectly elastic

A key assumption in the kinetic theory of ideal gases is that all collisions, whether between molecules themselves or between molecules and the container walls, are perfectly elastic. This means that both kinetic energy and momentum are conserved during these molecular interactions and collisions.

19. The pressure exerted by an ideal gas is proportional to:

- (A) The average velocity of the molecules
- (B) The average kinetic energy of the molecules
- (C) The average potential energy of the molecules
- (D) The total number of molecules directly

Ans :

(B) The average kinetic energy of the molecules

The pressure exerted by an ideal gas is directly related to the average translational kinetic energy of its molecules. Specifically, pressure arises from the constant bombardment of the container walls

by these rapidly moving molecules, transferring momentum to the walls during collisions frequently.

20. The expression for the pressure of an ideal gas according to kinetic theory is $P = (1/3)nm\overline{v^2}$. What does 'n' represent here?

(A) Number of moles
(B) Avogadro's number
(C) Number density of molecules
(D) Boltzmann constant

Ans :

(C) Number density of molecules

In the pressure equation $P = (1/3)nm\overline{v^2}$ derived from kinetic theory, 'n' represents the number density of molecules, which is the number of molecules per unit volume. This factor, along with molecular mass and mean square speed, determines the gas pressure value.

21. Which of the following statements about the kinetic interpretation of temperature is incorrect?

(A) Average kinetic energy of a molecule is proportional to the absolute temperature.
(B) Temperature is independent of the nature of the ideal gas.
(C) Internal energy of an ideal gas depends only on temperature.
(D) Lighter molecules have smaller rms speed at the same temperature.

Ans :

(D) Lighter molecules have smaller rms speed at the same temperature.

According to the kinetic interpretation of temperature, the average kinetic energy ($\frac{1}{2}m\overline{v^2}$) is proportional to absolute temperature (t). This implies that for a given temperature, lighter molecules (smaller m) must have a higher root mean square (rms) speed to maintain the same average kinetic energy.

22. The root mean square (rms) speed of gas molecules is defined as:

(A) The average of the speeds of all molecules
(B) The square root of the average of the squared speeds
(C) The most probable speed of the molecules
(D) The speed of the fastest molecule in the gas

Ans :

(B) The square root of the average of the squared speeds

The root mean square (rms) speed is a statistical measure of the speed of particles in a gas. It is

calculated by taking the square root of the mean (average) of the squares of the speeds of the individual molecules, providing an important parameter.

23. A flask contains argon (atomic mass 39.9 u) and chlorine (molecular mass 70.9 u) at the same temperature. The ratio of the average kinetic energy per molecule of argon to chlorine is:

(A) 39.9 : 70.9
(B) 70.9 : 39.9
(C) 1 : 1
(D) $\sqrt{70.9} : \sqrt{39.9}$

Ans :

(C) 1 : 1

The average kinetic energy per molecule of any ideal gas depends only on the absolute temperature ($E/N = (3/2)k_B T$). Since both argon and chlorine are in the same flask and thus at the same temperature, their average kinetic energies per molecule will be identical.

24. Arrange the following steps in the correct sequence for deriving the pressure of an ideal gas using kinetic theory, starting from a molecule hitting a wall:

1. Calculate momentum transferred to the wall by molecules hitting it in time Δt .
2. Calculate the change in momentum of a single molecule upon elastic collision with a wall.
3. Determine the number of molecules hitting an area A of the wall in time Δt .
4. Divide the total rate of momentum transfer by the area to get pressure.

(A) 2, 3, 1, 4
(B) 1, 2, 3, 4
(C) 2, 1, 3, 4
(D) 3, 2, 1, 4

Ans :

(A) 2, 3, 1, 4

The derivation begins with the momentum change of one molecule (2). Then, it considers how many such molecules hit the wall in a given time (3). Next, the total momentum transferred by these molecules is calculated (1). Finally, pressure is obtained from the rate of momentum change per unit area (4).

25. If the temperature of an ideal gas is doubled, what happens to the average translational kinetic energy of its molecules?

(A) It is halved
(B) It remains the same
(C) It is doubled
(D) It becomes four times

Ans :

(C) It is doubled

The average translational kinetic energy of the molecules of an ideal gas is directly proportional to its absolute temperature ($E/N = (3/2)k_B T$). Therefore, if the absolute temperature of the gas is doubled, the average translational kinetic energy of its molecules will also be doubled accordingly.

26. Which of the following statements correctly describes a feature of an ideal gas's internal energy based on kinetic theory?

(A) Internal energy depends on pressure and volume.
 (B) Internal energy depends only on temperature.
 (C) Internal energy is zero at absolute zero.
 (D) Internal energy is equally due to kinetic and potential energies.

Ans :

(B) Internal energy depends only on temperature.

For an ideal gas, whose molecules are assumed to have no intermolecular forces (and thus no potential energy due to these forces), the internal energy is solely the sum of the kinetic energies of its molecules. This total kinetic energy is directly proportional to the absolute temperature of the gas.

27. The law of equipartition of energy states that in thermal equilibrium, the total energy is equally distributed among all possible energy modes, with each mode having an average energy equal to:

(A) $k_B T$
 (B) $(1/2)k_B T$
 (C) $(3/2)k_B T$
 (D) $2k_B T$

Ans :

(B) $(1/2)k_B T$

The law of equipartition of energy states that for a system in thermal equilibrium, the average energy associated with each quadratic term in the expression for energy (each degree of freedom) is $(1/2)k_B T$. This principle is fundamental to understanding how energy is shared among molecular motions.

28. How many translational degrees of freedom does a molecule free to move in space have?

(A) 1
 (B) 2
 (C) 3
 (D) 0

Ans :

(C) 3

A molecule that is free to move in three-dimensional space requires three coordinates (e.g., x, y, z) to specify its location. Each of these independent directions of motion corresponds to a translational degree of freedom, contributing to its kinetic energy as it moves around.

29. For a diatomic molecule treated as a rigid rotator (at moderate temperatures), how many rotational degrees of freedom does it typically possess?

(A) 1
 (B) 2
 (C) 3
 (D) 0

Ans :

(B) 2

A diatomic molecule, like a dumbbell, can rotate about two independent axes perpendicular to the line joining the two atoms. Rotation about the axis joining the atoms has negligible moment of inertia. Thus, it has two rotational degrees of freedom contributing to its energy normally.

30. According to the law of equipartition of energy, how does the energy contribution of one vibrational mode compare to one translational degree of freedom?

(A) Vibrational mode contributes half the energy.
 (B) Vibrational mode contributes the same energy.
 (C) Vibrational mode contributes double the energy.
 (D) Vibrational mode contributes no energy.

Ans :

(C) Vibrational mode contributes double the energy. Each translational degree of freedom contributes $(1/2)k_B T$ to the average energy. A vibrational mode has two squared terms in its energy expression (kinetic and potential energy), so it contributes $2 \times (1/2)k_B T = k_B T$. Thus, a vibrational mode contributes double that of one translational degree.

31. For a monatomic ideal gas, the molar specific heat at constant volume (C_v) is:

(A) $(1/2)R$
 (B) R
 (C) $(3/2)R$
 (D) $(5/2)R$

Ans :

(C) $(3/2)R$

A monatomic gas has 3 translational degrees of freedom. According to the law of equipartition of energy, the total internal energy of one mole is

mass ratios of the constituent elements involved in chemical interactions then.

7. What does Avogadro's law state?

- (A) Equal volumes of all gases at any temperature and pressure have the same number of atoms.
- (B) Equal volumes of all gases at equal temperature and pressure have the same number of molecules.
- (C) Equal masses of all gases at equal temperature and pressure have the same number of molecules.
- (D) Equal volumes of all gases at equal temperature and pressure have the same mass of molecules.

Ans :

(B) Equal volumes of all gases at equal temperature and pressure have the same number of molecules.

Avogadro's law posits that under the same conditions of temperature and pressure, equal volumes of different gases contain an equal number of molecules. This law is crucial for understanding molar quantities and the relationship between gas volume and the amount of substance, a key element.

8. The approximate size of an atom is about:

- (A) 10^{-6} m
- (B) 10^{-8} m
- (C) 10^{-10} m
- (D) 10^{-12} m

Ans :

(C) 10^{-10} m

The typical size of an atom is on the order of an angstrom, which is 10^{-10} meters. This extremely small dimension is a fundamental characteristic of atoms and influences how they interact to form molecules and larger structures, which in turn affects material properties.

9. In gases, the average distance a molecule can travel without colliding with another molecule is called the:

- (A) Molecular diameter
- (B) Interatomic spacing
- (C) Mean free path
- (D) Collision radius

Ans :

(C) Mean free path

The mean free path is defined as the average distance a molecule in a gas travels between successive collisions with other molecules. This parameter is important in kinetic theory as it influences transport properties like viscosity, thermal conductivity, and diffusion in gases generally.

10. The inter-atomic forces important for solids and liquids typically exhibit:

- (A) Only short-range attraction always
- (B) Only short-range repulsion always
- (C) Long-range attraction and short-range repulsion
- (D) Long-range repulsion and short-range attraction

Ans :

(C) Long-range attraction and short-range repulsion
Inter-atomic forces, crucial for understanding the properties of condensed matter like solids and liquids, are characterized by attraction when atoms are a few angstroms apart and strong repulsion when they are squeezed very close together. This balance dictates their structure and behavior typically.

11. The perfect gas equation can be written as $PV = \mu RT$. What does μ represent in this equation?

- (A) The mass of the gas
- (B) The number of molecules
- (C) The number of moles
- (D) The molar mass

Ans :

(C) The number of moles

In the ideal gas equation $PV = \mu RT$, the symbol μ represents the number of moles of the gas. A mole is a specific quantity of substance containing Avogadro's number of particles, allowing macroscopic properties like pressure, volume, and temperature to be related systematically.

12. What is the value of Boltzmann constant (k_B) in SI units?

- (A) $1.38 \times 10^{-23} \text{ JK}^{-1}$
- (B) $6.02 \times 10^{23} \text{ JK}^{-1}$
- (C) $8.314 \text{ Jmol}^{-1} \text{ K}^{-1}$
- (D) $6.62 \times 10^{-34} \text{ Js}$

Ans :

(A) $1.38 \times 10^{-23} \text{ JK}^{-1}$

The Boltzmann constant, denoted as k_B , is a fundamental physical constant that relates the average kinetic energy of particles in a gas with the absolute temperature of the gas. Its value in SI units is approximately $1.38 \times 10^{-23} \text{ JK}^{-1}$ Joules per Kelvin, a critical parameter.

13. Real gases deviate from ideal gas behavior primarily because ideal gas theory neglects:

- (A) Molecular motion and kinetic energy
- (B) Molecular size and intermolecular forces
- (C) Gravitational forces between molecules
- (D) The effects of gas pressure only

also have vibrational modes. The pair is incorrect as it omits rotational modes.

- 38.** The law of equipartition of energy is well verified experimentally for many gases at:

(A) Very low temperatures only
(B) Very high temperatures only
(C) Ordinary temperatures
(D) Absolute zero temperature

Ans :

(C) Ordinary temperatures

The law of equipartition of energy generally provides good predictions for the specific heats of gases at ordinary temperatures. However, deviations occur at very low temperatures where quantum effects become significant and some degrees of freedom may “freeze out,” and also at high temperatures for vibrational modes.

- 39.** What is the primary reason that gases leaking from a cylinder take considerable time to diffuse across a room, despite molecules having high speeds?

(A) Low molecular speeds
(B) Gravitational pull on molecules
(C) Finite size and frequent collisions
(D) High viscosity of air

Ans :

(C) Finite size and frequent collisions

Although gas molecules travel at high speeds (comparable to the speed of sound), they have a finite size and undergo frequent collisions with other gas molecules. These collisions cause their paths to be incessantly deflected, hindering straight-line motion and slowing down the overall diffusion process considerably.

- 40.** The mean free path (l) of a gas molecule is inversely proportional to:

(A) The temperature of the gas
(B) The square of the molecular diameter
(C) The average speed of the molecules
(D) The volume of the container

Ans :

(B) The square of the molecular diameter

The formula for mean free path is $l = 1/(\sqrt{2} n \pi d^2)$, where ‘d’ is the molecular diameter and ‘n’ is the number density. This shows that the mean free path is inversely proportional to d^2 (the collision cross-section πd^2) and the number density.

- 41.** Which of the following pairs correctly matches a quantity from kinetic theory with its dependence?

(A) RMS speed : Inversely proportional to absolute temperature
(B) Mean free path : Directly proportional to number density
(C) Pressure : Proportional to mean square speed of molecules
(D) Average K.E. per molecule : Depends on molecular mass

Ans :

(C) Pressure : Proportional to mean square speed of molecules

The pressure exerted by an ideal gas is given by $P = (1/3) nm \overline{v^2}$, showing it is directly proportional to the mean square speed ($\overline{v^2}$) of the molecules. Other options are incorrect: RMS speed is proportional to $\sqrt{T}$, mean free path is inversely proportional to number density, and average K.E. per molecule only depends on T .

- 42.** Consider the development of the atomic/molecular view of matter. Arrange the following contributors/ideas in approximate chronological order, from earliest to latest:

1. John Dalton’s Atomic Theory
2. Kanada’s Vaiseshika school (Paramanu concept)
3. Democritus’s atomic hypothesis
4. Development of Kinetic Theory by Maxwell and Boltzmann

(A) 2, 3, 1, 4

(B) 3, 2, 1, 4

(C) 1, 2, 3, 4

(D) 2, 1, 3, 4

Ans :

(A) 2, 3, 1, 4

The earliest ideas came from ancient India (Kanada - 2) and Greece (Democritus - 3). Much later, John Dalton (1) proposed a scientific atomic theory. The detailed kinetic theory (4) was developed in the 19th century by scientists like Maxwell and Boltzmann based on these ideas.

- 43.** The mean free path in gases is typically of the order of thousands of angstroms, while inter-atomic distances in liquids are a few angstroms. This difference primarily explains why:

(A) Gases are less dense than liquids
(B) Gases can disperse away if not enclosed
(C) Liquids can flow but solids cannot
(D) Gases exert pressure but liquids do not

Ans :

(B) Gases can disperse away if not enclosed

The large mean free path in gases means molecules travel significant distances before colliding, allowing them to spread out and fill any available volume, or disperse if unconfined. In liquids, the short interatomic distances and mean free paths restrict such free movement extensively and overall.

44. Assertion (A): The internal energy of an ideal gas depends only on its absolute temperature.
Reason (R): For an ideal gas, intermolecular forces are assumed to be negligible, so there is no intermolecular potential energy.
- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The internal energy of an ideal gas consists only of the kinetic energy of its molecules, as intermolecular forces (and thus potential energy associated with them) are neglected. Since the average kinetic energy is directly proportional to the absolute temperature, the internal energy depends solely on temperature.

45. Assertion (A): At the same temperature, lighter gas molecules have a higher root mean square (rms) speed than heavier gas molecules.
Reason (R): The average kinetic energy per molecule is the same for all ideal gases at the same temperature.
- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

The average kinetic energy per molecule is $(1/2)mv_{rms}^2 = (3/2)k_B T$. Since $(3/2)k_B T$ is constant for all gases at the same temperature (Reason R), if mass 'm' is smaller (lighter molecule), then v_{rms}^2 (and thus v_{rms}) must be larger to maintain the equality (Assertion A).

46. Assertion (A): The ratio of specific heats (γ) is greater for a monatomic gas than for a diatomic gas (treated as rigid).
Reason (R): Monatomic gases have fewer degrees of freedom compared to diatomic gases, leading to a smaller C_v .
- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

(A) Both A and R are true and R is the correct explanation of A.

Monatomic gases (3 degrees of freedom) have $C_v = (3/2)R$, giving $\gamma = 5/3 \approx 1.67$. Rigid diatomic gases (5 degrees of freedom) have $C_v = (5/2)R$, giving $\gamma = 7/5 = 1.40$. Since the denominator C_v is smaller for monatomic gases due to fewer degrees of freedom, γ is larger.

47. Consider the following statements regarding the kinetic theory of gases:
1. It assumes molecules are in continuous, random motion.
 2. It assumes collisions between molecules are perfectly inelastic.
 3. It relates macroscopic properties like pressure to microscopic molecular behavior.
 4. It accurately predicts behavior of real gases at all temperatures and pressures.

Which of the above statements are correct?

- (A) 1 and 3 only
(B) 1, 2, and 3 only
(C) 1, 3, and 4 only
(D) 2 and 4 only

Ans :

(A) 1 and 3 only

Statement 1 is a basic postulate. Statement 3 describes the success of the theory. Statement 2 is incorrect; collisions are assumed perfectly elastic. Statement 4 is incorrect; kinetic theory is for ideal gases and real gases deviate, especially at high pressures and low temperatures.

48. Regarding the law of equipartition of energy, identify the correct statements:
1. It applies only to translational degrees of freedom.
 2. Each vibrational mode contributes $k_B T$ to the average energy.
 3. It predicts that specific heat of solids is approximately $3R$ per mole.

4. It is valid at all temperatures, including very low temperatures.

Which combination of statements is true?

- (A) 1 and 4 only
(B) 2 and 3 only
(C) 1, 2, and 3 only
(D) 2, 3, and 4 only

Ans :

- (B) 2 and 3 only

Statement 1 is false; it applies to translational, rotational, and vibrational modes. Statement 2 is true. Statement 3 is a correct prediction based on the law for solids (Dulong-Petit law). Statement 4 is false; the law fails at low temperatures due to quantum effects.

49. Which of the following statements concerning the mean free path (l) are true?

1. It increases if the number density of molecules decreases.
2. It decreases if the diameter of the gas molecules increases.
3. It is directly proportional to the absolute temperature of the gas.
4. It is the exact distance every molecule travels before a collision.

Select the correct combination:

- (A) 1 and 2 only
(B) 1, 2, and 3 only
(C) 3 and 4 only
(D) 1, 2, and 4 only

Ans :

- (A) 1 and 2 only

Mean free path $l = 1/(\sqrt{2} n \pi d^2)$. Statement 1 is true (inversely proportional to n). Statement 2 is true (inversely proportional to d^2). Statement 3 is false; temperature affects speed, but l depends on n and d . Statement 4 is false; it's an average distance.

50. Match the physical concept in Column I with its corresponding characteristic or formula in Column II.

	Column I		Column II
1.	Boyle's Law	P.	$PV = \mu RT$
2.	Ideal Gas Equation	Q.	Proportional to $\sqrt{T/m}$
3.	RMS speed	R.	$P \propto 1/V$ at const T
4.	Mean Free Path	S.	$1/(n\pi d^2 \sqrt{2})$

- (A) 1-R, 2-P, 3-Q, 4-S
(B) 1-P, 2-R, 3-Q, 4-S

- (C) 1-R, 2-Q, 3-P, 4-S

- (D) 1-S, 2-P, 3-Q, 4-R

Ans :

- (A) 1-R, 2-P, 3-Q, 4-S

Boyle's Law states $P \propto 1/V$ at constant T (1-R). The Ideal Gas Equation is $PV = \mu RT$ (2-P). RMS speed is proportional to $\sqrt{T/m}$ (3-Q). The Mean Free Path formula is $1/(n\pi d^2 \sqrt{2})$ (4-S). These pairings correctly link the concepts with their established definitions and formulas.

51. Match the type of gas molecule in Column I with its predicted molar specific heat at constant volume (C_v) in Column II, ignoring vibrational modes.

	Column I		Column II
1.	Monatomic	P.	$(7/2)R$
2.	Rigid Diatomic	Q.	$(5/2)R$
3.	Non-rigid Diatomic (with vibration)	R.	$(3/2)R$
4.	Solid (per mole of atoms)	S.	$3R$

- (A) 1-R, 2-Q, 3-P, 4-S

- (B) 1-Q, 2-R, 3-P, 4-S

- (C) 1-R, 2-P, 3-Q, 4-S

- (D) 1-P, 2-Q, 3-S, 4-R

Ans :

- (A) 1-R, 2-Q, 3-P, 4-S

For a monatomic gas, $C_v = (3/2)R$ (1-R). For a rigid diatomic gas (3 translational, 2 rotational d.o.f.), $C_v = (5/2)R$ (2-Q). For a non-rigid diatomic gas (adding 1 vibrational mode = 2 d.o.f. for energy calculation, so $5+2=7$ effective $(1/2)RT$ contributions), $C_v = (7/2)R$ (3-P). For a solid, $C_v \approx 3R$ (4-S).

52. Match the scientist in Column I with their significant contribution to the understanding of gases or atomic theory in Column II.

	Column I		Column II
1.	Avogadro	P.	Kinetic theory development, statistical mechanics
2.	Dalton	Q.	Atomic theory explaining chemical laws
3.	Boltzmann	R.	Law of combining gaseous volumes
4.	Gay Lussac	S.	Equal volumes, equal molecules (fixed T,P)

- 82.** What are the two factors on which the degrees of freedom of a gas depends ?

Ans :

The degree of freedom of a gas depends upon (i) atomicity and (ii) temperature.

- 83.** What is the lowest temperature attainable according to Charles' law ?

Ans :

-273.15°C

- 84.** What is one mole of a substance ?

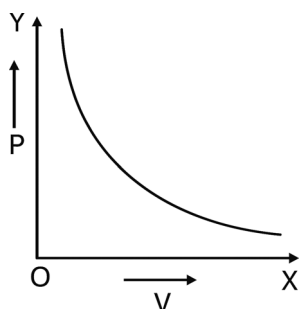
Ans :

One mole of a substance is the amount, which contains its atoms (or molecules) equal to Avogadro number. $(= 6.02 \times 10^{23})$

- 85.** What is the nature of graph between pressure P and volume V for a gas at constant temperature ?

Ans :

The P - V graph of a gas at constant temperature is as shown in figure and it is a rectangular parabola.



- 86.** What is the relation between pressure P and density ρ of a gas at constant temperature ?

Ans :

$$P \propto \rho \text{ (at constant temperature)}$$

i.e., $\frac{P}{\rho} = \text{constant}$

- 87.** What is the value of gas constant R ?

Ans :

$$8.31 \text{ J mol}^{-1}\text{K}^{-1}$$

- 88.** When a gas expands at constant temperature, why does the pressure decrease ?

Ans :

It is because of Boyle's law, according to which,

pressure is inversely proportional to volume at constant temperature.

- 89.** Can the temperature of a gas be increased keeping its pressure and volume constant ?

Ans :

No, we cannot increase the temperature of a gas without changing P or V .

- 90.** What is the physical significance of molar gas constant R ?

Ans :

Molar gas constant is the work done in increasing the temperature of 1 mole of gas through 1K.

- 91.** At what temperature, the gas loses all its energy (i.e. molecular motion ceases)

Ans :

At OK i.e. at -273°C , a gas loses all its energy. (or the molecular motion ceases)

- 92.** Why temperature less than absolute zero is not possible ?

Ans :

Because below absolute zero, pressure and volume of a gas became negative, which is meaningless.

- 93.** When a gas is suddenly compressed temperature rises. Why ?

Ans :

Since kinetic energy of a gas is directly proportional to its temperature, and when a gas is suddenly compressed, its KE increases which raises the temperature of the gas.

- 94.** At 273 K, water solidifies into ice. What happens to the kinetic energy of water molecules ?

Ans :

A part of KE of water molecules is converted into the binding energy of ice.

- 95.** How do you distinguish between average speed and root mean square speed of the molecules of a gas ?

Ans :

Average speed, $v_{av} = \frac{v_1 + v_2 + \dots + v_n}{n}$

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}}$$

Boyle's Law states that for a fixed mass of an ideal gas at constant temperature, the pressure is inversely proportional to its volume ($P \propto 1/V$). Therefore, if the volume is halved, the pressure must double to keep the product PV constant as per requirements.

- 58.** The internal energy of one mole of an ideal monatomic gas is given by $U = (3/2)RT$. This expression implies that internal energy is a function of:

(A) Volume only
(B) Pressure only
(C) Temperature only
(D) Pressure and volume

Ans :

(C) Temperature only

The formula $U = (3/2)RT$ for a monatomic ideal gas clearly shows that the internal energy (U) depends directly on the absolute temperature (T) and the universal gas constant (R). It does not explicitly depend on pressure or volume, which can vary.

- 59.** All of the following are assumptions of the kinetic theory of ideal gases EXCEPT:

(A) Gas molecules are in constant random motion.
(B) There are significant intermolecular attractive forces.
(C) Collisions between molecules are perfectly elastic.
(D) The size of molecules is negligible compared to intermolecular distances.

Ans :

(B) There are significant intermolecular attractive forces.

The kinetic theory of ideal gases assumes that there are no intermolecular forces of attraction or repulsion between gas molecules, except during the brief moments of collision. Real gases do experience these forces, leading to deviations from ideal behavior observed at times.

- 60.** The average kinetic energy of a molecule in an ideal gas is independent of:

(A) The absolute temperature of the gas
(B) The pressure of the gas
(C) The Boltzmann constant value
(D) The nature of the gas

Ans :

(B) The pressure of the gas

The average kinetic energy of a molecule in an ideal gas is given by $(3/2)k_B T$. This expression shows

that it depends on the absolute temperature (T) and the Boltzmann constant (k_B), but it is independent of the pressure, volume, or the specific nature (e.g., mass or type) of the ideal gas molecules themselves.

- 61.** Gas exerts pressure on the walls of the container because:

(a) Gas has weight
(b) Gas molecules have momentum
(c) Gas molecules collide with each other
(d) Gas molecules collide with walls of the container.

Ans :

(d) Gas molecules collide with walls of the container.

When the gas molecules collide with the walls of the container, there is change in momentum and hence pressure is exerted by the gas on the walls of the container.

- 62.** R.M.S. velocity of nitrogen molecules at N.T.P. is :

(a) 33 ms^{-1} (b) 493 ms^{-1}
(c) 517 ms^{-1} (d) 546 ms^{-1}

Ans :

(b) 493 ms^{-1}

$$c = \sqrt{\frac{3RT}{m}}$$

$$= \sqrt{\frac{3 \times 8.31 \times 273}{28 \times 10^{-3}}} = 493 \text{ ms}^{-1}$$

- 63.** For a gas, the r.m.s. speed at 800 K :

(a) Four times the value at 200 K
(b) Half the value at 200 K
(c) Twice the value at 200 K
(d) Same as at 200 K

Ans :

(c) Twice the value at 200 K

$$\frac{c_2}{c_1} = \sqrt{\frac{800}{200}} = 2$$

$$c_2 = 2c_1$$

- 64.** The mean translation kinetic energy of a perfect gas molecule at temperature T is (k = Boltzmann constant):

(a) $\frac{1}{2} kT$ (b) kT
(c) $\frac{3}{2} kT$ (d) $2kT$

Ans :

(c) $\frac{3}{2} kT$

65. According to kinetic theory of gases the absolute temperature of a gas is directly related to the average of :

- (a) Internal energy of gas molecules
- (b) Kinetic energy of gas molecules
- (c) Potential energy of gas molecules
- (d) Gravitational energy of gas molecules

Ans :

- (b) Kinetic energy of gas molecules

66. The rms speed of the molecules of a gas in a vessel is 400 ms^{-1} . If half of the gas leaks out at constant temperature, the rms speed of the remaining molecules will be:

- (a) 800 ms^{-1}
- (b) $400\sqrt{2} \text{ ms}^{-1}$
- (c) 400 ms^{-1}
- (d) 200 ms^{-1}

Ans :

- (c) 400 ms^{-1}

Since,
$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

So it depends only on T and is independent of P .

67. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ?

- (a) 80 K
- (b) -73 K
- (c) 3 K
- (d) 20 K

Ans :

- (d) 20 K

$$\sqrt{\frac{3kT}{MH}} = \sqrt{\frac{3R(47 + 273)}{M_O}}$$

So
$$T = \frac{320 M_H}{M_O} = \frac{320 \times 1}{16} = 20 \text{ K}$$

68. The kinetic energy of 1 g for hydrogen at 27°C will be:

- (a) $1.87 \times 10^3 \text{ J}$
- (b) $1.57 \times 10^3 \text{ J}$
- (c) $1.81 \times 10^3 \text{ J}$
- (d) $1.73 \times 10^3 \text{ J}$

Ans :

- (a) $1.87 \times 10^3 \text{ J}$

$$E = \frac{3}{2} \times \frac{RT}{T} = \frac{3}{2} \times \frac{8.31 \times 300}{2} = 1.87 \times 10^3 \text{ J}$$

69. The rate of diffusion is :

- (a) Faster in solids than in liquids and gases.
- (b) Faster in liquids, than in solids and gases.

- (c) Equal to solids, liquids and gases.
- (d) Faster in gases than in liquids and solids.

Ans :

- (d) Faster in gases than in liquids and solids.

Since,
$$r \propto \frac{1}{\rho}$$

As ρ is least in gases than in liquids and solids, hence rate of diffusion r is faster in gases than in liquids and solids.

70. The unit of universal gas constant in S.I. is :

- (a) Calories per degree celsius
- (b) joule per mole
- (c) $\frac{\text{joule}}{\text{K} \times \text{mole}}$
- (d) joule per k.g.

Ans :

- (c) $\frac{\text{joule}}{\text{K} \times \text{mole}}$

71. The r.m.s. velocity of gas molecules is 300 ms^{-1} . The r.m.s. velocity of molecules of gas with twice the molecular weight and half the absolute temperature is:

- (a) 300 ms^{-1}
- (b) 600 ms^{-1}
- (c) 75 ms^{-1}
- (d) 150 ms^{-1}

Ans :

- (d) 150 ms^{-1}

in 1st case
$$c = \sqrt{\frac{3RT}{M}}$$

in 2nd case
$$c' = \sqrt{\frac{3R\left(\frac{T}{2}\right)}{2M}} = \frac{1}{2} \sqrt{\frac{3RT}{M}} = \frac{c}{2} = \frac{300}{2} = 150$$

72. If the pressure in a closed vessel is reduced by drawing out some gas, the mean free path of the molecules:

- (a) is decreased
- (b) is increased
- (c) remains unchanged
- (d) increases or decreases according to the nature of the gas.

Ans :

- (b) is increased

On reducing the pressure, the gas molecules will find more distance to travel before colliding with other molecules and hence their mean free path increases.

73. At the same temperature the mean kinetic energies of molecules of hydrogen and oxygen are in the ratio:

- (a) 1 : 1 (b) 1 : 16
(c) 8 : 1 (d) 16 : 1

Ans :

- (a) 1 : 1

Since, $E = \frac{3}{2}kT$

So for same temperature the mean kinetic energy per molecule E is independent of mass of the gas molecule.

74. Temperature of a gas is a measure of :

- (a) The average kinetic energy of the gaseous molecules.
(b) The average potential energy of the gaseous molecules.
(c) The average distance between the molecules of the gas.
(d) The size of the molecules of the gas.

Ans :

- (a) The average kinetic energy of the gaseous molecules.

75. If the mass of all molecules of a gas are halved and their speeds doubled then the ratio of initial and final pressure will be:

- (a) 2 : 1 (b) 1 : 2
(c) 4 : 1 (d) 1 : 4

Ans :

- (b) 1 : 2

In 1st case $P_1 = \frac{1}{3} \frac{mnc^2}{v}$

In 2nd case $P_2 = \frac{1}{3} \frac{\left(\frac{m}{2}\right)n(2c)^2}{v}$

$$= 2 \frac{1}{3} \frac{mnc^2}{v} = 2P_1$$

$$\frac{P_1}{P_2} = \frac{1}{2}$$

ONE MARK QUESTIONS

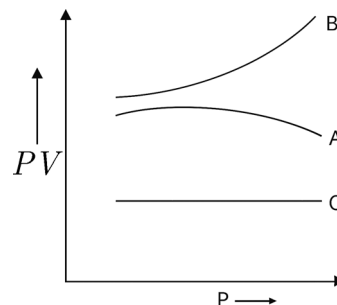
76. The temperature of a gas increases on heating. Explain it on the basis of kinetic theory of gases.

Ans :

Since $v_{rms} \propto \sqrt{T}$. So when a gas is heated, the root

mean square velocity of its molecules increase, hence the temperature of the gas is also increased.

77. The given figure shows the variation of the product PV with respect to the pressure (P) of given masses of three gases A, B and C. The temperature is kept constant. State with proper arguments which of these gases is ideal.



Ans :

Gas C is ideal, because PV is constant for this gas and the gas C obeys Boyle's law at all pressures.

78. On what factors does the average kinetic energy of gas molecules depend : Nature of the gas, temperature, volume ?

Ans :

Since $E_{av} \propto T$, so the average kinetic energy of the molecules depends only on temperature of the gas.

79. The ratio of vapour densities of two gases at the same temperature is 8 : 9. Compare the r.m.s. velocities of their molecules.

Ans :

Since, $\frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}$
 $= \sqrt{\frac{9}{8}} = 3:2\sqrt{2}$

80. What is average velocity of the molecules of an ideal gas ?

Ans :

Since the molecules of an ideal gas passes all types of velocities in random directions, hence their average velocity is zero.

81. Sometimes a cycle with well inflated tyre left in the sun has its tube burst open. Why ?

Ans :

Due to sun, the pressure of air inside the tube increases due to increase in temperature.

- (A) 1-S, 2-Q, 3-P, 4-R
- (B) 1-S, 2-P, 3-Q, 4-R
- (C) 1-R, 2-Q, 3-S, 4-P
- (D) 1-Q, 2-S, 3-P, 4-R

Ans :

- (A) 1-S, 2-Q, 3-P, 4-R

Avogadro proposed that equal volumes of gases at the same temperature and pressure contain equal numbers of molecules (1-S). Dalton developed the atomic theory to explain laws of definite and multiple proportions (2-Q). Boltzmann was a key figure in developing kinetic theory and statistical mechanics (3-P). Gay Lussac formulated the law of combining gaseous volumes (4-R).

53. Identify the correct pair from the following regarding degrees of freedom and energy contribution per mole according to equipartition theorem:
- (A) Rotational (linear molecule) : 3 degrees, $(3/2)RT$ energy
 - (B) Vibrational mode : 1 degree, $(1/2)RT$ energy
 - (C) Translational : 3 degrees, $(3/2)RT$ energy
 - (D) Rotational (non-linear molecule) : 2 degrees, RT energy

Ans :

- (C) Translational : 3 degrees, $(3/2)RT$ energy

A molecule has 3 translational degrees of freedom, and each contributes $(1/2)RT$ to the internal energy per mole. Thus, the total translational energy per mole is $3 \times (1/2)RT = (3/2)RT$. Other pairs are incorrect regarding the number of degrees or the energy contributed according to equipartition.

54. Which of the following gas properties and their typical behavior relationship is correctly stated?
- (A) Pressure : Inversely proportional to absolute temperature at constant volume.
 - (B) Volume : Directly proportional to pressure at constant temperature (Boyle's Law).
 - (C) Number of molecules : Directly proportional to volume at constant temperature and pressure (Avogadro's Law).
 - (D) RMS speed : Inversely proportional to the square root of molar mass at constant pressure.

Ans :

- (C) Number of molecules : Directly proportional to volume at constant temperature and pressure (Avogadro's Law).

Avogadro's Law states that equal volumes of all gases, at the same temperature and pressure, have the same number of molecules. This implies a direct

proportionality between volume and the number of molecules (or moles) when temperature and pressure are held constant for all comparisons.

55. Identify the incorrect pair relating a scientist and their associated concept or law from the options given.
- (A) Robert Boyle : $PV = \text{constant}$ for fixed mass and temperature of gas.
 - (B) Jacques Charles : $V/T = \text{constant}$ for fixed mass and pressure of gas.
 - (C) John Dalton : Total pressure of gas mixture is sum of partial pressures of components.
 - (D) Amedeo Avogadro : All atoms are indivisible and identical for a given element.

Ans :

- (D) Amedeo Avogadro : All atoms are indivisible and identical for a given element.

While Avogadro made significant contributions with his hypothesis on equal volumes and molecules, the statement about atoms being indivisible and identical for an element is more directly a central tenet of John Dalton's atomic theory, not Avogadro's primary law, which focuses more on molecules.

56. The kinetic theory of gases assumes that the time spent in a collision between molecules is:
- (A) Very long compared to the time between collisions
 - (B) Equal to the time between collisions always
 - (C) Negligible compared to the time between collisions
 - (D) Dependent on the temperature of the gas

Ans :

- (C) Negligible compared to the time between collisions

One of the simplifying assumptions in the kinetic theory of ideal gases is that the duration of a collision between molecules is very short, or negligible, when compared to the average time interval a molecule spends traveling freely between successive collisions, thus making it simpler.

57. If the volume of an ideal gas is halved at constant temperature, its pressure according to Boyle's Law will:

- (A) Be halved
- (B) Remain unchanged
- (C) Be doubled
- (D) Be quadrupled

Ans :

- (C) Be doubled

96. What is law of equipartition of energy ?

Ans :

It states that in equilibrium, the total energy of the system is equally divided among all the degrees of freedom and energy associated with each degree of freedom is $\frac{1}{2}kT$.

97. A glass of water is stirred and then allowed to stand until water stops moving. What has happened to kinetic energy of the moving water ?

Ans :

The kinetic energy of moving water is dissipated into internal energy. The temperature of water thus increases.

98. Define mean free path of a molecule.

Ans :

Mean free path of a molecule is the average distance travelled by a molecule between two successive collisions.

TWO MARKS QUESTIONS

99. What are the different methods of increasing the number of molecular collisions per second in a gas ?

Ans :

- By increasing the number of molecules.
- By increasing the temperature of the gas.
- By decreasing the volume of the container in which the gas is enclosed.

100. Gases show large deviation from ideal behaviour at low temperature and high pressure. Explain.

Ans :

Because at low temperature and high pressure,

- the interatomic attraction becomes appreciable and
- size of the molecules is not negligible small as compared to the distance between them.

Hence the gases show large deviation from ideal behaviour.

101. In the light of kinetic theory of gases, why pressure of container increases when the gas is heated ?

Ans :

When temperature of a gas is increased, the number of collisions made by molecules per second on the

walls of the container also increased, hence pressure on the wall also increased.

102. What is Boltzmann constant. What is its value in SI units ?

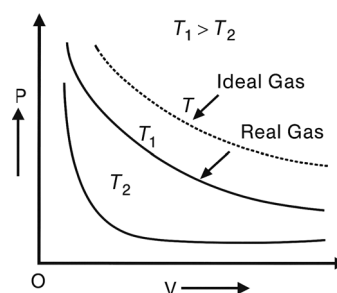
Ans :

The ratio of gas constant and Avogadro's number is called Boltzmann constant

$$\begin{aligned} \text{i.e., } k_B &= \frac{R}{N} = \frac{8.31 \text{ J mol}^{-1} \text{K}^{-1}}{6.023 \times 10^{23} \text{ mole}^{-1}} \\ &= 1.38 \times 10^{-23} \text{ J K}^{-1} \end{aligned}$$

103. Draw P - V curves showing deviations from ideal behaviour for a given mass of a gas for two different temperatures.

Ans :



104. The volume of a gas sample is increased. Why does the pressure which is exerted by the gas decrease ?

Ans :

On increasing the volume of a gas sample, the pressure exerted by the gas decreases due to following reasons :

- The molecules have to travel a longer distance between collisions on the wall of the container.
- These collisions are distributed over a large area.

105. Why cooling is caused by evaporation ?

Ans :

From kinetic theory of gases, we find that the molecules of a liquid are always in random motion, and the molecules near the surface of a liquid have enough energy so as to overcome the inter-molecular attraction of the other molecules on the surface and escape from the surface. Thus the liquid is left with molecules having lower speeds i.e. average kinetic energy of the remaining liquid decreases, thus the temperature of the liquid decreases. So we find that cooling is caused by evaporation.

106. Explain the basis of kinetic theory of gases.

Ans :

Kinetic theory explains the behaviour of gases by assuming that gases consist of a large number of rapidly moving atoms or molecules. These particles are so far apart that inter-atomic forces, important for solids and liquids, can be neglected.

107. Describe Richard Feynman's "Atomic Hypothesis".

Ans :

Feynman's Atomic Hypothesis states that all things are made of atoms. These are little particles that move around in perpetual motion, attracting each other when slightly apart but repelling upon being squeezed into one another.

108. How did John Dalton explain the laws of definite and multiple proportions?

Ans :

Dalton suggested that the smallest constituents of an element are atoms. Atoms of one element are identical but differ from those of other elements. A small number of atoms of each element combine to form a molecule of a compound.

109. State Avogadro's law and its relation to Gay Lussac's law.

Ans :

Avogadro's law states that equal volumes of all gases at equal temperature and pressure have the same number of molecules. This law, when combined with Dalton's atomic theory, explains Gay Lussac's law regarding the volumes of combining gases.

110. Compare the interatomic distances in solids, liquids, and gases.

Ans :

In solids, atoms are spaced about a few angstroms apart. In liquids, the separation is similar, but atoms are not rigidly fixed. In gases, interatomic distances are much larger, typically tens of angstroms.

111. What is meant by 'mean free path' for gas molecules?

Ans :

The mean free path is the average distance a molecule in a gas can travel without colliding with another molecule. In gases, this distance is of the order of thousands of angstroms, allowing considerable free movement.

112. Describe the nature of interatomic forces between gas molecules.

Ans :

Interatomic forces in gases are generally negligible due to large distances between molecules. The forces are short-range, involving attraction at a few angstroms and repulsion when molecules are squeezed closer.

113. Why are properties of gases easier to understand than those of solids and liquids?

Ans :

Properties of gases are simpler because gas molecules are far from each other, and their mutual interactions are negligible except during collisions. This allows for simpler models of their behaviour.

114. Explain the ideal gas law, $PV = \mu RT$.

Ans :

The ideal gas law states that for a given amount of gas, the product of its pressure (P) and volume (V) is proportional to its absolute temperature (T). Here, μ is the number of moles and R is the universal gas constant.

115. How is Boltzmann constant (k_B) related to the universal gas constant (R)?

Ans :

The Boltzmann constant (k_B) is the universal gas constant (R) divided by Avogadro's number (N_A). It represents the constant of proportionality in the ideal gas law when expressed in terms of the number of molecules.

116. What does Avogadro's hypothesis imply about the number of molecules in equal volumes of gases?

Ans :

Avogadro's hypothesis implies that at the same temperature and pressure, equal volumes of all gases contain the same number of molecules. This is justified by kinetic theory.

117. Under what conditions does a real gas behave like an ideal gas? Explain why.

Ans :

A real gas behaves like an ideal gas at low pressures and high temperatures. Under these conditions, molecules are far apart, and molecular interactions become negligible, similar to an ideal gas.

THREE MARKS QUESTIONS

- 118.** What is an ideal gas ? Why do the real gases show deviations from ideal behaviour ? Show these deviations graphically.

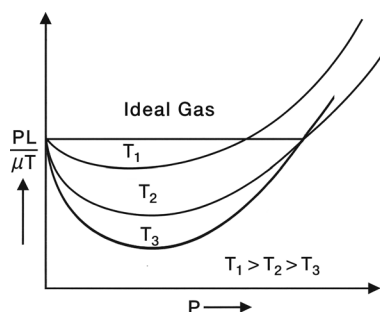
Ans :

Ideal gas. A gas which obeys the ideal gas equation,

$$PV = \mu RT$$

at all temperatures and pressures is called an ideal gas. In an ideal gas (i) the size of the molecule is negligibly small and (ii) there is no force of interaction among the molecules of the gas.

But no real gas possesses these conditions and the behaviour of real gas differs from that of an ideal gas. But at high temperature and low pressure some real gases do behave like an ideal gas.



Graphical representation

The given figure shows a graph between $\frac{PV}{\mu T}$ and P for three temperatures T_1 , T_2 and T_3 ($T_1 > T_2 > T_3$). From the graph we find that for an ideal gas,

$$\frac{PV}{\mu T} = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

But here we find departures from ideal gas behaviour of three gases. But we find that all of them approach the ideal gas behaviour for low pressures and high temperatures.

- 119.** At what temperature, gas molecules are devoid of all motions ?

Ans :

Since average kinetic energy per molecule of a gas is given by

$$\frac{1}{2} mc^2 = \frac{3}{2} kT$$

$$c^2 \propto T$$

If $T = 0$, $c = 0$

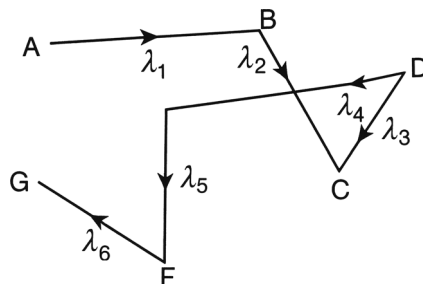
Thus at absolute zero, molecules of a gas stop moving.

- 120.** What do you mean by mean free path of a gas molecule ?

Ans :

Mean free path of a gas molecule.

It is defined as the average distance travelled by a molecule between two successive collisions with each other and is denoted by λ .



If a molecule starts from A and moving along AB, suffers collisions at B, C, D and so on having free paths,

$\lambda_1, \lambda_2, \lambda_3, \dots$ as shown in the figure, then

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots}{\text{Total number of collisions}}$$

- 121.** Obtain the dimensional formula for R used in the ideal gas equation $PV = RT$.

Ans :

Since,

$$PV = RT$$

So

$$R = \frac{PV}{T} = \frac{FV}{AT}$$

$$R = \frac{[MLT^{-2}][L^3]}{[L^2][K]}$$

$$R = [ML^2T^{-2}K^{-1}]$$

- 122.** Outline the key contributions of ancient Indian scholars to the atomic hypothesis.

Ans :

Ancient Indian scholars, particularly in the Vaiseshika school founded by Kanada, developed a detailed atomic picture.

- They proposed that atoms (Paramanu) were eternal, indivisible, and the ultimate parts of matter.
- They postulated four kinds of atoms: Bhoomi (Earth), Ap (water), Tejas (fire), and Vayu (air), each with characteristic mass and attributes.
- They also conceptualized that atoms combine to form molecules, with properties depending on the constituent atoms.

- 123.** Explain John Dalton's atomic theory and its relation to laws of chemical combination.

Ans :

John Dalton proposed his atomic theory about 200 years ago to explain chemical laws.

- He suggested that the smallest constituents of an element are atoms.
- Atoms of one element are identical but differ from those of other elements.
- A small number of atoms of each element combine to form a molecule of the compound, explaining the laws of definite and multiple proportions.

- 124.** Describe the typical sizes and separations of atoms/molecules in solids, liquids, and gases.

Ans :

The molecular nature of matter varies significantly across different states.

- The size of an atom is about an angstrom (10^{-10} m).
- In solids, atoms are tightly packed, spaced about a few angstroms apart.
- In liquids, the separation is similar, but atoms can move around.
- In gases, interatomic distances are much larger, typically tens of angstroms, with mean free paths being even larger.

- 125.** Define translational, rotational, and vibrational degrees of freedom for a molecule.

Ans :

Degrees of freedom specify the number of independent ways a molecule can possess energy.

- **Translational degrees of freedom** correspond to motion of the molecule's center of mass from one point to another, typically three for motion in space.
- **Rotational degrees of freedom** correspond to the rotation of the molecule about its center of mass around independent axes.
- **Vibrational degrees of freedom** correspond to the oscillation of atoms within the molecule relative to each other, each involving kinetic and potential energy.

- 126.** Calculate the molar specific heat at constant volume (C_v) and constant pressure (C_p) for a monatomic ideal gas.

Ans :

A monatomic gas molecule has 3 translational degrees of freedom.

According to the law of equipartition of energy, the average energy per molecule is $3 \times (1/2) k_B T$.

The total internal energy per mole is

$$U = (3/2) N_A k_B T = (3/2) RT.$$

Thus, $C_v = (dU/dT) = (3/2) R$.

Using $C_p - C_v = R$, we get

$$C_p = (3/2) R + R = (5/2) R.$$

- 127.** Calculate the molar specific heat at constant volume (C_v) and constant pressure (C_p) for a rigid diatomic ideal gas.

Ans :

A rigid diatomic molecule has 3 translational and 2 rotational degrees of freedom (total 5).

The average energy per molecule is $5 \times \frac{1}{2} k_B T$.

The total internal energy per mole is

$$U = (5/2) N_A k_B T = (5/2) RT.$$

Thus, $C_v = (dU/dT) = (5/2) R$.

Using $C_p - C_v = R$, we get

$$C_p = (5/2) R + R = (7/2) R.$$

- 128.** How does the specific heat capacity of solids relate to the law of equipartition of energy?

Ans :

For a solid, atoms vibrate about their mean positions in three dimensions.

- Each atom has 3 vibrational modes, and each mode has two squared terms (kinetic and potential energy) contributing to internal energy.
- By the law of equipartition, average energy per atom is $3 \times 2 \times (1/2) k_B T = 3k_B T$.
- For one mole (N_A atoms), total internal energy $U = 3N_A k_B T = 3RT$.
- The molar specific heat capacity at constant volume is

$$C_v \approx \frac{dU}{dT} = 3R.$$

- 129.** Define mean free path and derive an approximate expression for it.

Ans :

The mean free path (λ) is the average distance a molecule travels between successive collisions.

- Consider a molecule of diameter 'd' moving with average speed $\bar{v}$. In time Δt , it sweeps a collision volume $\pi d^2 \bar{v} \Delta t$.
- If 'n' is the number of molecules per unit volume, the number of collisions is $n\pi d^2 \bar{v} \Delta t$.
- The average time between collisions

$$\tau = \frac{1}{(n\pi\bar{v}d^2)}.$$

Thus,

$$\lambda = \bar{v}\tau = \frac{1}{(n\pi d^2)}.$$

A more exact treatment gives

$$\lambda = \frac{1}{(\sqrt{2} n\pi d^2)}.$$

FIVE MARKS QUESTIONS

- 130.** Derive relation between pressure and kinetic energy.
or

Give the Kinetic Interpretation of Temperature.

Ans :

Kinetic Interpretation of temperature.

Consider 1 mole of gas at temperature T .

If M is the molecular weight of the gas and V is its molar volume then, according to kinetic theory of gases, we have

$$P = \frac{1}{3}\rho C^2$$

But,

$$\rho = \frac{M}{V}$$

Hence,

$$P = \frac{1}{3}\frac{M}{V}C^2$$

i.e.,

$$PV = MC^2$$

According to ideal gas equation for one mole of the gas,

$$PV = RT$$

Hence,

$$RT = \frac{1}{3}MC^2$$

$$\frac{1}{2}MC^2 = \frac{3}{2}RT \quad \dots(1)$$

But $\frac{1}{2}MC^2$ denotes the kinetic energy (due to translational disorderly motion) of one mole of the gas, so that

$$\text{Kinetic energy per mole (E)} = \frac{3}{2}RT \quad \dots(2)$$

and Average kinetic energy of random motion per molecule

$$= \frac{3}{2}\frac{R}{N}T \quad \dots(3)$$

i.e. Average kinetic energy per molecule

$$= \frac{3}{2}kT \quad \dots(4)$$

Where, $k = \frac{R}{N} = \frac{8.31 \text{ mole}^{-1}\text{K}^{-1}}{6.023 \times 10^{23}}$

This constant (k) is called Boltzmann constant.

Its value is $3.38 \times 10^{-16} \text{ erg molecule}^{-1}\text{K}^{-1}$ or $1.38 \times 10^{-23} \text{ joule molecule}^{-1}\text{K}^{-1}$.

It is clear from equation (i) that kinetic energy (of random motion) per mole is proportional to absolute temperature of the gas. From equation (ii) we can conclude that kinetic energy (of random motion) per molecule is also proportional to the absolute temperature. Thus kinetic energy of random motion varies directly as the absolute temperature of gas. This is called kinetic interpretation of temperature. According to this, molecular motion of an ideal gas should cease altogether at 0 K.

- 131.** (a) State and explain Boyle's law, Charles' law.
(b) Deduce perfect gas law.

Ans :

- (a) Boyle's law. It states that temperature remaining constant, volume of a given mass of a gas is inversely proportional to the pressure applied.

i.e., $P \propto \frac{1}{V}$ (at constant temperature)

$$PV = \text{constant}$$

Figure(a) shows the variation of pressure with volume of 1 mole of steam at different temperatures.

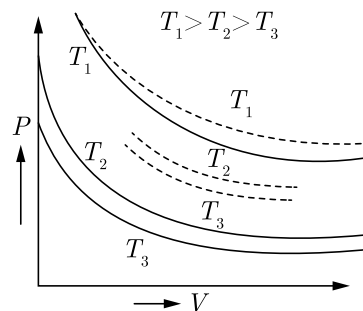


Figure: (a)

The solid lines are experimental P - V curves for steam at the temperatures compared with Boyle's law shown by dotted lines. P is in unit of atm and V is in units of litres. From the graph we find that at high temperature and low pressure, the agreement between the observed pressure-volume relationship and Boyle's law is in good agreement.

If n = number of molecules per unit volume, then

$$\frac{P}{n} = \text{constant}$$

At low densities molecules are far apart on the average, so that any effect due to forces

between them are small. Thus all gases should behave alike in this region.

As the temperature is decreases, gas tends to become liquid.

Charles' law. It states that the pressure remaining constant, volume of given mass of gas is directly proportional to absolute temperature.

i.e., $V \propto T$

$$\frac{V}{T} = \text{constant}$$

Figure (b) shows the comparison between experimental T - V curves (solid lines) and the theoretical curves predicted by Charles' law (dotted lines).

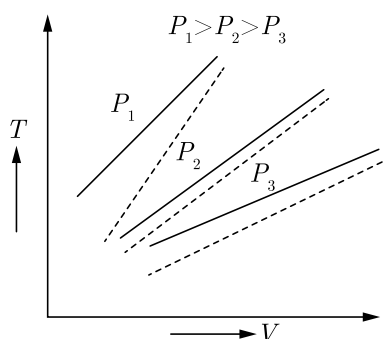


Figure: (b)

The graph shows that the behaviour of a real gas approaches the ideal gas behaviour for low pressures and high temperatures.

(b) Perfect gas law

From Boyle's law,

$$PV = \text{constant} \quad \dots(1)$$

From Charles' law,

$$\frac{V}{T} = \text{constant} \quad \dots(2)$$

Combining Equations (1) and (2), we get

$$\frac{PV}{T} = \text{constant}$$

The value of constant depends upon the mass of gas. If μ is the number of moles

$$\text{Then, } PV = \mu RT \quad \dots(3)$$

where R is a constant called universal gas constant and $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.

Equation (3), which relates P , V and T of gas is called Equation of state.

132. (a) What is kinetic theory of gases ? Give its postulates.
(b) Give qualitative definition of pressure on the basis of kinetic theory of gases.
(c) Derive expression for pressure exerted by gas.

Ans :

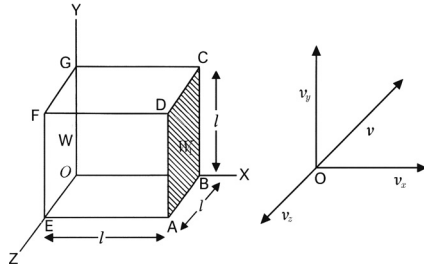
- (a) Kinetic theory of gases : A theory which explains physical properties like temperature, heat, pressure of gas is called kinetic theory of gases.

Assumptions of Kinetic Theory of Gases

1. A gas consists of a very large number of minute particles, called molecules. These are perfectly elastic spheres which are identical in all respects for a given gas and are different for different gases.
2. The molecules of a gas are in a state of continuous, rapid and random motion. They move in all directions with different speeds.
3. The size of the gas molecules is very small as compared to the distance between them.
4. The molecules do not exert any force of attraction or repulsion on each other, except during collision.
5. The collision of the molecules with themselves and with the walls of the vessel are perfectly elastic.
6. The molecular density is uniform throughout the gas.
7. Collisions are instantaneous.

- (b) Pressure due to an Ideal Gas : The molecules of a gas are in a state of continuous, rapid, random motion. Therefore, the molecules often collide against the walls of the container. As the collisions are perfectly elastic, the molecules rebound with exactly the same speed, with which they strike the walls of the container. One c.c. of a gas under normal conditions contains about 2.68×10^{19} molecules. Evidently, billions of molecules would strike against the wall of the container every second. The blows exerted by such a large number of molecules combine into a steady force, which is responsible for the pressure exerted by the gas.

- (c) Expression for Pressure of a Gas : Consider a vessel, having each side of length l . Its edges are parallel to three Cartesian co-ordinate axes OX , OY and OZ as shown in the figure . It contains n molecules of a gas, each molecule being of mass m .



Let v be velocity of any molecule at any instant and v_x , v_y and v_z be the components of velocity v along x , y and z axes respectively.

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

This molecule moves with momentum mv_x along x -axis having momentum mv_x . After it has suffered an elastic collision with wall W_1 (ABCD) its momentum becomes $-mv_x$ (as direction of motion is reversed).

Hence, Change in momentum of the molecule

$$= -mv_x - mv_x = -2mv_x \quad \dots(1)$$

From law of conservation of linear momentum, The momentum imparted to the wall of the cube in one collision $= +2mv_x$

Let us find the number of molecules hitting the face ABCD in time Δt .

A molecule with velocity v_x hit the wall ABCD in time Δt , if it is within a distance $v_x \Delta t$ from the wall. So all the molecules lying in the volume $A v_x \Delta t$ can hit the wall in time Δt .

Hence, Total number of gas molecules contained in volume $A v_x \Delta t = (A v_x \Delta t) n$

(n is the total number of molecules for unit volume)

Half of these molecules are moving along $+ve$ X -axis and other half along $-ve$ X -axis.

Hence, Effective number of molecules hitting the planar wall in time $\Delta t = \frac{1}{2} A v_x \Delta t n$,

So total momentum transferred to the wall in time Δt

$$= \left(\frac{1}{2} A v_x \Delta t n \right) 2mv_x \\ = mn A v_x^2 \Delta t$$

Since rate of change of momentum is called force.

Hence, Force exerted by the gas molecules along x -axis

Pressure exerted by gas molecules along x -axis

$$P_x = \frac{F_x}{A} = m A v_x^2$$

Since all molecules have different velocities, so we can replace v_x by $\bar{v}_x$, called mean square component speeds

$$\text{Hence, } P_x = mn \bar{v}_x^2$$

Similarly pressure exerted by gas molecules along Y and Z axes are

$$P_y = mn \bar{v}_y^2, P_z = mn \bar{v}_z^2$$

As there is no preferred direction (i.e. gases exert equal pressure in all directions)

$$\text{Hence, } P_x = P_y = P_z = P (\text{say})$$

Hence,

$$P_x + P_y + P_z = 3P$$

$$P = \frac{1}{3} (P_x + P_y + P_z)$$

$$= \frac{1}{3} = mn (\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2)$$

where $c^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$ is the mean square speed of the gas molecules.

Since,

mn = mass of one molecule

$\times$ number of molecules per unit volume

= total mass per unit volume

$$= \frac{M}{V}$$

$$\text{Hence, } P = \frac{1}{3} \frac{M}{V} c^2$$

But $\frac{M}{V}$, density of gas (ρ)

$$\text{Hence, } P = \frac{1}{3} \rho c^2 \quad \dots(4)$$

If $V = 1$ unit

$$\text{From (3), } P = \frac{1}{3} M c^2 = \frac{2}{3} \times \frac{1}{2} \times M c^2$$

$$P = \frac{2}{3} E$$

Thus pressure exerted by gas is $\frac{2}{3}$ of translational K.E. per unit volume of gas.

133. What is meant by degrees of freedom ? Find degrees of freedom for mono, di and tri-atomic gas molecule.

Ans :

The total number of independent quantities or coordinates required to describe the configuration of a dynamic system is called its degrees of freedom.

1. A particle moving along a straight line can be described by its displacement along the axis of motion, so it has one degree of freedom.
2. A particle moving in a plane can be described by its displacement along two axis, so it has two degrees of freedom.
3. Similarly, a particle moving in space has three degrees of freedom.

The number of degrees of freedom of a dynamic

- (ii) The value of Avogadro's number can also be calculated from Brownian motion of the particles.
- (iii) The value of N can also be found from the study of crystals. The distance between the two consecutive atoms of a crystal may be determined by diffraction of X-rays by the crystal. If the arrangement of atoms in the crystal is known, we may calculate the number of atoms in a given volume of the crystal, from where the number of atoms in one mole of the crystal can be calculated.

Using equation (i),

$$N = \frac{12}{m_c}$$

where m_c = mass of carbon atom (12_C) in g.

The absolute mass of a carbon atom can be measured by using a mass spectrograph and its value is 2.0×10^{-26} kg

$$\begin{aligned} \text{i.e.,} \quad m_c &= 2.0 \times 10^{-26} \text{ kg} \\ &= 2.0 \times 10^{-23} \text{ g} \end{aligned}$$

Hence,

$$\text{Avogadro's number} = \frac{12}{2 \times 10^{-23}} = 6 \times 10^{+23}$$

More accurate measurements give the value of N as 6.0225×10^{23} for Avogadro number. It is noted that Avogadro number is independent of element considered.

Importance of Avogadro Number.

Avogadro no. is useful for making certain calculations as discussed below :

- (i) It helps us to determine the mass of an atom or the mass of a molecule as given by the relation:
Mass of an atom (in grams)

$$= \frac{\text{Mass of a gram atom}}{\text{Avogadro number}}$$

and mass of a molecule (in grams)

$$= \frac{\text{Mass of a mole in gram}}{\text{Avogadro Number}}$$

- (ii) It helps us to determine the number of molecules or atoms in a given mass of an element.

Number of atoms

$$= \frac{\text{Avogadro number} \times \text{Mass of the element in grams}}{\text{Mass of a gram atom}}$$

Similarly, Number of molecules

$$= \frac{\text{Avogadro Number} \times \text{Mass of the element in grams}}{\text{Gram molecular mass}}$$

- (iii) It helps us to calculate the number of molecules present in a given volume of gas at S.T.P., using the relation.

$$\frac{\text{Number of molecules in the given volume}}{\text{Avogadro's number}}$$

$$= \frac{\text{Volume of the gas at S.T.P. (in litres)}}{22.4 \text{ litres}}$$

- (iv) It enables us to calculate the size of the atom or a molecule e.g. let us calculate the size of a molecule of water. As the density of water is 1 gm/cc., the volume of 18 g of water is 18 c.c. But 18 gm is one mole of water and, therefore, contains 6.023×10^{23} molecules.

Hence, Volume occupied by one molecule of water

$$\begin{aligned} &= \frac{18}{6.023 \times 10^{23}} \text{ c.c.} \\ &= 2.988 \times 10^{-23} \text{ c.c.} \end{aligned}$$

If we assume the molecules of the water to be spherical, the volume $= \frac{4}{3}\pi r^3$, where r is the radius of the molecule.

$$\text{Thus, } \frac{4}{3}\pi r^3 = 2.988 \times 10^{-23}$$

$$r^3 = \frac{2.988 \times 3}{4 \times \pi} \times 10^{-23}$$

$$r = \left(\frac{2.988 \times 3 \times 10^{-23}}{4\pi} \right)^{\frac{1}{3}}$$

$$= 1.925 \times 10^{-8} \text{ cm}$$

$$= 1.925 \text{ \AA}, \text{ where } 1 \text{ \AA} = 10^{-8} \text{ cm}$$

- 136.** State and explain the law of equipartition of energy of a dynamic system and use it to find the value of the ratio of the two specific heats of a monoatomic, a diatomic and a triatomic gas molecule.

Ans :

Law of Equipartition of Energy

It states that for any dynamical system in thermal equilibrium, the total energy is equally distributed among all the degrees of freedom and energy associated with each degree of freedom is $\frac{1}{2} kT$. (k is Boltzmann constant).

Consider a monoatomic gas. The kinetic energy of its molecule is due to translational motion only. From kinetic theory of gases,

$$\frac{1}{2} m v^2 = \frac{3}{2} kT \quad \dots(1)$$

If v_x , v_y and v_z are rectangular components of velocity of any molecule having velocity v , the

$$= n \times \text{volume of cylinder}$$

$$= n\pi d^2 v$$

This is equal to number of collisions per second.

Mean free path,

$$\lambda = \frac{\text{Total distance travelled in 1 second}}{\text{Total number of collisions in 1 second}}$$

$$\lambda = \frac{v}{\pi d^2 v n} = \frac{1}{\pi d^2 n}$$

$$\lambda = \frac{1}{\pi d^2 n} \quad \dots(1)$$

Equation (1) is approximate relation for the mean free path because it is based on the assumption that the moving molecules colliding with stationary molecules.

If we take into consideration the motion of all the molecules, the expression for mean free path becomes

$$\lambda = \frac{1}{\sqrt{2} n \pi d^2} \quad \dots(2)$$

Discussion

(i) From equation (2), we have

$$\lambda \propto \frac{1}{d^2}$$

i.e. larger the size of the molecule, smaller will be its mean free path.

(ii) If m is the mass of the molecule, then $mn = \rho$, the density of the gas

$$n = \frac{\rho}{m}$$

So equation (2) becomes

$$\lambda = \frac{m}{\sqrt{2} \rho n d^2} \quad \dots(3)$$

$$\lambda \propto \frac{1}{\rho}$$

i.e. larger the density of gas, smaller will be the mean free path.

(iii) For 1 mole of a perfect gas, we have

$$PV = RT$$

$$\frac{PV}{N} = \frac{R}{N} T = kT$$

$$\frac{P}{n} = kT \quad \left[\text{Since, } \frac{N}{V} = n \right]$$

So from equation (1), we have

$$\lambda = \frac{kT}{\sqrt{2} \pi P d^2} \quad \dots(4)$$

i.e.,
$$\lambda \propto \frac{1}{P}$$

i.e. higher the pressure of gas, lesser will be mean free path

(iv) Also from equation (4),

$$\lambda \propto T$$

i.e. higher the temperature of gas, greater will be mean free path.

135. Explain the term Avogadro's number and mention some methods for determination of Avogadro's number. What is significance of Avogadro's number?

Ans :

Avogadro's number (or Avogadro constant)

Avogadro's number may be defined as the number of molecules in one mole of a substance. The constant has the value 6.023×10^{23} . We can also express Avogadro's number in terms of the number of atoms in one gram-atom of a substance, where one gram atom of a substance is as many grams of a substance as its atomic weight. The atomic mass of a substance is also compared with the mass of a carbon atom, taken as 12 a.m.u., it can be shown that

$$= \frac{\text{gram atom of a substance}}{\text{mass of one atom}}$$

$$= \frac{\text{gram molecule of a substance}}{\text{mass of one molecule}}$$

$$= N \text{ (Avogadro's number.)}$$

Since the molecular mass is directly proportional to the atomic mass.

Thus 1 gram-atom of hydrogen contains 6.023×10^{23} atom and their mass is 1 g. Similarly, 1 mole of hydrogen contains 6.023×10^{23} molecules and their mass is 2 g.

Determination of Avogadro's number

(i) We know that :

Avogadro's number,

$$N = \frac{M}{m} \quad \dots(1)$$

where M is the mass of 1 gram molecule of the substance and m is its molecular mass. The absolute masses (m) of the molecules of different substances can be measured by an instrument called mass spectrometer. Thus N can be calculated

e.g. mass of hydrogen molecule,

$$m = 3.32 \times 10^{-27} \text{ kg}$$

$$= 3.32 \times 10^{-24} \text{ g}$$

1 mole of hydrogen = 2 g

Hence,
$$N = \frac{2}{3.32 \times 10^{-24}} = \frac{20 \times 10^{23}}{3.32}$$

$$= 6.023 \times 10^{23}$$

system can be obtained by subtracting the number of independent relations from the total number of coordinates required to specify the positions of constituent particles of the system.

If a system has N particles and the particles possess k independent relations between them, then number of degrees of freedom.

$$f = 3N - k \text{ e.g.,}$$

- (i) For mono-atomic gas molecule

Given, $N = 1, k = 0$

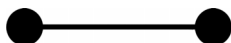
$$f = 3.1 - 0 = 3$$

So, a monoatomic gas molecule has three degrees of freedom.

- (ii) For diatomic gas molecule

Given, $N = 2, k = 1$

Hence, $f = 3.2 - 1 = 5$



So a diatomic gas molecule has five degrees of freedom.

- (iii) For triatomic gas molecule

- (a) In linear molecule

Given, $N = 3, k = 2$

$$f = 3.3 - 2 = 7$$



Figure: (a)

- (b) In non-linear molecule

Given, $N = 3, k = 3$

$$f = 3.3 - 3 = 6$$

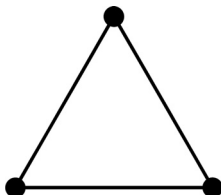


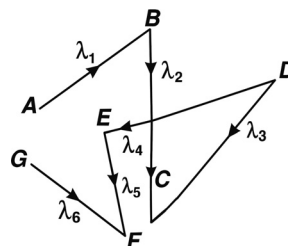
Figure: (b)

- 134.** What is mean free path of a gas molecule ? Show that the mean free path is inversely proportional to the pressure of the gas. Does the mean free path depend upon the temperature of the gas ?

Ans :

Mean free path of a gas molecule. According to kinetic theory of gases, the molecules of gases behave as hard spheres of diameter $\cong 2.5 \times 10^{-10} m$ and have finite size. These molecules are in a state of random motion and are frequently colliding with each other. Between two successive collisions a molecule travels in a straight line with uniform velocity called free path. The average of free paths taken over a long interval of time is called mean free path.

The mean free path can also be defined as the average distance travelled by a molecule between two successive collisions with each other and is denoted by λ .



If a molecule starting from A and moving along AB, suffer collisions at B, C, D and so on having free paths,

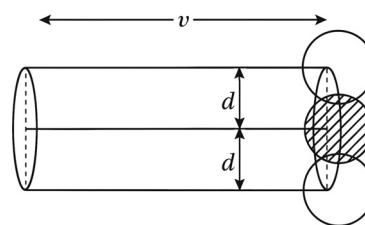
$\lambda_1, \lambda_2, \lambda_3, \dots$ as shown in Fig. 13.18, then Mean free path,

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots}{\text{Total number of collisions}}$$

Expression for mean free path

To simplify the expression, we suppose that

- (i) Only one molecule under consideration is in motion and other molecules are at rest.



- (ii) Each molecule has a sphere of influence equal to the diameter of the molecule around it.

Let a molecule A be moving with velocity v so in one second it will collide with all those molecules the centre of which lie within a cylinder of radius d and length v

$$\text{Volume of the cylinder} = \pi d^2 v$$

If n is the number of molecules per c.c. then
No. of molecules in the cylinder

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

So Equation (1) becomes

$$\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 = \frac{3}{2}kT \quad \dots(2)$$

Since the molecular motion is random, so same mean amount of the kinetic energy should be associated in each direction.

$$\text{i.e.,} \quad \frac{1}{2}mv_x^2 = \frac{1}{2}mv_y^2 = \frac{1}{2}mv_z^2 \quad \dots(3)$$

From Equation (2) and (3), we get

$$\frac{1}{2}mv_x^2 = \frac{1}{2}mv_y^2 = \frac{1}{2}mv_z^2 = \frac{1}{2}kT$$

Since factors $\frac{1}{2}mv_x^2$, $\frac{1}{2}mv_y^2$ and $\frac{1}{2}mv_z^2$ stand for energy associated with a particular degree of freedom, so, Mean Kinetic energy per molecule per degree of freedom $= \frac{1}{2}kT$.

This is known as Boltzmann's law of equipartition of energy.

Ratio of specific heats of a gas

(1) For mono-atomic gas : A mono-atomic gas has three degrees of freedom i.e. $f=3$

$$\begin{aligned} \text{K.E. per molecule per degree of freedom} \\ = \frac{1}{2}kT \end{aligned}$$

$$\begin{aligned} \text{K.E. per molecule with three degrees of freedom} \\ = \frac{3}{2}kT \end{aligned}$$

Total energy with one mole of a gas

$$U = \frac{3}{2}NkT = \frac{3}{2}RT$$

[Hence, $Nk = R$, gas constant]

$$\text{So} \quad C_v = \frac{dU}{dT} = \frac{3}{2}R$$

$$\text{Hence, } C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$$

$$\text{And} \quad \gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.67$$

(2) For diatomic gas. For diatomic gas, $f=5$

$$\text{Hence, } U = \frac{5}{2}RT$$

$$\text{So} \quad C_v = \frac{dU}{dT} = \frac{5}{2}R$$

$$\text{Hence, } C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$$

$$\text{And} \quad \gamma = \frac{C_p}{C_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.4$$

(3) For triatomic gas

(a) For linear molecule

$$f = 7$$

$$\text{Since, } U = \frac{7}{2}RT$$

$$\text{So} \quad C_v = \frac{dU}{dT} = \frac{7}{2}R$$

$$\text{Hence, } C_p = C_v + R = \frac{7}{2}R + R = \frac{9}{2}R$$

$$\text{And} \quad \gamma = \frac{C_p}{C_v} = \frac{9}{7} = 1.28$$

(b) For non-linear molecule

$$f = 6$$

$$\text{Since, } U = \frac{6}{2}RT = 3RT$$

$$\text{So} \quad C_v = \frac{dU}{dT} = 3R$$

$$\text{Hence, } C_p = C_v + R = 3R + R = 4R$$

$$\text{And} \quad \gamma = \frac{4R}{3R} = \frac{4}{3} = 1.33$$

137. Explain the law of equipartition of energy and discuss its application to monatomic gases.

Ans :

The law of equipartition of energy is a principle of classical statistical mechanics concerning energy distribution in a system at thermal equilibrium.

- **Statement:** In thermal equilibrium at absolute temperature T , the total energy of a system is equally distributed among all its possible energy modes (degrees of freedom), with each mode having an average energy of $\frac{1}{2}k_B T$.
- Each quadratic term in the energy expression (e.g., $(1/2)mv_x^2$ or $(1/2)I\omega^2$) corresponds to one degree of freedom.

Application to Monatomic Gases:

- A monatomic gas molecule has 3 translational degrees of freedom (for motion along x, y, and z axes).
- Therefore, the average energy per molecule is $3 \times (1/2)k_B T = (3/2)k_B T$.
- The total internal energy of one mole of a monatomic gas is

$$U = N_A \times (3/2)k_B T = (3/2)RT.$$

138. Apply the law of equipartition of energy to determine the molar specific heats (C_v and C_p) for a rigid diatomic gas.

Ans :

For a rigid diatomic gas, the law of equipartition of energy helps predict its molar specific heats.

- A rigid diatomic molecule (like O_2 or N_2) has:
- 3 translational degrees of freedom.
- 2 rotational degrees of freedom (rotation about two axes perpendicular to the line joining the atoms; rotation along the line has negligible moment of inertia).
- Total degrees of freedom $= 3 + 2 = 5$.

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- 147.** A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

Ans :

From kinetic theory of gases, pressure exerted by a gas is :

$$P = \frac{1}{3} \rho C^2 = \frac{1}{2} \frac{M}{V} C^2$$

As temperature T is kept constant, therefore, C^2 is constant. Also, V is constant

Hence, $P \propto M$

$$\frac{P_2}{P_1} = \frac{M_2}{M_1}$$

$$\frac{P_2}{76} = \frac{\left(M_1 + \frac{50}{100} M_1\right)}{M_1} = \frac{3}{2}$$

$$P_2 = \frac{3}{2} \times 76 = 114 \text{ cm of Hg.}$$

- 148.** Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at NTP. Take the size of an oxygen molecule to be roughly $3 \text{ \AA}$.

Ans :

Radius of the oxygen molecule,

$$r = \frac{3}{2} \text{ \AA} = 1.5 \times 10^{-10} \text{ m}$$

Volume of the molecules in one mole of oxygen gas

$$\begin{aligned} &= \left(\frac{4}{3} \pi r^3\right) N \\ &= \frac{4\pi}{3} \times (1.5 \times 10^{-10})^3 \times 6.0225 \times 10^{23} \\ &= 8.5 \times 10^{-6} \text{ m}^3 = 8.5 \times 10^{-3} \text{ litre} \end{aligned}$$

[Actual volume occupied

by oxygen molecules
Now $\frac{\text{Volume of oxygen molecules}}{\text{Volume of oxygen gas at NTP}}$

$$= \frac{8.5 \times 10^{-3} \text{ litre}}{22.4 \text{ litre}} = 3.8 \times 10^{-4}$$

- 149.** A gas has molar heat capacity $C = 37.55 \text{ J mole}^{-1} \text{ K}^{-1}$, in the process $PT = \text{constant}$. Find the number of degrees of freedom of the molecules of the gas.

Ans :

Given, $C = 37.55 \text{ J mole}^{-1} \text{ K}^{-1}$;

and $PT = K$ (constant) ... (1)

According to standard gas equation

$$PV = RT$$

$$P = \frac{RT}{V}$$

From (1), $\frac{RT}{V} \times T = K$

$$V = \frac{RT^2}{K}$$

Hence, $\frac{dV}{dT} = \frac{2RT}{K}$

But $\frac{T}{K} = \frac{1}{P}$ from (1)

Hence, $\frac{dV}{dT} = \frac{2R}{P}$... (2)

As $C = C_v + P \frac{dV}{dT}$, therefore,

using (2), $C = C_v + P \times \frac{2R}{P} = C_v + 2R$

$$C_v = C - 2R$$

As, $C_v = \frac{n}{2} R$

Hence, $\frac{n}{2} R = C - 2R$

$$\begin{aligned} n &= \frac{2(C - 2R)}{R} \\ &= \frac{2(37.55 - 2 \times 8.3)}{8.3} = 5 \end{aligned}$$

- 150.** How can γ be determined from the number of degrees of freedom ?

or

Prove that $\gamma = 1 + \frac{2}{n}$ where n is number of degrees of freedom.

Ans :

Let a gas have n degrees of freedom.

Energy per molecule per degree of freedom

$$= \frac{1}{2} kT$$

Energy per molecule for n degrees of freedom

$$= \frac{n}{2} kT$$

$$\text{Energy per mole} = \frac{n}{2} kTN = \frac{n}{2} \frac{R}{N} TN = \frac{n}{2} RT$$

$$C_v = \frac{d}{dT} \left(\frac{n}{2} RT \right) = \frac{n}{2} R$$

$$C_p = C_v + R$$

$$= \frac{n}{2} R + R = \left(\frac{n}{2} + 1 \right) R$$

$$\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{n}{2} + 1 \right) R}{\frac{n}{2} R} = 1 + \frac{2}{n}$$

$$\gamma = 1 + \frac{2}{n}$$

ordinary temperatures. So, there are 2 rotational degrees of freedom. Each contributes $\frac{1}{2}k_B T$.

Vibrational Degrees of Freedom (Non-Rigid):

- If the molecule is not rigid, the atoms can vibrate along the internuclear axis. This vibrational mode has two energy terms: kinetic energy and potential energy. Each contributes $\frac{1}{2}k_B T$, so one vibrational mode contributes $k_B T$ to the average energy.

143. Derive the relation $C_p - C_v = R$ for an ideal gas.

Ans :

The relation $C_p - C_v = R$ connects the molar specific heats at constant pressure (C_p) and constant volume (C_v) for an ideal gas, where R is the universal gas constant.

- From the first law of thermodynamics, for one mole, $\Delta Q = \Delta U + P\Delta V$.

At constant volume ($\Delta V = 0$):

- Heat supplied $\Delta Q_v = \Delta U$.
- So, $C_v = (\Delta Q/\Delta T)_v = (\Delta U/\Delta T)_v$.
Since U for an ideal gas depends only on T , we can write

$$C_v = \frac{dU}{dT}.$$

At constant pressure :

$$\begin{aligned} C_p &= (\Delta Q/\Delta T)_p \\ &= (\Delta U/\Delta T)_p + P(\Delta V/\Delta T)_p. \end{aligned}$$

- Since U depends only on T for an ideal gas,
 $\left(\frac{\Delta U}{\Delta T}\right)_p = \frac{dU}{dT} = C_v$.
- For one mole of an ideal gas, $PV = RT$. At constant pressure, $P\Delta V = R\Delta T$, so
 $P\left(\frac{\Delta V}{\Delta T}\right)_p = R$.
- Substituting these into the equation for C_p :
 $C_p = C_v + R$, which gives $C_p - C_v = R$.

NUMERICAL QUESTIONS

144. If the density of nitrogen at N.T.P. is 0.00125 g cm^3 , calculate the root mean square velocity of its molecules.

Ans :

$$\begin{aligned} P &= 76 \text{ cm of Hg} = 1.013 \times 10^5 \text{ Nm}^{-2} \\ \rho &= 0.00125 \text{ g cm}^{-3} \\ &= 1.25 \text{ kg m}^{-3} \end{aligned}$$

Since, $C = \sqrt{\frac{3P}{\rho}}$

so
$$\begin{aligned} C &= \sqrt{\frac{3 \times 1.013 \times 10^5}{1.25}} \\ &= 4.93 \times 10^2 \text{ m s}^{-1} \end{aligned}$$

145. Calculate r.m.s. velocity of nitrogen molecules at 27°C and 76 cm of mercury pressure.

Ans :

Here $P = 76 \text{ cm Hg}$

$$\begin{aligned} &= 0.76 \times 13.6 \times 10^3 \times 9.8 \times 1 \text{ m}^{-2} \\ &= 1.013 \times 10^5 \text{ Nm}^{-2} \\ T &= 27^\circ\text{C} = 27 + 273 = 300 \text{ K} \\ R &= 8.31 \text{ J mol}^{-1} \text{ K}^{-1} \\ M &= 28 \text{ g} = 0.028 \text{ kg} \end{aligned}$$

So
$$\begin{aligned} C &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{0.028}} \\ &= 5.2 \times 10^2 \text{ m s}^{-1} \end{aligned}$$

146. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27°C . How much heat has to be transferred to the gas to double the r.m.s. velocity of its molecules ?

Ans :

As
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

i.e.,
$$v_{\text{rms}} \propto \sqrt{T}$$

So, to double rms speed, the temperature must be increased to four times.

Initial temperature = $27^\circ\text{C} = 300 \text{ K}$

Final temperature = $300 \times 4 = 1200 \text{ K}$

As volume of vessel is constant,

$$W = \int PdV = 0$$

Change in internal energy = $nC_v \Delta T$

No. of moles,
$$n = \frac{0.014 \times 10^3}{28} = 0.5$$

C_v for diatomic gas = $\frac{5}{2}R$

Hence,
$$\begin{aligned} \Delta U &= nC_v \Delta T \\ &= 0.5 \times \frac{5}{2}R \times (1200 - 300) \\ &= 0.5 \times \frac{5}{2} \times 8.3 \times 900 \\ &= 9337.5 \text{ joules} \\ &= \frac{9337.5}{4.2} \text{ cal} = 2223.2 \text{ cal.} \end{aligned}$$

- According to equipartition, the average energy per molecule is $5 \times (1/2) k_B T = (5/2) k_B T$.
- The total internal energy of one mole is

$$U = N_A \times (5/2) k_B T = (5/2) RT.$$

- The molar specific heat at constant volume is $C_v = dU/dT = (5/2) R$.
- The molar specific heat at constant pressure is $C_p = C_v + R = (5/2) R + R = (7/2) R$.
- The ratio of specific heats is $\gamma = C_p/C_v = (7/2) R / (5/2) R = 7/5 = 1.40$.

- 139.** Discuss the specific heat capacity of solids based on the law of equipartition of energy.

Ans :

The law of equipartition of energy can be applied to estimate the specific heat capacity of solids.

- In a solid, atoms vibrate about their mean equilibrium positions in three dimensions.
- Each atom can be considered as an oscillator with three vibrational modes, one for each dimension.
- Each vibrational mode has two degrees of freedom: one for kinetic energy (e.g., $(1/2) mv_x^2$) and one for potential energy (e.g., $(1/2) kx^2$).
- Thus, each atom has $3 \times 2 = 6$ degrees of freedom, each contributing $\frac{1}{2} k_B T$ to the average energy.
- Average energy per atom

$$= 6 \times (1/2) k_B T = 3 k_B T.$$

- For one mole of a solid (N_A atoms), the total internal energy is $U = N_A \times 3 k_B T = 3RT$.
- The molar specific heat at constant volume (since ΔV is negligible for solids) is $C \approx dU/dT = 3R$. This is known as the Dulong-Petit law.

- 140.** Define the mean free path of a gas molecule and derive the expression $l = 1/(\sqrt{2} n \pi d^2)$.

Ans :

The mean free path (λ) is the average distance a molecule in a gas travels between two successive collisions.

- Consider a molecule of diameter 'd' moving with average speed $\bar{v}$. It will collide with any molecule whose center is within a cylinder of radius 'd' (cross-sectional area πd^2) around its path. In time Δt , it sweeps a volume $\pi d^2 \bar{v} \Delta t$.
- If 'n' is the number of molecules per unit volume, the number of collisions is $n \pi d^2 \bar{v} \Delta t$.
- The rate of collisions is $n \pi d^2 \bar{v}$, so the average time between collisions

$$\tau = \frac{1}{(n \pi \bar{v} d^2)}.$$

- An initial approximation for ' λ ' is

$$\bar{v} \tau = \frac{1}{(n \pi d^2)}.$$

- This derivation assumes other molecules are at rest. When their motion is considered, the relative velocity becomes important. A more exact treatment, accounting for the relative velocities of all molecules, introduces a factor of $\sqrt{2}$, leading to $\lambda = \frac{1}{(\sqrt{2} n \pi d^2)}$.

- 141.** Explain how the kinetic theory of gases accounts for Boyle's law and Charles' law.

Ans :

The kinetic theory of gases provides a microscopic basis for the empirical gas laws.

Boyle's Law (PV = constant at constant T):

- From kinetic theory, $P = \frac{1}{3} n m \bar{v}^2$. Also, the average kinetic energy $\frac{1}{2} m \bar{v}^2$ is proportional to T.
- So, $\frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$. Thus, $m \bar{v}^2 = 3 k_B T$.
- Substituting this into the pressure equation, $P = (1/3) n (3 k_B T) = n k_B T$.
- Since $n = \frac{N}{V}$ (number of molecules N in volume V), $P = (N/V) k_B T$, so $PV = N k_B T$.
- If N (amount of gas) and T are constant, then PV is constant.

Charles' Law (V/T = constant at constant P):

- From $PV = N k_B T$, if P and N are constant, then

$$\frac{V}{T} = \frac{N k_B}{P}, \text{ which is a constant.}$$

- 142.** Discuss the concept of degrees of freedom for diatomic molecules, considering both rigid and non-rigid (vibrating) cases.

Ans :

Degrees of freedom for a diatomic molecule depend on whether it's treated as rigid or allowed to vibrate.

Translational Degrees of Freedom:

- Like any molecule, a diatomic molecule has 3 translational degrees of freedom, corresponding to motion of its center of mass along the x, y, and z axes. Each contributes $\frac{1}{2} k_B T$ to average energy.

Rotational Degrees of Freedom (Rigid Rotator):

- A diatomic molecule can rotate about two independent axes perpendicular to the line joining the two atoms. Rotation about the internuclear axis has negligible moment of inertia and is usually not considered active at

CASE BASED QUESTIONS

- 151.** A sealed container of fixed volume V contains N molecules of an ideal gas at a temperature T and pressure P . The gas molecules are in incessant random motion and undergo elastic collisions with each other and with the walls of the container. The kinetic theory provides a model to relate these microscopic properties to the macroscopic pressure. Questions:

1. What are the basic assumptions of the kinetic theory for an ideal gas regarding molecular interactions and collisions?
2. How is the pressure exerted by the gas on the walls of the container explained by the kinetic theory?
3. The pressure P of an ideal gas is given by

$$P = \frac{1}{3} n m \bar{v}^2.$$

What do 'n', 'm', and $\bar{v}^2$ represent in this equation?

4. If the number of molecules (N) in the container is doubled, keeping the volume V and temperature T constant, how would the pressure P change according to the ideal gas equation ($PV = Nk_B T$)? How does this relate to the kinetic theory formula for pressure?

Ans :

1. Kinetic theory assumes that interaction between molecules is negligible (they move freely) except during collisions. All collisions between molecules or with walls are elastic.
2. Pressure is explained as the result of molecules colliding with the walls of the container, transferring momentum to the walls. The force on the wall is the rate of momentum transfer, and pressure is this force per unit area.
3. In the equation $P = \frac{1}{3} n m \bar{v}^2$:
'n' is the number of molecules per unit volume (number density).
'm' is the mass of a single molecule.
 $\bar{v}^2$ is the mean of the squared speeds of the molecules.
4. According to $PV = Nk_B T$, if N is doubled while V and T are constant, the pressure P must also double. From $P = \frac{1}{3} n m \bar{v}^2$, if N doubles, n (number density N/V) also doubles. If T is constant, $\bar{v}^2$ (which is related to T by $\frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$) remains constant. Thus, pressure P also doubles, consistent with the ideal gas equation.

- 152.** A flask contains a mixture of two ideal gases, Argon (monatomic, atomic mass = 39.9 u) and Chlorine (diatomic, molecular mass = 70.9 u), at a room temperature of 27°C (300 K). The molecules of both gases are in random motion.

Questions:

1. According to the kinetic theory, what does the absolute temperature of an ideal gas measure at the molecular level?
2. Will the average translational kinetic energy per molecule be different for Argon and Chlorine in this mixture? Why or why not?
3. Calculate the root mean square (rms) speed of an Argon atom at 300 K. ($k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$, $1u = 1.66 \times 10^{-27} \text{ kg}$).
4. Will the rms speed of a Chlorine molecule be greater than, less than, or equal to the rms speed of an Argon atom in this mixture? Explain your reasoning.

Ans :

1. The absolute temperature of an ideal gas is a measure of the average translational kinetic energy of its molecules.
2. No, the average translational kinetic energy per molecule will be the same for both Argon and Chlorine because it depends only on the temperature of the gas ($\frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$) and is independent of the nature of the ideal gas.
3. Mass of Argon atom

$$m_{Ar} = 39.9u = 39.9 \times 1.66 \times 10^{-27} \text{ kg}$$

$$\approx 6.62 \times 10^{-26} \text{ kg}.$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ J K}^{-1} \times 300 \text{ K}}{6.62 \times 10^{-26} \text{ kg}}}$$

$$\approx \sqrt{\frac{1242 \times 10^{-23}}{6.62 \times 10^{-26}}}$$

$$\approx \sqrt{187.6 \times 10^3} \approx \sqrt{187600} \approx 433 \text{ m/s}.$$

4. The rms speed of a Chlorine molecule will be less than that of an Argon atom. Since $\frac{1}{2} m v_{rms}^2$ is the same for both, and a Chlorine molecule ($m_{Cl_2} = 70.9u$) is heavier than an Argon atom ($m_{Ar} = 39.9u$), the Chlorine molecule must have a smaller v_{rms} .

- 153.** Consider a diatomic gas like Oxygen (O_2) at a moderate temperature. Its molecules can undergo translational motion, rotational motion (about axes perpendicular to the line joining the atoms), and at higher temperatures, vibrational motion along the interatomic axis.

Questions:

1. What is the law of equipartition of energy?

Ans :

(a) Given, $x(t) = A \cos(\omega t + \phi)$

At $t = 0, x = 1 \text{ cm}$

$$1 = A \cos(\omega \cdot 0 + \phi) = A \cos \phi$$

$$A \cos \phi = 1 \quad \dots(1)$$

And $v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$

At $t = 0, v = \omega \text{ cm s}^{-1}$

$$\omega = -A\omega \sin(\omega \cdot 0 + \phi) = -A\omega \sin \phi$$

$$A \sin \phi = -1 \quad \dots(2)$$

From (1) and (2), we have

$$A^2(\cos^2 \phi + \sin^2 \phi) = 1^2 + (-1)^2 = 2$$

$$A = \sqrt{2} \text{ cm} \quad \dots(3)$$

Also $\frac{A \sin \phi}{A \cos \phi} = -1$

$$\tan \phi = -1$$

$$\tan \phi = -\tan \frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right)$$

$$\phi = \frac{3\pi}{4}$$

(b) Given $x = B \sin(\omega t + a)$

At $t = 0, x = 1 \text{ cm}$

So $1 = B \sin(\omega \cdot 0 + a)$

$$B \cos a = 1 \quad \dots(4)$$

And $v = \frac{dx}{dt} = B\omega \cos(\omega t + a)$

At $t = 0, v = \omega \text{ cm s}^{-1}$

$$\omega = B\omega \cos(\omega \cdot 0 + a)$$

$$B \cos a = 1 \quad \dots(5)$$

From (4) and (5), we have

$$B^2(\cos^2 a + \sin^2 a) = 1^2 + 1^2$$

$$B = \sqrt{2} \text{ cm}$$

Also $\frac{B \sin a}{B \cos a} = 1$

$$\tan a = 1$$

$$\tan a = \tan \frac{\pi}{4}$$

$$a = \frac{\pi}{4}$$

8. A spring balance has a scale that reads 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Ans :

Given, $m = 50 \text{ kg}, x = 20 \text{ cm} = 0.20 \text{ m}$

$$M = ?$$

$$g = 9.8 \text{ ms}^{-1}$$

$$T = 0.60 \text{ s}$$

$$T = 2\pi \sqrt{\frac{Mx}{mg}}$$

$$\begin{aligned} M &= \frac{T^2 mg}{4\pi^2 x} \\ &= \frac{(0.60)^2 \times 50 \times 9.8}{4 \times (3.14)^2 \times 0.20} \\ &= 22.38 \text{ kg} \end{aligned}$$

$$\text{Weight of the body} = 22.38 \times 9.8 = 219 \text{ N}$$

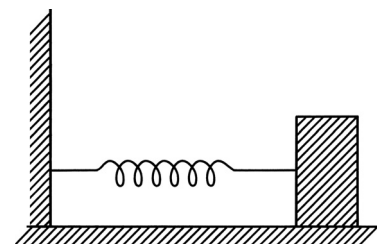
9. A spring of force constant 1200 Nm^{-1} is mounted on a horizontal table as shown in figure. A mass of 3.0 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.

Determine

- the frequency of oscillations,
- maximum acceleration of the mass, and
- the maximum speed of the mass.

Ans :

(i) $T = 2\pi \sqrt{\frac{m}{\kappa}} = 2\pi \sqrt{\frac{3}{1200}} = \frac{\pi}{10} \text{ s}$



Frequency, $v = \frac{1}{T} = \frac{10}{\pi} \text{ s}^{-1} = 3.19 \text{ s}^{-1}$

(ii) $\omega = 2\pi v = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20 \text{ s}^{-1}$

Maximum acceleration,

$$\alpha_{\max} = \omega^2 A = (20)^2 (0.02) = 8.0 \text{ m s}^{-2}$$

(iii) Maximum speed,

$$v_{\max} = \omega A = (20) (0.02) = 0.4 \text{ m s}^{-1}$$

4. If the temperature of the water is increased, the water molecules would move faster and have higher average kinetic energy (since temperature is a measure of average kinetic energy of molecules). This would lead to more energetic and frequent collisions with the pollen grains, causing their Brownian motion to become more vigorous or agitated.

156. Air molecules at standard temperature and pressure (STP) move at high speeds, yet the smell of perfume from one corner of a room takes some time to reach the other corner. This is because molecules undergo frequent collisions with each other, and the average distance a molecule travels between collisions is its mean free path.

Questions:

1. Define the mean free path (l) of a molecule in a gas.
2. On what factors does the mean free path depend, according to the formula

$$l = 1/(\sqrt{2} n \pi d^2)?$$

3. If the number density 'n' of gas molecules in a container is halved (e.g., by increasing volume at constant T, or by removing half the gas), how does the mean free path change?
4. The mean free path for air molecules at STP to be about 2.9×10^{-7} m (or $1500d$ where d is molecular diameter). How does this compare to the typical size of an air molecule (diameter $\approx 2\text{\AA}$)? Does this support the assumption in ideal gas theory that molecules travel freely for significant distances between collisions?

Ans :

1. The mean free path is the average distance a molecule can travel without colliding.
2. The mean free path ' l ' depends inversely on the number density 'n' (number of molecules per unit volume) and inversely on the square of the molecular diameter ' d '.
3. Since $l \propto 1/n$, if the number density 'n' is halved, the mean free path ' l ' will be doubled.
4. The molecular diameter

$$d \approx 2\text{\AA} = 2 \times 10^{-10} \text{ m.}$$

The mean free path $l \approx 2.9 \times 10^{-7}$ m.

The ratio

$$l/d \approx (2.9 \times 10^{-7}) / (2 \times 10^{-10})$$

$$= 1.45 \times 10^3 = 1450.$$

The mean free path is about 1500 times the molecular diameter. This large ratio supports the ideal gas assumption that molecules travel freely for significant distances relative to their size before colliding.

- 157.** A vessel of fixed volume V contains a mixture of μ_1 moles of Neon (gas 1) and μ_2 moles of Oxygen (gas 2) at a temperature T . The gases are non-reactive. The total pressure exerted by the mixture is P .

Questions:

1. State Dalton's law of partial pressures.
2. What is meant by the "partial pressure" of a gas in a mixture?
3. If P_1 is the partial pressure of Neon and P_2 is the partial pressure of Oxygen, how is the total pressure P related to P_1 and P_2 ?
4. If the temperature T and volume V are the same for both gases, how is the ratio of their partial pressures (P_1/P_2) related to the ratio of their number of moles (μ_1/μ_2)?

Ans :

1. Dalton's law of partial pressures states that the total pressure of a mixture of ideal gases is the sum of their partial pressures.
2. The partial pressure of a gas in a mixture is the pressure it would exert at the same conditions of volume and temperature if it alone occupied the vessel.
3. The total pressure $P = P_1 + P_2$.
4. Since $P_1 V = \mu_1 RT$ and $P_2 V = \mu_2 RT$, dividing these equations gives $P_1/P_2 = \mu_1/\mu_2$.

- 158.** The density of liquid water is 1000 kg/m^3 . The density of water vapour at 100°C and 1 atm pressure is 0.6 kg/m^3 . We want to estimate the fraction of the molecular volume to the total volume occupied by water vapour under these conditions.

Questions:

1. If the density of water molecules themselves is assumed to be roughly the same as bulk liquid water, what does the large difference in density between liquid water and water vapour imply about the spacing between molecules in the vapour phase?
2. For a given mass of water, how many times larger is the volume occupied by water vapour compared to liquid water under the given conditions?
3. If the fraction of molecular volume to total volume in the liquid state is approximately 1 (molecules closely packed), what is the estimated fraction of molecular volume to total volume in the vapour state?
4. Why is the actual volume occupied by molecules much smaller than the total volume of a gas at STP, supporting a key assumption of kinetic theory?

Ans :

1. The large difference in density implies that the spacing between molecules in the vapour phase

- How many translational degrees of freedom does an O_2 molecule have? How many rotational degrees of freedom does it have if treated as a rigid rotator?
- According to the law of equipartition of energy, what is the average energy associated with each translational degree of freedom? What about each rotational degree of freedom?
- If the O_2 molecule also has one mode of vibration, how many “squared terms” does this vibrational mode contribute to the total energy expression, and what is the average energy associated with this vibrational mode at temperature T ?

Ans :

- The law of equipartition of energy states that in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2}k_B T$.
- An O_2 molecule has 3 translational degrees of freedom. As a rigid rotator, it has 2 rotational degrees of freedom.
- The average energy associated with each translational degree of freedom is $\frac{1}{2}k_B T$. The average energy associated with each rotational degree of freedom is also $\frac{1}{2}k_B T$.
- One vibrational mode contributes two squared terms (one for kinetic energy and one for potential energy) to the total energy expression. The average energy associated with this vibrational mode is

$$2 \times \frac{1}{2}k_B T = k_B T.$$

- 154.** Helium (He) is a monatomic gas, and Nitrogen (N_2) is a diatomic gas. We want to compare their molar specific heat capacities at constant volume (C_v) based on kinetic theory, ignoring vibrational modes for Nitrogen.

Questions:

- For a monatomic ideal gas, what is the expression for the total internal energy (U) of one mole in terms of R and T ?
- How is the molar specific heat at constant volume (C_v) related to the internal energy U ?
- Calculate the predicted C_v for one mole of Helium. What is its value for Nitrogen (treated as a rigid rotator)?
- Why is the molar specific heat capacity (C_v) generally higher for diatomic gases like Nitrogen compared to monatomic gases like Helium at the same temperature (ignoring vibration)?

Ans :

- For one mole of a monatomic ideal gas, the total internal energy $U = \frac{3}{2}RT$.
- The molar specific heat at constant volume C_v is given by

$$C_v = \frac{dU}{dT}.$$

- For Helium (monatomic, 3 translational degrees of freedom): $U = \frac{3}{2}RT$, so $C_v = \frac{3}{2}R$.
For Nitrogen (diatomic rigid rotator, 3 translational + 2 rotational = 5 degrees of freedom): $U = \frac{5}{2}RT$, so $C_v = \frac{5}{2}R$.
- Diatomic gases have more degrees of freedom (translational + rotational) than monatomic gases (only translational). According to the law of equipartition of energy, each degree of freedom contributes to the internal energy. Therefore, more heat is required to raise the temperature of a diatomic gas by one unit, resulting in a higher C_v .

- 155.** In 1827, Robert Brown observed that pollen grains suspended in water execute a continuous, random, zigzag motion. This phenomenon, known as Brownian motion, provided early evidence for the molecular nature of matter.

Questions:

- What is the basic idea of the “Atomic Hypothesis” as described by Feynman?
- How does the kinetic theory explain the pressure exerted by a gas?
- Based on the kinetic theory, what causes the random, zigzag motion of the pollen grains (Brownian motion)?
- If the temperature of the water is increased, how would you expect the Brownian motion of the pollen grains to change? Explain using the kinetic interpretation of temperature.

Ans :

- Feynman’s Atomic Hypothesis states: “All things are made of atoms - little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.”
- Kinetic theory explains gas pressure as the result of continuous bombardment of the container walls by rapidly moving gas molecules.
- The random, zigzag motion of pollen grains is caused by the unequal and random bombardment by the much smaller, rapidly moving molecules of the surrounding fluid (water) [This is a standard explanation, the chapter implies it with “molecules ... in incessant random motion”].

CHAPTER 13

Oscillations

SUMMARY

1. OSCILLATORY MOTION

The to and fro (or back and forth) motion on the same path is called oscillatory motion or vibratory motion.

2. PERIODIC MOTION

If the motion is repeated again and again after equal intervals of time, the motion of a body is called periodic.

3. IMPORTANT TERMS

Some important terms related to wave motion are as follows :

3.1 Phase (ϕ)

Phase of a vibrating particle at any instant is the position and direction of motion of the particle at that instant w.r.t. its mean position.

3.2 Displacement (y)

The displacement of a particle executing S.H.M. from its mean position.

i.e. $y = r \sin \omega t$

3.3 Amplitude (A)

The maximum value of the displacement on either side of its mean position is called its amplitude.

3.4 Time Period (T)

The time taken by a vibrating body to complete one full vibration is called its time period. It is denoted by symbol T.

3.5 Frequency (ν)

The number of oscillations completed per second is called frequency and is denoted by ν . Its unit is hertz.

4. SIMPLE HARMONIC MOTION

A harmonic motion in which the restoring force is directly proportional to the displacement and is always directed towards the mean position is called simple harmonic motion (S.H.M.).

The displacement of a S.H.M. at any time t is given by

$$y = r \sin(\omega t + \phi) \text{ or } y = r \cos(\omega t + \phi),$$

Where ω is the angular frequency related to the time period by

$$\omega = \frac{2\pi}{T} \text{ and } \theta \text{ is called phase}$$

If $\phi = 0$

Then, $y = r \sin \omega t$ or $y = r \cos \omega t$

4.1 Velocity of the Particle Executing S.H.M.

Velocity of particle, which is executing S.H.M. is given by

$$v = \omega \sqrt{r^2 - y^2}$$

At the mean position,

$$y = 0$$

$$v = \omega r$$

At the extreme position,

$$y = r$$

$$v = 0$$

Acceleration of the particle executing S.H.M.

$$a = -\omega^2 y$$

Here negative sign shows that acceleration is always directed to its mean position.

5. OSCILLATION

One oscillation is the to and fro motion of a particle between two consecutive passages in the same direction.

is much larger than in the liquid phase; the molecules in vapour are far apart.

2. The volume of vapour is

$$= 1000 \text{ kg/m}^3 / 0.6 \text{ kg/m}^3$$

$$= 1 / (6 \times 10^{-4}) \approx 1667$$

times larger than the volume of the same mass of liquid water.

3. If the volume in the vapour state has increased by a factor of $1 / (6 \times 10^{-4})$, the fractional volume occupied by the molecules themselves would be approximately 6×10^{-4} .
4. The actual volume of the molecules is negligible compared to the total volume occupied by the gas. This supports the kinetic theory assumption that gas molecules are point masses and the intermolecular forces are negligible except during collisions because they are far apart.

- 159.** A diatomic molecule, like N_2 , can be treated as a rigid rotator at ordinary temperatures. This means it has translational degrees of freedom and rotational degrees of freedom, but vibrational modes are not excited.

Questions:

- How many rotational degrees of freedom does a rigid diatomic molecule possess? Why not three?
- For one mole of such a diatomic gas (rigid rotator), what is the total internal energy U in terms of R and T , according to the law of equipartition of energy?
- What are the predicted values for the molar specific heat at constant volume (C_v) and molar specific heat at constant pressure (C_p) for this gas?
- If at higher temperatures, this diatomic molecule also exhibits one vibrational mode, how would its C_v change?

Ans :

- A rigid diatomic molecule has 2 rotational degrees of freedom. Rotation along the line joining the atoms has a very small moment of inertia and is not considered a degree of freedom for energy storage due to quantum mechanical reasons.
- A rigid diatomic molecule has 3 translational and 2 rotational degrees of freedom (total 5). Each contributes $\frac{1}{2} k_B T$ to average energy. For one mole,

$$U = 5 \times \left(\frac{1}{2} k_B T \times N_A \right) = \frac{5}{2} RT.$$

$$3. \quad C_v = \frac{dU}{dT} = \frac{5}{2} R.$$

Since $C_p - C_v = R$ for an ideal gas,

$$C_p = C_v + R$$

$$= \frac{5}{2} R + R = \frac{7}{2} R.$$

4. If one vibrational mode is also present, it adds $k_B T$ to the average energy per molecule (or RT per mole) because a vibrational mode has both kinetic and potential energy modes. So,

$$U = \left(\frac{5}{2} RT + RT \right) = \frac{7}{2} RT.$$

Then C_v would become $\frac{7}{2} R$.

- 160.** Uranium hexafluoride gas is used in the process of enriching uranium. It consists of molecules made with two isotopes of uranium, ^{235}U and ^{238}U . To separate these isotopes, the gas is allowed to diffuse through a porous barrier.

Questions:

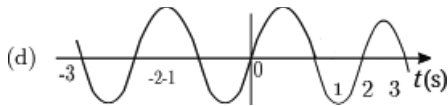
- How is the root mean square (rms) speed of gas molecules related to their mass 'm' and the absolute temperature T ?
- If molecules of $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ are at the same temperature, which ones will have a larger average speed?
- When gases diffuse, their rate of diffusion is inversely proportional to the square root of their masses. How does this relate to their average speeds?
- Explain briefly how the difference in average speeds of $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ molecules is exploited in separating the uranium isotopes using diffusion through a porous cylinder.

Ans :

1. The rms speed

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}.$$

- At the same temperature, the average kinetic energy ($\frac{1}{2} m \overline{v^2}$) is the same. Therefore, the lighter molecules ($^{235}\text{UF}_6$) will have a larger average speed than the heavier molecules ($^{238}\text{UF}_6$).
- Since $v_{rms} \propto 1/\sqrt{m}$, and the rate of diffusion is related to the average speed of molecules, the rate of diffusion is also inversely proportional to the square root of their masses.
- The lighter $^{235}\text{UF}_6$ molecules, having a slightly higher average speed, will collide more frequently with the pores of the barrier and are more likely to pass through than the heavier $^{238}\text{UF}_6$ molecules. This results in the gas emerging from the barrier being slightly enriched in the lighter isotope. This process is repeated many times for sufficient enrichment.



Ans :

- (a) It is not periodic.
 (b) It is periodic with a period = 2 s.
 (c) It is not periodic. Repetition of merely one portion is not enough for periodic motion. The entire motion during one period should be repeated successively.
 (d) It is periodic with a period = 2 s.

4. Which of the following functions of time represent :

- (a) simple harmonic.
 (b) periodic but not simple harmonic, and
 (c) non-periodic motion ? Give the period for each case of periodic motion : (ω is any positive constant)
 (a) $\sin \omega t - \cos \omega t$
 (b) $\sin^3 \omega t$
 (c) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$
 (d) $\cos \omega t + \cos 3 \omega t + \cos 5 \omega t$
 (e) $\exp(-\omega^2 t^2)$
 (f) $1 + \omega t + \omega^2 t^3$

Ans :

- (a) Let $x = \sin \omega t - \cos \omega t$

$$\frac{dx}{dt} = \omega \cos \omega t + \omega \sin \omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 \sin \omega t + \omega^2 \cos \omega t$$

$$= -\omega^2 (\sin \omega t - \cos \omega t)$$

$$= -\omega^2 x$$

or acceleration is proportional to displacement,
 Thus it is simple harmonic. Its time period is

$$T = \left(\frac{2\pi}{\omega}\right)$$

- (b) Since $\sin^3 \omega t = \frac{1}{4} (3 \sin \omega t - \sin 3 \omega t)$

$$[\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta]$$

Here each term i.e. $\sin \omega t$ and $\sin 3 \omega t$ individually represents S.H.M. but the combination is a periodic motion but not S.H.M. having $T = \frac{2\pi}{\omega}$.

- (c) Here $3 \cos\left(\frac{\pi}{4} - 2\omega t\right) = 3 \cos\left(2\omega t - \frac{\pi}{4}\right)$,

so it is simple harmonic and its time period is

$$T = \frac{2\pi}{2\omega} = \left(\frac{\pi}{\omega}\right)$$

- (d) It is periodic ($T = \frac{2\pi}{\omega}$) but not simple harmonic.

- (e) $\exp(-\omega^2 t^2)$ is an exponential function and decreases with increasing value of t and becomes zero at $t = \infty$. Hence it never repeats itself So it is not periodic.

- (f) It is non-periodic (physically unacceptable because the function $\rightarrow \infty$ as $t \rightarrow \infty$).

5. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

- (a) at the end A,
 (b) at the end B,
 (c) at the mid-point of AB going towards A.
 (d) at 2 cm away from B going towards A.
 (e) at 3 cm away from A going towards B and
 (f) at 4 cm away from A going towards A.

Ans :

- (a) At the end A 0, +, +
 (b) At the end B 0, -, -
 (c) at the mid-point of AB going towards A -, 0, 0
 (d) at 2 cm away from B going towards A -, -, -
 (e) at 3 cm away from A going towards B +, +, +
 (f) at 4 cm away from A going towards A (-, -, -)
 (Here 0 stands for the magnitude of the quantity)

6. Which of the following relationship between the acceleration a and the displacement x of a particle involves simple harmonic motion :

- (a) $a = 0.7 x$ (b) $a = -200x^2$
 (c) $a = -10x$ (d) $a = 100x^2$

Ans :

For a simple harmonic motion acceleration = - (constant) displacement.

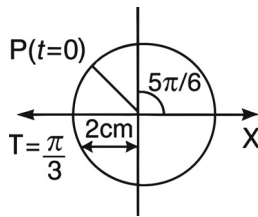
7. The motion of a particle executing S.H.M. is described by the displacement function

$$x(t) = A \cos(\omega t + \phi)$$

If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is π cm s⁻¹ what are its amplitude and initial phase angle ? The angular frequency of the particle is π s⁻¹. If instead of the cosine function, we choose the sine function to describe the S.H.M. : $x = B \sin(\omega t + a)$, what are the amplitude and initial phase of the particle with the above initial conditions?

Comparing with $x = r \cos (\omega t + \phi)$, we get

$$r = 2, \omega = 3 \text{ and } \phi = \frac{5\pi}{6}$$

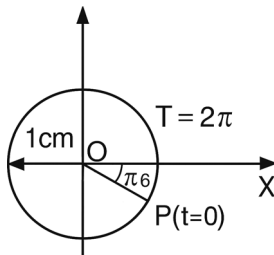


(b)
$$x = \cos\left(\frac{\pi}{6} - t\right)$$

$$x = \cos\left(t - \frac{\pi}{6}\right)$$

Comparing with $x = A \cos (\omega t + \phi)$, we get

$$r = 1, \omega = 1 \text{ and } \phi = -\frac{\pi}{6}$$

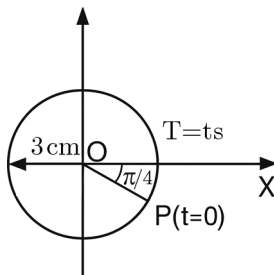


(c)
$$x = 3 \sin\left(2\pi t + \frac{\pi}{4}\right)$$

$$= 3 \cos\left[\frac{\pi}{2} - \left(2\pi t + \frac{\pi}{4}\right)\right]$$

$$= 3 \cos\left[2\pi t + \frac{\pi}{4} - \frac{\pi}{2}\right]$$

$$= 3 \cos\left[2\pi t - \frac{\pi}{4}\right]$$



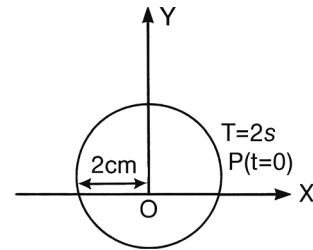
Comparing with $x = A \cos (\omega t + \phi)$, we get

$$r = 3, \omega = 2\pi, \text{ and } \theta = -\frac{\pi}{4}$$

(d)
$$x = 2 \cos \pi t$$

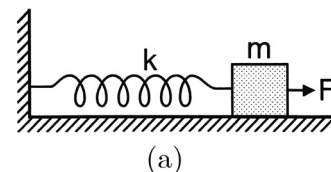
Comparing with $x = r \cos (\omega t + \phi)$, we get

$$r = 2, \omega = \pi; \text{ and } \phi = 0$$

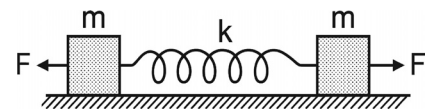


13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end stretches the spring. Figure (b) shows these same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure (b) is stretched by the same force F .

- (i) What is the maximum extension of the spring in the two cases ?
 (ii) If the mass in (a) and the two masses in (b) are released free, what is the period of oscillation in each case ?



(a)



(b)

Ans :

- (i) Maximum extension of the spring in case (a)

$$y = \frac{F}{k} \quad [\text{since, } F = ky]$$

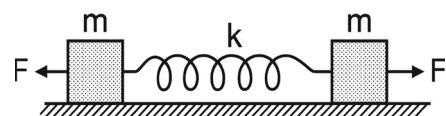
Maximum extension of the spring in case (b)

$$y = \frac{F}{k}$$

- (ii) Period of oscillation for case (a)

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Period of oscillation for case (b) can be determined by considering it to be equivalent to the two springs



(b)

10. In QUESTION 9, let us take the position of mass when the spring is unstretched as $x = 0$ and the direction from left to right as the positive direction of x -axis. Given x as a function of time t for the oscillating mass if at the moment we start the stopwatch ($t = 0$), the mass is

- at the mean position
- at the maximum stretched position, and
- at the maximum compressed position.

In what way do these functions for SHM differ from each other in frequency, in amplitude or the initial phase?

Ans :

The equation of displacement of mass in x direction is given by

$$x = A \sin(\omega t + \phi) \quad \dots(1)$$

- (a) When the mass is at mean position,

$$\phi = 0, \omega = 20 \text{ s}^{-1}$$

$$A = 0.02 \text{ m} = 2 \text{ cm}$$

$$\text{So } x = A \sin(20t + 0) = 2 \sin 20t \quad \dots(2)$$

- (b) When the mass is at the maximum stretched position,

$$\phi = \frac{\pi}{2}$$

$$x = A \sin\left(20t + \frac{\pi}{2}\right) = 2 \cos 20t$$

$$x = 2 \cos 20t \quad \dots(3)$$

- (c) When the mass is at the maximum compressed position,

$$\phi = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

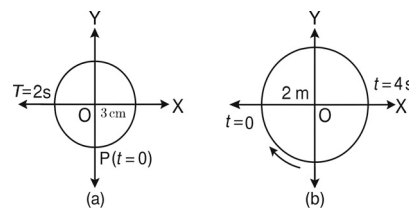
$$x = A \sin(20t + \pi)$$

$$= -A \sin 20t$$

$$x = -2 \cos 20t \quad \dots(4)$$

Here x is in cm. We find that these functions differ neither in amplitude nor frequency. They differ in initial phase.

11. Figure below correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the period of revolution (*i.e.* clockwise or anticlockwise) are indicated on each figure. Obtain the corresponding simple harmonic motion of the x -projection of the radius vector of the revolving particle P in each case.



Ans :

- (a) At $t = 0$, OP makes an angle of $270^\circ = \frac{3\pi}{2}$ with the axis. After a time t , it covers an angle of $\frac{2\pi}{T}t$ in the anticlockwise sense and makes an angle of $\frac{2\pi}{T}t + \frac{3\pi}{2}$ with the x -axis.

The x - projection of OP at time t is given by

$$x(t) = 3 \cos\left(\frac{2\pi}{T}t + \frac{3\pi}{2}\right)$$

$$x(t) = -3 \sin t \frac{2\pi}{T}$$

$$\text{For } T = 2 \text{ s;}$$

$$x(t) = -3 \sin \pi t$$

where x is in cm.

- (b) In this case at $t = 0$, OP makes angle of $180^\circ = \frac{\pi}{2}$ with the x -axis. After a time t , it covers an angle $\frac{2\pi}{T}t$ clock-wise sense, and makes an angle of $(\pi - \frac{2\pi}{T}t)$ with the x -axis. The x -projection of OP at time t is given by

$$x(t) = 2 \cos\left(\pi - \frac{2\pi}{T}t\right) = -2 \cos \frac{2\pi}{T}t$$

$$\text{For } T = 4 \text{ s,}$$

$$x(t) = -2 \cos \frac{\pi}{2}t, \text{ where } x \text{ is in m.}$$

12. Plot the corresponding reference circle for each of the following S.H.M. Indicate the initial $t = 0$ position of the particle, the radius of circle and angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to the anticlockwise in every case : (x is in cm and t is in s)

(a) $x = -\sin\left(3t + \frac{\pi}{5}\right)$

(b) $x = \cos\left(\frac{\pi}{6} - t\right)$

(c) $x = 3 \sin\left(2\pi t + \frac{\pi}{4}\right)$

(d) $x = 2 \cos \pi t$

Ans :

(a) $x = -2 \sin\left(3t + \frac{\pi}{3}\right)$

$$= 2 \cos\left(\frac{\pi}{2} + 3t + \frac{\pi}{3}\right)$$

$$= 2 \cos\left(3t + \frac{5\pi}{6}\right)$$

6. APPLICATIONS OF SIMPLE HARMONIC MOTION

6.1 Simple Pendulum

A heavy point mass attached to a weightless, inextensible string suspended from a rigid support about which it is free to oscillate is called an ideal simple pendulum.

In practice, it is not possible to have an ideal simple pendulum, hence in practice, we take a heavy metallic bob suspended by a thin silken thread suspended from a rigid support.

Time period of simple pendulum

$$T = 2\pi\sqrt{l/g}$$

6.2 Seconds Pendulum

A simple pendulum whose time period is 2 seconds is called a seconds' pendulum.

Time period of a spring loaded with mass m is given by

$$T = 2\pi\sqrt{\frac{m}{\kappa}}$$

where m is called inertia factor and κ is called spring factor.

6.3 Energy of a Harmonic Oscillator (E)

The total energy possessed by a harmonic oscillator is the sum of its K.E. and P.E.

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}m\omega^2(r^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$E = \frac{1}{2}m\omega^2 r^2$$

$$E = 2x^2 m\omega^2 r^2 = \frac{2\pi^2 m r^2}{T^2}$$

NCERT TEXT-BOOK QUESTIONS

1. Which of the following examples represent periodic motion ?

- A swimmer completing one (return) trip from one bank of a river to the other and back.
- A freely suspended bar magnet displaced from its N-S direction and released.
- A hydrogen molecule rotating about its centre of mass.
- An arrow released from a bow.

Ans :

- A freely suspended bar magnet displaced from its N-S direction and released.
- A hydrogen molecule rotating about its centre of mass.

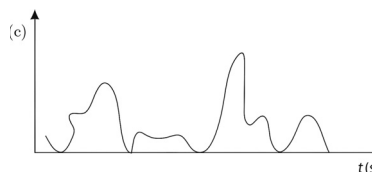
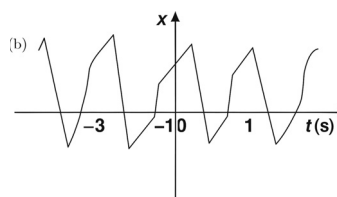
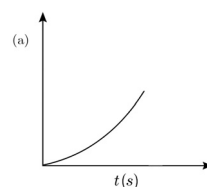
2. Which of the following examples represent (nearly) simple harmonic motion, and which represents periodic but not simple harmonic motion :

- The rotation of the earth about its axis.
- The motion of an oscillatory mercury column in a U-tube.
- The motion of a ball-bearing inside a smooth curved bowl, when released from a point slightly above the lowermost position.
- General vibration of a polyatomic molecule about its equilibrium configuration position.

Ans :

- The rotation of the earth about its axis represents periodic but not S.H.M.
- The motion of an oscillatory mercury column in a U-tube is S.H.M.
- The motion of a ball-bearing inside a smooth curved bowl when released from a point slightly above the lowermost position represent S.H.M.
- A polyatomic molecule has a number of natural frequencies, so in general, its vibration is a superposition of S.H.M.'s of a number of different frequencies. The superposition is periodic but not S.H.M.

3. Given below are $x-t$ plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion in case of periodic motion ?



each of length $\frac{l}{2}$ and joined at point O at the centre of the spring shown in figure. If the force constant of each half is k' then $k' = 2k$, so

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

- 14.** The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with angular frequency of 200 rad/min. What is its minimum speed?

Ans :

Given, $2r = 1.0 \text{ m}$

So $r = \frac{1}{2} \text{ m}$

$$\omega = 200 \text{ rad/min}$$

Since, $v_{\max} = r\omega$

$$= \frac{1}{2} \times 200 = 100 \text{ m/min}$$

$$= \frac{100}{60} \text{ m s}^{-1} = 1.67 \text{ m s}^{-1}$$

- 15.** The acceleration due to gravity on the surface of the moon 1.7 m s^{-2} . What is the time period of a simple pendulum on the surface of moon if time-period on the surface of earth is 3.5 s ? (g on the surface of earth = 9.8 m s^{-2})

Ans :

On the surface of earth,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$l = \frac{T^2 \times g}{4\pi^2} = \frac{(3.5)^2 \times 9.8}{4 \times 3.14 \times 3.14}$$

$$= 3.045 \text{ km}$$

On the surface of moon,

$$T = 2\pi\sqrt{\frac{l}{g'}}$$

$$T = 2 \times 3.14 \sqrt{\frac{3.045}{1.7}} = 8.4 \text{ s}$$

- 16.** A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v . If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?

Ans :

Time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

In this case, the pendulum has two accelerations :

- (i) acceleration due to gravity ' g '
(ii) centripetal acceleration, $a_c = \frac{v^2}{R}$

Since g and $\frac{v^2}{R}$ are at right angle to each other, so net acceleration,

$$a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

$$= \left(g^2 + \frac{v^4}{R^2}\right)^{1/2}$$

Time period of oscillation of this pendulum

$$T = 2\pi \sqrt{\frac{l}{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}}$$

- 17.** A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1 . The cork is depressed slightly and released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$, where ρ is the density of the cork.

(Ignore damping due to viscosity etc.)

Ans :

In equilibrium, weight of the cork equals upthrust
When the cork is depressed slightly by an amount x , net upward force is $Ax\rho_1 g$

Thus the force constant, $k = A\rho_1 g$

and $m = A\rho g$, where ρ is the density of the cork

Now $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{A h \rho}{A \rho_1 g}}$$

$$T = 2\pi\sqrt{\frac{h \rho}{\rho_1 g}}$$

- 18.** One end of a U-tube containing mercury is connected to a suction pump and the other end is open to the atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the liquid column in the U-tube executes simple harmonic motion.

Ans :

Let a U-tube contains some liquid upto level A_1, A_2 . Let L be the total length of liquid in the pipe and ρ be the density of liquid.

Mass of liquid in the tube $m = AL\rho$, where A is the area cross-section of U-tube.

- 36.** Assertion: All oscillatory motions are necessarily periodic.

Reason: Periodic motion is defined as a motion that repeats itself after a fixed interval of time, which is inherent in to-and-fro motion.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

Oscillatory motion, being a to-and-fro movement about a mean position, inherently repeats itself over time. This repetition at regular intervals is the definition of periodic motion. Therefore, all oscillatory motions are a subset of periodic motions.

- 37.** Assertion: In simple harmonic motion, the acceleration is zero when the velocity is maximum.

Reason: For a particle in SHM, acceleration $a = -\omega^2 x$ and velocity $v = \pm \omega \sqrt{A^2 - x^2}$.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

Velocity is maximum at the mean position ($x = 0$). According to $a = -\omega^2 x$, when $x = 0$, acceleration $a = 0$. The reason provides the correct formulae that lead to this conclusion, thus correctly explaining the assertion.

- 38.** Assertion: The total mechanical energy of a simple harmonic oscillator is constant.

Reason: The restoring force in simple harmonic motion is a conservative force.

- (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
 (C) Assertion is true, but Reason is false.
 (D) Assertion is false, but Reason is true.

Ans :

(A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

The restoring force in SHM ($F = -kx$) is a conservative force, meaning that the work done by it is path-independent and depends only on initial and final positions. For motion under a conservative force, the total mechanical energy (sum of kinetic and potential energy) is conserved, i.e., remains constant.

- 39.** Consider the following statements about simple harmonic motion:

1. The displacement is a sinusoidal function of time.
2. The velocity is maximum at the extreme positions.
3. The acceleration is always directed towards the equilibrium position.
4. The total energy of the particle is proportional to the square of the amplitude.

Which of the above statements are correct?

- (A) 1, 2, and 3
 (B) 1, 3, and 4
 (C) 2, 3, and 4
 (D) 1, 2, and 4

Ans :

(B) 1, 3, and 4

Statement 1 is the definition of SHM. Statement 3 is correct as $a = -\omega^2 x$. Statement 4 is correct as $E = \frac{1}{2} kA^2$. Statement 2 is incorrect; velocity is zero at extreme positions and maximum at the mean position.

- 40.** Regarding the period (T) of a simple pendulum, which of the following statements are true?

1. It is proportional to the square root of its length.
2. It is inversely proportional to the square root of the acceleration due to gravity.
3. It depends on the mass of the bob.
4. It depends on the amplitude of oscillation for small angles.

- (A) 1 and 2 only
 (B) 1, 2, and 3
 (C) 3 and 4 only
 (D) 1, 2, and 4

Ans :

(A) 1 and 2 only

The period of a simple pendulum for small oscillations is $T = 2\pi\sqrt{L/g}$. Thus, T is proportional to $\sqrt{L}$ (Statement 1) and inversely proportional to

- 20.** You are riding an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Ans :

- (a) Mass supported by each wheel, $m = 750$ kg

$$\text{Displacement } y = 15 \text{ cm} = 0.15 \text{ m}$$

$$\text{and } a = g$$

$$\text{Since, } mg = ky$$

$$\text{Hence, } k = \frac{mg}{y} = \frac{750 \times 9.8}{0.15}$$

$$k = 4.9 \times 10^4 \text{ Nm}^{-2} \\ = 5 \times 10^4 \text{ Nm}^{-1}$$

- (b) Since time period

$$T = 2\pi \sqrt{\frac{m}{g}} \quad \dots(1)$$

If the amplitude drops to half of its initial value in time period $T_{1/2}$, then

$$T_{1/2} = \frac{\ln\left(\frac{1}{2}\right)}{\frac{b}{2m}} \quad \dots(2)$$

Given that $T = T_{1/2} = \text{time of one oscillation}$

$$\text{Hence, } 2\pi \sqrt{\frac{m}{k}} = \frac{\ln\left(\frac{1}{2}\right)}{\frac{b}{2m}}$$

$$b = \frac{\ln\left(\frac{1}{2}\right) 2m}{2\pi \sqrt{\frac{m}{k}}} \\ = \frac{0.693 \times 2 \times 750}{2 \times 3.142 \times \sqrt{\frac{750}{5 \times 10^4}}} \\ = \frac{0.693 \times 1500}{6.284 \times 0.1225} \\ = 1350.37 \text{ kg s}^{-1}$$

- 21.** Show that for a particle in linear S.H.M. the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Ans :

Average kinetic energy over a period of oscillation

$$= \frac{1}{T} \int_0^T \frac{1}{2} mv^2 dt = \frac{1}{2T} mv^2 [t]_0^T \\ = \frac{1}{2T} mv^2 \times T = \frac{mv^2}{2}$$

Average potential energy

$$= \frac{1}{T} \int_0^T \frac{1}{2} kx^2 dt = \frac{1}{2T} kx^2 [t]_0^T \\ = \frac{1}{2T} kx^2 \cdot T = \frac{1}{2} kx^2$$

$$\text{Since, } \omega = \sqrt{\frac{k}{m}}$$

$$\text{Hence } \omega^2 = k/m$$

$$k = m\omega^2$$

Hence, average potential energy

$$= \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} mv^2$$

Therefore, for a particle in linear S.H.M. the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

- 22.** A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant a is defined by the relation $J = -a\theta$, where J is the restoring couple and θ the angle of twist).

Ans :

The time period of a torsional pendulum

$$T = 2\pi \sqrt{\frac{I}{a}}$$

where I is the moment of inertia about the axis of rotation.

$$I = \frac{1}{2} MR^2$$

where M is the mass of the disc and R its radius.

$$I = \frac{1}{2} \times 10 \times (0.15)^2 \\ = 0.1125 \text{ kg m}^2$$

$$T = 2\pi \sqrt{\frac{I}{a}}$$

$$T^2 = 4\pi^2 \frac{I}{a}$$

$$a = 4\pi^2 \frac{I}{T^2}$$

$$\text{Hence, } a = 4 \times \frac{22}{7} \times \frac{22}{7} \times \frac{0.1125}{(1.5)^2}$$

Depress liquid through distance y in left side of the tube, the liquid will rise by y in the other tube and then release.

Before release, the difference in level in the two tubes = $2y$

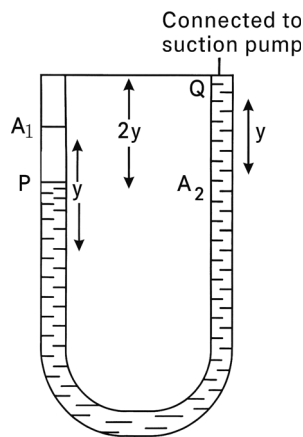
Weight of liquid in column of length $2y$ in U tube will provide the restoring force to liquid.

Restoring force, $F = -(2yA)\rho g = -(2A\rho g)y$

We find that, $F \propto y$

Liquid will execute S.H.M.

Spring factor, $K = 2A\rho g$



$$T = 2\pi \sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$$

$$T = 2\pi \sqrt{\frac{LA\rho}{2A\rho g}}$$

$$T = 2\pi \sqrt{\frac{L}{2g}}$$

or frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{2g}{L}}$

If h is the height of the liquid column in each limb, then $L = 2h$

Hence, $T = 2\pi \sqrt{\frac{2h}{2g}} = 2\pi \sqrt{\frac{h}{g}}$

Hence the motion of liquid column in the U-tube is simple harmonic motion.

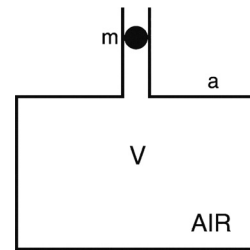
19. An air chamber of volume V has a neck of area cross-section a into which a ball of mass m just fits and can move up and down without any friction. Show that when the ball is pressed down a little and released, it executes S.H.M. Obtain an expression for the time period of the S.H.M. assuming pressure-volume variations of the air to be isothermal.

Ans :

Decrease in volume,

$$\Delta V = ax$$

$$\text{Volume strain} = \frac{\Delta V}{V}$$



Since bulk modulus of elasticity

$$K = -\frac{\text{Hydrostatic strain}}{\text{Volumestrain}}$$

$$= -\frac{P}{\Delta V/V}$$

Hence, $P = -K \frac{\Delta V}{V}$

But, $P = \frac{F}{a}$

Hence, $\frac{F}{a} = -K \frac{\Delta V}{V}$

$$F = -K \frac{\Delta V}{V} a$$

$$F = -K \frac{Ka^2x}{V} \quad [\Delta v = ax]$$

$$m \frac{d^2x}{dt^2} = \frac{Ka^2}{V} x$$

$$\frac{d^2x}{dt^2} = \frac{Ka^2}{mV} x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \left[\text{Where } \omega^2 = \frac{Ka^2}{mV} \right]$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

which is the differential equation of S.H.M.

Thus the motion of ball vibrating in the neck of a chamber is S.H.M. Its time period is given by

Since, $\omega = \frac{Ka^2}{mV}$

$$T = 2\pi \sqrt{\frac{Vm}{Ka^2}}$$

where K is Bulk modulus of air

For isothermal case, $K = P$

$$T = 2\pi \sqrt{\frac{Vm}{Pa^2}}$$

oscillatory in this sense. Oscillations occur due to restoring forces.

3. Which physical phenomenon is responsible for bringing an oscillating body eventually to rest in practice?

(A) The constant application of a restoring force.
 (B) The generation of undamped oscillations.
 (C) Damping due to friction and dissipative causes.
 (D) An increase in the system's total energy.

Ans :

(C) Damping due to friction and dissipative causes. In real-world scenarios, oscillating bodies eventually stop due to damping forces like friction and air resistance. These forces dissipate the system's energy, usually as heat, leading to a decrease in amplitude over time until the motion ceases.

4. What is the smallest interval of time after which a periodic motion is repeated called?

(A) Frequency
 (B) Amplitude
 (C) Period
 (D) Phase

Ans :

(C) Period

The period (T) of a periodic motion is defined as the smallest interval of time after which the motion repeats itself. Frequency is the number of repetitions per unit time, amplitude is the maximum displacement, and phase describes the state of motion.

5. How is the frequency (ν) of a periodic motion related to its period (T)?

(A) $\nu = T$
 (B) $\nu = 1/T$
 (C) $\nu = 2\pi T$
 (D) $\nu = T/(2\pi)$

Ans :

(B) $\nu = 1/T$

Frequency (ν) is the reciprocal of the period (T). It represents the number of oscillations or cycles completed per unit of time. The period is the time taken for one complete oscillation, so frequency is one divided by this time.

6. What is the SI unit of frequency?

(A) Second (s)
 (B) Meter (m)
 (C) Hertz (Hz)

(D) Radian (rad)

Ans :

(C) Hertz (Hz)

The SI unit of frequency is hertz (Hz), named after Heinrich Rudolph Hertz. One hertz is equivalent to one oscillation or cycle per second (s^{-1}). Second is the unit of period, meter is for displacement, and radian is for angle.

7. In simple harmonic motion (SHM), displacement refers to the change in which physical property with time?

(A) Only the position vector from a fixed origin.
 (B) Any physical property under consideration that changes.
 (C) Specifically the angular velocity of the object.
 (D) Solely the total energy of the oscillating system.

Ans :

(B) Any physical property under consideration that changes.

In the context of oscillations, displacement is used more generally to refer to the change with time of any physical property being observed, such as position, angle, pressure, or voltage, from its equilibrium value.

8. Which of the following statements about the function $f(t) = A \cos(\omega t)$ representing periodic motion is incorrect?

(A) The function is periodic with a period $T = 2\pi/\omega$.
 (B) The value of the function remains the same if ωt is increased by 2π .
 (C) A linear combination of such sine and cosine functions is not periodic.
 (D) The function $f(t) = A \sin(\omega t)$ is also periodic with the same period t .

Ans :

(C) A linear combination of such sine and cosine functions is not periodic.

A linear combination of sine and cosine functions of the same angular frequency ω , such as $f(t) = A \sin(\omega t) + B \cos(\omega t)$, is also a periodic function with the same period $T = 2\pi/\omega$. The other statements are correct descriptions of periodic functions.

9. Identify the correct statement regarding simple harmonic motion (SHM).

(A) All periodic motions are simple harmonic motions.
 (B) SHM is a motion where displacement is a sinusoidal function of time.

Hence, $\alpha = 1.976$
 $= 2.0 \text{ Nm rad}^{-1}$

- 23.** A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when displacement is (a) 5 cm (b) 3 cm (c) 0 cm.

Ans :

Given, $r = 5 \text{ cm} = 0.05 \text{ m}$, $T = 0.2 \text{ s}$

So $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi \text{ rad s}^{-1}$

(a) Here $y = 5 \text{ cm} = 0.05 \text{ m}$

So $v = \omega\sqrt{r^2 - y^2}$
 $= 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0$

And $a = -\omega^2 y = -(10\pi)^2 \times 0.05$
 $= -5\pi^2 \text{ m s}^{-2}$

(b) Here $y = 3 \text{ cm} = 0.03 \text{ m}$

So $v = \omega\sqrt{r^2 - y^2}$
 $= 10\pi\sqrt{(0.05)^2 - (0.03)^2}$
 $= 0.4\pi \text{ m s}^{-1}$

and $a = -\omega^2 y = -(10\pi)^2 \times 0.03$
 $= -3\pi^2 \text{ m s}^{-2}$

(c) Here $y = 0$

So $v = \omega\sqrt{r^2 - r^2} = \omega r = 10\pi \times 0.05$
 $= 0.5\pi \text{ m s}^{-1}$

And $a = -\omega^2 y = 0$

- 24.** A mass attached to a spring is free to oscillate, with angular velocity ω , in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω , x_0 and v_0 [Hint : Start with the equation $x = a \cos(\omega t + \theta)$ and note that the initial velocity is negative].

Ans :

Let displacement x of the particle at any time t is given by

$$x = a \cos(\omega t + \theta) \quad \dots(1)$$

At $t = 0, x = x_0$

So $x_0 = a \cos \theta \quad \dots(2)$

The velocity of particle at time t is given by

$$v = \frac{dx}{dt} = -a\omega \sin(\omega t + \theta) \quad \dots(3)$$

At $t = 0, v = v_0$

So $v_0 = -a\omega \sin \theta$
 $= -\omega(a \sin \theta)$
 $v_0 = -\omega\sqrt{a^2(1 - \cos \theta)}$
 $v_0 = -\omega\sqrt{a^2 - a^2 \cos \theta}$
 $= -\omega\sqrt{a^2 - x_0^2} \quad \dots(4)$

From eq. (iv), we find that initial velocity is negative. Equation (iv) can also be written as

$$v_0^2 = \omega^2(a^2 - x_0^2)$$

$$a^2 = x_0^2 + \frac{v_0^2}{\omega^2}$$

$$a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

NODIA

OBJECTIVE QUESTIONS

- 1.** Which of the following best describes a motion that repeats itself at regular intervals of time?

- (A) Rectilinear motion
 (B) Projectile motion
 (C) Periodic motion
 (D) Random motion

Ans :

- (C) Periodic motion

Periodic motion is defined as any motion that repeats itself after a fixed interval of time. Examples include the motion of planets around the sun or the swinging of a pendulum. Rectilinear motion is along a straight line, projectile motion follows a curved path under gravity, and random motion has no specific pattern.

- 2.** What is the primary characteristic of oscillatory motion?

- (A) It always follows a circular path continuously.
 (B) It moves to and fro about a mean position.
 (C) It occurs only in the absence of any force.
 (D) It is a non-repetitive type of movement.

Ans :

- (B) It moves to and fro about a mean position.

Oscillatory motion is characterized by a repetitive to-and-fro movement of an object around a central point, known as the mean or equilibrium position. Circular motion is periodic but not necessarily

- (C) $x(t) = A \tan(\omega t + \phi)$
 (D) $x(t) = \omega A \cos(\omega t + \phi)$

Ans :

(B) $x(t) = A \cos(\omega t + \phi)$

If the particle P is in uniform circular motion, its angular position at time t is $(\omega t + \phi)$. The x-projection of its position vector OP (where O is the center) is given by $A \cos(\omega t + \phi)$, which is the defining equation for SHM.

17. Which of the following is NOT a characteristic of the velocity of a particle in SHM, given by $v(t) = -\omega A \sin(\omega t + \phi)$?
- (A) Velocity is maximum when displacement is zero.
 (B) Velocity is zero at the extreme positions of displacement.
 (C) Velocity is always in the same direction as displacement.
 (D) Velocity varies sinusoidally with time.

Ans :

(C) Velocity is always in the same direction as displacement.

The velocity in SHM is given by $v(t) = -\omega A \sin(\omega t + \phi)$, while displacement is $x(t) = A \cos(\omega t + \phi)$. The velocity and displacement are out of phase by $\pi/2$; velocity is maximum when displacement is zero, and zero at extremes. Velocity is not always in the same direction as displacement.

18. How does the phase of acceleration in SHM compare with the phase of displacement?
- (A) Acceleration is in phase with displacement.
 (B) Acceleration leads displacement by $\pi/2$.
 (C) Acceleration lags displacement by $\pi/2$.
 (D) Acceleration is out of phase with displacement by π .

Ans :

(D) Acceleration is out of phase with displacement by π .

Displacement is $x(t) = A \cos(\omega t + \phi)$ and acceleration is $a(t) = -\omega^2 A \cos(\omega t + \phi)$. The negative sign indicates a phase difference of π radians (180°). When displacement is positive maximum, acceleration is negative maximum.

19. What is the instantaneous acceleration $a(t)$ of a particle in SHM if its displacement is $x(t)$ and angular frequency is ω ?
- (A) $a(t) = \omega x(t)$
 (B) $a(t) = -\omega x(t)$
 (C) $a(t) = \omega^2 x(t)$

(D) $a(t) = -\omega^2 x(t)$

Ans :

(D) $a(t) = -\omega^2 x(t)$

The acceleration of a particle in simple harmonic motion is directly proportional to its displacement from the mean position and is always directed towards the mean position. Mathematically, this is expressed as $a(t) = -\omega^2 x(t)$.

20. What is the maximum speed of a particle executing SHM with amplitude A and angular frequency ω ?

- (A) $A\omega^2$
 (B) A/ω
 (C) $A\omega$
 (D) A/ω^2

Ans :

(C) $A\omega$

The velocity in SHM is $v(t) = -\omega A \sin(\omega t + \phi)$. The maximum value of $\sin(\omega t + \phi)$ is 1. Therefore, the maximum speed (magnitude of velocity) is ωA . This occurs when the particle passes through its equilibrium position ($x=0$).

21. The force $F(t)$ acting on a particle of mass m in SHM is given by which expression, where K is the force constant?

- (A) $F(t) = kx(t)$
 (B) $F(t) = -kx(t)$
 (C) $F(t) = k/x(t)$
 (D) $F(t) = -k/x(t)$

Ans :

(B) $F(t) = -kx(t)$

The force law for simple harmonic motion states that the restoring force is directly proportional to the displacement $x(t)$ from the equilibrium position and acts in the opposite direction to the displacement. This is expressed as $F(t) = -kx(t)$, also known as Hooke's Law for an elastic spring.

22. How is the force constant K related to the mass m and angular frequency ω of a particle in SHM?

- (A) $k = m/\omega^2$
 (B) $k = m\omega^2$
 (C) $k = \omega/m$
 (D) $k = m\omega$

Ans :

(B) $k = m\omega^2$

From Newton's second law, $F = ma$. In SHM, $F = -kx$ and $a = -\omega^2 x$. Therefore, $-kx = m(-\omega^2 x)$, which simplifies to $k = m\omega^2$. The force constant K represents the stiffness of the system.

- (C) In SHM, the restoring force is independent of the displacement.
- (D) The period of SHM is directly proportional to its amplitude.

Ans :

(B) SHM is a motion where displacement is a sinusoidal function of time.
Simple harmonic motion (SHM) is a specific type of periodic motion where the displacement of the particle from its equilibrium position is described by a sinusoidal function (sine or cosine) of time. Not all periodic motions are SHM.

10. In the SHM equation $x(t) = A \cos(\omega t + \phi)$, what does the symbol 'A' represent?
- (A) Angular frequency
(B) Phase constant
(C) Amplitude
(D) Time period

Ans :

(C) Amplitude
In the equation for simple harmonic motion, $x(t) = A \cos(\omega t + \phi)$, 'A' represents the amplitude. The amplitude is the maximum displacement of the particle from its equilibrium position during the oscillation. It defines the extent of the motion.

11. What is the term $(\omega t + \phi)$ in the SHM equation $x(t) = A \cos(\omega t + \phi)$ called?
- (A) Angular displacement
(B) Phase of the motion
(C) Frequency of motion
(D) Initial displacement

Ans :

(B) Phase of the motion
The term $(\omega t + \phi)$ in the SHM equation $x(t) = A \cos(\omega t + \phi)$ is called the phase of the motion. It determines the state of the particle (its position and velocity) at any given time 't'.

12. What is the value of the phase $(\omega t + \phi)$ at $t=0$ known as in SHM?
- (A) Phase difference
(B) Angular phase
(C) Phase constant
(D) Zero phase

Ans :

(C) Phase constant
The value of the phase $(\omega t + \phi)$ at $t=0$ is ϕ . This constant ϕ is known as the phase constant or initial

phase (or phase angle). It specifies the initial state of motion of the oscillating particle.

13. How is the angular frequency (ω) of SHM related to its time period (T)?
- (A) $\omega = T/(2\pi)$
(B) $\omega = 2\pi/T$
(C) $\omega = T$
(D) $\omega = 1/(2\pi T)$

Ans :

(B) $\omega = 2\pi/T$
The angular frequency (ω) of simple harmonic motion is related to the time period (T) by the equation $\omega = 2\pi/T$. Since $T = 1/\nu$ (where ν is frequency), ω is also equal to $2\pi\nu$.

14. Which of the following functions represents a simple harmonic motion?
- (A) $x(t) = Ae^{-\omega t}$
(B) $x(t) = A \log(\omega t)$
(C) $x(t) = A \sin(\omega t - \pi/4)$
(D) $x(t) = A \tan(\omega t)$

Ans :

(C) $x(t) = A \sin(\omega t - \pi/4)$
Simple harmonic motion is defined by a displacement that is a sinusoidal function of time. The function $x(t) = A \sin(\omega t - \pi/4)$ fits this definition, representing SHM with amplitude A, angular frequency ω , and phase constant $-\pi/4$.

15. The projection of uniform circular motion on a diameter of the circle follows what type of motion?
- (A) Random motion
(B) Rectilinear motion
(C) Simple harmonic motion
(D) Projectile motion

Ans :

(C) Simple harmonic motion
When a particle undergoes uniform circular motion, its projection onto any diameter of the circle executes simple harmonic motion. The radius of the circle corresponds to the amplitude of the SHM, and the angular speed of the circular motion corresponds to the angular frequency of the SHM.

16. If a particle P moves uniformly on a circle of radius A with angular speed ω , and its initial angular position is ϕ , what is the x-projection of its position at time t?
- (A) $x(t) = A \sin(\omega t + \phi)$
(B) $x(t) = A \cos(\omega t + \phi)$

Ans :

(C) Simple harmonic motion

For a simple pendulum, the restoring torque is $\tau = -mgL\sin\theta$. For small angles, $\sin\theta \approx \theta$ (in radians), so $\tau \approx -mgL\theta$. Since torque is proportional to angular displacement, the motion is approximately simple harmonic for small angular displacements.

30. What provides the restoring torque for a simple pendulum?

- (A) The tension in the string only.
 (B) The component of gravitational force $mg\cos\theta$
 (C) The component of gravitational force $mg\sin\theta$
 (D) The air resistance acting upon the bob.

Ans :

(C) The component of gravitational force $mg\sin\theta$
 The restoring torque for a simple pendulum, which tends to bring the bob back to its mean position, is provided by the tangential component of the gravitational force, $mg\sin\theta$, acting at a perpendicular distance L (length of pendulum) from the point of suspension. So, $\tau = -(mg\sin\theta)L$.

31. The equation of angular motion for a simple pendulum is $I\alpha = -mgL\sin\theta$. For small θ , this approximates to:

- (A) $I\alpha = -mgL\theta^2$
 (B) $I\alpha = -mgL\theta$
 (C) $I\alpha = -mgL\cos\theta$
 (D) $I\alpha = -mgL/\theta$

Ans :

(B) $I\alpha = -mgL\theta$

For small angular displacements θ (expressed in radians), the approximation $\sin\theta \approx \theta$ is valid. Substituting this into the equation of rotational motion $I\alpha = -mgL\sin\theta$, we get $I\alpha \approx -mgL\theta$, which is the form required for angular SHM.

32. What is the moment of inertia (I) of a simple pendulum bob of mass m at a distance L from the support, assuming a massless string?

- (A) $I = mL$
 (B) $I = \frac{1}{2}mL^2$
 (C) $I = mL^2$
 (D) $I = m/L^2$

Ans :

(C) $I = mL^2$

For a simple pendulum, the bob of mass m is considered a point mass, and the string is massless of length L . The moment of inertia (I) of this point mass about the point of suspension is given by $I = mr^2$, where r is the distance from the axis of rotation. Here, $r = L$, so $I = mL^2$.

33. The time period of a simple pendulum of length L is given by $T = 2\pi\sqrt{L/g}$. This formula is accurate under which condition?

- (A) For any angle of displacement of the pendulum.
 (B) Only when the mass of the bob is very large.
 (C) For small angular displacements of the pendulum.
 (D) Only when the length of the string is very short.

Ans :

(C) For small angular displacements of the pendulum.

The formula $T = 2\pi\sqrt{L/g}$ is derived using the approximation $\sin\theta \approx \theta$, which is valid only for small angular displacements. For larger angles, the motion is periodic but not simple harmonic, and the period depends on the amplitude.

The time period of a simple pendulum does NOT depend on which of the following factors, for small oscillations?

- (A) Length of the pendulum (L)
 (B) Acceleration due to gravity (g)
 (C) Mass of the bob (m)
 (D) The location on Earth

Ans :

(C) Mass of the bob (m)

For small oscillations, the time period of a simple pendulum is $T = 2\pi\sqrt{L/g}$. This formula shows that the period depends on the length L and the acceleration due to gravity g , but it is independent of the mass of the bob. The location on Earth affects ' g '.

35. How does the period of a "seconds pendulum" (which ticks seconds) compare to one second?

- (A) The period is exactly 1 second.
 (B) The period is 2 seconds.
 (C) The period is 0.5 seconds.
 (D) The period is π seconds.

Ans :

(B) The period is 2 seconds.

A "seconds pendulum" is defined as a pendulum whose period is two seconds. This means it takes one second for the bob to swing from one extreme to the other (half a period), hence it "ticks" seconds.

- 23.** A particle oscillating under a force $F(t) = -kx(t)$ is called a:
- (A) Non-linear oscillator
 - (B) Damped oscillator
 - (C) Linear harmonic oscillator
 - (D) Forced oscillator

Ans :

(C) Linear harmonic oscillator

When the restoring force is directly proportional to the displacement (as in $F = -kx$), the oscillator is termed a linear harmonic oscillator. If the force includes terms like x^2 or x^3 , it becomes a non-linear oscillator.

- 24.** When is the kinetic energy of a particle in SHM maximum?
- (A) At the extreme positions of displacement.
 - (B) When the potential energy is also maximum.
 - (C) When the particle is at the mean position.
 - (D) When the acceleration of particle is maximum.

Ans :

(C) When the particle is at the mean position.

Kinetic energy $K = \frac{1}{2}mv^2$. The velocity v is maximum when the particle passes through the mean (equilibrium) position ($x=0$). Therefore, the kinetic energy is maximum at the mean position. At extreme positions, velocity is zero, so kinetic energy is zero.

- 25.** Which of the following statements regarding energy in SHM is incorrect?
- (A) Kinetic energy is $K = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$.
 - (B) Potential energy is $U = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$.
 - (C) Total mechanical energy $E = \frac{1}{2}kA^2$ and it varies with time.
 - (D) The period of kinetic energy and potential energy is $T/2$.

Ans :

(C) Total mechanical energy $E = \frac{1}{2}kA^2$ and it varies with time.

The total mechanical energy E in SHM is $E = K + U = \frac{1}{2}kA^2$. This total energy remains constant and does not vary with time, assuming no damping. The expressions for K and U , and their period ($T/2$) are correct.

- 26.** Choose the correct statement about the potential energy $U(x)$ of a particle in SHM.
- (A) Potential energy is always zero at the mean position.
 - (B) Potential energy is maximum when kinetic energy is maximum.

(C) Potential energy is $U(x) = \frac{1}{2}m\omega^2 x$.

(D) Potential energy is always negative at extreme positions.

Ans :

(A) Potential energy is always zero at the mean position.

The potential energy in SHM is given by $U(x) = \frac{1}{2}kx^2$. At the mean position, $x = 0$, so the potential energy $U(0) = 0$. Potential energy is maximum at extreme positions ($x = \pm A$), where kinetic energy is zero.

- 27.** What is the total mechanical energy (E) of a particle of mass m executing SHM with amplitude A and angular frequency ω ?

(A) $E = \frac{1}{2}m\omega A^2$

(B) $E = \frac{1}{2}m\omega^2 A$

(C) $E = \frac{1}{2}m\omega^2 A^2$

(D) $E = m\omega^2 A^2$

Ans :

(C) $E = \frac{1}{2}m\omega^2 A^2$

The total mechanical energy in SHM is the sum of kinetic and potential energies and is constant. It can be expressed as $E = \frac{1}{2}kA^2$. Since $k = m\omega^2$, the total energy is $E = \frac{1}{2}m\omega^2 A^2$.

- 28.** How often do kinetic energy and potential energy peak during one period (T) of SHM?

(A) Once each

(B) Twice each

(C) Four times each

(D) Never peak

Ans :

(B) Twice each

Both kinetic energy ($K \propto \sin^2(\omega t + \phi)$) and potential energy ($U \propto \cos^2(\omega t + \phi)$) involve squared sinusoidal terms. These terms go from zero to maximum twice within one full period (T) of the displacement. Thus, K and U each peak twice during one period T of SHM, having a period of $T/2$.

- 29.** For small angular displacements (θ), the motion of a simple pendulum is considered to be:

(A) Uniform circular motion

(B) Projectile motion

(C) Simple harmonic motion

(D) Non-periodic motion

$\sqrt{g}$ (Statement 2). It is independent of the mass of the bob (Statement 3 is false) and for small angles, it is independent of amplitude (Statement 4 is false).

41. Which statements correctly describe the phase relationships in SHM?

1. Displacement and velocity are in phase.
2. Velocity leads displacement by $\pi/2$.
3. Acceleration leads velocity by $\pi/2$.
4. Displacement and acceleration are out of phase by π .

- (A) 1 and 3
(B) 2 and 4
(C) 2 and 3
(D) 1 and 4

Ans :

(B) 2 and 4

If displacement $x = A \cos(\omega t + \phi)$, then velocity

$$v = -\omega A \sin(\omega t + \phi)$$

$$= \omega A \cos(\omega t + \phi + \pi/2)$$

so velocity leads displacement by $\pi/2$. Acceleration

$$a = -\omega^2 A \cos(\omega t + \phi)$$

$$= \omega^2 A \cos(\omega t + \phi + \pi)$$

so acceleration is out of phase with displacement by π . Also, acceleration leads velocity by $\pi/2$.

42. Match the physical quantities in Column I with their corresponding expressions in Column II for a particle in SHM described by $x(t) = A \cos(\omega t)$.

	Column I		Column II
1.	Velocity at $x = 0$	P.	0
2.	Acceleration at $x = A$	Q.	ωA
3.	Kinetic Energy at $x = A$	R.	$-\omega^2 A$
4.	Potential Energy at $x = 0$	S.	$\frac{1}{2} m \omega^2 A^2$

- (A) 1-Q, 2-R, 3-P, 4-P
(B) 1-Q, 2-R, 3-S, 4-P
(C) 1-P, 2-Q, 3-R, 4-S
(D) 1-R, 2-P, 3-S, 4-Q

Ans :

(A) 1-Q, 2-R, 3-P, 4-P

(1) At $x = 0$ (mean position), velocity is maximum, $v_{max} = \omega A$. (2) At $x = A$ (extreme position), acceleration $a = -\omega^2 A$. (3) At $x = A$, velocity is zero, so kinetic energy is zero. (4) At $x = 0$, potential energy $U = \frac{1}{2} kx^2 = 0$.

43. Match the type of motion or concept in Column I with its characteristic feature in Column II.

	Column I		Column II
1.	Periodic Motion	P.	Projection of uniform circular motion
2.	Oscillatory Motion	Q.	To and fro about a mean position
3.	Simple Harmonic Motion	R.	Repeats at regular time intervals
4.	Damping	S.	Decrease in amplitude due to resistive forces

(A) 1-R, 2-Q, 3-P, 4-S

(B) 1-Q, 2-R, 3-S, 4-P

(C) 1-R, 2-P, 3-Q, 4-S

(D) 1-S, 2-Q, 3-P, 4-R

Ans :

(A) 1-R, 2-Q, 3-P, 4-S

(1) Periodic motion repeats at regular intervals. (2) Oscillatory motion involves to-and-fro movement. (3) SHM can be represented as the projection of UCM. (4) Damping refers to the decrease in amplitude due to resistive forces.

44. Match the terms related to SHM in Column I with their SI units in Column II.

	Column I		Column II
1.	Amplitude	P.	Hertz (Hz)
2.	Angular Frequency	Q.	Joule (J)
3.	Frequency	R.	Meter (m)
4.	Total Energy	S.	Radian/second (rad/s)

(A) 1-R, 2-S, 3-P, 4-Q

(B) 1-R, 2-P, 3-S, 4-Q

(C) 1-P, 2-S, 3-R, 4-Q

(D) 1-Q, 2-S, 3-P, 4-R

Ans :

(A) 1-R, 2-S, 3-P, 4-Q

(1) Amplitude is a displacement, so its unit is meter. (2) Angular frequency is angle per unit time, so rad/s. (3) Frequency is cycles per second, so Hertz. (4) Total energy is measured in Joules.

45. Which of the following pairs correctly matches a physical quantity in SHM with its value at the mean position ($x = 0$)?

- (A) Potential Energy : Maximum
(B) Kinetic Energy : Zero
(C) Acceleration : Maximum
(D) Velocity : Maximum

97. Is restoring force necessary in S.H.M. ?

Ans :

Yes

98. Can a motion be periodic and not oscillatory ?
or

Are all periodic motions oscillatory ?

Ans :

Uniform circular motion is periodic but not oscillatory.

99. Are all oscillatory motions periodic ?

Ans :

Yes

100. In an S.H.M., can velocity and displacement be in the same direction ?

Ans :

Yes, when the particle moves from mean position to extreme position the velocity and displacement are in the same direction.

101. What is the (a) distance moved (b) displacement of a particle executing S.H.M. ?

Ans :

- (a) 4 times the amplitude
- (b) Zero

102. What is the basic condition for the motion of a particle to be S.H.M. ?

Ans :

The basic condition (i.e. necessary and sufficient conditions) for a motion to be S.H.M. is that the restoring force must be linear i.e. $F = -ky$ (or $\tau = -c\theta$).

103. Can a body have acceleration without velocity ?

Ans :

In S.H.M., at extreme positions velocity is zero but acceleration is maximum.

104. What is force constant and write its SI unit ?

Ans :

Force constant is the force per unit displacement and its SI unit is Nm^{-1} .

105. Why a point on a rotating wheel cannot be considered as executing S.H.M. ?

Ans :

The motion of the rotating point on the wheel is periodic but not oscillatory.

106. Which trigonometrical functions are suitable for expressing periodic motion and why ?

Ans :

Sine and cosine functions are simple periodic and bounded functions. Hence they are suitable for expressing periodic motion.

107. What is the effect on the time period of a simple pendulum if the mass of the bob is doubled ?

Ans :

No effect, because time period T of simple pendulum is independent of mass of the bob.

108. Two simple pendulums of equal length cross each other at mean position. What is their phase difference?

Ans :

π radian i.e. 180° .

109. What will be the change in time period of a loaded spring when taken to moon ?

Ans :

No change, because $T = 2\pi\sqrt{\frac{m}{\kappa}}$ i.e. T is independent of g .

110. Which quantity remains constant throughout the vibration of a pendulum ?

Ans :

The total energy

111. A simple pendulum moves from one end to the other in $1/2$ second. What is its frequency ?

Ans :

Since time period, $T = \frac{1}{2} + \frac{1}{2} = 1\text{s}$

$$\nu = \frac{1}{T} = 1\text{Hz}$$

112. Is second's pendulum a simple pendulum ?

Ans :

Yes

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51. A child on a swing is an example of which type of motion, primarily?

(A) Uniform circular motion
(B) Oscillatory motion
(C) Rectilinear motion
(D) Random motion

Ans :

(B) Oscillatory motion

A child on a swing moves back and forth about a central, lowest point (the mean position). This to-and-fro movement is characteristic of oscillatory motion. While the path is an arc of a circle, the overall motion is repetitive and oscillatory.

52. The quartz crystal in a watch vibrates with a very precise frequency. This vibration is an example of:

(A) Damped oscillation
(B) Simple harmonic motion
(C) Forced oscillation
(D) Periodic motion

Ans :

(D) Periodic motion

The vibrations of a quartz crystal are highly regular and repeat at fixed intervals, making it an excellent example of periodic motion. While it might be a specific type of oscillation (like SHM or maintained), its fundamental characteristic for timekeeping is its precise periodicity.

53. If the displacement of a particle is given by $x = A \sin(\omega t) + B \cos(\omega t)$, its motion is:

(A) Not periodic.
(B) Periodic but not SHM.
(C) Simple harmonic motion.
(D) Oscillatory but not periodic.

Ans :

(C) Simple harmonic motion.

A linear combination of sine and cosine functions with the same angular frequency (ω) results in another sinusoidal function of time, which is the definition of simple harmonic motion. The amplitude of this SHM would be $\sqrt{A^2 + B^2}$.

54. The angular frequency (ω) in SHM represents:

(A) The rate of change of phase.
(B) The number of oscillations per second.
(C) The time taken for one oscillation.
(D) The maximum angular displacement.

Ans :

(A) The rate of change of phase.

The phase of SHM is $(\omega t + \phi)$. The rate of change of this phase with respect to time is $\frac{d}{dt}(\omega t + \phi) = \omega$. Thus, angular frequency is the rate at which the phase of the oscillation changes.

55. If a particle in SHM has displacement $x(t) = 5 \cos(2\pi t + \pi/4)$ meters, what is its amplitude?

(A) 5 m
(B) 2π m
(C) $\pi/4$ m
(D) 10π m

Ans :

(A) 5 m

The general equation for SHM is $x(t) = A \cos(\omega t + \phi)$, where A is the amplitude. Comparing this with the given equation $x(t) = 5 \cos(2\pi t + \pi/4)$, the amplitude A is clearly 5 meters.

56. What is the phase difference between the velocity and acceleration of a particle in SHM?

(A) 0 radians
(B) $\pi/2$ radians
(C) π radians
(D) $3\pi/2$ radians

Ans :

(B) $\pi/2$ radians

If displacement is $x = A \cos(\omega t + \phi)$, then

$$v = -\omega A \sin(\omega t + \phi) \\ = \omega A \cos(\omega t + \phi + \pi/2)$$

and $a = -\omega^2 A \cos(\omega t + \phi)$

$$= \omega^2 A \cos(\omega t + \phi + \pi)$$

Thus, acceleration leads velocity by $\pi/2$ (or velocity leads acceleration by $-\pi/2$ or $3\pi/2$).

57. If two springs with force constants k_1 and k_2 are connected in parallel to a mass m , what is the effective force constant k_{eff} of the combination?

(A) $k_{eff} = (k_1 k_2) / (k_1 + k_2)$
(B) $k_{eff} = k_1 + k_2$
(C) $k_{eff} = (k_1 + k_2) / 2$
(D) $k_{eff} = \sqrt{k_1 k_2}$

Ans :

(B) $k_{eff} = k_1 + k_2$

When springs are connected in parallel, the effective force constant is the sum of the individual force constants. This is because for a given displacement, both springs exert a force, and these forces add up.

- 125.** At a certain speed of bus, its whole body starts vibrating strongly. Explain.

Ans :

The frame of the bus begins to vibrate strongly on account of resonance. At a particular speed, the frequency of bus engine becomes equal to the frequency of its frame.

- 126.** If instantaneous velocity at a point is zero, is instantaneous acceleration necessarily be zero ?

Ans :

No, when the body is moving along a line with S.H.M., its instantaneous velocity at extreme point is zero whereas its instantaneous acceleration is maximum.

- 127.** What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ?

Ans :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

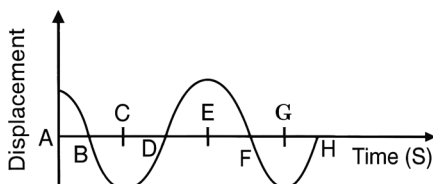
$$v = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

Since, $g = 0$ when it is freely falling, hence

$$\text{Frequency} = \nu = 0$$

- 128.** Displacement versus time curve for a particle executing S.H.M. is shown in figure.

Identify the points marked at which (i) velocity of the oscillator is zero, (ii) speed of the oscillator is maximum.



Ans :

- (i) The points A, E, G are the points on the mean position and hence the displacement is zero.
(ii) The points B, D, F and H are the points at which the oscillator has maximum speed.

- 129.** A girl is swinging in a swing in the sitting position. How will the period of the swing be affected if she stands up ?

Ans :

When the girl stands up, her centre of gravity is raised up. This results in the decrease in the length

(l) of the pendulum (formed by the girl and swing). As a result of this, period of the swing is decreased.

- 130.** Explain the difference between periodic motion and oscillatory motion using examples.

Ans :

Periodic motion repeats itself at regular intervals, like uniform circular motion. Oscillatory motion is a type of periodic motion where the body moves to and fro about a mean position, like a swinging pendulum. All oscillatory motions are periodic, but not all periodic motions are oscillatory.

- 131.** Define the terms 'period' and 'frequency' in the context of oscillatory motion and state their relationship.

Ans :

The period (T) is the smallest interval of time after which an oscillatory motion repeats itself. Frequency (ν) is the number of repetitions per unit time. They are related by $\nu = \frac{1}{T}$.

- 132.** What are the key characteristics of simple harmonic motion (SHM)?

Ans :

Simple harmonic motion is an oscillatory motion where displacement is a sinusoidal function of time. The restoring force is directly proportional to the displacement from the mean position and is always directed towards it.

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- 133.** Describe how the displacement of a particle in SHM is represented mathematically.

Ans :

The displacement $x(t)$ of a particle in SHM is represented as $x(t) = A \cos(\omega t + \phi)$. Here, A is the amplitude, ω is the angular frequency, and ϕ is the initial phase constant.

- 134.** Explain 'amplitude' and 'phase constant' in simple harmonic motion.

Ans :

Amplitude (A) is the magnitude of the maximum displacement of the particle from its equilibrium position. The phase constant (ϕ) is the value of the phase at $t = 0$ and determines the initial state of motion.

Ans :

(c) $\frac{\pi}{\omega}$

Given, $v = \frac{dx}{dt} = -A\omega \sin\left(\omega t + \frac{\pi}{4}\right)$

Now, v will be maximum

If $\omega t + \frac{\pi}{4} = \frac{\pi}{2}$

$$\omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega}$$

66. A body executes S.H.M. The P.E. and K.E. and total energy (T.E.) are measured as a function of displacement x . Which of the following statement is true ?

- (a) K.E. is maximum when x is maximum.
 (b) P.E. is maximum when $x = 0$.
 (c) K.E. is maximum when $x = 0$.
 (d) T.E. is zero when $x = 0$.

Ans :

- (c) K.E. is maximum when
- $x = 0$

Since velocity is maximum when $x = 0$, so K.E. is maximum when $x = 0$.

67. The potential energy of a simple harmonic oscillator, when the particle is half way to its end point is:

- (a) $\frac{2}{3}E$ (b) $\frac{E}{8}$
 (c) $\frac{E}{4}$ (d) $\frac{E}{2}$

Ans :

(c) $\frac{E}{4}$

Since, $E = \frac{1}{2}m\omega^2 r^2$ and

$$E_p = \frac{1}{2}m\omega^2 \left(\frac{a}{2}\right)^2 = \frac{E}{4}$$

68. Pendulum after some time becomes slow in motion and finally stops due to :

- (a) Air friction (b) Earth's gravity
 (c) Mass of pendulum (d) None of these

Ans :

- (a) Air friction

69. If a metal bob of a simple pendulum is replaced by wooden bob, then its time period will:

- (a) Increase
 (b) Decreases

- (c) Remain same

- (d) First 'a' and then 'b'

Ans :

- (c) Remain same

Time period is independent of mass of the bob.

70. To make the frequency double of an oscillator, we have to:

- (a) Double the mass
 (b) Half the mass
 (c) Quadruple of mass
 (d) Reduce the mass to one-fourth.

Ans :

- (d) Reduce the mass to one-fourth.

Since, $\nu = \frac{1}{2\pi} \frac{k}{m}$

So $\nu \propto \frac{1}{\sqrt{m}}$

71. In case of a forced vibration, the resonance wave becomes very sharp when the:

- (a) Damping force is small
 (b) Restoring force is small
 (c) Applied periodic force is small
 (d) Quality factor is small

Ans :

- (a) Damping force is small

72. A child swinging on a swing in a sitting position stands up, then the time period of the swing will

- (a) Increase
 (b) Decrease
 (c) Remain same
 (d) Increase if the child is long and decrease if the child is short

Ans :

- (b) Decrease

Since, $t = 2\pi\sqrt{\frac{l}{g}}$

i.e., $T \propto \sqrt{l}$, so when child stands up, the C.G. of the child shift upwards, i.e., l decreases and hence l decreases

73. The total energy of the particle performing SHM depends on:

- (a) k, A, m (b) k, A
 (c) k, A, x (d) k, x

Ans :

(c) k, A, x

$$\begin{aligned}\text{Since, } E &= \frac{1}{2} m \omega^2 A^2 \\ &= \frac{1}{2} k A^2\end{aligned}$$

74. The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times, then its period becomes:

- (a) 16s (b) 12s
(c) 8s (d) 4s

Ans :

(d) 4s

$$\begin{aligned}\text{Since, } T &\propto \sqrt{l} \\ \text{So, } \frac{T_1}{T} &= \sqrt{\frac{l}{4l}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \\ T_1 &= 2 \times T = 2 \times 2 = 4s\end{aligned}$$

75. A system exhibiting S.H.M. must possess:

- (a) Elasticity as well as inertia
(b) Elasticity, inertia and an external force
(c) Elasticity only
(d) Inertia only

Ans :

- (a) Elasticity as well as inertia
The system must possess elasticity and inertia.

76. The acceleration of a particle in S.H.M. is :

- (a) Always constant
(b) Maximum at mean position
(c) Maximum at equilibrium position
(d) Always zero

Ans :

- (b) Maximum at mean position

$$a_{\max} = -\omega^2 r \text{ i.e. } a \propto r$$

77. For a body of mass m attached to the spring, the spring factor is given by (ω , the angular frequency):

- (a) $\frac{m}{\omega^2}$ (b) $m\omega^2$
(c) $m^2\omega$ (d) $m^2\omega^2$

Ans :

- (b) $m\omega^2$

$$\begin{aligned}\text{Since, } \nu &= \frac{1}{2\pi} \sqrt{\frac{k}{m}} \\ \frac{k}{m} &= (2\pi \nu)^2 = \omega^2\end{aligned}$$

$$k = m\omega^2$$

78. When a mass is attached to a spring, its length is increased by 20 cm. It is now further lowered and releases. The time period is:

- (a) $\frac{2\pi}{7}$ Sec
(b) 7 Sec
(c) 2π Sec
(d) Enough data not available

Ans :

- (a) $\frac{2\pi}{7}$ Sec

$$\begin{aligned}T &= 2\pi \sqrt{\frac{l}{g}} \\ &= 2\pi \sqrt{\frac{20}{980}} = \frac{2\pi}{7} \text{ sec}\end{aligned}$$

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79. A spring of force constant K is cut into three equal parts. The force constant of each part will be:

- (a) K (b) $3K$
(c) $\frac{K}{3}$ (d) $9K$

Ans :

- (b) $3K$

If K is the force constant of each part, then

$$\begin{aligned}\frac{1}{K} &= \frac{1}{K} + \frac{1}{K} + \frac{1}{K} = \frac{3}{K} \\ K &= 3K\end{aligned}$$

80. A small mass executes linear S.H.M. about a point O with amplitude r and period T . Its displacement from O at time $\frac{T}{8}$ after passing through O is:

- (a) $\frac{r}{\sqrt{2}}$ (b) $\frac{r}{2\sqrt{2}}$
(c) $\frac{r}{2}$ (d) $\frac{r}{8}$

Ans :

- (a) $\frac{r}{\sqrt{2}}$

$$\begin{aligned}y &= r \sin \omega t = r \sin -\frac{2\pi}{T} \times \frac{T}{8} \\ &= r \sin \frac{\pi}{4} = \frac{r}{\sqrt{2}}\end{aligned}$$

ONE MARK QUESTIONS

- 81.** A spring has a mass ' m ' suspended from it. What shall happen to the frequency of oscillation if the spring is cut into two halves ?

Ans :

The spring factor (k) becomes double when the spring is cut into two halves as $k = \frac{F}{l}$.

As $\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$, the frequency of oscillation becomes $\sqrt{2}$ times the original frequency.

- 82.** What is time period of a pendulum in a spaceship ?

Ans :

In spaceship, $g = 0$, therefore

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{l}{0}} = \infty$$

Time period will be definite and frequency will be zero.

- 83.** Why the motion of a satellite around a planet cannot be taken as S.H.M. ?

Ans :

For S.H.M., the motion must be both periodic and oscillatory. The motion of a satellite around a planet is periodic but not oscillatory, hence it is not a S.H.M.

- 84.** What will be the change in time period of a loaded spring, when taken to moon ?

Ans :

The time period of load spring will remain same because the time period of loaded spring is independent of acceleration due to gravity (a). Time period T is given as

$$T = 2\pi \sqrt{\frac{m}{k}}$$

- 85.** The bob of an oscillating pendulum is made of ice. How will the time period change when ice starts melting ?

Ans :

It will remain unchanged because the time period is independent of mass.

- 86.** What is the time period of second's pendulum ?

Ans :

Its time period is 2s.

- 87.** Is it true that P.E. at mean position is zero ?

Ans :

No, P.E. has minimum value at mean position so it, may or may not be zero at mean position.

- 88.** A pendulum clock is in a lift that descends at a constant velocity. Does it keep correct time ?

Ans :

Yes, because the acceleration of lift is zero.

- 89.** Is there any relation between uniform circular motion and S.H.M. ?

Ans :

The motion of the projection of uniform circular motion on any diameter of the circle is S.H.M.

- 90.** Are all oscillatory motions periodic ?

Ans :

Yes

- 91.** Are all periodic motions oscillatory ?

Ans :

No

- 92.** What is the length of a seconds pendulum ?

Ans :

99.3 cm

- 93.** Is spring constant a dimensionless constant ?

Ans :

No

- 94.** What is periodic motion?

Ans :

Periodic motion is a motion which repeats itself after equal interval of time.

- 95.** What is oscillatory motion ?

Ans :

Oscillatory motion is a motion, in which a body moves to and fro or back and forth repeatedly about a mean position.

- 96.** A particle has maximum velocity at mean position and zero velocity at the extreme position. Is it sure test for S.H.M. ?

Ans :

No

Ans :

(D) Velocity : Maximum

At the mean position ($x = 0$) in SHM: Potential energy ($U = \frac{1}{2}kx^2$) is zero. Kinetic energy ($K = \frac{1}{2}mv^2$) is maximum because velocity is maximum. Acceleration ($a = -\omega^2x$) is zero. Velocity is indeed maximum.

46. Identify the correct pair regarding the period (T) of oscillation.

(A) Spring-mass system : $T = 2\pi\sqrt{k/m}$

(B) Simple pendulum : $T = 2\pi\sqrt{g/L}$

(C) Seconds pendulum : $T = 1\text{s}$

(D) SHM : $T = 2\pi/\omega$

Ans :

(D) SHM : $T = 2\pi/\omega$

For a spring-mass system, $T = 2\pi\sqrt{m/k}$. For a simple pendulum, $T = 2\pi\sqrt{L/g}$. A seconds pendulum has a period of $T = 2\text{s}$. The general relation for SHM is that the period $T = 2\pi/\omega$, where ω is the angular frequency.

47. Which of the following pairs incorrectly matches a concept with its description?

(A) Phase constant (ϕ) : Determines initial state of motion.

(B) Angular frequency (ω) : 2π times the linear frequency.

(C) Restoring force (F) : Always directed away from mean position.

(D) Amplitude (a) : Maximum displacement from mean position.

Ans :

(C) Restoring force (F) : Always directed away from mean position.

The restoring force in SHM is always directed towards the mean (equilibrium) position, not away from it. This force is what causes the object to oscillate. The other pairs correctly describe their respective concepts.

48. Identify the incorrect pair of physical quantity in SHM and its characteristic at an extreme position ($x = \pm A$).

(A) Displacement : $\pm A$

(B) Velocity : Zero

(C) Acceleration : Zero

(D) Potential Energy : Maximum

Ans :

(C) Acceleration : Zero

At an extreme position ($x = \pm A$): Displacement is maximum ($\pm A$). Velocity is zero. Acceleration ($a = -\omega^2x$) is maximum in magnitude ($-\omega^2(\pm A)$) and directed towards the mean position, so it is not zero. Potential energy ($U = \frac{1}{2}kx^2$) is maximum.

49. Arrange the following stages of energy transformation for a particle in SHM starting from the mean position and moving towards a positive extreme:

1. Kinetic energy is maximum, Potential energy is zero.

2. Kinetic energy decreases, Potential energy increases.

3. Kinetic energy is zero, Potential energy is maximum.

(A) 1, 2, 3

(B) 2, 1, 3

(C) 3, 2, 1

(D) 1, 3, 2

Ans :

(A) 1, 2, 3

Starting at the mean position ($x = 0$): Kinetic energy is maximum, potential energy is zero (Stage 1). As the particle moves towards the positive extreme, its speed decreases (kinetic energy decreases) and its displacement increases (potential energy increases) (Stage 2). At the extreme position ($x = A$): Kinetic energy becomes zero, and potential energy is maximum (Stage 3).

50. A particle in SHM starts from its positive extreme position. What is the correct sequence of positions it passes through in the first full cycle? (O is the mean position, +A is positive extreme, -A is negative extreme)

1. O (moving towards -A)

2. -A

3. O (moving towards +A)

4. +A

(A) 4, 1, 2, 3, 4

(B) 4, 3, 2, 1, 4

(C) 4, 2, 1, 3, 4

(D) 4, 1, 3, 2, 4

Ans :

(A) 4, 1, 2, 3, 4

Starting at +A (labeled 4 for initial state): The particle moves towards the mean position O (Stage 1, moving towards -A), then to the negative extreme -A (Stage 2), then back through the mean position O (Stage 3, moving towards +A), and finally returns to the positive extreme +A (completing one cycle, labeled 4 again).

- 113.** What factors determine the natural frequency of an oscillator ?

Ans :

The dimensions and the elastic properties of an oscillator.

- 114.** Why are vibrations of a simple pendulum damped ?

Ans :

Because the energy of the pendulum is used to overcome air resistance.

- 115.** What determines the natural frequency of a body ?

Ans :

- (i) Elastic properties of the material of the body.
- (ii) Dimensions of the body.

- 116.** How many times KE and PE of an oscillator in one vibration become maximum ?

Ans :

In one vibration the KE and PE become maximum two time.

- 117.** When two unequal springs of same material are loaded with same load, which one will have the larger value of time period ?

Ans :

The longer spring when loaded with weight mg , will have more extension l

$$\text{As } T = 2\pi\sqrt{\frac{l}{g}}$$

So, the longer spring will have a large value of the time period.

TWO MARKS QUESTIONS

- 118.** For an oscillating simple pendulum is the tension constant throughout the oscillation ? If not when it is (a) the least (b) the greatest.

Ans :

No, the tension is not constant throughout the oscillation. It is least at extreme positions and greatest at mean position.

- 119.** Water in U-tube executes S.H.M. Will the time period for mercury filled up to the same height in the U-tube be lesser or greater than that in case of water ?

Ans :

The time period of liquid executing S.H.M. in a U-tube is independent of the nature of the liquid. So time period with mercury filled up to the same height in the U-tube will remain same.

- 120.** What are two basic characteristics of an oscillatory system ?

Ans :

- (i) The oscillatory motion should repeats it self over and over again about its mean position.
- (ii) The oscillatory motion should remain confined within two extreme limits on either side of the mean position.

- 121.** Can a motion be oscillatory but not simple harmonic?

Ans :

The motion of a ball thrown from a height on a perfectly elastic plane is an oscillatory motion but not simple harmonic. Here $F = mg = \text{constant}$ but not $F \propto -y$ (displacement), Which is an essential condition for S.H.M.

- 122.** What are the two basic characteristics of an oscillating system ?

Ans :

- (i) It repeats itself after a fixed interval. Sine and cosine functions have a period of 2π radian.
- (ii) It oscillates to and fro about its mean position.

- 123.** At what point along the path of a simple pendulum is the tension maximum ?

Ans :

At the mean position of simple pendulum, the tension in its string is maximum and is equal to the weight (mg) of the bob attached to the lower end of string.

- 124.** Justify the statement, "sometimes a wine glass is broken by the powerful voice of celebrated singer."

Ans :

When the natural frequency of wine glass becomes the same as that of the singer, the resulting resonant vibrations due to powerful voice of the singer may break the wine glass.

58. The potential energy of a simple harmonic oscillator is zero when the particle is:

(A) At its positive extreme position.
 (B) At its negative extreme position.
 (C) At its equilibrium position.
 (D) Midway between mean and extreme.

Ans :

(C) At its equilibrium position.

Potential energy in SHM is given by $U = \frac{1}{2}kx^2$, where x is the displacement from the equilibrium position. At the equilibrium position, $x = 0$, so the potential energy $U = 0$.

59. For a simple pendulum executing SHM, the tension in the string is maximum when the bob is:

(A) At its highest point (extreme position).
 (B) At its mean position (lowest point).
 (C) Midway between mean and extreme.
 (D) Tension is always constant in SHM.

Ans :

(B) At its mean position (lowest point).

At the mean position, the speed of the bob is maximum. The tension must provide the gravitational force component mg and the centripetal force mv^2/L . At extremes, speed is zero, tension supports $mg\cos\theta$. Thus, tension is maximum at the mean position.

60. A clockmaker needs to design a pendulum that has a period of 1 second for a special clock. What approximate length should the pendulum have if $g \approx \pi^2 \text{m/s}^2$?

(A) 0.25 m
 (B) 0.50 m
 (C) 1.00 m
 (D) 4.00 m

Ans :

(A) 0.25 m

The period of a simple pendulum is $T = 2\pi\sqrt{L/g}$. We are given $T = 1$ s and $g \approx \pi^2 \text{m/s}^2$. So, $1 = 2\pi\sqrt{L/\pi^2}$. Squaring both sides: $1 = 4\pi^2(L/\pi^2) = 4L$. Therefore, $L = 1/4 = 0.25$ m.

61. A particle executes S.H.M. Then the graph of velocity as a function of displacement is:

(a) A straight line
 (b) A circle
 (c) An ellipse
 (d) A hyperbola

Ans :

(c) An ellipse

An ellipse, because $v^2 = \omega^2(r^2 - y^2)$ an equation of an ellipse.

62. The instantaneous acceleration of a particle executing S.H.M. given by $y = a \sin \omega t$ is:

(a) $+\omega^2 y$ (b) $+\omega y$
 (c) $-\omega y^2$ (d) $-\omega^2 y$

Ans :

(c) $-\omega y^2$

$$y = a \sin \omega t$$

$$v = \frac{dy}{dt} = a \cos \omega t$$

So

$$a = \frac{dv}{dt} = -\omega^2 a \sin \omega t = -\omega^2 y$$

63. When the potential energy of a particle executive S.H.M. is one-fourth of the maximum value during the oscillation, its displacement from the equilibrium position in terms of its amplitude a is:

(a) $\frac{a}{4}$ (b) $\frac{a}{3}$
 (c) $\frac{a}{2}$ (d) $\frac{2a}{2}$

Ans :

(c) $\frac{a}{2}$

$$\text{Given, } \frac{1}{2} m\omega^2 y^2 = \frac{1}{4} \frac{1}{2} m\omega^2 a^2$$

$$\text{Hence, } y = \frac{a}{2}$$

64. If the equation of SHM is $y = a \sin (4\pi t + \phi)$, how much is its frequency ?

(a) 2 (b) $\frac{1}{2}$
 (c) 2π (d) $\frac{1}{2\pi}$

Ans :

(c) 2π

Given,

$$\omega = 4\pi$$

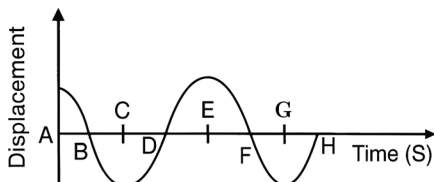
$$2\pi\nu = 4\pi$$

$$\nu = 2$$

65. An instantaneous displacement of a simple harmonic oscillator is $x = A \cos \left(\omega t + \frac{\pi}{4} \right)$ Its speed will be maximum at time:

(a) $\frac{\pi}{4\omega}$ (b) $\frac{\pi}{2\omega}$
 (c) $\frac{\pi}{\omega}$ (d) $\frac{2\pi}{\omega}$

- 135.** Displacement versus time curve for a particle executing S.H.M. is shown in figure. Identify the points marked at which (i) velocity of the oscillator is zero, (ii) speed of the oscillator is maximum.



Ans :

- (i) The points A, C, E are the points on the mean position and hence the displacement is zero.
 (ii) The points B, D, F and H are the points at which the oscillator has maximum speed.
- 136.** How does the projection of uniform circular motion relate to simple harmonic motion?

Ans :

The projection of a particle undergoing uniform circular motion onto any diameter of the circle executes simple harmonic motion. The radius of the circle corresponds to the amplitude of the SHM.

- 137.** Describe the velocity of a particle executing SHM at its mean position and extreme positions.

Ans :

The velocity of a particle in SHM is maximum at the mean position ($x = 0$). At the extreme positions ($x = \pm A$), the velocity is zero as the particle momentarily stops before reversing direction.

- 138.** Describe the acceleration of a particle executing SHM. How is it related to displacement?

Ans :

The acceleration $a(t)$ of a particle in SHM is given by $a(t) = -\omega^2 x(t)$. It is directly proportional to the displacement $x(t)$ and always directed towards the mean position, opposite to the displacement.

- 139.** State and explain the force law for simple harmonic motion.

Ans :

The force law for SHM is $F(t) = -kx(t)$, where $F(t)$ is the restoring force, k is the force constant, and $x(t)$ is the displacement. This indicates the force is proportional to displacement and opposes it.

- 140.** Discuss the kinetic energy of a particle in SHM. How does it vary with time and position?

Ans :

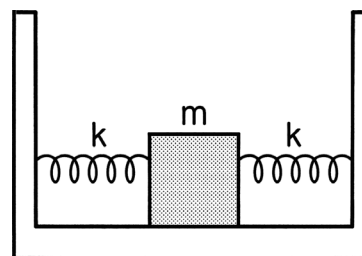
Kinetic energy

$$K = (1/2)mv^2 = (1/2)kA^2\sin^2(\omega t + \phi).$$

It is maximum at the mean position (zero displacement) and zero at extreme positions. It varies periodically with a period of $T/2$.

THREE MARKS QUESTIONS

- 141.** Two identical springs of spring constant K are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right, find the restoring force.



Ans :

If F_1 and F_2 be the restoring forces developed in the two springs. And x is the displacement, then

$$F_1 = -kx \text{ and } F_2 = -kx$$

Hence, Total restoring force,

$$F = F_1 + F_2 = -kx - kx = -2kx$$

Thus the total restoring force $= 2kx$ towards left.

- 142.** Write any two main characteristics of S.H.M.

Ans :

The two characteristics of S.H.M. are as follows :

- (i) A particle is said to be executing S.H.M. if its acceleration at any instant is directly proportional to its displacement from the mean position and is always directed towards mean position.
 (ii) S.H.M. is the projection of a uniform circular motion on any diameter of a circle of reference.

- 143.** Can we use pendulum watch in an artificial satellite?

Ans :

In an artificial satellite, a body is in a state of weightlessness and as such the effective value of g for it is zero. The time period of a pendulum is given by

$T = 2\pi\sqrt{\frac{l}{g}}$ As $g = 0$, $T = \infty$. Thus inside the satellite, the pendulum watch does not oscillate at all. Hence, we cannot use a pendulum watch in an artificial satellite.

- 144.** When soldiers cross a suspension bridge, they are advised to break the steps. Why ?

Ans :

While crossing the bridge if the soldiers move in steps, the frequency of their marching steps may match the natural frequency of oscillation of the suspended bridge *i.e.*, the bridge may vibrate in resonance with marching steps. This will result in enormous increase in amplitude and bridge may crack. Hence, soldiers are advised to move out of steps.

- 145.** A hollow sphere is filled with water through a small hole in it. It is hung by a long thread and as water slowly flows out of the hole at the bottom, one finds that the periods of oscillation first increases and then decreases. Explain why ?

Ans :

As the water flows out of the hole of the sphere the equivalent length of pendulum increases due to lowering of centre of gravity upto centre of sphere and then the equivalent length of the pendulum decreases due to lowering of water level from the centre of sphere to its bottom, hence the time period of oscillation first increases and then decreases.

- 146.** Define and explain seconds pendulum.

Ans :

A simple pendulum whose time period is 2 seconds is called a seconds pendulum.

Since for a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \dots(1)$$

For Seconds pendulum,

$$T = 2s$$

So equation (1) becomes

$$2 = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{g}{\pi^2} = \frac{9.8}{9.87} = 0.993 \text{ m}$$

$$l = 99.3 \text{ cm}$$

Hence a simple pendulum whose length is 99.3 cm is a seconds pendulum.

- 147.** A pendulum is taken to moon. Will it go faster or slower in comparison with the earth ?

Ans :

When the pendulum is transported to moon, it will go slow. It is on account of the fact that the value of acceleration due to gravity (g) on moon is less than its value on earth. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. With the decrease in value of ' g ', T increases and as such the pendulum goes slow.

- 148.** Discuss whether a simple pendulum experiment can be done inside a satellite.

Ans :

Inside a satellite, a body is in the state of weightlessness and as such the effective value of ' g ' for it is zero. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ As $g = 0$, $T = \infty$. Thus inside a satellite, the pendulum does not oscillate at all. Hence simple pendulum experiment cannot be performed inside a satellite.

- 149.** What change in mass is required to double the frequency of a harmonic oscillator ?

Ans :

The time period of a harmonic oscillator is given by $T = 2\pi\sqrt{\frac{m}{k}}$, Where ' k ' is force constant of spring.

If ν is frequency of harmonic oscillator, then

$$\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\nu \propto \frac{1}{\sqrt{m}}$$

Thus, to double the frequency of oscillator, the mass should be reduced to $\frac{1}{4}$ th of its original value.

- 150.** Is spring constant a dimensional or non-dimensional quantity ?

Ans :

In a spring, the magnitude of restoring force is given by

$$F = kx$$

where, x is extension

$$k = \frac{F}{x} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$$

Thus, spring constant is a dimensional quantity.

- 151.** Velocity and displacement of a particle executing S.H.M. are out of phase by $\frac{\pi}{2}$. Explain why ?

Ans :

The velocity of a particle executing S.H.M. is given by

$$v = r\omega \cos \omega t$$

$$v = r\omega \sin \left(\omega t + \frac{\pi}{2} \right) \quad \dots(1)$$

Displacement of a particle executing S.H.M. is given by

$$y = r \sin \omega t \quad \dots(2)$$

From equation (1) and (2), we find that velocity and displacement of a particle executing S.H.M. differ in phase by $\frac{\pi}{2}$.

- 152.** The amplitude of a simple harmonic oscillator is doubled. How does this affect:
- the period
 - the total energy
 - the maximum velocity of the oscillator ?

Ans :

- (i) Since,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So,
$$T = 2\pi\sqrt{\frac{1}{\text{acceleration per unit displacement}}}$$

Since acceleration per unit displacement is a constant quantity, T does not vary on changing the amplitude.

(ii)
$$E = 2\pi^2 m r^2 \nu^2$$

Therefore, if amplitude is doubled, the energy will become four times.

(iii)
$$v_{\max} = r\omega$$

Therefore, if amplitude is doubled, the value of maximum velocity will also get doubled.

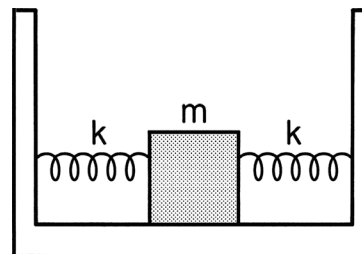
- 153.** Define periodic motion and illustrate it with two distinct examples.

Ans :

Periodic motion is a type of motion that repeats itself at regular intervals of time. The smallest such interval is known as the period of the motion.

- An insect climbing up a ramp and falling down repeatedly follows a periodic motion.
- A child climbing up and down a step identically also demonstrates periodic motion.

- 154.** Two identical springs of spring constant K are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right, find the restoring force.



Ans :

If F_1 and F_2 be the restoring forces developed in the two springs. And x is the displacement, then

$$F_1 = -kx \text{ and } F_2 = -kx$$

Hence, Total restoring force,

$$F = F_1 + F_2 = -kx - kx = -2kx$$

Thus the total restoring force $= 2kx$ towards left.

- 155.** Explain the terms 'period (T)' and 'frequency (v)' of periodic motion, including their SI units and interrelation.

Ans :

In periodic motion:

- The **period (T)** is the smallest interval of time after which the motion is repeated. Its SI unit is the second (s).
- The **frequency (v)** is the number of repetitions of the motion that occur per unit time. Its SI unit is hertz (Hz), where $1 \text{ Hz} = 1 \text{ s}^{-1}$. They are related by the formula $v = 1/T$.

- 156.** What is displacement in the context of oscillatory motion? Provide examples beyond just position.

Ans :

In oscillatory motion, displacement refers to the change with time of any physical property under consideration from its equilibrium or mean value.

- For a block attached to a spring, it is its distance from the equilibrium position.
- For an oscillating pendulum, it can be the angle from the vertical.
- It can also be the voltage across a capacitor in an AC circuit or pressure variations in a sound wave.

- **Restoring Force:** In the spring, $F = -kx$. In the pendulum, the tangential component $F_t \approx -mg\theta = -(mg/L)s$.
- **Period:** For spring, $T = 2\pi\sqrt{m/k}$. For pendulum, $T = 2\pi\sqrt{L/g}$.
- **Dependence:** Spring period depends on mass and spring constant. Pendulum period depends on length and gravity, independent of mass (for small angles).

170. Describe an experiment to show that the projection of uniform circular motion on a diameter is SHM.

Ans :

To demonstrate this:

- Take a ball and tie it to a string.
- Make the ball move in a horizontal circle with a constant angular speed. This is uniform circular motion.
- View this motion edge-on, or observe the shadow of the ball cast on a vertical wall by a distant light source placed in the plane of the circle.
The ball (or its shadow) will appear to move to and fro along a straight line, which is simple harmonic motion.

FIVE MARKS QUESTIONS

- 171.** (a) How S.H.M. is represented by a function ? Describe it from Fourier series.
(b) What is meant by amplitude and phase ? Deduce from Fourier series.

Ans :

- (a) **Simple harmonic motion :** Periodic motion for which the displacement can be mathematically represented by the function

$$F(t) = A \cos\left(\frac{2\pi t}{T} + \phi\right)$$

where A and ϕ are constants, is called simple harmonic motion.

For simple Harmonic motion

$$F(t) = b_1 \cos \frac{2\pi t}{T} + a_1 \sin \frac{2\pi t}{T}$$

Let $b_1 = A \cos \phi$
and $a_1 = -A \sin \phi$... (1)

$$F(t) = A \cos \phi \cos \frac{2\pi t}{T} - A \sin \phi \sin \frac{2\pi t}{T}$$

where, $A = (b_1^2 + a_1^2)^{1/2}$... (2)

then, $F(t) = A \left(\cos \phi \cos \frac{2\pi t}{T} - \sin \phi \sin \frac{2\pi t}{T} \right)$

Using the trigonometric identity

We get, $F(t) = A \cos\left(\frac{2\pi t}{T} + \phi\right)$

and $\phi = -\tan^{-1}\left(\frac{a_1}{b_1}\right)$

- (b) **Amplitude :** For a S.H.M., A is called the amplitude. The maximum value of cosine function is one. Therefore, the amplitude A is the maximum value of the displacement variable of the S.H.M. represented by the function,

$$F(t) = A \cos\left(\frac{2\pi t}{T} + \phi\right)$$

Phase : The argument of the cosine function is $\frac{2\pi t}{T} + \phi$ called the phase at time t

Another form of function of S.H.M.

Let $b_1 = A \sin \phi'$

$$a_1 = A \cos \phi'$$

Such that, $A = (b_1^2 + a_1^2)^{1/2}$

and $\phi' = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

the function described by equation

$$F(t) = b_1 \cos \frac{2\pi t}{T} + a_1 \sin \frac{2\pi t}{T}$$

may be expressed as

$$F(t) = A \sin\left(\frac{2\pi t}{T} + \phi'\right)$$

If we replace ϕ by $\phi + \frac{\pi}{2}$ the function represented

by $F(t) = A \sin\left(\frac{2\pi t}{T} + \phi'\right)$

and $F(t) = A \cos\left(\frac{2\pi t}{T} + \phi\right)$ are identical

Therefore, the representation of S.H.M. given by the function equation

$$F(t) = A \sin\left(\frac{2\pi t}{T} + \phi\right) \text{ is not independent}$$

- 172.** Define S.H.M. Show that S.H.M. is the motion of the projection of uniform circular motion.

Ans :

Simple harmonic motion is a special kind of periodic motion in which a particle moves to and fro about a fixed point such that :

- Acceleration of the particle at any point is directly proportional to the displacement of the particle from the fixed point.
- Acceleration is always directed towards the mean position.

- 164.** Discuss the kinetic energy (K) of a particle in SHM, its expression, and its variation with time and position.

Ans :

The kinetic energy of a particle of mass m and velocity v in SHM is given by

$$K = \frac{1}{2}mv^2.$$

Substituting $v(t) = -\omega A \sin(\omega t + \phi)$:

$$K = \frac{1}{2}m[-\omega A \sin(\omega t + \phi)]^2$$

$$K = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

- It is maximum ($\frac{1}{2}kA^2$) at the mean position ($x = 0$).
- It is zero at the extreme positions ($x = \pm A$).
- It varies periodically with time, having a period of $T/2$.

- 165.** Discuss the potential energy (U) of a particle in SHM, its expression, and its variation.

Ans :

The restoring force in SHM, $F = -kx$, is a conservative force. The potential energy U associated with this force is given by

$$U = \frac{1}{2}kx^2.$$

Substituting

$$x(t) = A \cos(\omega t + \phi):$$

$$U(x) = \frac{1}{2}k[A \cos(\omega t + \phi)]^2$$

$$= \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

- It is maximum ($\frac{1}{2}kA^2$) at the extreme positions ($x = \pm A$).
- It is zero at the mean position ($x = 0$).
- It also varies periodically with time, having a period of $T/2$.

- 166.** Derive the expression for the total mechanical energy (E) of a particle in SHM and show it is conserved.

Ans :

The total mechanical energy E of a particle in SHM is the sum of its kinetic energy (K) and potential energy (U):

$$E = K + U$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t + \phi) + \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

Since $\sin^2\theta + \cos^2\theta = 1$,

$$E = \frac{1}{2}kA^2$$

This expression is independent of time, showing that total mechanical energy is conserved in SHM.

- 167.** Explain the working principle of a simple pendulum and derive its time period for small oscillations.

Ans :

A simple pendulum consists of a bob of mass m tied to an inextensible string of length L , fixed at the other end.

When displaced by an angle θ , the restoring force is $mg \sin\theta$ acting tangentially.

The restoring torque about the support is

$$\tau = -L(mg \sin\theta).$$

For small θ , $\sin\theta \approx \theta$, so $\tau \approx -mgL\theta$.

Also,

$$\tau = I\alpha = mL^2\alpha.$$

Equating, $mL^2\alpha = -mgL\theta$, giving

$$\alpha = -(g/L)\theta.$$

Comparing with $\alpha = -\omega^2\theta$, we get

$$\omega = \sqrt{(g/L)},$$

$$\text{so } T = \frac{2\pi}{\omega} = 2\pi\sqrt{(L/g)}.$$

- 168.** What are the characteristics of displacement, velocity, and acceleration in SHM? How do they differ in phase?

Ans :

In SHM, displacement, velocity, and acceleration all vary sinusoidally with time and have the same period T .

- Displacement $x(t) = A \cos(\omega t + \phi)$ varies between $-A$ and $+A$.
- Velocity $v(t) = -\omega A \sin(\omega t + \phi)$ varies between $-\omega A$ and $+\omega A$. It leads displacement by a phase of $\frac{\pi}{2}$ radians.
- Acceleration $a(t) = -\omega^2 A \cos(\omega t + \phi)$ varies between $-\omega^2 A$ and $+\omega^2 A$. It leads displacement by a phase of π radians and velocity by $\frac{\pi}{2}$ radians.

- 169.** Compare the oscillations of a block attached to a spring with the motion of a simple pendulum.

Ans :

Both a spring-mass system (on a frictionless surface) and a simple pendulum (for small angles) execute SHM.

$$\begin{aligned}\text{And frequency, } \nu &= \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi} \sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}\end{aligned}$$

- 179.** Show that motion of a massless loaded spring is simple harmonic Motion.

Ans :

Consider a spring AB of negligible mass suspended from a rigid support as shown in the figure(a).

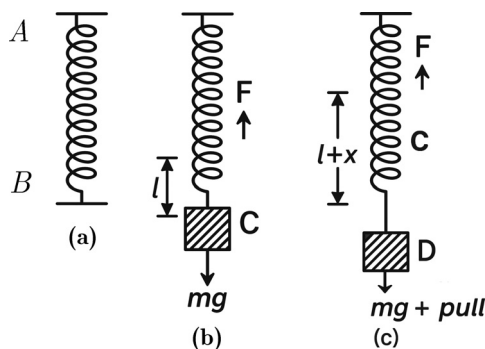
When the spring is loaded with a mass m , it is pulled down *i.e.*, it is elongated to the point C as shown in the figure(b).

Let the extension in length,

$$BC = l$$

By Hooke's law, $F \propto l$

$$F = -kl, \quad \dots(1)$$



where k is a force constant.

Putting $F = -mg$, we get

$$\begin{aligned}-mg &= -kl \\ m &= \frac{kl}{g} \quad \dots(2)\end{aligned}$$

Now displace the mass m to the point D through a further distance x . If F' is the restoring force in this position of the spring as shown in the figure(c), then

$$F' - k(l+x)$$

Net restoring force,

$$\begin{aligned}F' - F &= -k(l+x) - (-kl) \\ &= -kl - kx + kl \\ F' - F &= -kx\end{aligned}$$

If a is the acceleration, then $F = ma$

$$\begin{aligned}ma &= -kx \\ a &= \frac{-k}{m}x \quad \dots(3)\end{aligned}$$

Acceleration is proportional to displacement. Negative sign indicates that motion is directed towards the mean position. Thus, motion of a loaded spring is S.H. motion.

So time period of oscillation,

$$T = 2\pi \sqrt{\frac{\text{disp}}{\text{acc}}} = 2\pi \sqrt{\frac{x}{a}}$$

Using equation (3), we get

$$T = 2\pi \sqrt{\frac{x \cdot m}{kx}} = 2\pi \sqrt{\frac{m}{k}}$$

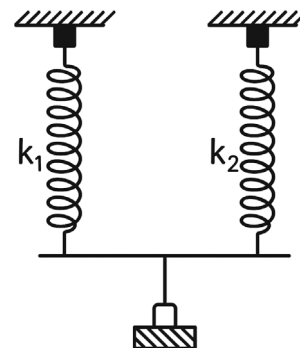
Using equation (2), we get

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots(4)$$

And frequency of oscillation,

$$\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad \dots(5)$$

- 180.** Two identical springs, one of spring factor k_1 and other of spring factor k_2 are first connected in (i) parallel and then in (ii) series with same rigid support. Deduce the spring constant, when a body of weight mg is connected to each of combination.



Ans :

- (i) Let S_1 and S_2 be the springs connected first in parallel as shown. In this case, the displacement set up in each of the spring is the same *i.e.* x , while restoring force is different depending upon the force constant,

$$F_1 = -k_1 x$$

$$\text{and } F_2 = -k_2 x$$

Let K_P be the spring factor of the combined system which is also displaced through a displacement x by a combined restoring force $(F_1 + F_2)$, then

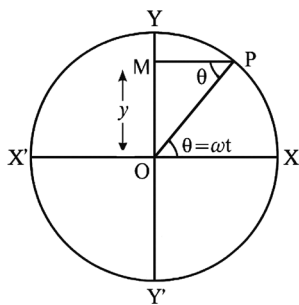
$$F_1 + F_2 = -K_P x$$

Putting the values of F_1 and F_2 , we get

$$-k_1 x + (-k_2 x) = -K_P x$$

$$K_P = k_1 + k_2$$

These are also known characteristics of S.H.M.



Let us imagine a particle P moving along the circumference of a circle with a constant speed v in anticlockwise direction. As P moves from X to Y, the projection M of P on diameter YY' moves from O to Y; when P moves from Y to X', M moves from Y to O. Similarly as P moves from X' to Y' and Y' to X, the projection of P i.e. M moves from O to Y' and back from Y' to O.

Thus it is seen that M moves to and fro along the diameter YY' as the point P moves round the circle (as shown in the figure). The point M is said to execute S.H.M. Thus simple harmonic motion is defined as the projection of uniform circular motion upon the diameter of the circle. The circle is called circle of reference and the particle P, particle of reference. M is called, the vibrating particle.

Amplitude : The maximum distance covered by the vibrating particle on either side of the mean position is called its amplitude.

In above example, OY or OY' i.e. the radius of circle of reference is called amplitude.

Vibration : It is the to and fro motion between two consecutive passages of the particle about a point in the same direction. In above example, motion between O to Y, from Y to Y' and again to O is one complete vibration.

Time period : The time taken to complete one vibration is called time period.

Frequency : The number of vibrations made by a vibrating body in one second is called frequency.

Frequency, $\nu = 1/T$

Phase : The phase of a vibrating particle at any instant is its value as regards its position and direction of motion at that instant.

It is measured in terms of fraction of time period T or as a fraction of 2π radian. If the vibrating particle M starts from O upwards, its phase at Y is $\pi/2$ or $T/4$.

Phase difference : Between the two particles is the extent by which the two particles are out of step

with each other. It is also measured as a fraction of time period T or of 2π radian.

Displacement : Let a particle starts from X and reach the point P at time t so that $\angle XOP = \theta$

During this time the vibrating particle reaches M. Let the displacement of vibrating particle = y .

Hence, $\frac{OM}{OP} = \sin \theta$

$$OM = OP \sin \theta$$

$$y = a \sin \theta = a \sin \omega t$$

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Draw time-displacement curve for a body moving in S.H.M.

Ans :

To draw time-displacement curve divide the circle of reference into eight equal parts, each being covered in $T/8$ second. Draw two lines XX' and YY' .

Now displacement of particle executing S.H.M. is

$$y = a \sin \omega t$$

At $t = 0, y = a \sin 0 = 0$

At $t = T/8$ or $t = \frac{\pi}{8} = \frac{\pi}{4}$

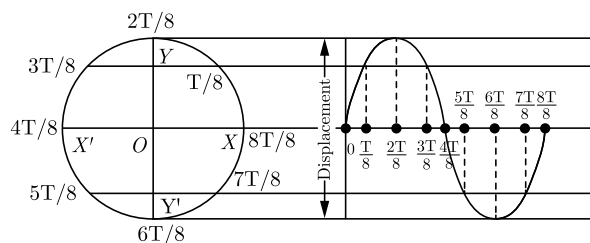
$$y = a \sin \frac{\pi}{4} = \frac{a}{\sqrt{2}}$$

At $t = 2T/8$ or $\omega t = \pi/2, y = a \sin \pi/2 = a$

Similarly at $t = 3T/8$ or $\omega t = 3\pi/4$

$$y = A \sin 3\pi/4 = a/\sqrt{2} \text{ and soon}$$

Plotting y along y -axis and t along x -axis, get curve as shown in figure.



174. Define and calculate velocity of a particle executing S.H.M. and show that it is maximum at the mean position and minimum (i.e. zero) at the extreme position.

Ans :

The time rate of change of displacement of a particle is called its velocity.

The displacement y of a particle executing S.H.M. at any instant t having angular velocity ω is given by

- 157.** Describe simple harmonic motion (SHM) with its defining displacement equation.

Ans :

Simple harmonic motion (SHM) is a specific type of oscillatory motion where the displacement of the particle from its origin or mean position varies sinusoidally with time.

The displacement $x(t)$ is given by the equation:

$$x(t) = A \cos(\omega t + \phi)$$

Here, A is the amplitude, ω is the angular frequency, and ϕ is the phase constant.

- 158.** Explain the terms amplitude (A), angular frequency (ω), and phase constant (ϕ) in the SHM equation $x(t) = A \cos(\omega t + \phi)$.

Ans :

In the SHM equation $x(t) = A \cos(\omega t + \phi)$:

- **Amplitude (A)** is the magnitude of the maximum displacement of the particle from its equilibrium position.
- **Angular frequency (ω)** is related to the period T by $\omega = 2\pi/T$ and determines how rapidly the oscillations occur. Its SI unit is radians per second.
- **Phase constant (ϕ)** is the initial phase at $t = 0$, determining the particle's initial state.

- 159.** Illustrate the relationship between uniform circular motion and simple harmonic motion with an experiment.

Ans :

The projection of uniform circular motion on a diameter of the circle exhibits simple harmonic motion. This can be visualized by:

- Tying a ball to a string and moving it in a horizontal circle with constant angular speed.
- Observing the ball's motion from the side (edge-on to the plane of motion) or its shadow on a perpendicular wall.

The ball or its shadow will appear to execute a to-and-fro SHM along a line.

- 160.** Derive the expression for the velocity $v(t)$ of a particle in SHM whose displacement is $x(t) = A \cos(\omega t + \phi)$.

Ans :

The velocity of a particle in SHM is the time derivative of its displacement.

Given $x(t) = A \cos(\omega t + \phi)$

$$v(t) = \frac{d}{dt}x(t)$$

$$= \frac{d}{dt}[A \cos(\omega t + \phi)]$$

$$= A[-\sin(\omega t + \phi)] \cdot \omega$$

$$= -\omega A \sin(\omega t + \phi)$$

This equation gives the instantaneous velocity of the particle.

- 161.** Derive the expression for the acceleration $a(t)$ of a particle in SHM and relate it to displacement $x(t)$.

Ans :

Acceleration in SHM is the time derivative of velocity. Given $v(t) = -\omega A \sin(\omega t + \phi)$:

$$a(t) = \frac{d}{dt}v(t)$$

$$= \frac{d}{dt}[-\omega A \sin(\omega t + \phi)]$$

$$= -\omega A [\cos(\omega t + \phi)] \cdot \omega$$

$$= -\omega^2 A \cos(\omega t + \phi)$$

Since $x(t) = A \cos(\omega t + \phi)$, we can write $a(t) = -\omega^2 x(t)$. This shows acceleration is proportional to displacement and oppositely directed.

- 162.** Explain the force law governing simple harmonic motion and define the force constant.

Ans :

The force law for simple harmonic motion is derived from Newton's second law, $F(t) = ma$. Using the acceleration $a(t) = -\omega^2 x(t)$, the force is:

$$F(t) = m(-\omega^2 x(t)) = -(m\omega^2)x(t)$$

This is written as $F(t) = -kx(t)$, where $k = m\omega^2$ is the force constant. The force is a restoring force, proportional to displacement and directed towards the mean position.

- 163.** A block of mass m is attached to two identical springs (spring constant k) fixed to supports on a frictionless surface. Show its motion is SHM.

Ans :

Consider the block displaced by x from its equilibrium position.

- The spring on the left, if elongated by x , exerts a restoring force $F_1 = -kx$.
- The spring on the right, if compressed by x , exerts a restoring force $F_2 = -kx$ (in the same direction as F_1).

The net force on the mass is

$$F = F_1 + F_2 = -kx - kx = -2kx.$$

Since the net force is proportional to displacement and directed towards the mean position, the motion is simple harmonic.

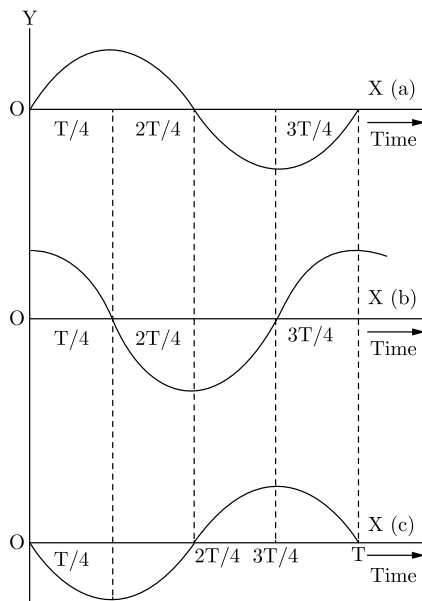
where $x(t)$ is the displacement of particle in S.H.M.
 $\dots(1)$

Differentiating both sides of (i) w.r.t. t , we get

$$\begin{aligned}\frac{dx}{dt} &= r[-\sin(\omega t + \phi_0)]\omega \\ \text{i.e. Velocity, } v &= \frac{dx}{dt} \\ &= r\omega[-\sin(\omega t + \phi_0)] \quad \dots(2) \\ &= r\omega \cos[90^\circ + \omega t + \phi_0]\end{aligned}$$

Differentiating both sides of (ii) w.r.t. t , we get

$$\begin{aligned}\frac{d^2x}{dt^2} &= -r\omega \cos(\omega t + \phi_0)\omega \\ \text{i.e. acceleration, } a &= \frac{d^2x}{dt^2} \\ &= -\omega^2 \cos(\omega t + \phi_0) \\ &= \omega^2 r \cos(\omega t + \phi_0) \\ &= \omega^2 r \cos[180^\circ + (\omega t + \phi_0)] \quad \dots(3)\end{aligned}$$



The variation of displacement, velocity and acceleration with time, as obtained from equations (1), (2) and (3) is shown in figure.

- (i) At, $t = 0$ displacement is zero (i.e. particle is at mean position), velocity is max. and acceleration is zero.
- (ii) $t = \frac{T}{4}$, displacement is max. i.e. particle is at the extreme position, velocity is zero and acceleration is max. but directed opposite to the displacement and so on.

Clearly, the velocity of the particle in S.H.M. is ahead of the displacement by a phase $\pi/2$ again, acceleration is ahead of velocity by a phase of $\pi/2$

acceleration of the particle in S.H.M. is ahead of the displacement by a phase angle π .

- 178.** Obtain an expression for the frequency of a mass attached to a mass-less spring when it vibrates in a horizontal direction.

Ans :

Consider a spring of negligible mass lying on a frictionless table horizontally with its one end attached to a rigid support and other end is attached to mass m as shown in figure.

Let an external force $\vec{F}_{ext}$ displaced the mass from O to A through a displacement $\vec{x}$ then a restoring force $\vec{F}$ acts on the mass.

$$\begin{aligned}\vec{F} &= \vec{F}_{ext} \\ \text{Since, } \vec{F}_{ext} &\propto \vec{x} \\ \text{Hence, } -\vec{F} &\propto \vec{x} \\ \vec{F} &= -k\vec{x} \\ \vec{F} &= -kx \quad \dots(1)\end{aligned}$$

Where k is constant of proportionality and is called spring constant.

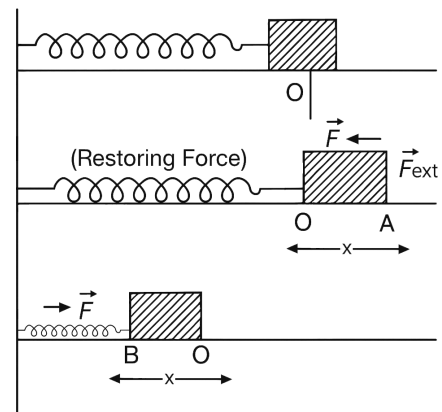
Now if the external force is reduced to zero, the mass m will oscillate about its mean position.

If a is the acceleration of the particle, then

$$\begin{aligned}a &= \frac{F}{m} = -\frac{k}{m}x \\ a &\propto -x \quad \dots(2)\end{aligned}$$

Acceleration $\propto$ displacement

Hence the motion is S.H.M.



Time period, $T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

Using equation (2), we get

$$T = 2\pi \sqrt{\frac{xm}{kx}} = 2\pi \sqrt{\frac{m}{k}}$$

$$y = r \sin \omega t \quad \dots(1)$$

Hence, Velocity of the particle

$$v = \frac{dy}{dt}$$

$$v = \frac{d}{dt}(r \sin \omega t)$$

$$v = r \omega \cos \omega t \quad \dots(2)$$

$$v = r \omega \sqrt{1 - \sin^2 \omega t}$$

$$[\sin^2 \theta + \cos^2 \theta = 1]$$

$$v = \omega \sqrt{r^2 - r^2 \sin^2 \omega t}$$

Using equation (1), we get

$$v = \omega \sqrt{r^2 - y^2} \quad \dots(3)$$

At the mean position, $y = 0$

Hence, $v = \omega \sqrt{r^2 - 0} = \omega r = \text{maximum}$

At the extreme position, $y = r$

Hence, $v = \omega \sqrt{r^2 - r^2} = 0$

So the velocity of a particle executing S.H.M. is maximum at the mean position and minimum (*i.e.* zero) at the extreme position.

- 175.** Define acceleration and derive expressions for accelerations, for acceleration and time period in S.H.M.

Ans :

The time rate of change of velocity is called acceleration.

The velocity of a particle executing S.H.M. at any instant t having angular velocity ω and amplitude ' r ' is given by

$$v = r \omega \cos \omega t \quad \dots(1)$$

Hence, Acceleration of the particle,

$$a = \frac{dv}{dt} = \frac{d}{dt} (r \omega \cos \omega t)$$

$$= r \omega \frac{d}{dt} (\cos \omega t)$$

$$a = r \omega (-\sin \omega t) \omega$$

$$= -\omega^2 r \sin \omega t$$

$$a = -\omega^2 y \quad [y = r \sin \omega t]$$

In magnitude, $a = \omega^2 y \quad \dots(2)$

At the mean position, $y = 0$

$$a = 0$$

At the extreme position, $y = r$

Hence, $a = \omega^2 r = \text{maximum}$

Hence the acceleration of a particle executing S.H.M. is maximum at the extreme position and zero at the mean position.

Time period (T)

From equation (2), we have

$$\omega^2 = \frac{a}{y}$$

$$\omega = \sqrt{\frac{a}{y}}$$

$$\frac{2\omega}{T} = \sqrt{\frac{a}{y}}$$

$$T = 2\pi \sqrt{\frac{y}{a}}$$

$$= 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}} \quad \dots(3)$$

- 176.** The velocity of a particle executing S.H.M. is v_1 when displacement is x_1 and v_2 when displacement is x_2 . What is the amplitude of vibration of the particle ?

Ans :

We know that the velocity v of a particle executing S.H.M. having amplitude r and displacement x from the mean position is given by

$$v = \omega(r^2 - x^2)^{1/2}$$

Here $v = v_1$ at $x = x_1$

and $v = v_2$ at $x = x_2$

Hence, $v_1 = \omega(r^2 - x_1^2)^{1/2}$

$$v_2 = \omega(r^2 - x_2^2)^{1/2}$$

Hence, $\frac{v_1}{v_2} = \frac{(r^2 - x_1^2)^{1/2}}{(r^2 - x_2^2)^{1/2}}$

$$\frac{v_1^2}{v_2^2} = \frac{r^2 - x_1^2}{r^2 - x_2^2}$$

$$v_1^2 r^2 - v_1^2 x_2^2 = r^2 v_2^2 - v_2^2 x_1^2$$

$$r^2 (v_2^2 - v_1^2) = v_2^2 - v_2^2 x_1^2$$

$$r^2 = \frac{v_2^2 x_1^2 - x_1^2 x_2^2}{v_2^2 - v_1^2}$$

$$r = \left[\frac{v_2^2 x_1^2 - x_1^2 x_2^2}{v_2^2 - v_1^2} \right]^{1/2}$$

- 177.** Give the graphical representation of displacement, velocity and acceleration of particle in S.H.M.

Ans :

General equation of harmonic oscillation is

$$x(t) = r \cos[\omega t + \phi]$$

That is, the spring constant for the parallel combination is sum of the spring constants for individual springs.

- (ii) When the springs are connected in series, the displacements produced in two springs are different while the restoring force in each as well as for combination is the same. Let x_1 and x_2 be the displacement and F be the restoring force, then

For spring, $S_1 F = -k_1 x_1$

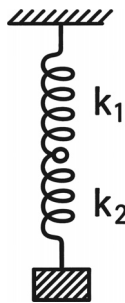
$$x_1 = -F/k_1$$

For spring $S_2, F = -k_2 x_2$

$$x_2 = -F/k_2$$

For combination in series

$$F = -k_s (x_1 + x_2)$$



where k_s is spring factor for combination in series.

Putting the values of x_1 and x_2 , we get

$$F = -k_s \left(-\frac{F}{k_1} - \frac{F}{k_2} \right)$$

$$\frac{F}{k_s} = -\frac{F}{-k_s} = \left(-\frac{F}{k_1} - \frac{F}{k_2} \right)$$

$$\frac{1}{k_s} = \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

Thus, for series arrangement the reciprocal of the spring factor of combination is sum of the reciprocals of the spring factor for individual springs.

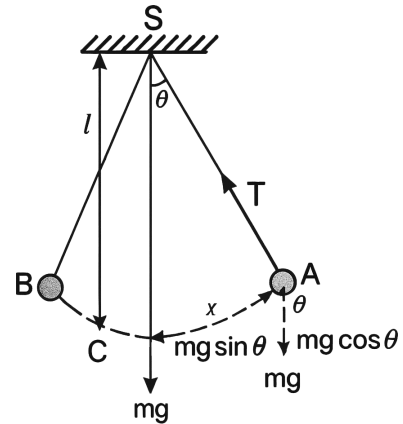
- 181.** (a) What is a simple pendulum ? Show that motion of simple pendulum is S.H.M.
(b) Calculate its time period and its drawbacks.

Ans :

- (a) An ideal simple pendulum consists of a heavy point mass suspended from a rigid support by a massless, inextensible and perfectly elastic string.

In actual practice, it is not possible to have a simple pendulum as defined above, because we cannot have a point heavy mass and a

weightless and inextensible string, so we take a heavy metallic bob suspended by a thin silken thread from a rigid support.



When the bob is displaced from the mean position O through a small angle and is released at A, the pendulum starts vibrating about O. As is clear from figure, the component $mg \cos \theta$ balances the tension T in the string. The tangential component $mg \sin \theta$ provides the restoring force F .

Hence, $F = -mg \sin \theta$

If θ is small, $\sin \theta \approx \theta$

$$F = -mg \theta \quad \dots(1)$$

Let $OA = x$, the displacement and $SO = SA = l$, the length of the pendulum

Hence, $\theta = \frac{x}{l}$

From (1), $F = -mg = \frac{x}{l}$

$$ma = -\frac{mg}{l} x$$

$$a = -\frac{g}{l} x = -\omega^2 x$$

where $\omega^2 = \frac{g}{l}$

Clearly acceleration is directly proportional to the displacement x and is directed opposite to it.

Motion of simple pendulum is simple harmonic.

- (b) **Time period :** The time period of the pendulum would be

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/l}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Hence the time period T of a simple pendulum is independent of the mass of the bob and

4. The maximum speed of the particle P' in SHM is $v_{max} = \omega A$. This is equal to the speed of the reference particle P in uniform circular motion.

205. A particle executes SHM described by $x(t) = A \cos(\omega t)$. Its velocity at any time t is $v(t) = -\omega A \sin(\omega t)$ and its acceleration is $a(t) = -\omega^2 A \cos(\omega t)$.

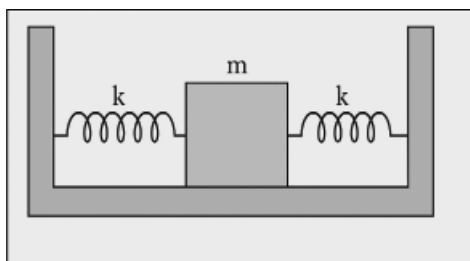
Questions:

- How is the acceleration $a(t)$ of a particle in SHM related to its displacement $x(t)$?
- At what points in its motion (displacement x) is the particle's speed maximum? At what points is it zero?
- What is the phase difference between the displacement $x(t)$ and the velocity $v(t)$ in SHM?
- What is the phase difference between the displacement $x(t)$ and the acceleration $a(t)$ in SHM?

Ans :

- The acceleration $a(t)$ is related to the displacement $x(t)$ by $a(t) = -\omega^2 x(t)$.
- The particle's speed is maximum at the mean position ($x = 0$). Its speed is zero at the extreme positions ($x = \pm A$).
- $x(t) = A \cos(\omega t)$ and $v(t) = -\omega A \sin(\omega t) = \omega A \cos(\omega t + \pi/2)$. The phase difference between displacement and velocity is $\pi/2$ (velocity leads displacement by $\pi/2$).
- $x(t) = A \cos(\omega t)$ and $a(t) = -\omega^2 A \cos(\omega t) = \omega^2 A \cos(\omega t + \pi)$. The phase difference between displacement and acceleration is π .

206. Two identical springs, each of spring constant k , are attached to a block of mass m and to fixed supports as shown in Fig. (one spring on each side of the mass, connecting it to two opposite fixed supports). The mass is displaced from its equilibrium position on a frictionless surface.



Questions:

- If the mass 'm' is displaced by a small distance 'x' to the right, what is the direction and magnitude of the force exerted by the left spring (F_1) and the right spring (F_2) on the mass?

- What is the net force (F) acting on the mass 'm' when it is displaced by 'x'?
- Show that this net force leads to simple harmonic motion. What is the effective spring constant of this system?
- What is the period of oscillation for this system?

Ans :

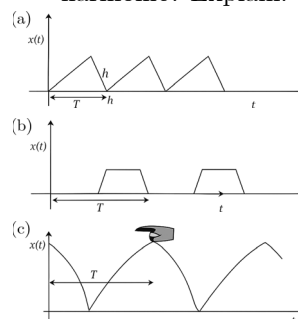
- If the mass is displaced by 'x' to the right, the left spring is elongated by 'x' and exerts a force $F_1 = -kx$ (pulling left). The right spring is compressed by 'x' and exerts a force $F_2 = -kx$ (pushing left).
- The net force
$$F = F_1 + F_2 = (-kx) + (-kx) = -2kx.$$
- Since the net force $F = -(2k)x$, it is proportional to the displacement 'x' and directed towards the mean position. This is the condition for SHM. The effective spring constant of this system is $k_{eff} = 2k$.
- The period of oscillation is
$$T = 2\pi\sqrt{m/k_{eff}} = 2\pi\sqrt{m/(2k)}.$$

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207. Consider the motion of an insect climbing up a ramp, falling down, and then repeating the process identically. Its height above the ground versus time is plotted.

Questions:

- What is the smallest interval of time after which a periodic motion is repeated called?
- Is all periodic motion necessarily oscillatory? Give an example.
- Is the motion of the insect described (Fig. a) periodic?
- Compare the insect's motion (Fig. a) with the motion of a ball bouncing between one's palm and the ground (Fig. c). Are both simple harmonic? Explain.



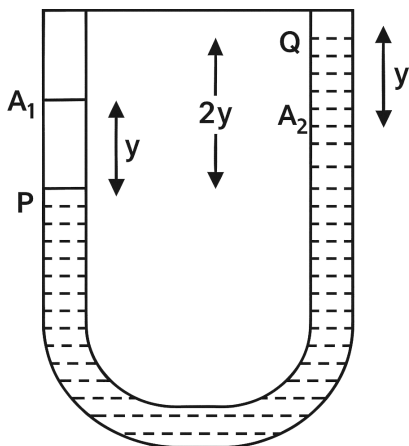
Ans :

- The smallest interval of time after which the motion is repeated is called its period.
- No, every periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory.

- 184.** Explain the oscillation of a liquid in a U-tube. Find its time period.

Ans :

Let a U-tube contains some liquid upto level A_1 , A_2 . Let L be the total length of liquid in the pipe and ρ be the density of liquid.



Mass of liquid in the tube $m = AL\rho$, where A is the area of cross-section of U-tube.

Depress liquid through distance y in left side of the tube, the liquid will rise by y in the other tube and then release.

Before release, the difference in level in the two tubes $= 2y$

Weight of liquid in column of length $2y$ in U-tube will provide the restoring force to liquid.

Restoring force, $F = -(2yA)\rho g = (2A\rho g)y$

We find that, $F \propto y$

Liquid will execute S.H.M.

Spring factor, $K = 2A\rho g$

$$T = 2\pi \sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$$

$$T = 2\pi \sqrt{\frac{LA\rho}{2A\rho g}}$$

$$T = 2\pi \sqrt{\frac{L}{2g}}$$

Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{2g}{L}}$

If h is the height of the liquid column in each limb, then $L = 2h$

Hence, $T = 2\pi \sqrt{\frac{2h}{2g}} = 2\pi \sqrt{\frac{h}{g}}$

And $\nu = \frac{1}{2\pi} \sqrt{\frac{g}{h}}$

- 185.** Derive an expression for total of a body at any instant.

or

Show that the total energy of a body moving with S.H.M. is conserved.

Ans :

Let us first find the K.E. and P.E. at any instant say when displacement is y .

Potential energy : Consider a particle of mass m executing S.H.M. with amplitude a and a constant angular frequency ω .

Its acceleration at any instant is

$$a = -\omega^2 y$$

$$\text{Restoring force} = ma = -m\omega^2 y = -ky,$$

where $k = m\omega^2 = \text{force constant}$

Work done for displacement dy against the restoring force,

$$dW = -F dy$$

$$= -(-ky) dy = ky dy$$

Total work done in displacing the particle from mean position to displacement y is

$$W = \int_0^y ky dy = k \int_0^y y^1 dy$$

$$W = \frac{1}{2} k y^2 = \frac{1}{2} m \omega^2 y^2$$

This work done is stored in the form of P.E. (E_p)

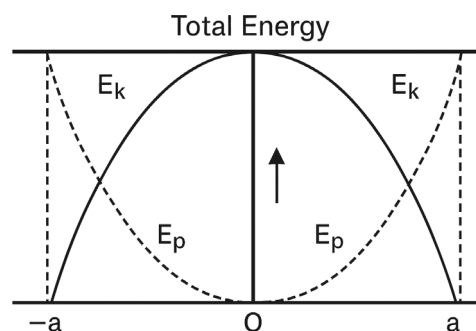
$$E_p = \frac{1}{2} m \omega^2 y^2$$

Kinetic Energy

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m (\omega \sqrt{r^2 - y^2})^2$$

$$= \frac{1}{2} m \omega^2 (r^2 - y^2)$$

$$E_k = \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 y^2$$



- 191.** A mass of 0.5 kg is suspended from a fixed point by a light string. The mass is extended by 0.05 m in the equilibrium position. The mass is then pulled down by 0.03 m and released. Find the time period and maximum kinetic energy of the mass.

Ans :

Given, $m = 0.05 \text{ kg}$, $x = 0.05 \text{ m}$

and $a = 0.03 \text{ m}$

Force constant, $k = \frac{F}{x} = \frac{mg}{x} = \frac{0.05 \times 9.8}{0.05}$

$k = 98 \text{ N m}^{-1}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.05}{98}}$

$= 2 \times \frac{22}{7} \times \frac{1}{14} = 0.4489 \text{ s}$

The maximum velocity of the mass in the mean position is,

$v_s = a \omega$

$= a \times \frac{2\pi}{T} = \frac{0.03 \times 2 \times 3.14}{0.4489}$

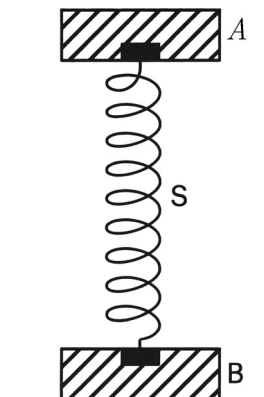
$= 0.41963 \text{ m s}^{-1}$

Maximum K.E. $= \frac{1}{2} m v_s^2$

$= \frac{1}{2} \times 0.05 (0.41963)^2$

$= 0.044035 \text{ J}$

- 192.** Two bodies A and B (as shown in the figure) of mass 1 kg and 2 kg respectively are rigidly connected to a spring of force constant 400 N m^{-1} . The body B rests on a horizontal table. From the rest position the body A is compressed by a distance of 2 cm and then released. Deduce (i) the frequency of oscillation, (ii) the maximum velocity of the body A.



Ans :

Given that, $m = 1 \text{ kg}$ (for body A)

$k = 400 \text{ N m}^{-1}$

$a = 2 \text{ cm} = 0.02 \text{ m}$

(i) Frequency of oscillations is given by

$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

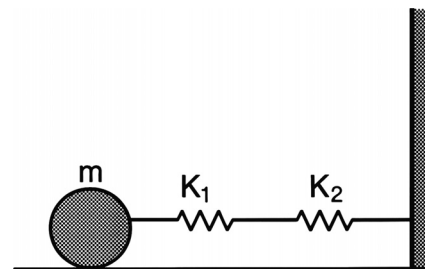
$= \frac{1}{2 \times 22} \sqrt{\frac{400}{1}} = 3.18 \text{ Hz}$

(ii) Maximum velocity of the body A

$= a \omega = a 2\pi \nu$

$= 0.02 \times 2 \times \frac{22}{7} \times 3.18 = 0.4 \text{ m s}^{-1}$

- 193.** Two springs are joined and connected to a mass m as shown in the figure. If the spring force constants are k_1 and k_2 show that the frequency of oscillation of mass m is



$\nu = \frac{1}{2\pi} \left(\frac{k_1 k_2}{(k_1 + k_2) m} \right)$

Ans :

Let a force F applied on the system produce displacements x_1 and x_2 is the two springs.

$F_1 = -k_1 x_1$

$F_2 = -k_2 x_2$

Now, $x_1 = -\frac{F}{k_1}, x_2 = -\frac{F}{k_2}$

Hence total displacement,

$$x = x_1 + x_2 = -\frac{F}{k_1} - \frac{F}{k_2}$$
$$= -F \left[\frac{1}{k_1} + \frac{1}{k_2} \right] = -F \left[\frac{k_1 + k_2}{k_1 k_2} \right]$$

Hence, $F = -\frac{k_1 k_2}{k_1 + k_2} x$

As $F = -k x$ where k is the spring constant of the system

Hence, $k = \frac{k_1 k_2}{k_1 + k_2}$

NUMERICAL QUESTIONS

189. Which of the following functions of time represent

- (a) simple harmonic
- (b) periodic but not simple harmonic and
- (c) non-periodic motion ?

Give the period for each case of periodic motion (ω is any positive constant).

- (i) $\sin \omega t + \cos \omega t$
- (ii) $\sin \omega t + \cos 2 \omega t + \sin 3 \omega t$
- (iii) $e^{-\omega t}$
- (iv) $2 \cos\left(\frac{\omega t}{2} + \frac{\pi}{6}\right)$
- (v) $\sin^2 \omega t + \cos^2 \omega t$
- (vi) $\log \omega t$

Ans :

- (i) $\sin \omega t + \cos \omega t$ represents simple harmonic motion. It can be written as $\sqrt{2} \sin\left(\omega t + \frac{\pi}{4}\right)$

$$\begin{aligned} \text{Now } \sqrt{2} \sin\left(\omega t + \frac{\pi}{4}\right) &= \sqrt{\sin^2\left(\omega t + \frac{\pi}{4} + 2\pi\right)} \\ &= \sqrt{2} \sin\left\{\omega\left(t + \frac{2\pi}{\omega}\right) + \frac{\pi}{4}\right\} \end{aligned}$$

The period of the function is $\frac{2\pi}{\omega}$.

- (ii) This is an example of a periodic function which is not of simple harmonic type. Note, each term separately represents S.H.M. but the sum does not. Since period is the least interval of time after which a function repeats its value. $\sin \omega t$ has period $T = \frac{2\pi}{\omega}$, $\cos 2 \omega t$ has a period, $\frac{\pi}{\omega} = \frac{T}{2}$ $\sin 3 \omega t$ has a period $= \frac{2\pi}{3\omega} = \frac{T}{3}$. The last two

terms of course, repeat their values after any integer multiple of the period. Thus each term in the sum repeats its value after T and the sum is a periodic function with a period $= \frac{2\pi}{\omega}$.

- (iii) The function $e^{-\omega t}$ (where e is a number nearly equal to 2.71828) is known as exponential function. It decreases monotonically to zero as $t \rightarrow \infty$, and thus never repeats its value. It therefore represents non-periodic function.
- (iv) This represents simple harmonic motion. The period is

$$\frac{2\pi}{\omega/2} = \frac{4\pi}{\omega}$$

- (v) $\sin^2 \omega t + \cos^2 \omega t$

$$= \frac{1}{2}(1 - \cos 2 \omega t) + \frac{1}{2}(\cos 2 \omega t + 1)$$

This is similar to case (ii). The constant term can be assigned any period. The function is periodic with a period equal to $\frac{2\pi}{\omega}$.

- (vi) The function $\log \omega t$ increases monotonically with t . It, therefore, never repeats its value and is a non-periodic function. But as $t \rightarrow \infty$, $\log \omega t$ diverges to ∞ . It cannot therefore, represent any kind of physical displacement.

190. A displacement wave is represented by $y = 0.25 \times 10^{-3} \sin(500t - 0.025x)$. Deduce (i) amplitude; (ii) period; (iii) angular frequency (iv) wavelength; (v) amplitude of particle velocity (vi) amplitude of particle acceleration. y , t and x are in cm, s and m respectively.

Ans :

We know that, $y = r \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Comparing it with

$$y = 0.25 \times 10^{-3}$$

$\sin(500t - 0.025x)$, we have

- (i) Amplitude, $r = 0.25 \times 10^{-3} \text{ cm}$

(ii) $\frac{2\pi}{T} = 500$

or $T = \frac{2\pi}{500} = \frac{\pi}{250} \text{ s}$

- (iii) Angular frequency,

$$\omega = \frac{2\pi}{T} = 500 \text{ Hz}$$

(iv) $\frac{2\pi}{\lambda} = 0.025$

or $\lambda = \frac{2\pi}{0.025} = 80\pi \text{ metre}$

- (v) Amplitude of particle velocity

$$= \text{Maximum velocity} = \omega r$$

$$= 500(0.25 \times 10^{-3}) = 0.125 \text{ cm s}^{-1}$$

- (vi) Amplitude of particle acceleration

$$= \text{Maximum acceleration} = \omega^2 r$$

$$= (500)^2 (0.25 \times 10^{-3})$$

$$= 62.5 \text{ cm s}^{-2}$$

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depends only on length of the simple pendulum and acceleration due to gravity at the place of experiment.

Drawbacks of simple pendulum

1. Ideal simple pendulum cannot be achieved in practice.
2. The motion of the bob is not linear.
3. Air friction and buoyancy is not taken into account.

182. Derive expression for vertical oscillations of a floating body.

Ans :

Consider a cylindrical body floating in a liquid. Let l , A and ρ be respectively length, area of cross-section and density of the body. If σ is density of the liquid as shown in the figure, then

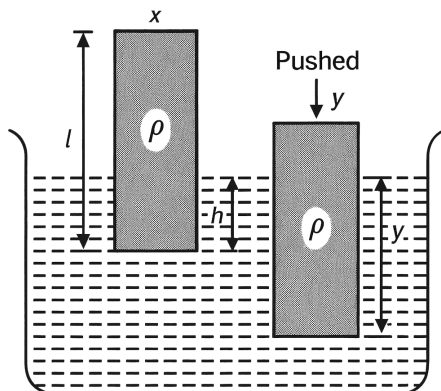
Weight of liquid displaced by the immersed part of the body = $mg = Vag = Ahag$

Weight of whole cylindrical body = $Al\rho g$

In equilibrium,

$$Al\rho g = Ahag$$

$$l = \frac{ha}{\rho} \quad \dots(1)$$



Hence, the motion is S.H.M.

$$\begin{aligned} \text{Time period, } T &= 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}} \\ &= 2\pi \sqrt{\frac{y}{\frac{y}{l} \frac{\rho}{\sigma}}} = 2\pi \sqrt{\frac{yl\rho}{\sigma y}} \\ T &= 2\pi \sqrt{\frac{l\rho}{\sigma g}} \quad \dots(2) \end{aligned}$$

Using equation (1), we get

$$T = \sqrt{\frac{ha\rho}{\sigma ag}} = 2\pi \sqrt{\frac{h}{g}}$$

$$\text{And frequency, } \nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{h}}$$

183. A hole is dug in the earth and passing through its centre. A body is dropped into the hole. Show that it will execute S.H.M. Also determine its time period. Given $g = 10 \text{ m s}^{-2}$ and radius of the earth to be 6400 km. Consider the earth to be a homogeneous sphere.

Ans :

Let a mass m be dropped into the hole and reaches point P at a depth d below the surface of the earth. If g' is the acceleration due to gravity, then

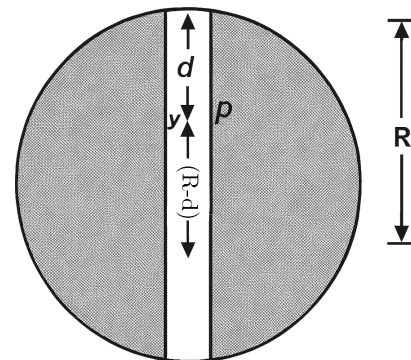
$$g' = \left(1 - \frac{d}{R}\right)g = g \left(\frac{R-d}{R}\right)$$

$$g' = \frac{g}{R}(R-d) = \frac{g}{R}(y)$$

[where $R-d=y$ the displacement from O]

We find that acceleration is proportional to displacement y and hence the body will execute S.H.M.

$$\text{Force on the body} = mg' = \left(\frac{mg}{R}\right)y$$



$$\text{Spring factor, } k = \frac{mg}{R}$$

$$\text{Time period, } T = 2\pi \sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$$

$$\begin{aligned} T &= 2\pi \sqrt{\frac{m}{mg/R}} \\ &= 2\pi \sqrt{\frac{R}{g}} \end{aligned}$$

$$T = 2 \times 3.14 \sqrt{\frac{6400 \times 1000}{10}}$$

$$T = 6.28 \times 800$$

$$T = 628 \times 8$$

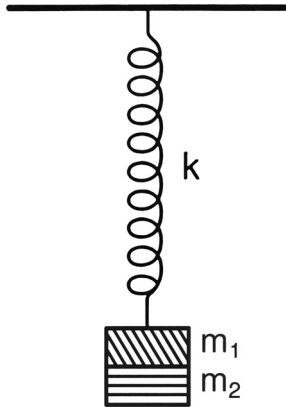
$$T = 5,024 \text{ s}$$

If ν is the frequency of oscillation of mass m , then

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k_1 k_2}{(k_1 + k_2) m}}$$

- 194.** Two masses m_1 and m_2 together are suspended from a massless spring of constant k . When masses are in equilibrium, m_2 is removed without disturbing the system. Find the amplitude of oscillation and angular frequency of m_1 .



Ans :

Suppose individually m_1 and m_2 produce extension l_1 and l_2 . Thus when m_1 and m_2 are together the extension is $(l_1 + l_2)$. In equilibrium,

$$(m_1 + m_2)g = k(l_1 + l_2) \quad \dots(1)$$

when m_2 is gently removed and the system is again in equilibrium then

$$m_1 g = k l_1 \quad \dots(2)$$

substrate equation (2) from equation (1), we get

$$m_2 g = k l_2 \quad \dots(3)$$

$$l_2 = \frac{m_2 g}{k} \quad \dots(4)$$

This l_2 is the amplitude of oscillation. Now the time period of oscillation is given by

Angular frequency of oscillation,

$$\omega = \frac{1}{2\pi} \sqrt{\frac{k}{m_1}}$$

- 195.** A spring has mass m suspended from it. What will happen to the frequency of oscillations if the spring is cut into two halves ?

Ans :

Time period of a spring loaded with mass m is

$$T = 2\pi \sqrt{\frac{m}{k}}$$

And frequency, $\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

When the spring is cut into two halves

$$k' = 2k \quad \left[k = \frac{F}{l} \right]$$

So new frequency

$$\nu' = \frac{1}{2\pi} \sqrt{\frac{k'}{m}} = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$

$$= \sqrt{2} \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\nu' = \sqrt{2} \nu$$

i.e. Frequency of oscillations becomes $\sqrt{2}$ times the original frequency.

- 196.** How will the period of a simple pendulum change when its length is double ?

Ans :

Time period of a simple pendulum is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

When,

$$l' = 2l$$

$$T' = 2\pi \sqrt{\frac{l'}{g}} = 2\pi \sqrt{\frac{2l}{g}}$$

$$T' = \left(2\pi \sqrt{\frac{l}{g}} \right) \sqrt{2} = \sqrt{2} T$$

- 197.** At what distance from mean position is K.E. equal to P.E. ?

Ans :

At any instant,

$$\text{K.E.} = \frac{1}{2} m \omega^2 (r^2 - y^2)$$

and

$$\text{P.E.} = \frac{1}{2} m \omega^2 y^2$$

If

$$\text{K.E.} = \text{P.E.} \text{ when displacement is } y,$$

$$\text{Then, } \frac{1}{2} m \omega^2 (r^2 - y^2) = \frac{1}{2} m \omega^2 y^2$$

$$r^2 - y^2 = y^2 \text{ or } r^2 = 2y^2$$

$$y^2 = \frac{r^2}{2} \text{ or } y = \frac{r}{\sqrt{2}}$$

- 198.** The displacement of a particle in S.H.M. may be given by $y = r \sin (\omega t + \phi_0)$. Show that if the time t is increased by $\frac{2\pi}{\omega}$, the value of y remains unchanged.

Ans :

Displacement at any time t ,

$$y = r \sin (\omega t + \phi_0)$$

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Sum of K.E. and P.E. at any instant is the total energy during S.H.M.,

$$E = \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 y^2 + \frac{1}{2} m \omega^2 y^2$$

$$E = \frac{1}{2} m \omega^2 r^2$$

which is independent of y . Thus we find the sum of K.E. and P.E. at any instant is independent of y . Thus total energy during S.H.M. is conserved.

- 186.** Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Ans :

Average kinetic energy over a period of oscillation

$$\begin{aligned} &= \frac{1}{T} \int_0^T \frac{1}{2} m v^2 dt = \frac{1}{2T} m v^2 [t]_0^T \\ &= \frac{1}{2T} m v^2 \times T = \frac{m v^2}{2} \end{aligned}$$

Average potential energy

$$\begin{aligned} &= \frac{1}{T} \int_0^T \frac{1}{2} k x^2 dt = \frac{1}{2T} k x^2 [t]_0^T \\ &= \frac{1}{2T} m v^2 \times T = \frac{1}{2} k x^2 \end{aligned}$$

Now

$$\omega = \sqrt{k/m}$$

$$\omega^2 = k/m$$

$$k = m \omega^2$$

Hence, average potential energy

$$= \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} m v^2$$

Therefore, for a particle in linear S.H.M., the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

- 187.** Define periodic motion, oscillatory motion, and simple harmonic motion, highlighting the differences between them.

Ans :

Periodic motion, oscillatory motion, and simple harmonic motion are related but distinct types of motion:

- **Periodic Motion:** This is the broadest category. A motion is periodic if it repeats itself at regular intervals of time. The smallest such interval is called the period. Examples include uniform circular motion or the motion of planets.
- **Oscillatory Motion:** This is a specific type of periodic motion where the body moves to and fro about a mean or equilibrium position. Every

oscillatory motion is periodic. An example is the swinging of a pendulum.

- **Simple Harmonic Motion (SHM):** This is the simplest form of oscillatory motion. It occurs when the restoring force acting on the body is directly proportional to its displacement from the mean position and is always directed towards that mean position ($F = -kx$). The displacement in SHM is a sinusoidal function of time.

Key Differences:

- All SHM is oscillatory, and all oscillatory motion is periodic.
- However, not all periodic motion is oscillatory (e.g., uniform circular motion).
- Not all oscillatory motion is simple harmonic (e.g., oscillations with non-linear restoring forces). SHM requires a specific linear restoring force.

- 188.** Explain the terms displacement, velocity, and acceleration in SHM using sinusoidal functions. Discuss their phase relationships.

Ans :

In Simple Harmonic Motion (SHM), the displacement, velocity, and acceleration of a particle can be described by sinusoidal functions of time.

If displacement is given by $x(t) = A \cos(\omega t + \phi)$:

- **Displacement, $x(t)$:** This represents the instantaneous position of the particle from its mean position. It varies between $+A$ and $-A$.
- **Velocity, $v(t)$:** This is the rate of change of displacement.

$$v(t) = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi).$$

It can also be written as

$$v(t) = A\omega \cos\left(\omega t + \phi + \frac{\pi}{2}\right).$$

Velocity leads displacement by $\frac{\pi}{2}$ radians. Its maximum value is $A\omega$.

- **Acceleration, $a(t)$:** This is the rate of change of velocity.

$$a(t) = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \phi).$$

It can also be written as

$$a(t) = A\omega^2 \cos(\omega t + \phi + \pi).$$

Acceleration leads displacement by π radians and leads velocity by $\frac{\pi}{2}$ radians. Its maximum value is $A\omega^2$.

Also, $a(t) = -\omega^2 x(t)$.

- 202.** A particle of mass 1 kg is performing SHM with an amplitude of 10 cm (0.1 m) and an angular frequency of 7.07 rad/s. The spring constant is 50 N/m. We are interested in its energy when it is 5 cm (0.05 m) away from the mean position.

Questions:

1. What is the formula for the total mechanical energy (E) of a harmonic oscillator?
2. How do the kinetic energy (K) and potential energy (U) of the particle vary during SHM? When is K maximum and U maximum?
3. Calculate the total mechanical energy of the block.
4. Calculate the kinetic and potential energies of the block when it is 5 cm away from the mean position. Do these values add up to the total energy? (Refer to Example 13.7 calculations)

Ans :

1. The total mechanical energy (E) of a harmonic oscillator is $E = \frac{1}{2}kA^2$.
2. Kinetic energy is maximum at the mean position ($x = 0$) and zero at the extreme positions ($x = \pm A$). Potential energy is zero at the mean position (by choice) and maximum at the extreme positions. They vary such that their sum remains constant.

3. Total energy

$$\begin{aligned} E &= \frac{1}{2}kA^2 \\ &= \frac{1}{2}(50\text{N/m})(0.1\text{m})^2 = \frac{1}{2} \times 50 \times 0.01 \\ &= 0.25\text{J}. \end{aligned}$$

4. When $x = 0.05\text{m}$:
Potential Energy

$$\begin{aligned} U &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}(50\text{N/m})(0.05\text{m})^2 = 0.0625\text{J}. \end{aligned}$$

Kinetic Energy

$$\begin{aligned} K &= E - U \\ &= 0.25\text{J} - 0.0625\text{J} = 0.1875\text{J} \end{aligned}$$

(or 0.19 J as in Ex 13.7).

Yes, these values add up to the total energy (0.0625J + 0.1875J = 0.25J).

- 203.** A simple pendulum consists of a small bob of mass 'm' tied to an inextensible massless string of length L, fixed at the other end. It oscillates in a vertical plane. For small angular displacements (θ), its motion is approximately simple harmonic.

Questions:

1. What is the restoring torque (τ) acting on the bob of a simple pendulum when it is displaced by a small angle θ ?

2. For small angles, how can $\sin \theta$ be approximated? What does this approximation lead to in the equation of motion for the pendulum?
3. What is the formula for the time period (T) of a simple pendulum for small oscillations?
4. If a simple pendulum that "ticks seconds" has a period of 2 s on Earth (where $g \approx 9.8\text{m/s}^2$), what would be its approximate length?

Ans :

1. The restoring torque $\tau = -L(mg\sin\theta)$.
2. For small angles θ (in radians), $\sin\theta \approx \theta$. This approximation leads to the equation of angular acceleration

$$\alpha = -\frac{mgL}{I}\theta,$$

which is the condition for SHM.

3. The time period $T = 2\pi\sqrt{L/g}$.
4. Given $T = 2\text{s}$ and $g = 9.8\text{m/s}^2$.
From $T = 2\pi\sqrt{L/g}$, we get

$$\begin{aligned} L &= \frac{gT^2}{4\pi^2} \\ &= \frac{(9.8\text{m/s}^2)(2\text{s})^2}{4\pi^2} \approx \frac{9.8 \times 4}{4 \times (3.14)^2} \\ &\approx \frac{9.8}{9.86} \approx 0.994\text{m} \text{ or about 1 meter} \end{aligned}$$

- 204.** A particle P moves uniformly on a circle of radius A with angular speed ω . Its initial position vector **OP** at $t = 0$ makes an angle ϕ with the positive x-axis. The projection of this particle's motion onto the x-axis, say P', executes SHM.

Questions:

1. What is the displacement $x(t)$ of the projection P' on the x-axis as a function of time?
2. What does the term "reference circle" refer to in this context?
3. If the particle P moves with an angular speed $\omega = 2\text{rad/s}$ on a circle of radius $A = 5\text{cm}$, and its initial phase $\phi = \pi/2$, write the equation for $x(t)$. What is the velocity of the particle P in its circular motion?
4. How is the maximum speed of the particle P' executing SHM related to the speed of particle P in uniform circular motion?

Ans :

1. The displacement of the projection P' on the x-axis is $x(t) = A \cos(\omega t + \phi)$.
2. The particle P and the circle on which it moves are sometimes referred to as the reference particle and the reference circle, respectively.
3. $x(t) = 5 \cos(2t + \pi/2)\text{cm}$. The speed of particle P in circular motion is

$$v = \omega A = (2\text{rad/s})(5\text{cm}) = 10\text{cm/s}.$$

Displacement at any time $t + \frac{2\pi}{\omega}$ will be

$$\begin{aligned} y' &= r \sin \left[\omega \left(t + \frac{2\pi}{\omega} \right) + \phi_0 \right] \\ &= r \sin [(\omega t + \phi_0) + 2\pi] \\ &= r \sin (\omega t + \phi_0) \end{aligned}$$

Hence, displacement at times t and $t + \frac{2\pi}{\omega}$ are the same.

CASE BASED QUESTIONS

199. A child is swinging on a garden swing. Her motion is repetitive, moving back and forth about a central lowest point (mean position). The time taken for one complete to-and-fro movement is observed to be approximately 3 seconds.

Questions:

1. What is periodic motion?
2. Is the motion of the child on the swing an example of oscillatory motion? Explain why.
3. What is the period (T) of the child's swing motion based on the observation?
4. Calculate the frequency (ν) of the child's swing motion. What does this frequency represent?

Ans :

1. Periodic motion is a motion that repeats itself at regular intervals of time.
2. Yes, the motion of the child on the swing is an example of oscillatory motion because it is a periodic to-and-fro motion about a mean position.
3. The period (T) of the child's swing motion is approximately 3 seconds, as this is the time for one complete to-and-fro movement.
4. The frequency

$$\nu = 1/T = 1/3s \approx 0.33Hz.$$

This represents the number of complete swings (oscillations) the child makes per second.

200. A particle is executing Simple Harmonic Motion (SHM). Its displacement from the origin at any time t is given by the equation $x(t) = A \cos(\omega t + \phi)$, where A is the amplitude, ω is the angular frequency, and ϕ is the phase constant.

Questions:

1. What is the amplitude (A) of SHM?
2. What does the term $(\omega t + \phi)$ represent, and what is the significance of ϕ ?
3. If the period of this SHM is T , how is ω related to T ?

4. If two SHMs have the same amplitude and period but differ in their initial displacement at $t = 0$, which of the parameters (A , ω , or ϕ) would be different for them?

Ans :

1. The amplitude A of SHM is the magnitude of the maximum displacement of the particle from its mean position.
2. The term $(\omega t + \phi)$ is called the phase of the motion and it determines the state of motion (position and velocity) at any time t . ϕ is the phase constant or phase angle, representing the phase at $t = 0$.
3. The angular frequency ω is related to the period T by $\omega = 2\pi/T$.
4. If the initial displacement at $t = 0$ is different (and assuming A and ω are the same), the phase constant ϕ would be different for them, as ϕ can be determined from the displacement at $t = 0$.

201. A block of mass ' m ' is attached to a light spring with spring constant ' k '. The other end of the spring is fixed to a rigid wall. The block is pulled to a distance x_0 from its equilibrium position on a frictionless horizontal surface and then released. The block then oscillates back and forth.

Questions:

1. What is the force law for a particle executing SHM?
2. What provides the restoring force in this spring-mass system?
3. What is the angular frequency (ω) of oscillation for this spring-mass system?
4. If the mass ' m ' is quadrupled, how would the period of oscillation (T) change?

Ans :

1. The force law for SHM is $F(t) = -kx(t)$, where k is the force constant and $x(t)$ is the displacement.
2. The spring itself provides the restoring force, which is proportional to the displacement from the equilibrium position and directed towards it (Hooke's Law).
3. The angular frequency of oscillation for this spring-mass system is $\omega = \sqrt{k/m}$.
4. The period of oscillation is $T = 2\pi\sqrt{m/k}$. If the mass ' m ' is quadrupled ($4m$), the new period

$$T' = 2\pi\sqrt{4m/k} = 2 \times (2\pi\sqrt{m/k}) = 2T.$$

The period would double.

3. Yes, the motion of the insect described in Fig. 13.1a is periodic as it repeats itself at regular intervals of time.
4. Neither the insect's motion (which has sharp changes in direction/velocity) nor the bouncing ball's motion (where sections are parabolic under gravity) are simple harmonic. SHM requires the displacement to be a sinusoidal function of time, and the restoring force to be directly proportional to displacement.

208. A body oscillates with SHM according to the equation (in SI units): $x = 5 \cos(2\pi t + \pi/4)$. We need to find its displacement at $t = 1.5\text{s}$.

Questions:

1. In the equation $x(t) = A \cos(\omega t + \phi)$, what do A , ω , and ϕ represent?
2. What is the unit of the argument of the cosine function ($\omega t + \phi$)?
3. Identify the amplitude, angular frequency, and initial phase constant for the given SHM equation.
4. Calculate the displacement of the body at $t = 1.5\text{s}$. (Refer to Example 13.5).

Ans :

1. A is the amplitude, ω is the angular frequency, and ϕ is the phase constant (or initial phase).
2. The argument of the cosine function ($\omega t + \phi$) is an angle, and its natural unit is radians.
3. From $x = 5 \cos(2\pi t + \pi/4)$:
Amplitude $A = 5\text{ m}$.
Angular frequency $\omega = 2\pi\text{ rad/s}$.
Initial phase constant $\phi = \pi/4\text{ rad}$.
4. At $t = 1.5\text{s}$:
 $x = 5 \cos(2\pi(1.5) + \pi/4) = 5 \cos(3\pi + \pi/4)$
 $3\pi + \pi/4$ is in the third quadrant where cosine is negative. It's equivalent to

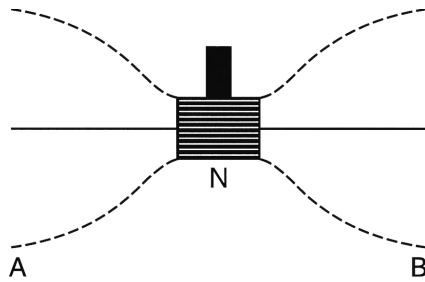
$$\cos(\pi + \pi/4) = -\cos(\pi/4).$$

$$\begin{aligned} x &= 5 \times (-\cos(\pi/4)) = 5 \times (-1/\sqrt{2}) \\ &= 5 \times (-0.707) = -3.535\text{m}. \end{aligned}$$

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Frequency of the note = 2.53 kHz

$$= 2.53 \times 1000 \text{ Hz}$$

Velocity of sound in rod, $v_r = \nu \lambda$

$$= 2 \times 2.53 \times 1000 \text{ ms}^{-1}$$

$$= 5.06 \text{ kms}^{-1}$$

17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will this same source be in resonance with the pipe if its both ends are open ? (Speed of sound in air is 340 ms^{-1})

Ans :

Given, $l = 20 \text{ cm} = 0.20 \text{ m}$

For fundamental note,

$$v_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$$

$$\text{L.H.S } v_1 = 430 \text{ Hz}$$

$$\text{R.H.S } \frac{v}{4l} = \frac{v}{4 \times 2} = \frac{340 \times 10}{8}$$

$$= 425 \text{ Hz}$$

Hence first harmonic or fundamental node of the pipe is reasonably excited by a 430 Hz source when the pipe is open at both ends,

For fundamental mode,

$$v_1 = \frac{v}{l} = \frac{v}{2l} = 850 \text{ Hz}$$

Hence the same source will not be in resonance with the pipe.

18. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B ?

Ans :

Given,

$$\text{Frequency of the fork} = 324 \text{ Hz}$$

$$\text{Beat per second} = 6$$

Frequency of the B is either $324 + 6 = 330 \text{ Hz}$

$$324 - 6 = 318 \text{ Hz.}$$

When the tension of A is reduced, its frequency decreases by 6 Hz.

Frequency of A becomes $324 - 6 = 318$

$$\text{Frequency of B} = 318 \text{ Hz}$$

19. Explain why (or how)

- In a sound wave, a displacement node is a pressure anti-node and vice versa.
- Bats can ascertain distances, directions, nature and size of the obstacles without any "eye".
- A violin note and a sitar note may have the same frequency, yet we can distinguish between the two notes.
- Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases.
- The shape of a pulse gets distorted during propagation in a dispersive medium.

Ans :

- Because at displacement node the pressure variation is maximum.
- 'Echo' produced in the reflection of ultrasonics emitted by bats helps the bats to ascertain distances.
 - The phase difference produced between the waves repeated from different directions helps to find the direction.
 - Phase difference helps to ascertain the shape and size. Hence reverberation time is reduced.
- This is due to presence of overtones. Even if the fundamental frequency is the same, yet their overtones are different. Hence they are distinguishable.
- Gases have no elasticity of shapes. Therefore no transverse wave can pass through it.
- Because the wavelengths in different media are different.

OBJECTIVE QUESTIONS

- What is the defining characteristic of a mechanical wave?
 - It can travel through a perfect vacuum always.
 - It requires a material medium for its propagation.
 - It involves the transfer of matter over distance.
 - It always travels at the constant speed of light.

4. NEWTON'S FORMULA

According to Newton, the velocity of sound (or longitudinal wave) is given by

$$v = \sqrt{\frac{P}{\rho}}$$

5. LAPLACE'S FORMULA

According to Laplace, the velocity of sound wave is given by $v = \sqrt{\frac{\gamma P}{\rho}}$, where γ is the ratio of two specific heat capacities of a gas.

Temperature Coefficient of Velocity of Sound

It is defined as the change in velocity of sound for 1°C change in temperature of the medium.

i.e.,
$$a = \frac{v - v_0}{t}$$

6. TRAVELLING OR PROGRESSIVE WAVE

A wave travelling in a medium in given direction with constant amplitude is called a travelling or progressive wave.

Mathematically, the displacement of a travelling wave moving along X-axis at any time t is given by

$$y = r \sin \frac{2\pi}{\lambda} (vt - x)$$

7. PRINCIPLE OF SUPERPOSITION

If two or more waves reach at a point, the resulting displacement at that point is the algebraic sum of the displacements of individual waves at that point.

8. STATIONARY WAVES

When two waves of the same frequency moving in the opposite direction superpose, they give rise to stationary waves.

9. BEATS

When two waves of slightly different frequencies moving in the same direction superpose, the amplitude of the resultant waves alternately becomes maximum and minimum and beats are said to be produced.

Each rise and fall in intensity of sound is called one beat.

10. FUNDAMENTAL FREQUENCY

When a stretched string vibrates in one segment as a whole, its frequency is called fundamental

frequency and is given by

$$v = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

10.1 Overtones

Notes of higher frequency are called overtones.

10.2 Nodes

In stationary waves, nodes are the points where the displacement of particles is minimum. The change of pressure and density is maximum at these points.

10.3 Anti-nodes

Anti-nodes are the points of maximum displacement. The strain is minimum at these points.

The distance between two consecutive nodes or anti-nodes is $\lambda/2$.

11. ORGAN PIPES

In an organ pipe (open or closed), the air column vibrates producing nodes and anti-nodes due to reflection from open end (for open organ pipe) and from closed end (for closed organ pipe).

In an open pipe, all the harmonics are present, whereas in closed pipe only odd harmonics are present. **For an open organ pipe** of length l , the fundamental frequency, $\nu = v/2l$ while for n th harmonic or $(n-1)$ th overtone, $\nu_{n-1} = n(v/2l)$ where $n = 1, 2, 3, \dots$

The situation is the same as for a string fixed at both ends.

For a closed organ pipe of length l , the fundamental frequency $\nu = (2n-1)(v/4l)$ while for n th overtone, $\nu = (2n+1)(\frac{v}{4l})$ where $n = 1, 2, \dots$. Only odd harmonics ($v, 3v, 5v, \dots$) are present whereas even harmonics ($2v, 4v, 6v, \dots$) are missing.

The fundamental frequency of an open organ pipe is double that of a closed organ pipe of the same length.

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NCERT TEXT-BOOK QUESTIONS

1. A string of mass 2.50 kg is under a tension of 200N. The length of the stretched string is 20.0 m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Ans :

Given,

Tension,

$$T = 200 \text{ N},$$

10. For the travelling harmonic wave $y(x, t) = 2.0 \cos 2\pi(10t - 0.0080x + 0.35)$, where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m, (b) 0.5 m, (c) $\lambda/2$, (d) $\frac{3\lambda}{4}$?

Ans :

The given equation of a travelling wave is

$$y(x - t) = 2.0 \cos 2\pi(10t - 0.0080x + 0.35) \quad \dots(1)$$

The standard equation of a travelling wave is

$$y(x, t) = A \cos \left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x - \phi_0 \right] \quad \dots(2)$$

Comparing equation (1) and (2), we get

$$\begin{aligned} \frac{2\pi}{\lambda} &= 2\pi \times 0.008 \text{ cm}^{-1} \\ &= 2\pi \times 0.8 \text{ m}^{-1} \end{aligned}$$

(a) Here Path difference = 4 m

$$\begin{aligned} \text{Phase difference} &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= 2\pi \times 0.8 \times 4 = 6.4 \pi \text{ rad} \end{aligned}$$

(b) Path difference = 0.5 m

$$\begin{aligned} \text{Phase difference} &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= 2\pi \times 0.8 \times 0.5 = 0.8 \pi \text{ rad} \end{aligned}$$

(c) Phase difference = $\frac{\lambda}{2}$

$$\begin{aligned} \text{Path difference} &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ rad} \end{aligned}$$

(d) Path difference = $\frac{3\lambda}{4}$

$$\begin{aligned} \text{Phase difference} &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \text{ rad} \\ &= \left(\pi + \frac{\pi}{2} \right) \end{aligned}$$

Since, $\cos(x + \theta) = \cos \theta$

Hence, effective path difference = $\frac{\pi}{2}$

11. The transverse displacement of a string (clamped at its two ends) is given by

$$y(x, t) = 0.060 \sin \frac{2\pi}{3} \cos(120 \pi t)$$

where x, y are in m and t is in s. The length of the string is 1.5 m and mass is 3.0×10^{-2} kg.

Answer the following :

- Does the function represent a travelling or a stationary wave ?
- Interpret the waves as a superposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave ?
- Determine the tension in the string.

Ans :

The transverse displacement of a string is given by

$$y(x, t) = 0.060 \sin \frac{2\pi}{3} x \cos(120 \pi t)$$

- It is the equation of a stationary wave.
- Compare it with $y = 2a \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{T} t$

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$$\lambda = 3 \text{ m}$$

$$\frac{2\pi}{T} = 120 \pi$$

$$T = \frac{2\pi}{120\pi} = \frac{1}{60}$$

So

$$\nu = 60 \text{ Hz}$$

Hence,

$$v = \nu \lambda = 60 \times 3 = 180 \text{ ms}^{-1}$$

(c)

$$\nu = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$$

Here

$$\mu = \frac{3.0 \times 10^{-2} \text{ Kg}}{1.5 \text{ m}}$$

$$= 2 \times 10^{-2} \text{ Kg m}^{-1}$$

$$60 = \frac{1}{2 \times 1.5} \sqrt{\frac{T}{2 \times 10^{-2}}}$$

Squaring both sides,

$$60 \times 60 = \frac{1}{4 \times 1.5 \times 1.5} \times \frac{T}{2 \times 10^{-2}}$$

$$T = 648 \text{ N}$$

12. (i) For the wave on a string described in Question 11, do all the points on the string oscillate with the same (a) frequency (b) phase (c) amplitude ? Explain your answers.
- (ii) What is the amplitude of a point 0.375 m away from one end ?

Ans :

- All the points except the nodes on the string have the same frequency and phase but not the same amplitude because amplitude is zero at node and maximum at anti-node.

5. You have learnt that a travelling wave in one dimension is represented by function $y = f(x, t)$ where x and t must appear in the combination $x - vt$ or $x + vt$ i.e. $y = f(x \pm vt)$. Examine if the following functions of y can possibly represent a travelling wave :

- (a) $(x - vt)^2$
 (b) $\log[x + vt]/x_0$
 (c) $\frac{1}{x + vt}$

Ans :

The converse is not true. An obvious requirement for an acceptable function for a travelling wave is that it should be finite everywhere and at all times. Only function (c) satisfies this condition. The remaining functions cannot possibly represent a wave.

6. A bat emits ultrasonic sound of frequency 1000 kHz in air. "If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound ? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1} .

Ans :

Given,

$$v = 1000 \times 1000 \text{ Hz}, v = 340 \text{ ms}^{-1}$$

$$\lambda_{\text{air}} = \frac{v}{\nu} = \frac{340}{1000 \times 1000} \\ = 3.40 \times 10^{-4} \text{ m}$$

In water, the frequency remains constant.

$$\lambda_w = \frac{v_w}{\nu} = \frac{1486}{1000 \times 1000} \\ = 1.486 \times 10^{-3} \text{ m} = 1.49 \times 10^{-3} \text{ m}$$

7. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 kms^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Ans :

Given,

$$\text{Speed of sound, } v = 1.7 \text{ km s}^{-1} = 1.7 \times 10^3 \text{ ms}^{-1}$$

Frequency of the scanner

$$= 4.9 \text{ MHz} = 4.2 \times 10^6 \text{ Hz}$$

$$\text{As we know that, } \lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6}$$

$$= 4.05 \times 10^{-4} \text{ m}$$

$$= 4.1 \times 10^{-4} \text{ m}$$

8. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin(36t + 0.018x + \frac{\pi}{4})$

where x, y are in cm and t in s. The positive direction of x is from left to right.

- (a) Is this a travelling wave or a stationary wave? If it is travelling. What are the speed and direction of its propagation ?
 (b) What are its amplitude and frequency ?
 (c) What is the initial phase at the origin ?
 (d) What is the least distance between two successive crests in the wave ?

Ans :

It is a travelling wave comparing with

$$y(x, t) = A \sin \left[\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right) + \phi \right]$$

- (a) Speed, $v = \frac{36}{0.018} = 2000 \text{ cm s}^{-1}$
 $= 20 \text{ ms}^{-1}$

Thus it is travelling with a speed of 20 ms^{-1} from right to left.

Hence, as x increases, phase also increases.

- (b) Amplitude = 3.0 cm

$$2\pi\nu = 3.6$$

$$\text{Frequency, } \nu = \frac{3.6}{2\pi} = \frac{18}{3.14} = 5.7 \text{ s}^{-1}$$

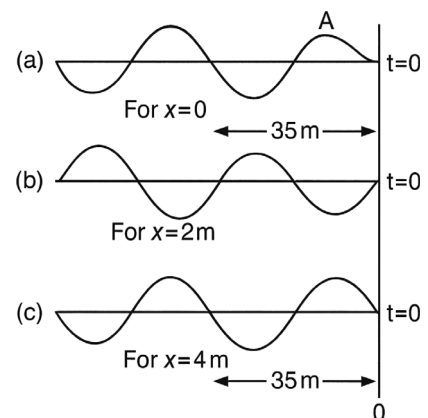
- (c) Initial phase at the origin = $\frac{\pi}{4}$ radian

- (d) Least distance between two successive crests is λ and is given by

$$\lambda = \frac{2\pi}{0.018} = \frac{2 \times 3.14}{0.018} = 3.5 \text{ m}$$

9. For the wave described in Question number 8, plot the displacement (y) versus time (t) graphs for $x = 0.2$ and 4 cm. What are the shapes of these graphs ? In which aspects does the oscillatory motion in a travelling wave differ from one point to another amplitude, frequency or phase ?

Ans :



CHAPTER 14

WAVES

SUMMARY

1. WAVE MOTION

It is a kind of disturbance which travels through a medium due to repeated motion of particles of the medium, the disturbance being handed over from particle to particle. Waves transport energy and information.

1.1 Longitudinal Wave Motion

In these types of waves, particles of the medium vibrate to and fro about their mean position along the direction of propagation of wave. These are also called pressure waves.

1.2 Transverse Wave Motion

In this case, particles of the medium vibrate up and down perpendicular to the direction of propagation of wave.

2. IMPORTANT TERMS RELATED TO WAVE MOTION

2.1 Compression

A compression of a region of medium in which the particles of the medium come to distances less than the normal distance between them.

2.2 Rarefaction

A rarefaction is a region of a medium in which the particles of the medium get apart to distances more than the normal distance between them.

2.3 Wavelength (λ) (for Longitudinal Wave)

It is the distance between two successive compressions or rarefactions.

2.4 Crest

The hump or elevation in a medium in which a transverse wave is passing is called crest.

2.5 Trough

The hollow or depression in a medium in which a transverse wave is passing is called trough.

2.6 Wavelength (λ) (for Transverse Wave)

The distance between two successive crests or troughs is called wavelength of the wave.

2.7 Relation between Wave Velocity u , Wavelength λ and Wave Frequency v

Wave velocity = frequency $\times$ wavelength

$$v = u\lambda$$

3. VELOCITIES OF DIFFERENT WAVES

(i) In general

Velocity of an elastic wave is given by

$$v = \sqrt{\frac{\text{Elasticity of the medium}}{\text{Density of the medium}}} \\ = \sqrt{\frac{E}{\rho}}$$

(ii) For transverse wave

(a) In solids

$$v = \sqrt{\frac{\eta}{\rho}} \quad (\eta \text{ is modulus of rigidity})$$

(b) In a stretched string

$$u = \sqrt{\frac{T}{\mu}} \quad (T \text{ is tension in the string})$$

(iii) For longitudinal wave

(a) In solids

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

(K is bulk modulus of elasticity, η is modulus of rigidity)

(b) Solids in the form of long rods

$$v = \sqrt{\frac{Y}{\rho}} \quad (Y \text{ is Young's modulus})$$

(c) In liquids or gases

$$v = \sqrt{\frac{K}{\rho}} \quad (K \text{ is Bulk modulus})$$

$$(ii) \quad y(x, t) = 0.060 \sin \frac{2x\pi}{3}$$

$$\text{When,} \quad x = 0.375 \text{ m}$$

$$y(x, t) = 0.060 \times \sin \frac{2\pi}{3} \times 0.375$$

$$\text{Hence,} \quad a = 0.042 \text{ m}$$

- 13.** Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent

- (i) a travelling wave,
 (ii) a stationary wave or
 (iii) none of all

$$(a) \quad y = 2 \cos(3x) \sin(10t)$$

$$(b) \quad y = 2\sqrt{x - vt}$$

$$(c) \quad y = 3 \sin(5x - 0.5t) + 4 \cos(5x - 0.5t)$$

$$(d) \quad y = \cos x \sin t + \cos 2x \sin 2t$$

Ans :

- (a) Stationary wave because it has two harmonic functions of each x and t separately.
 (b) Unacceptable function for any wave because it contains no harmonic function.
 (c) Travelling harmonic wave along positive x direction.
 (d) Superposition of two stationary waves.

- 14.** A wire stretched between two rigid supports vibrates in its fundamental node with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m⁻¹. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string ?

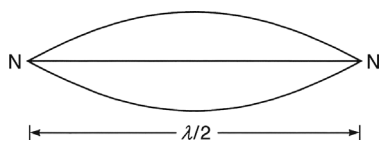
Ans :

$$\lambda = 45 \text{ Hz}$$

$$\rho = 4.0 \times 10^{-2} \text{ kg m}^{-1}$$

$$\text{Mass of the wire} = 3.5 \times 10^{-2} \text{ Kg}$$

$$\rho = \frac{\text{Mass}}{\text{Length}}$$



$$\text{Length of wire} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875 \text{ m}$$

$$\text{Speed,} \quad v = ?, l = \frac{\lambda}{2}, \lambda = 2l$$

$$v = \nu \lambda = \nu \times 2l$$

$$= 45 \times 2 \times 0.875 = 78.08 \text{ ms}^{-1}$$

$$= 79 \text{ ms}^{-1}$$

$$v = \sqrt{\frac{T}{\rho}}$$

$$T = v^2 \times \rho = (78.8)^2 \times 40 \times 10^{-2}$$

$$= 248.8 \text{ N} = 248 \text{ N}$$

- 15.** A metre long tube at one end with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Ans :

$$\nu_n = \frac{(2n-1)v}{4l}$$

$$n = 1, 2, \dots \text{ for a pipe with one end closed}$$

$$340 = \frac{(2n_1-1)v}{4l_1} = \frac{(2n_2-1)v}{4l_2}$$

$$\text{Now,} \quad l_2 = 3l_1$$

$$\text{Thus,} \quad \frac{(2n_1-1)}{(2n_2-1)} = \frac{1}{3}$$

$$n_1 = 1, n_2 = 2$$

Thus the fixed source of frequency 340 Hz is in resonance with the fundamental mode of the pipe for $l = 25.5$ cm, and with the third harmonic of the pipe for $l = 79.3$ cm

$$\nu_r = \frac{(2n-1)v}{4l}$$

$$340 = \frac{v}{4 \times 25.5 \times 10^{-2}}$$

$$v = 340 \times 4 \times 25.5 \times 10^{-2}$$

$$v = 347 \text{ ms}^{-1}$$

- 16.** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel ?

Ans :

$$\text{Here, length of rod,} \quad l_r = 100 \text{ cm}$$

As the rod is clamped in the middle, the wavelength of sound in the material of the rod is given by

$$\lambda_r = 2l_r = 200 \text{ cm} = 2 \text{ m}$$

As we know that,

$$\text{Mass/length, } \mu = \frac{2.50}{20} \text{ kg m}^{-1}$$

$$\text{Velocity of wave, } v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200 \times 30}{2.50}} = 40 \text{ ms}^{-1}$$

$$\begin{aligned} \text{Time} &= \text{Distance/velocity} \\ &= 20/40 = 0.5 \text{ s} \end{aligned}$$

2. A stone dropped from the top of a tower 300 m high splashes to the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms⁻¹ (g = 9.8 ms⁻²).

Ans :

Given

$$u = 0, a = 9.8 \text{ ms}^{-2}, t = ?$$

As we know that,

$$x = ut + \frac{1}{2}at^2$$

$$300 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$300 = 4.9t^2 \text{ or } t^2 = \frac{300}{4.9}$$

$$\text{Hence, } t = 7.82 \text{ s}$$

Time taken by sound to travel a distance of 300 m upward (= $t' - 7.82$)

$$\text{Since, } h = vt$$

$$\text{Now, } 300 = 340(t' - 7.82)$$

$$0.88 = t' - 7.82$$

$$t' = 8.7 \text{ s}$$

3. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C (= 343 ms⁻¹) ?

Ans :

Here μ (mass per unit length of the wire)

$$\mu = \frac{2.10}{12},$$

$$v = 343 \text{ ms}^{-1}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$343 = \sqrt{\frac{T}{2.10} \times 12}$$

Squaring both sides, we get

$$343 \times 343 = \frac{12T}{2.10}$$

$$\text{Hence, } T = 2.06 \times 10^{-4} \text{ N}$$

4. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air
- is independent of pressure
 - increases with temperature
 - increases with humidity ?

Ans :

$$v = \sqrt{\gamma P / \rho}$$

$$\text{For an ideal gas } PV = RT, \text{ or } \frac{PM}{\rho} = RT, \frac{P}{\rho} = \frac{RT}{M}$$

Where R is the gas constant, M the molecular mass and T the absolute temperature.

$$\text{Hence, } v = \left(\frac{\gamma RT}{M} \right) \quad \dots(1)$$

This shows that v is

- independent of pressure,
- increases with temperature as $v \propto \sqrt{T}$
- The presence of water vapours in air changes its density apart from a slight change in the value of γ . That is why the velocity of sound changes with humidity of air.

$$\text{Let, } \rho_m = \text{density of moist air,}$$

$$\rho_d = \text{density of dry air,}$$

$$\gamma_m \text{ and } \gamma_d = \text{values of } \gamma \text{ for moist and dry air respectively}$$

$$v_m = \text{velocity of sound in moist air,}$$

$$\gamma_d = \text{velocity of sound in dry air.}$$

Using relation (1), we get

$$v_m = \sqrt{\frac{\gamma_m \rho}{\rho_m}} \text{ and } v_d = \sqrt{\frac{\gamma_d \rho}{\rho_d}}$$

$$\text{Dividing, we get } \frac{v_m}{v_d} = \sqrt{\frac{\gamma_m}{\gamma_d} \cdot \frac{\rho_d}{\rho_m}} \quad \dots(2)$$

Obviously, dry air is heavier than water vapours. The presence of water vapours, therefore, reduces the density of air.

$$\text{i.e. } \rho_m < \rho_d$$

$$\text{Also } \frac{\gamma_d}{\gamma_m} = \frac{1.30}{1.41} = 0.922 \approx 1$$

$$\text{As } \rho_m < \rho_d \text{ therefore, from (2) } v_m < v_d$$

Hence velocity of sound in moist air is greater than the velocity of sound in dry air. That is why sound travels faster on a rainy day than on a dry day.

Ans :

(B) It requires a material medium for its propagation. Mechanical waves, unlike electromagnetic waves, necessitate a medium (like air, water, or a solid) to transfer energy through the oscillation of its constituent particles. They cannot propagate in a vacuum environment completely.

- 2.** In which type of wave do the constituents of the medium oscillate perpendicular to the direction of wave propagation?

(A) Longitudinal wave motion.
(B) Transverse wave motion.
(C) Surface wave formation.
(D) Compressional wave form.

Ans :

(B) Transverse wave motion.

Transverse waves are characterized by particle oscillations occurring at a right angle (perpendicular) to the direction in which the wave itself is travelling. Examples include waves on a string or light waves.

- 3.** What are waves called that involve oscillations of constituent particles along the direction of wave propagation?

(A) Longitudinal waves.
(B) Transverse waves.
(C) Electromagnetic waves.
(D) Non-mechanical waves.

Ans :

(A) Longitudinal waves.

Longitudinal waves are those where the particles of the medium vibrate back and forth in the same direction (parallel) as the wave's energy transfer. Sound waves serve as a common example of this.

- 4.** Which of the following wave types does NOT require a material medium for its propagation?

(A) Seismic waves.
(B) Sound waves.
(C) Water waves.
(D) Light waves.

Ans :

(D) Light waves.

Light waves are electromagnetic waves and can travel through the vacuum of space, unlike mechanical waves such as sound or water waves which need a medium to propagate their energy effectively.

- 5.** What term describes a wave pattern that moves without the actual physical transfer of matter as a whole?

(A) Particle current flow.
(B) Material displacement.
(C) Wave propagation.
(D) Medium convection.

Ans :

(C) Wave propagation.

Wave propagation refers to the process by which a disturbance travels through a medium or vacuum, transferring energy from one point to another without the net movement of the medium's matter itself.

- 6.** What is the typical nature of particle motion in water waves, such as those in an ocean?

(A) Purely transverse only.
(B) Purely longitudinal only.
(C) Combination of longitudinal and transverse.
(D) Neither longitudinal nor transverse type.

Ans :

(C) Combination of longitudinal and transverse.

Particles in ocean waves exhibit a complex motion, moving both up and down (transverse component) and back and forth (longitudinal component), resulting in a combined or orbital-like movement as wave passes.

- 7.** Why can transverse waves be propagated in solids but generally not in fluids like gases or liquids (except on the surface)?

(A) Fluids lack the necessary density for transverse waves.
(B) Fluids cannot sustain the shearing stresses required.
(C) Fluids have particles that are too far apart always.
(D) Fluids only allow for electromagnetic wave types.

Ans :

(B) Fluids cannot sustain the shearing stresses required.

Transverse waves involve shearing deformation. Solids can resist shear, allowing these waves to propagate. Fluids (gases and liquids, except at their surface) typically do not sustain shear stresses and flow instead.

- 8.** The minimum distance between two points in a wave having the same phase is known as:

(A) Amplitude.
(B) Wavelength.
(C) Frequency.

(D) Wave period.

Ans :

(B) Wavelength.

Wavelength, denoted by λ , is a fundamental property of a wave. It represents the spatial period of the wave, or the distance over which the wave's shape repeats itself distinctly.

9. What does the term 'amplitude' of a wave refer to?

- (A) The total distance the wave travels each second.
- (B) The maximum displacement from equilibrium position.
- (C) The number of wave cycles completed per unit time.
- (D) The time taken for one complete wave oscillation.

Ans :

(B) The maximum displacement from equilibrium position.

Amplitude (a) signifies the peak displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position. It indicates the intensity or energy of wave.

10. The SI unit of angular wave number (k) is:

- (A) radian per second.
- (B) radian per metre.
- (C) hertz per second.
- (D) metre per radian.

Ans :

(B) radian per metre.

Angular wave number (k) is defined as $2\pi/\lambda$, where λ is the wavelength. Since λ is in metres, k has units of radians per metre, quantifying wave oscillations in space.

11. The time it takes for an element of the medium to complete one full oscillation as a wave passes is called the:

- (A) Wave speed.
- (B) Wave period.
- (C) Wave phase.
- (D) Wave crest.

Ans :

(B) Wave period.

The period (T) of a wave is the duration for one complete cycle of oscillation of a particle in the medium. It is inversely related to the frequency of the wave.

12. How is the angular frequency (ω) of a wave related to its period (T)?

- (A) $\omega = 2\pi T$
- (B) $\omega = T/(2\pi)$
- (C) $\omega = 2\pi/T$
- (D) $\omega = T^2/(2\pi)$

Ans :

(C) $\omega = 2\pi/T$

Angular frequency (ω) measures the rate of change of phase angle and is related to the period (T) by $\omega = 2\pi/T$, representing radians per unit time for the wave.

13. A wave is described by the equation $y(x, t) = a \sin(kx + \omega t + \phi)$. In which direction is this wave travelling?

- (A) Positive x-direction.
- (B) Negative x-direction.
- (C) Positive y-direction.
- (D) Negative y-direction.

Ans :

(B) Negative x-direction.

In the wave equation $y(x, t) = a \sin(kx + \omega t + \phi)$, a plus sign between the kx and ωt terms indicates that the wave is propagating in the negative x-direction over time.

14. If the frequency of a wave is doubled, while its speed remains constant, what happens to its wavelength?

- (A) It is doubled too.
- (B) It is halved now.
- (C) It remains unchanged.
- (D) It becomes fourfold.

Ans :

(B) It is halved now.

The relationship between wave speed (v), frequency (ν), and wavelength (λ) is $v = \nu\lambda$. If v is constant and ν is doubled, λ must be halved to maintain the equality.

15. A wave travelling along a string is described by $y(x, t) = 0.002 \sin(50.0x - 2.0t)$, with numerical constants in SI units. What is the amplitude of this wave?

- (A) 0.002 m
- (B) 50.0 m
- (C) 2.0 m
- (D) 0.004 m

Ans :

(A) 0.002 m

Comparing the given equation $y(x, t) = 0.002 \sin(50.0x - 2.0t)$ with the standard form $y(x, t) = a \sin(kx - \omega t)$, the amplitude 'a' is directly identified as 0.002 metres.

16. The speed of a transverse wave on a stretched string is primarily determined by which properties of the string?
- Its length and its color properties.
 - Its tension and its linear mass density.
 - Its amplitude and its given frequency.
 - Its material conductivity and its shape.

Ans :

(B) Its tension and its linear mass density.

The speed of a transverse wave on a string is given by $v = \sqrt{T/\mu}$, where T is the tension in the string and μ is its linear mass density (mass per unit length).

17. Which of the following statements about the speed of sound is correct?
- Sound travels faster in gases than in solids generally.
 - Sound speed is independent of the medium's temperature.
 - Sound travels faster in hotter air than in colder air.
 - Newton's formula for sound speed was perfectly accurate.

Ans :

(C) Sound travels faster in hotter air than in colder air.

The speed of sound in a gas increases with temperature. This is because higher temperatures lead to faster molecular motion, facilitating quicker propagation of the sound wave's energy through it.

18. Which of the following statements regarding Newton's formula for the speed of sound in a gas is incorrect?
- It assumed the process of sound propagation was isothermal perfectly.
 - It predicted a speed of sound about 15% lower than observed.
 - It considered the adiabatic bulk modulus for the gas correctly.
 - It used the relationship $v = \sqrt{P/\rho}$ for ideal gases.

Ans :

(C) It considered the adiabatic bulk modulus for the gas correctly.

Newton's formula assumed isothermal conditions (constant temperature), meaning it used the isothermal bulk modulus ($B = P$). Laplace corrected this by assuming adiabatic conditions (no heat exchange), using the adiabatic bulk modulus ($B_{ad} = \gamma P$).

19. What is the primary difference between Newton's formula for the speed of sound and Laplace's correction?
- Newton used density, Laplace used specific volume of gas.
 - Newton assumed adiabatic, Laplace assumed isothermal processes.
 - Newton assumed isothermal, Laplace assumed adiabatic processes.
 - Newton considered pressure, Laplace considered only gas type.

Ans :

(C) Newton assumed isothermal, Laplace assumed adiabatic processes.

Newton incorrectly assumed that the compressions and rarefactions during sound propagation in air occur isothermally. Laplace corrected this by proposing that these changes are rapid and occur adiabatically (without heat transfer).

20. The principle of superposition of waves states that the resultant displacement of overlapping waves is the:
- Product of individual displacements only.
 - Algebraic sum of individual displacements.
 - Average of individual displacements always.
 - Larger of the individual displacements.

Ans :

(B) Algebraic sum of individual displacements.

The principle of superposition states that when two or more waves meet at a point in a medium, the net displacement at that point is the vector sum (algebraic sum for 1D waves) of the displacements due to each individual wave.

21. What phenomenon occurs when two waves meet such that their crests align, resulting in a larger amplitude?
- Destructive interference.
 - Constructive interference.
 - Wave diffraction always.
 - Wave polarization only.

Ans :

(B) Constructive interference.

76. When an open pipe is producing third harmonic, number of nodes is :

(a) 1 (b) 2
(c) 3 (d) 4

Ans :

(c) 3

No. of harmonics in 3rd harmonic = 3 in an open organ pipe.

77. As an empty vessel is filled with water, its frequency:

(a) Increases (b) Decreases
(c) Remains the same (d) None of these

Ans :

(a) Increases

$\sin n = \frac{v}{2l}$ so when l decreases, n increases.

78. When a source of sound moves towards stationary observer, the wavelength of sound received by the observer:

(a) Increases (b) Decreases
(c) Remains the same (d) Cannot say

Ans :

(a) Increases

When source is in motion, the wavelength does not change.

79. In case of a moving source of sound approaching an observer.

(a) The wavelength of sound appears to be less
(b) The wavelength of sound appears to be more
(c) The frequency appears to be less
(d) None of these.

Ans :

(a) The wavelength of sound appears to be less

80. If an open pipe of length l vibrates in fundamental mode, then pressure variation is maximum at:

(a) Ends of pipe
(b) Middle of pipe
(c) One fourth from ends of pipe
(d) One third from ends of pipe

Ans :

(b) Middle of pipe

In an open pipe in fundamental mode both the ends have anti-nodes and node is formed at the middle of the pipe, hence the displacement is minimum and pressure is maximum at the middle.

ONE MARK QUESTIONS

81. A wave transmits energy. Can it transmit momentum?

Ans :

Yes.

82. What is the relation between path difference and phase difference ?

Ans :

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

i.e. $\phi = \frac{2\pi}{\lambda} x$

83. Which property of the medium enables the transverse waves to pass through it ?

Ans :

Modulus of rigidity.

84. What is the relation between wavelength (λ), wave velocity (u) and frequency (v) ?

Ans :

$$v = u\lambda$$

85. What are pressure waves ?

Ans :

Pressure waves are longitudinal waves in which the pressure and volume changes during their propagation through a medium.

86. Which waves do not require medium for propagation?

Ans :

Electromagnetic waves.

87. Can mechanical waves travel through vacuum ?

Ans :

No.

88. What are the two main properties a material must possess for the propagation of waves through it.

Ans :

- (i) Property of elasticity.
(ii) Inertia.

- 29.** An air column in a pipe closed at one end and open at the other has a length L . What is the wavelength of its fundamental mode of vibration?

(A) $L/2$
 (B) L
 (C) $2L$
 (D) $4L$

Ans :

(D) $4L$.

For a pipe closed at one end, the closed end is a displacement node and the open end is an antinode. In the fundamental mode, one-quarter of a wavelength fits into the length L , so $\lambda = 4L$.

- 30.** Which of the following statements about standing waves is correct?

(A) Energy is continuously transferred along the wave direction.
 (B) All particles in the wave oscillate with same amplitude.
 (C) The wave pattern moves with the speed of sound.
 (D) There are points of permanent zero displacement called nodes.

Ans :

(D) There are points of permanent zero displacement called nodes.

Standing waves, or stationary waves, are characterized by fixed points called nodes where the displacement is always zero, and antinodes where the displacement is maximum. Energy is confined between nodes.

- 31.** Which of the following statements about beats is incorrect?

(A) Beats are produced by two waves of nearly equal frequencies.
 (B) The beat frequency is the sum of the two original frequencies.
 (C) Beats involve a periodic variation in the sound's intensity.
 (D) The amplitude of the resultant wave varies with time regularly.

Ans :

(B) The beat frequency is the sum of the two original frequencies.

Beats arise from the interference of two waves with slightly different frequencies. The beat frequency is equal to the absolute difference between the two original frequencies ($\nu_{beat} = |\nu_1 - \nu_2|$), not their sum.

- 32.** The phenomenon of 'beats' arises primarily due to which wave property?

(A) Diffraction.
 (B) Polarization.
 (C) Interference.
 (D) Refraction.

Ans :

(C) Interference.

Beats are a specific manifestation of wave interference, occurring when two sound waves of slightly different frequencies superpose. This interference leads to a periodic variation in the loudness of sound.

- 33.** If two sound waves have frequencies ν_1 and ν_2 (where $\nu_1 > \nu_2$ and they are close), what is the beat frequency produced?

(A) $(\nu_1 + \nu_2)/2$
 (B) $\nu_1 + \nu_2$
 (C) $\nu_1 - \nu_2$
 (D) $\sqrt{\nu_1 \nu_2}$

Ans :

(C) $\nu_1 - \nu_2$

The beat frequency is the rate at which the loudness of the sound fluctuates. It is mathematically equal to the absolute difference between the frequencies of the two interfering sound waves.

- 34.** How do progressive waves differ from stationary waves in terms of energy propagation?

(A) Progressive waves store energy, stationary waves transmit energy.
 (B) Progressive waves transmit energy, stationary waves confine energy.
 (C) Both wave types always transmit energy equally and effectively.
 (D) Neither wave type is capable of any energy propagation.

Ans :

(B) Progressive waves transmit energy, stationary waves confine energy.

Progressive waves transfer energy from one point to another in the direction of propagation. Stationary waves, however, do not transport energy; energy is localized between the nodes of the pattern.

- 35.** A string fixed at both ends is vibrating in its third harmonic. How many nodes are present between the two fixed ends?

(A) One node.
 (B) Two nodes.

- (B) 2 : 1
(C) 1 : 2
(D) 4 : 1

Ans :

- (B) 2 : 1.

For an open pipe, $v_o = v/(2L)$. For a closed pipe, $v_c = v/(4L)$.

Therefore, the ratio

$$\begin{aligned} v_o/v_c &= (v/(2L))/(v/(4L)) \\ &= (v/(2L)) \times (4L/v) \\ &= 4L/(2L) \\ &= 2/1 \text{ or } 2 : 1. \end{aligned}$$

58. The phenomenon of interference of light provides evidence for which of the following?
(A) The particle nature of the light source.
(B) The wave nature of the light source.
(C) The quantum nature of the light source.
(D) The thermal nature of the light source.

Ans :

- (B) The wave nature of the light source.

Interference, characterized by constructive and destructive superposition of waves, is a hallmark of wave behavior. It demonstrates that light exhibits wave properties, not particle or thermal primarily.

59. What primarily determines the speed of mechanical waves in a specific elastic medium?
(A) The frequency of the wave only chosen.
(B) The wavelength of the wave only chosen.
(C) The amplitude of the wave only chosen.
(D) The inertial and elastic properties of medium.

Ans :

(D) The inertial and elastic properties of medium.
The speed of a mechanical wave is an inherent characteristic of the medium through which it propagates. It depends on how quickly the medium's particles can transfer energy (elasticity) and their resistance to motion (inertia).

60. The initial phase angle (ϕ) in the wave equation $y(x, t) = a \sin(kx - \omega t + \phi)$ determines the:
(A) Wave's amplitude at $x = 0$ and $t=0$ instant.
(B) Wave's displacement at $x = 0$ and $t=0$ instant.
(C) Wave's overall speed at $x = 0$ and $t=0$ instant.
(D) Wave's frequency at $x = 0$ and $t=0$ instant.

Ans :

- (B) Wave's displacement at $x = 0$ and $t=0$ instant.

The initial phase angle ϕ specifies the displacement $y(0, 0) = a \sin(\phi)$ of the wave at the origin ($x = 0$) at the initial time ($t = 0$). It sets the starting point of the oscillation.

61. Water waves are :

- (a) Longitudinal
(b) Transverse
(c) Both longitudinal and transverse
(d) Neither longitudinal nor transverse

Ans :

- (c) Both longitudinal and transverse

Water waves are longitudinal and transverse.

62. Velocity of sound is maximum in:

- (a) He (b) N_2
(c) H_2 (d) O_2

Ans :

- (a) He

$v = \frac{\gamma P}{\rho}$ He is monoatomic gas and $\gamma = \frac{5}{3} = \text{maxi}$.

Hence v in 'He' is maximum.

63. Through which character we can distinguish the light waves from sound waves?

- (a) Interference (b) Refraction
(c) Polarisation (d) Reflection

Ans :

- (c) Polarisation

Light waves can be polarised because they are transverse waves and sound waves cannot be polarised as they are longitudinal waves.

64. Two sound waves of slightly different frequencies propagating in the same direction produce beats due to:

- (a) Interference (b) Diffraction
(c) Reflection (d) Refraction

Ans :

- (c) Reflection

Beats are produced due to interference of two waves of slightly different frequencies.

65. A change in temperature affects which property of sound?

- (a) Frequency (b) Amplitude
(c) Wavelength (d) Loudness

Ans :

- (c) Wavelength

41. Consider the following statements about waves:
1. Mechanical waves can transfer energy through vacuum.
 2. In transverse waves, particle oscillation is parallel to wave velocity.
 3. Sound waves in air are examples of longitudinal waves typically.
 4. Electromagnetic waves require a medium for their propagation.

Which of the following combinations of statements is correct?

- (A) 1 and 2 only.
 (B) 3 only is correct.
 (C) 2 and 4 only.
 (D) 1 and 3 only.

Ans :

(B) 3 only is correct.

Statement 1 is false; mechanical waves need a medium. Statement 2 is false; it's perpendicular. Statement 3 is true. Statement 4 is false; electromagnetic waves can travel in vacuum. Thus, only statement 3 is correct.

42. Regarding the characteristics of a sinusoidal progressive wave given by $y(x, t) = a \sin(kx - \omega t + \phi)$:
1. 'a' represents the wavelength of the wave.
 2. 'k' is known as the angular frequency always.
 3. 'ω' is the angular wave number constantly.
 4. 'φ' is the initial phase angle at point $x = 0, t = 0$.
- Which of the statements given above is/are correct?

- (A) 4 only is correct.
 (B) 1 and 2 are correct.
 (C) 2 and 3 are correct.
 (D) 1, 3 and 4 are correct.

Ans :

(A) 4 only is correct.

'a' is amplitude, not wavelength. 'k' is angular wave number, not angular frequency. 'ω' is angular frequency, not angular wave number. 'φ' is correctly defined as the initial phase angle at $x = 0, t = 0$.

43. Identify the correct statements regarding stationary waves:
1. All particles oscillate with the same fixed frequency.
 2. The amplitude of oscillation is constant for all particles.
 3. There is a continuous transfer of energy across any section.
 4. Nodes are points of maximum displacement continuously.
- Which combination of statements is true for stationary waves?

- (A) 1 only is correct.
 (B) 1 and 4 are correct.
 (C) 2 and 3 are correct.
 (D) 3 and 4 are correct.

Ans :

(A) 1 only is correct.

Statement 1 is true. Statement 2 is false; amplitude varies with position. Statement 3 is false; energy is localized. Statement 4 is false; nodes are points of zero displacement, antinodes have maximum displacement.

Match the wave property in Column I with its corresponding characteristic in Column II.

	Column I		Column II
1.	Amplitude	P.	Number of oscillations per second.
2.	Frequency	Q.	Distance between two successive crests.
3.	Wavelength	R.	Maximum displacement from equilibrium.
4.	Wave speed	S.	Rate at which the wave pattern travels

- (A) 1-R, 2-P, 3-Q, 4-S
 (B) 1-P, 2-R, 3-Q, 4-S
 (C) 1-R, 2-Q, 3-P, 4-S
 (D) 1-S, 2-P, 3-Q, 4-R

Ans :

(A) 1-R, 2-P, 3-Q, 4-S

Amplitude is the maximum displacement (R). Frequency is oscillations per second (P). Wavelength is the distance between successive crests (Q). Wave speed is the rate the pattern travels (S).

45. Match the type of wave phenomenon in Column I with its description in Column II.

	Column I		Column II
1.	Reflection	P.	Resultant amplitude depends on phase difference.
2.	Superposition	Q.	Bouncing back of a wave from a boundary.
3.	Interference	R.	Algebraic sum of displacements of overlapping waves.
4.	Beats	S.	Periodic variation in intensity from two close frequencies.

Constructive interference happens when two or more waves superpose in phase (e.g., crest meets crest). This leads to a resultant wave with an amplitude that is the sum of the individual amplitudes.

22. If two identical waves travelling in the same direction interfere destructively, what is the phase difference between them?
- (A) 0 radians.
(B) $\pi/2$ radians.
(C) π radians.
(D) 2π radians.

Ans :

(C) π radians.

For complete destructive interference between two identical waves, they must be out of phase by π radians (or 180°), or any odd multiple of π . This means a crest meets a trough.

23. All of the following affect the speed of sound in air EXCEPT:
- (A) Temperature of the air.
(B) Humidity of the current air.
(C) Density of the present air.
(D) Amplitude of the sound wave.

Ans :

(D) Amplitude of the sound wave.

The speed of sound in air depends on properties of the medium like temperature, humidity (which affects density and molar mass), and density itself. It does not depend on the amplitude of the wave.

24. What is the phase change when a transverse wave pulse is reflected from a rigid boundary?
- (A) 0 radians phase change.
(B) $\pi/2$ radians phase change.
(C) π radians phase change.
(D) 2π radians phase change.

Ans :

(C) π radians phase change.

When a wave encounters a rigid boundary, it is reflected with a phase inversion. This means the reflected wave is 180° or π radians out of phase with the incident wave at that boundary.

25. In a stationary wave, what are the points of zero displacement called?
- (A) Antinodes.
(B) Nodes.
(C) Crests.
(D) Troughs.

Ans :

(B) Nodes.

Nodes are specific points along a stationary wave where the amplitude of oscillation is always zero. These points remain at rest as the wave pattern vibrates around them.

26. What are the points in a stationary wave where the amplitude of oscillation is maximum?
- (A) Nodes.
(B) Crests.
(C) Antinodes.
(D) Troughs.

Ans :

(C) Antinodes.

Antinodes are positions in a standing wave pattern where the amplitude of the oscillation is at its maximum. These points experience the largest displacement during the vibration of standing wave.

27. The distance between two consecutive nodes in a stationary wave is equal to:
- (A) $\lambda/4$
(B) $\lambda/2$
(C) λ
(D) 2λ

Ans :

(B) $\lambda/2$

In a stationary wave pattern, nodes are points of zero displacement. The distance from one node to the very next node is always half of the wavelength (λ) of the interfering waves.

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28. For a string of length L fixed at both ends, what is the wavelength of the fundamental mode (first harmonic)?
- (A) $L/2$
(B) L
(C) $2L$
(D) $4L$

Ans :

(C) $2L$.

In the fundamental mode for a string fixed at both ends, the string vibrates in a single segment, with nodes at the ends and an antinode in the middle. This corresponds to half a wavelength fitting into the length L , so $\lambda = 2L$.

(assuming adiabatic process), which is more accurate.

51. Arrange the following steps in the correct sequence for the generation and perception of sound when a person speaks to another:

1. Detection of sound waves by the listener's ear.
2. Production of sound waves by the speaker's vocal cords.
3. Propagation of sound waves through the air medium.
4. Conversion of sound waves into neural signals in the ear.

- (A) 2, 3, 1, 4
(B) 3, 2, 4, 1
(C) 2, 1, 3, 4
(D) 1, 2, 3, 4

Ans :

- (A) 2, 3, 1, 4

First, the speaker produces sound waves (2). These waves then propagate through the air (3). Next, the listener's ear detects these waves (1). Finally, the ear converts them into neural signals for the brain (4).

52. Consider a pulse travelling down a string towards a fixed (rigid) end. What is the sequence of events?

1. The pulse is inverted upon reflection from end.
2. The pulse exerts a force on the fixed end.
3. The pulse travels towards the fixed string end.
4. The fixed end exerts an equal and opposite force.

- (A) 3, 2, 4, 1
(B) 2, 3, 1, 4
(C) 3, 4, 2, 1
(D) 1, 3, 2, 4

Ans :

- (A) 3, 2, 4, 1

Initially, the pulse travels towards the fixed end (3). Upon reaching the end, it exerts a force on it (2). By Newton's third law, the fixed end exerts an equal and opposite force back on the string (4), which generates the reflected pulse, inverted (1).

53. If the tension in a stretched string is quadrupled, and its linear mass density is kept constant, how does the speed of transverse waves on the string change?

- (A) It becomes half of original.
(B) It becomes double original.
(C) It becomes four times original.
(D) It remains completely unchanged.

Ans :

- (B) It becomes double original.

The speed of a transverse wave on a string is $v = \sqrt{T/\mu}$. If tension T is quadrupled, the speed becomes $\sqrt{4T/\mu} = 2\sqrt{T/\mu}$, so the speed is doubled from original.

54. Two tuning forks produce 4 beats per second when sounded together. If the frequency of one fork is 256 Hz, what could be the frequency of the other fork?

- (A) 250 Hz or 262 Hz.
(B) 252 Hz or 260 Hz.
(C) 256 Hz or 260 Hz.
(D) 252 Hz or 256 Hz.

Ans :

- (B) 252 Hz or 260 Hz.

The beat frequency is the difference between the two frequencies. So, if $\nu_1 = 256\text{Hz}$ and $\nu_{\text{beat}} = 4\text{Hz}$, then ν_2 could be $\nu_1 + \nu_{\text{beat}} = 256 + 4 = 260\text{Hz}$ or $\nu_1 - \nu_{\text{beat}} = 256 - 4 = 252\text{Hz}$.

55. A wave has a wavelength of 2m and a frequency of 50 Hz. What is the speed of this wave?

- (A) 25 m/s
(B) 50 m/s
(C) 100 m/s
(D) 0.04 m/s

Ans :

- (C) 100 m/s

The speed of a wave (v) is related to its wavelength (λ) and frequency (ν) by the formula $v = \nu\lambda$. Given $\lambda = 2\text{m}$ and $\nu = 50\text{Hz}$, the speed $v = 50 \times 2 = 100\text{m/s}$.

56. Which statement about the propagation constant (k) is incorrect?

- (A) It is also called the angular wave number value.
(B) Its SI unit is radian per metre commonly.
(C) It is related to wavelength by $k = 2\pi/\lambda$ only.
(D) It is directly proportional to the wave period.

Ans :

- (D) It is directly proportional to the wave period.

The propagation constant $k = 2\pi/\lambda$. Since $\lambda = vT$, then $k = 2\pi/(vT)$. Thus, k is inversely proportional to the period T , not directly proportional, for a constant wave speed.

57. In comparing the fundamental frequency of a pipe of length L open at both ends (ν_o) to that of a pipe of the same length L closed at one end (ν_c), what is the ratio ν_o/ν_c ?

- (A) 1 : 1

- (A) 1-Q, 2-R, 3-P, 4-S
 (B) 1-R, 2-Q, 3-P, 4-S
 (C) 1-Q, 2-P, 3-R, 4-S
 (D) 1-P, 2-S, 3-R, 4-Q

Ans :

- (A) 1-Q, 2-R, 3-P, 4-S

Reflection is bouncing back (Q). Superposition is the algebraic sum of displacements (R). Interference results in amplitude depending on phase difference (P). Beats are periodic intensity variations (S).

46. Match the boundary condition for wave reflection in Column I with the outcome for the reflected wave in Column II.

	Column I		Column II
1.	Rigid Boundary	P.	No phase change from the incident one.
2.	Open Boundary	Q.	Phase change of π radians from incident.
3.	Incident Crest	R.	Becomes a trough if reflected from rigid end.
4.	Incident Trough	S.	Becomes a crest if reflected from rigid end.

- (A) 1-Q, 2-P, 3-R, 4-S
 (B) 1-P, 2-Q, 3-R, 4-S
 (C) 1-Q, 2-P, 3-S, 4-R
 (D) 1-P, 2-Q, 3-S, 4-R

Ans :

- (A) 1-Q, 2-P, 3-R, 4-S

Reflection from a rigid boundary causes a π phase change (Q), so a crest becomes a trough (R) and a trough becomes a crest (S). Reflection from an open boundary causes no phase change (P).

47. Identify the correctly matched pair regarding wave properties.
 (A) Wavelength (λ) : Unit is Hertz (Hz) usually.
 (B) Angular frequency (ω) : Relation is $2\pi\nu$ correctly.
 (C) Wave speed (v) : Formula is k/ω always.
 (D) Period (T) : Defined as $\omega/(2\pi)$ now.

Ans :

- (B) Angular frequency (ω) : Relation is $2\pi\nu$ correctly.

Angular frequency $\omega = 2\pi\nu$, where ν is the linear frequency. Wavelength unit is metre. Wave speed $v = \omega/k$. Period $T = 2\pi/\omega$. Thus, only option (B) states a correct relation.

48. Which of the following pairs correctly describes a characteristic of a wave type?

- (A) Sound waves : Always transverse in nature primarily.
 (B) Light waves : Require a material medium always.
 (C) Waves on a string : Purely longitudinal in all cases.
 (D) Longitudinal waves : Can propagate in liquids also.

Ans :

- (D) Longitudinal waves : Can propagate in liquids also.

Sound waves are longitudinal. Light waves do not require a medium. Waves on a string are transverse. Longitudinal waves, involving compressions, can propagate through solids, liquids, and gases effectively.

49. Identify the incorrectly matched pair concerning wave phenomena.

- (A) Constructive interference : Amplitudes add up generally.
 (B) Destructive interference : Waves are in phase always.
 (C) Nodes in stationary wave : Points of zero displacement.
 (D) Antinodes in stationary wave : Points of maximum displacement.

Ans :

- (B) Destructive interference : Waves are in phase always.

Destructive interference occurs when waves are out of phase (typically by π radians or odd multiples), causing amplitudes to subtract. Waves in phase lead to constructive interference normally.

50. Which one of the following pairs relating to wave speed formulas is incorrectly stated?

- (A) Transverse wave on string : $v = \sqrt{T/\mu}$ exactly.
 (B) Longitudinal wave in fluid : $v = \sqrt{B/\rho}$ correctly.
 (C) Sound in ideal gas (Newton) : $v = \sqrt{\gamma P/\rho}$ accurately.
 (D) Longitudinal wave in solid bar : $v = \sqrt{Y/\rho}$ precisely.

Ans :

- (C) Sound in ideal gas (Newton) : $v = \sqrt{\gamma P/\rho}$ accurately.

Newton's formula for the speed of sound in an ideal gas was $v = \sqrt{P/\rho}$ (assuming isothermal process). The formula $v = \sqrt{\gamma P/\rho}$ is Laplace's correction

A change in temperature produces a change in its velocity and hence a change in wavelength takes place ($\nu = v\lambda$).

- 66.** Human ear cannot hear those mechanical waves whose frequency lies in the frequency range:

- (a) Less than 1000 Hz but greater than 10000 Hz
(b) Between 1000 Hz and 5000 Hz
(c) Between 500 Hz and 20000 Hz
(d) Less than 20 Hz and more than 2000 Hz.

Ans :

- (d) Less than 20 Hz and more than 2000 Hz.

Audible range is 20Hz to 20000 Hz.

- 67.** At a given temperature, velocity of sound in oxygen and in hydrogen has the ratio:

- (a) 4 : 1 (b) 1 : 4
(c) 1 : 1 (d) 2 : 1

Ans :

- (b) 1 : 4

$$\frac{v_0}{v_H} = \frac{\rho_H}{\rho_0} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

- 68.** A wave along a string has the following equation (y, x in metre and t in sec) $y = 0.02 \sin (30t - 4.0x)$
The amplitude of the wave is:

- (a) 0.02 m (b) 0.04 m
(c) 4.0 m (d) 30 m

Ans :

- (a) 0.02 m

Compare the given equation with general equation.

i.e., $y = r \sin (\omega t - \kappa x)$

So $r = 0.02 \text{ m}$

- 69.** When a wave is reflected from a denser medium, the change in phase is :

- (a) 0 (b) π
(c) 2π (d) 3π

Ans :

- (b) π

When a wave is reflected from a denser medium, a phase change of π takes place.

- 70.** If the speed of sound in air at 0°C is 331 m s^{-1} . What will be its value at 35°C ?

- (a) 331 m s^{-1} (b) 366 m s^{-1}
(c) 351.6 m s^{-1} (d) 332 m s^{-1}

Ans :

- (c) 351.6 m s^{-1}

$$v_t = 331 \sqrt{\frac{273 + 35}{273 + 0}} = 351.6 \text{ m s}^{-1}$$

In a longitudinal wave, pressure variation and displacement variation are:

- (a) 180° out of phase (b) 70° out of phase
(c) 45° out of phase (d) In phase.

Ans :

- (a) 180° out of phase

In longitudinal wave, the pressure is minimum where displacement is maximum. So the two are 180° out of phase.

- 72.** In a stationary wave,

- (a) particle move periodically
(b) All particles are at rest
(c) Amplitude is same at all places
(d) Amplitude is maximum at some places and minimum at some other.

Ans :

- (d) Amplitude is maximum at some places and minimum at some other.

The amplitude is maximum at anti-nodes and minimum at nodes.

- 73.** In the principle of superposition, the characteristics that get add vectorial is:

- (a) Displacement (b) Velocity
(c) Amplitude (d) Frequency

Ans :

- (a, c) Displacement and Amplitude

In superposition principle, both displacement and amplitude get added.

- 74.** The phase difference between two particles of a medium lying just on the opposite sides of a node is:

- (a) Zero (b) $\frac{\pi}{3}$
(c) π (d) 2π

Ans :

- (c) π

- 75.** A tuning is made of :

- (a) Steel (b) Iron
(c) Invar (d) Elinvar

Ans :

- (d) Elinvar

- (C) Three nodes.
(D) Four nodes.

Ans :

- (B) Two nodes.

For the n th harmonic on a string fixed at both ends, there are n antinodes and $(n-1)$ nodes between the ends. The ends themselves are also nodes. So, for the 3rd harmonic ($n=3$), there are $3 - 1 = 2$ nodes between the ends.

36. For an air column in a pipe open at both ends, which of the following harmonics is NOT possible?
(A) First harmonic.
(B) Second harmonic.
(C) Third harmonic.
(D) All harmonics are possible.

Ans :

- (D) All harmonics are possible.

An air column open at both ends can sustain vibrations for all harmonics. The fundamental frequency is $v/(2L)$, and higher harmonics are integer multiples ($n\nu_1$) of this fundamental frequency.

37. The speed of sound in air is 340 m/s. If a pipe of length 0.85 m is open at both ends, what is its fundamental frequency?
(A) 100 Hz
(B) 200 Hz
(C) 300 Hz
(D) 400 Hz

Ans :

- (B) 200 Hz.

For a pipe open at both ends, the fundamental frequency $\nu_1 = v/(2L)$.

Given $v = 340\text{m/s}$ and $L = 0.85\text{m}$,

$$\begin{aligned}\nu_1 &= 340/(2 \times 0.85) \\ &= 340/1.7 \\ &= 200\text{Hz}\end{aligned}$$

38. Assertion (A): The speed of a wave in a medium depends on the wavelength of the wave.
Reason (R): For a given medium, the wave speed is constant, and the relation $v = \nu\lambda$ holds.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

- (D) A is false but R is true.

The speed of a wave in a non-dispersive medium is determined by the medium's properties (e.g., tension and linear density for a string, or bulk modulus and density for a fluid), not by the wavelength or frequency of the wave itself. The relation $v = \nu\lambda$ shows how wavelength and frequency are related for a given speed.

39. Assertion (A): When a wave reflects from a rigid boundary, the reflected wave has a phase change of π .
Reason (R): At a rigid boundary, the displacement of the particles must always be zero, which requires the incident and reflected waves to interfere destructively at the boundary.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

- (A) Both A and R are true and R is the correct explanation of A.

The condition of zero displacement at a rigid boundary implies that the incident and reflected waves must cancel each other out. This destructive interference occurs if the reflected wave is phase-shifted by π radians relative to the incident wave.

40. Assertion (A): In stationary waves, energy is not transported from one point to another.
Reason (R): The amplitude of oscillation varies from point to point in a stationary wave, being zero at nodes.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans :

- (B) Both A and R are true but R is NOT the correct explanation of A.

Stationary waves do not transport net energy; energy oscillates between kinetic and potential forms within each segment between nodes. While the varying amplitude and presence of nodes are characteristics, they don't fully explain why energy isn't transported, which is due to the superposition of two counter-propagating waves.

- 89.** When a stone is thrown on the surface of water, a wave travels out. From where does the energy came?

Ans :

The energy comes from the kinetic energy of the stone shared by the water molecules on which it falls.

- 90.** Sound can be heard over a long distance on a rainy day. Why ?

Ans :

On a rainy day humidity is high and speed of sound is higher in humid air than in dry air.

- 91.** What is the common property with all types of mechanical waves ?

Ans :

The medium does not move from one point to the other as the mechanical wave travels from one point to the other.

- 92.** Sound wave from a point source are propagating in all directions. What will be ratio of amplitudes at distances of 9 m and 25 m from the source ?

Ans :

$$\text{Intensity} \propto (\text{amplitude})^2 \propto \frac{1}{(\text{distance})^2}$$

$$\text{Amplitude} \propto \frac{1}{\text{distance}}$$

Hence,
$$\frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{9}$$

- 93.** The ratio of amplitude of two waves is 2 : 3. What is the ratio of intensities of these waves ?

Ans :

$$\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

- 94.** In which medium-solid, liquid or gas, do the sound waves travel fastest ?

Ans :

Solid

- 95.** Represent a harmonic wave in terms of wavelength and time period.

Ans :

$$y = r \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

- 96.** What is the distance between the two points which have a phase difference of 2π ?

Ans :

Wavelength.

- 97.** A flute has several holes in it. Why ?

Ans :

A flute is an open organ pipe and the length of air column can be changed by putting fingers on them and different frequencies can be produced.

- 98.** What is the distance between a node and an adjoining anti-node in a stationary wave ?

Ans :

The distance is $\lambda/4$ (or $\pi/4$)

- 99.** Is it possible to have interference between the waves produced by two violins ? Why ?

Ans :

No, because in order to produce interference between the two waves, they must have a constant phase difference.

- 100.** Will sound be louder at node or antinode in a stationary wave ?

Ans :

It will be louder at node due to large pressure variation there on $\Delta p = -\text{strain} \times \text{elasticity}$.

- 101.** When are the tones called harmonics ?

Ans :

The tones are called harmonics, if the frequencies of the fundamental tone and other overtones produced by a source of sound are in harmonic series.

- 102.** What is the difference between a tone and a note ?

Ans :

Tone : Sound of single frequency is called a tone.

Note : A combination of tones of different frequencies is called a note.

- 103.** Why stationary wave is so called ?

Ans :

A stationary wave is so called because there is no net propagation of energy.

particle, labelled by its equilibrium position x , is displaced some distance y in the transverse direction. The value of y depends on which particle of string is being described (that is on x) and also on the time t at which look at it. This y is a function of x and t : $y = f(x, t)$. If this function is known for a particular wave motion, it can be used to predict the position of any particle at any time.

The function $y = f(x, t)$ constitutes a complete description of the motion. Such a function is called a wave function.

- 152.** In which case is the speed of sound greater air, liquid and a solid ? Give example.

Ans :

Since solids are so much stiffer than gases and liquids, therefore speeds in solids are much higher. It is seen that speed of sound in a solid (say iron)

$$\approx 4 \times \text{speed of sound in liquid (say water).}$$

$$\approx 16 \times \text{speed of sound in air.}$$

- 153.** If we place our ear at the end of a long pipe and ask our friend to hammer at the other end, we will hear two distinct type of sound. Explain why ?

Ans :

On hammering the other end of the pipe, the sound produced by the hammer travels through the solid pipe as well as through the air which is inside the pipe. As velocity of sound in steel is 16 times greater than in air, so we hear two distinct sounds.

- 154.** Are particle velocity and wave velocity the same ?

Ans :

No, Particle velocity and wave velocity are different. Particle velocity is the velocity with which a particle executes S.H.M. It is zero at the extreme positions and maximum at the mean position wave velocity.

Wave velocity is the velocity with which progressive wave front travels forwards and in one medium it remains constant and is given by $v = \nu\lambda$.

- 155.** When sound travels through gases, is it under isothermal condition or adiabatic conditions ? And why ?

Ans :

Sound travels through gases under adiabatic condition, because sound being longitudinal wave travels in the form of compression and rarefaction. During compression, the compressed layers gain

heat and during rarefaction, the rarefied layers lose heat and the change in volume is so quick that this rise and fall in temperature do not get enough time to be equilised due to bad conducting and radiating power of air.

- 156.** Define temperature coefficient of velocity of sound and show that it is $0.61 \text{ ms}^{-1} \text{ } ^\circ\text{C}^{-1}$.

Ans :

Temperature coefficient of velocity of sound is defined as the change of velocity of sound for 1°C change in temperature of the medium.

Let v_0 = velocity of sound at 0°C

v_t = velocity of sound at $t^\circ\text{C}$

$$\begin{aligned} \frac{v_t}{v_0} &= \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}} \\ &= \left[1 + \frac{t}{273}\right]^{1/2} \end{aligned}$$

Since $\frac{t}{273} \ll 1$, so using Binomial theorem and neglecting the terms containing higher powers of $\frac{t}{273}$,

we get

$$\frac{v_t}{v_0} = \left(1 + \frac{t}{546}\right)$$

$$v_t = v_0 + \frac{v_0}{546} t$$

$$\frac{v_t - v_0}{t} = \frac{v_0}{546} = \frac{332}{546}$$

[since, $v_0 = 332 \text{ ms}^{-1}$]

$$\frac{v_t - v_0}{t} = 0.61 \text{ m s}^{-1} \text{ } ^\circ\text{C}$$

- 157.** What should be the minimum distance between source and reflector to hear the distinct echo of a sound ?

Ans :

The minimum distance between the source and the reflector should be 17 m.

Let d be distance between source and reflector and t be time taken by source to reach the listener after reflection.

Total distance travelled by sound from listener to reflector and back = $2d$

$$2d = v t \text{ (} v = \text{velocity of sound)}$$

$$d = \frac{vt}{2}$$

Since the time period of persistence of hearing is $\frac{1}{10} \text{ s}$

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TWO MARKS QUESTIONS

121. What is the use of stem of a tuning fork ?

Ans :

The stem provides a handle for the fork. Also since the vibrations in the stem are parallel to its length, they do not die out when it is held in hand. The stem also helps in transmitting its energy to sounding boards etc.

122. A wave transmits momentum. Can it transfer angular momentum ?

Ans :

A wave transmits momentum but cannot transmit angular momentum because angular momentum can be transferred by means of torque which causes rotatory motion.

123. A tuning fork has two prongs. Why ?

Ans :

Because the prongs of tuning fork generally vibrate in opposite phases, thus they keep the centre of gravity of the fork undisturbed and no external force is required to keep the stem in steady position.

124. Give two differences between progressive and stationary waves.

Ans :

- (i) Nodes and anti-nodes shift their positions with time in progressive waves whereas their positions remain same in stationary waves.
- (ii) Progressive waves are formed due to a single disturbance whereas stationary waves are formed due to two disturbance travelling in opposite directions.

125. Define wave and wave motion.

Ans :

Wave : When a medium is disturbed, the pattern of disturbance which moves without the actual physical transfer of matter or flow of the medium as a whole is called wave.

Wave motion : Wave motion is a sort of disturbance, which travels through the medium on account of repeated periodic vibrations of the particles about their mean position. The disturbance is handed on from one particle to the adjoining particle without any net transport of the medium.

126. Sound is produced by vibratory motion, explain why then a vibrating pendulum does not produce sound ?

Ans :

We can hear sound only if its frequency is between 20 Hz to 20 kHz called audible sound. The frequency of vibrating pendulum does not lie within the audible range and hence it does not produce sound.

127. What is the difference between wave velocity and particle velocity ?

Ans :

Wave velocity is given by $u = v\lambda$ and is constant for a given medium whereas particle velocity changes harmonically with time. It has maximum value at mean position and zero value at the extreme position.

128. Is it possible to have longitudinal waves on a string and on a transverse wave in a steel rod ?

Ans :

No, we cannot have longitudinal waves on a string, because string can neither be compressed nor rarefied. In steel rod, transverse waves are possible because steel possess elasticity ?

129. How is it that strings vibrate in transverse manner and produce longitudinal waves ?

Ans :

A string vibrating in a transverse manner, pushes the particles of the medium forward which due to elasticity of the medium start vibrating in longitudinal manner.

130. Distinguish between mechanical and electromagnetic waves with respect to their medium requirement.

Ans :

Mechanical waves require a material medium for their propagation, as they involve oscillations of constituent particles. Electromagnetic waves, however, do not necessarily require a medium and can travel through vacuum; they consist of oscillating electric and magnetic fields.

131. Explain the nature of matter waves and give an example of their application.

Ans :

Matter waves are associated with moving constituents of matter like electrons, protons, and atoms, arising in quantum mechanical descriptions.

properties of that medium. Their key characteristics include:

- They require a material medium for propagation and cannot travel through a vacuum.
- They involve oscillations of the constituent particles of the medium.
- Their speed depends on the elastic and inertial properties of the medium.
- Examples include sound waves in air, waves on a stretched string, and seismic waves.

165. Differentiate between transverse and longitudinal waves, giving one example of each.

Ans :

Transverse and longitudinal waves are two fundamental types of mechanical waves, distinguished by the motion of the medium's constituents relative to wave propagation:

- **Transverse waves:** The particles of the medium oscillate perpendicular to the direction of wave propagation. An example is a wave on a stretched string.
- **Longitudinal waves:** The particles of the medium oscillate along (parallel to) the direction of wave propagation. An example is a sound wave in air.

166. Define the terms amplitude, phase, and initial phase for a progressive wave.

Ans :

For a progressive sinusoidal wave described by

$$y(x, t) = a \sin(kx - \omega t + \phi):$$

- **Amplitude (a):** It is the maximum displacement of the constituents of the medium from their equilibrium position.
- **Phase ($kx - \omega t + \phi$):** It is the argument of the sine function, determining the displacement of the wave at any position and at any instant.
- **Initial phase (ϕ):** It is the phase of the wave at position $x = 0$ and time $t = 0$.

167. Explain wavelength and angular wave number, and state the relationship between them.

Ans :

Wavelength and angular wave number are important parameters describing the spatial characteristics of a wave:

- **Wavelength (λ):** It is the minimum distance between two points in a wave having the same phase, such as the distance between two consecutive crests or troughs.

- **Angular wave number (k):** It is 2π times the number of wavelengths that fit into a unit distance.
- The relationship between them is $k = 2\pi/\lambda$.

168. Define period, angular frequency, and frequency of a wave. How are they related?

Ans :

These terms describe the temporal characteristics of a wave:

- **Period (T):** The time taken for any element of the medium to complete one full oscillation.
- **Angular frequency (ω):** It is 2π times the frequency, related to the period by $\omega = 2\pi/T$. Its SI unit is rad s^{-1} .
- **Frequency (ν):** The number of oscillations per second, given by $\nu = 1/T$.

169. How does the displacement function $y(x, t) = a \sin(kx - \omega t)$ represent a travelling wave?

Ans :

The function $y(x, t) = a \sin(kx - \omega t)$ represents a travelling wave because:

- At a fixed instant $t = t_0$,
 $y(x, t_0) = a \sin(kx - \text{constan } t)$, showing a sinusoidal shape in space.
- At a fixed location $x = x_0$,
 $y(x_0, t) = a \sin(\text{constan } t - \omega t)$, showing simple harmonic motion in time.
- For the phase $(kx - \omega t)$ to remain constant, as t increases, x must also increase, indicating propagation in the positive x-direction.

170. Explain the factors determining the speed of a transverse wave on a stretched string.

Ans :

The speed of a transverse wave on a stretched string is determined by the physical properties of the string itself. These properties are:

- **Tension (T):** The restoring force in the string. Higher tension leads to a faster wave speed.
- **Linear mass density (μ):** The mass per unit length of the string, representing its inertia. Higher linear mass density leads to a slower wave speed.

The speed is given by $v = \sqrt{T/\mu}$.

171. Discuss Newton's formula for the speed of sound in a gas and Laplace's correction to it.

Ans :

Newton first derived a formula for the speed of sound in a gas assuming isothermal compressions and rarefactions, yielding $v = \sqrt{P/\rho}$. This formula

- 142.** Define period and frequency of a wave. How are they related to angular frequency?

Ans :

- **Period (T):** The time it takes for an element of the medium to complete one full oscillation. It's related by $T = 2\pi/\omega$.
- **Frequency (ν):** The number of oscillations per second. It's related by $\nu = \omega/2\pi$.

- 143.** How does the displacement function for a longitudinal wave differ in notation from a transverse wave?

Ans :

For a transverse wave, displacement perpendicular to propagation is often denoted by $y(x, t)$. For a longitudinal wave, displacement along the direction of propagation is typically denoted by $s(x, t)$. The mathematical form $a \sin(kx - \omega t + \phi)$ is similar.

- 144.** What determines the displacement of a wave at any position and instant? Explain.

Ans :

The displacement of a wave at any position x and instant t is determined by its phase, $(kx - \omega t + \phi)$, and its amplitude 'a'. The phase specifies the state of oscillation of a particle, while the amplitude gives the maximum displacement.

- 145.** Derive the relation $v = \omega/k$ for the speed of a travelling wave.

Ans :

For a point of constant phase on a wave, $kx - \omega t = \text{constant}$. Differentiating with respect to time: $k(dx/dt) - \omega = 0$. Since dx/dt is the wave speed v , we get $kv - \omega = 0$, which gives $v = \omega/k$.

- 146.** On what properties of the medium does the speed of a transverse wave on a string depend?

Ans :

The speed of a transverse wave on a stretched string depends on two properties of the medium:

- The tension (T) in the string.
- The linear mass density (μ) of the string.

- 147.** Explain Newton's formula for the speed of sound in an ideal gas. What was its limitation?

Ans :

Newton proposed $v = \sqrt{P/\rho}$ for the speed of sound in an ideal gas, assuming isothermal pressure variations. Its limitation was that the calculated value (e.g., 280 m/s for air at STP) was about 15% smaller than the experimental value.

- 148.** What correction did Laplace make to Newton's formula for the speed of sound in gases?

Ans :

Laplace corrected Newton's formula by suggesting that pressure variations in sound waves are adiabatic, not isothermal, due to their rapid nature. This replaced the bulk modulus B with γP , leading to $v = \sqrt{\gamma P/\rho}$.

- 149.** Why is the speed of sound generally higher in solids and liquids than in gases?

Ans :

Solids and liquids have much higher bulk moduli (B) compared to gases, indicating they are harder to compress. Although their densities (ρ) are also higher, the increase in B is significantly greater, leading to a higher $v = \sqrt{B/\rho}$.

THREE MARKS QUESTIONS

- 150.** What is the nature of sound waves ?

Ans :

When longitudinal waves are transmitted through a medium due to compressions and rarefactions, there is a change in the value of the medium. Thus formation of longitudinal wave is possible only if the medium offers a resistance to a change in its volume i.e. if it possesses **volumetric elasticity**. Since solids, liquids and gases all offer a resistance to a change in their volume (i.e. they possess volumetric elasticity), therefore longitudinal wave motion can be transmitted through all the three states of matter viz. solid, liquid and gas. No wave of any kind can be transmitted through a medium which does not possess either the elasticity of shape or the elasticity of volume.

Sound waves, as we know, can easily travel through air and since only longitudinal waves can be transmitted through air, we therefore, conclude that sound is carried through air by longitudinal wave i.e., sound waves are longitudinal.

- 151.** Define wave functions.

Ans :

The functions which can mathematically represent a moving wave pulse are called wave functions.

We consider a stretched string which in its equilibrium position lies along the x -axis of a coordinate system. During wave motion each

- 178.** Discuss the formation of normal modes in a stretched string fixed at both ends.

Ans :

When a string fixed at both ends is disturbed, waves travel along it and reflect from the ends, creating stationary waves. The boundary conditions (nodes at both ends) restrict the possible wavelengths.

- The length of the string L must be an integer multiple of $\lambda/2$, i.e., $L = n(\lambda/2)$.
- This leads to allowed wavelengths $\lambda_n = 2L/n$ and corresponding frequencies $\nu_n = nv/2L$, where $n = 1, 2, 3, \dots$. These are the normal modes.

- 179.** Explain the fundamental mode and harmonics for a stretched string fixed at both ends.

Ans :

For a stretched string fixed at both ends, the allowed frequencies of vibration are its normal modes:

- **Fundamental mode (First harmonic):** This is the lowest possible natural frequency, occurring when $n = 1$. Its frequency is $\nu_1 = v/2L$, and the string vibrates in one segment with an antinode at the center.
- **Harmonics:** Higher frequencies are integer multiples of the fundamental frequency. The n^{th} harmonic has frequency $\nu_n = n\nu_1$. For example, the second harmonic ($n = 2$) has frequency $\nu_2 = 2\nu_1 = v/L$.

- 180.** Describe the normal modes of an air column in a pipe closed at one end and open at the other.

Ans :

For an air column in a pipe closed at one end (node) and open at the other (antinode):

- The length L must satisfy $L = (n + 1/2)\lambda/2$, for $n = 0, 1, 2, \dots$
- The possible wavelengths are $\lambda_n = 2L/(n + 1/2)$ or $4L/(2n + 1)$.
- The natural frequencies (normal modes) are $\nu_n = (n + 1/2)v/2L = (2n + 1)v/4L$.
- This means only odd harmonics of the fundamental frequency ($v/4L$) are present.

- 181.** Compare the normal modes of vibration for a pipe open at both ends with a pipe closed at one end.

Ans :

The normal modes differ significantly:

- **Pipe open at both ends:** It has antinodes at both ends. It produces all harmonics of its fundamental frequency $\nu_1 = v/2L$. So, frequencies are $\nu_1, 2\nu_1, 3\nu_1, \dots$

- **Pipe closed at one end:** It has a node at the closed end and an antinode at the open end. It produces only odd harmonics of its fundamental frequency $\nu_1 = v/4L$. So, frequencies are $\nu_1, 3\nu_1, 5\nu_1, \dots$

- 182.** Explain the phenomenon of beats. How is beat frequency determined?

Ans :

Beats are an interference phenomenon resulting from the superposition of two sound waves having slightly different frequencies, say ν_1 and ν_2 .

- The listener perceives a sound of average frequency $(\nu_1 + \nu_2)/2$.
- The intensity of this sound waxes and wanes periodically.
- The beat frequency, which is the frequency of these intensity variations, is given by $\nu_{\text{beat}} = |\nu_1 - \nu_2|$.

FIVE MARKS QUESTIONS

- 183.** What is wave and wave motion ?

or

What are types of wave ?

Ans :

Wave is a periodic transmission of disturbance from one point to the other.

In a wave motion, disturbance travels through some medium but medium does not travel along with the disturbance. Thus we define wave motion as a kind of disturbance which travels through a material medium on account of repeated periodic vibrations of the particles of the medium about their mean position, the disturbance being handed on from one particle to the adjoining particle and so on without any net transport of the medium.

Wave motion is of two types :

- (i) Transverse wave motion is that in which the individual particles of the medium execute S.H.M. about mean position in a direction perpendicular to the direction of propagation of disturbance.
- (ii) Longitudinal wave motion is that in which the individual particles of the medium execute S.H.M. about their mean position in a direction of propagation of disturbance.

i.e., $t = \frac{1}{10} \text{ s}$

So $d = \frac{340}{2} \times \frac{1}{10} = 17 \text{ m}$

- 158.** What are the factors on which the frequency of a stretched vibrating string depends ?

Ans :

The frequency of fundamental note of a vibrating string depends upon following factors :

- (i) The frequency is inversely proportional to length

i.e. $v \propto \frac{1}{l}$

- (ii) The frequency is directly proportional to the square root of the stretching force

i.e. $v \propto \sqrt{T}$

- (iii) The frequency is inversely proportional to the square root of mass per unit length

i.e. $v \propto \frac{1}{\sqrt{m}}$

Combining these three factors, we get

$$v \propto \frac{1}{l} \sqrt{\frac{T}{m}}$$

$$v = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

(where $\frac{1}{2}$ is constant of proportionality)

- 159.** What are standing waves or stationary waves ? Why are they so called ?

Ans :

When two sets of waves of same type (longitudinal or transverse), having equal amplitudes and equal frequencies, travel with same speed, along the same straight line but in opposite directions, their superposition upon each other gives a set of resultant waves which do not travel either way with time. These waves are called **stationary waves or standing waves**.

They are so called because unlike progressive waves, they do not travel either way with time. This means that mode of vibration of each particle is fixed and which does not pass on to the next particles.

- 160.** What is practical importance of stationary waves in sound ?

Ans :

Velocity of sound is given by $v = \nu \lambda$ So knowing the frequency of sound, the knowledge of stationary waves is used to measure the distance between two consecutive nodes or antinodes and the value of

λ is calculated. Thus the velocity of sound in any medium can be determined.

- 161.** What is the difference between Interference and Beats?

Ans :

The difference between interference and beats are as follows :

	Interference	Beats
1.	It is due to superposition of two waves of same frequency, travelling in same direction.	It is due to superposition of two waves of slightly different frequency.
2.	The position of maxima and minima are permanent.	The intensity of wave changes at a given point with time.

- 162.** The fact that we have two ears enables us to sense the direction from which a sound is coming. This depends on the interference principle. Explain.

Ans :

There is always some phase difference between the two waves reaching the two ears which depends upon the direction of source of sound relative to the two ears. Since two waves reaching the two ears have same frequency and amplitude, differing only in phase, interference takes place. Thus the ability of two ears to detect the direction of source depends upon the phenomenon of interference.

- 163.** We cannot hear beats, when beat frequency is more than 10. Why ?

or

To hear beats, it is essential to have . two frequencies very close to each other. Why ?

Ans :

Beats are defined as periodic variation in intensity of sound due to superposition of two sound waves of nearly the same frequency. If the difference of frequencies of two notes is more than 10, the number of beats will not be heard by a human ear due to persistence of hearing. So to hear beats distinctly, the number of beats per second should not be more than 10.

- 164.** Explain the key characteristics of mechanical waves with examples.

Ans :

Mechanical waves are disturbances that propagate through a material medium due to the elastic

These waves demonstrate wave-particle duality. An application of matter waves associated with electrons is in electron microscopes.

- 132.** How does energy propagate in wave motion without mass transfer?

Ans :

In wave motion, energy propagates through the disturbance of particles in a medium or fields in space. The particles of the medium oscillate about their mean positions, transferring energy to adjacent particles, but the matter itself does not travel with the wave.

- 133.** Describe how particle motion differs in transverse and longitudinal waves.

Ans :

In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation. In longitudinal waves, the constituents of the medium oscillate along (parallel to) the direction of wave propagation.

- 134.** Why can transverse waves propagate in solids but not in fluids?

Ans :

Transverse waves involve shearing strain as particles oscillate perpendicular to wave motion. Solids can sustain shearing stress, allowing transverse wave propagation. Fluids typically do not sustain shearing stress and thus cannot support transverse waves, except sometimes at their surface.

- 135.** Explain why longitudinal waves can propagate through solids, liquids, and gases.

Ans :

Longitudinal waves propagate as compressions and rarefactions, involving changes in volume and density, and thus compressive strain. All elastic media—solids, liquids, and gases—can sustain compressive strain, allowing longitudinal waves to propagate through them.

- 136.** What are the two kinds of waves on the surface of water? Explain their restoring forces.

Ans :

Waves on the surface of water are of two kinds:

- **Capillary waves:** These are ripples of short wavelength, and the restoring force is the surface tension of water.
- **Gravity waves:** These have longer wavelengths, and the restoring force is the pull of gravity.

- 137.** Are sound waves in air transverse or longitudinal? Justify your answer.

Ans :

Sound waves in air are longitudinal waves. This is because they propagate as compressions and rarefactions of air, meaning the air particles oscillate back and forth in the same direction as the wave travels.

- 138.** Explain the formation of a pulse on a long string when given a single jerk.

Ans :

When one end of a long string is given a single up and down jerk, the disturbance causes a segment of the string to oscillate. This displacement and subsequent restoring force travels along the string as a pulse, with each element oscillating perpendicular to the string's length.

- 139.** Write the general equation for a sinusoidal travelling wave and label its components.

Ans :

The equation is $y(x, t) = a \sin(kx - \omega t + \phi)$.

- $y(x, t)$ is displacement.
- 'a' is amplitude.
- 'k' is angular wave number.
- ' ω ' is angular frequency.
- ' ϕ ' is the initial phase angle.
- $(kx - \omega t + \phi)$ is the phase.

- 140.** Define amplitude and initial phase angle for a progressive wave.

Ans :

- **Amplitude (a):** It is the maximum displacement of the constituents of the medium from their equilibrium position.
- **Initial phase angle (ϕ):** It is the phase of the wave at position $x = 0$ and time $t = 0$.

- 141.** Explain wavelength and angular wave number. How are they related?

Ans :

- **Wavelength (λ):** The minimum distance between two points having the same phase, such as between two consecutive crests or troughs.
- **Angular wave number (k):** It represents 2π times the number of waves per unit length. They are related by $k = 2\pi/\lambda$.

gave values about 15% lower than experimental results.

Laplace corrected this by proposing that sound wave propagations are adiabatic due to their rapidity, not allowing time for heat exchange. This leads to an adiabatic bulk modulus $B_{ad} = \gamma P$, and the corrected speed $v = \sqrt{\gamma P / \rho}$, which matches experimental values well.

- 172.** State and explain the principle of superposition of waves.

Ans :

The principle of superposition of waves states that when two or more wave pulses traverse the same medium simultaneously, the resultant displacement of any element of the medium is the algebraic sum of the displacements due to each individual wave. Key points are:

- Each wave moves as if the others are not present.
- The net displacement

$$y(x, t) = y_1(x, t) + y_2(x, t) + \dots$$

- This principle is fundamental to understanding phenomena like interference and beats.

- 173.** What is interference of waves? Explain constructive and destructive interference with respect to phase difference.

Ans :

Interference is a phenomenon that occurs when two or more waves superpose in a medium.

- **Constructive interference:** Occurs when waves are in phase (phase difference $\phi = 2n\pi$, where $n = 0, 1, 2, \dots$). Their amplitudes add up, resulting in a wave of maximum possible amplitude. For equal amplitudes 'a', the resultant is $2a$.
- **Destructive interference:** Occurs when waves are completely out of phase (phase difference $\phi = (2n + 1)\pi$). Their amplitudes subtract, and if equal, the resultant displacement is zero.

- 174.** Describe the reflection of a travelling wave at a rigid boundary and at an open boundary.

Ans :

When a travelling wave encounters a boundary, it gets reflected. The nature of reflection depends on the boundary type:

- **Rigid boundary:** The reflected wave suffers a phase change of π radians (or 180°) with respect to the incident wave. This ensures zero displacement at the boundary. If incident wave is $a \sin(kx - \omega t)$, reflected is $-a \sin(kx + \omega t)$.

- **Open boundary:** The reflected wave has the same phase as the incident wave; there is no phase change. If incident wave is $a \sin(kx - \omega t)$, reflected is $a \sin(kx + \omega t)$.

- 175.** What are stationary waves? How are they formed, and what are their key characteristics?

Ans :

Stationary waves, or standing waves, are wave patterns produced when two identical waves (same amplitude and wavelength) travelling in opposite directions superpose. Their key characteristics are:

- The wave pattern does not move; it appears stationary.
- Amplitude varies from point to point, with fixed points of zero amplitude called nodes and maximum amplitude called antinodes.
- All elements of the medium between two nodes oscillate in phase with the same frequency.

- 176.** Derive the equation for a stationary wave formed by the superposition of two identical waves travelling in opposite directions.

Ans :

Consider two identical waves travelling in opposite directions:

$$y_1(x, t) = a \sin(kx - \omega t) \text{ and } y_2(x, t) = a \sin(kx + \omega t)$$

By the principle of superposition, the resultant wave is

$$y(x, t) = y_1(x, t) + y_2(x, t).$$

$$y(x, t) = a[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

Using the trigonometric identity

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right),$$

we get:

$$y(x, t) = 2a \sin(kx) \cos(\omega t).$$

This is the equation of a stationary wave.

- 177.** Define nodes and antinodes. What is the distance between two consecutive nodes or antinodes?

Ans :

In a stationary wave pattern:

- **Nodes:** These are the points at which the amplitude of oscillation is always zero. The string elements at these points do not move.
- **Antinodes:** These are the points at which the amplitude of oscillation is the largest (equal to $2a$).

The distance between two consecutive nodes is $\lambda/2$. Similarly, the distance between two consecutive antinodes is also $\lambda/2$.

- 104.** What is the physical change occurs when the source of sound is stationary but the observer moves ?

Ans :

The number of sound waves received by the observer changes.

- 105.** What physical change occurs when source of sound moves and observer is stationary ?

Ans :

Due to motion of source, the wavelength of sound waves changes.

- 106.** If you set your watch by the sound of a distant siren will it go fast or slow ?

Ans :

It will go slow due to low value of the speed of sound in air.

- 107.** What is transported during wave motion in a medium?

Ans :

Waves transport energy and information (or a pattern of disturbance).

- 108.** Do all waves require a medium to travel through ? Give examples.

Ans :

No, all the waves do not require a medium to travel through e.g., sound waves require a medium and light waves do not.

- 109.** What types of waves are produced on water surface?

Ans :

Transverse waves

- 110.** How is velocity of sound affected by an increase of pressure ?

Ans :

Velocity of sound is not affected by increase of pressure.

- 111.** Why do we say that sound waves are mechanical in character ?

Ans :

Because sound waves can travel with the help of mechanical oscillations of the particles of the medium.

- 112.** What carries sound-longitudinal or transverse waves ?

Ans :

Longitudinal waves.

- 113.** What is the range of audible sound waves ?

Ans :

Audible sound waves have frequency between 20 Hz to 20 kHz.

- 114.** Can we concentrate the sound waves at a particular point ?

Ans :

Yes, by using converging lens e.g. CO₂ gas lens.

- 115.** What are pressure waves ?

Ans :

Longitudinal waves which evolve change in pressure and volume during their propagation through a medium are called pressure waves. Sound waves are pressure waves.

- 116.** Is the phenomenon of interference, characteristics of all wave motion ?

Ans :

Yes, both transversal and longitudinal waves can interfere.

- 117.** An explosion takes place at the bottom of a lake. What type of waves are produced inside the water ?

Ans :

Longitudinal waves.

- 118.** Which characteristic of the medium determine the velocity of longitudinal sound waves in a medium ?

Ans :

Elasticity and density of the medium.

- 119.** In which gas H₂ or O₂ will sound travel with greater speed under given conditions of temperature and pressure ?

Ans :

H₂

- 120.** What is the distance between two consecutive nodes?

Ans :

$\lambda/2$

- 184.** Derive a relation between wavelength, frequency and velocity of the wave.

Ans :

Consider a wave (transverse or longitudinal) transmitted through an elastic medium. The various particles start vibrating in S.H.M., transferring energy from one particle to the next.

Suppose, λ = Wavelength of the wave

T = Time period of vibration of each of the particle of medium.

ν = Frequency of vibration of each of the particle of the medium or frequency of the wave

v = Velocity of the wave.

We know that by the time the vibrating particle completes one vibration about its mean position the wave covers a distance equal to wavelength. (This result is true both for transverse as well as longitudinal wave). Distance covered by wave is λ in time T .

$$\text{Velocity of wave} = \frac{\text{Distance}}{\text{Time}}$$

$$v = \frac{\lambda}{T} \quad \left(\text{But } \nu = \frac{1}{T} \right)$$

Hence, $v = \nu \lambda$

Velocity of wave = frequency $\times$ wavelength

This equation is true for all wave motion, whatever be its origin, that is, it applied to sound waves, electromagnetic waves and mechanical waves.

- 185.** What are different types of wave motions ?

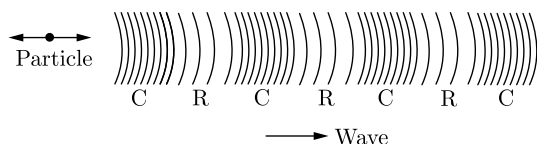
Ans :

The wave motion is of two types :

1. Longitudinal Wave Motion.
2. Transverse Wave Motion.

1. Longitudinal Wave Motion

Wave motion is said to be longitudinal, when the particles of the medium through which the disturbance travels vibrate about their mean positions in the same direction, along which the disturbance wave is propagated, as shown in the figure.



A longitudinal wave travels in a medium in the form of compression and rarefactions. Since pressure of the medium at different points changes when a longitudinal wave passes through a medium, therefore it is also called a pressure wave.

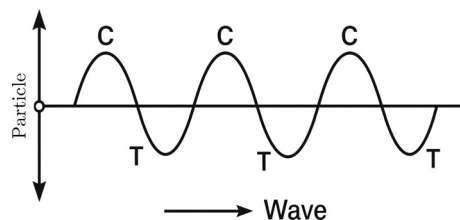
Since solids, liquids and gases, all possess volume elasticity so longitudinal waves can travel in all these media.

For examples :

- (i) Sound wave travels through air in the form of longitudinal wave.
- (ii) The prongs of vibrating tuning fork produce longitudinal wave.
- (iii) Waves produced in a spring suspended in a vertical position, when pulled along its length.
- (iv) Vibration of air column in organ pipes characteristics of medium for propagation of longitudinal wave.

2. Transverse Wave Motion

Wave motion is said to be transverse, when the particles of the medium through which the disturbance travels, vibrate about their mean positions in a direction perpendicular to the direction of propagation of the wave. This is shown in the figure.



For examples

- (i) Vibrations in a stretched string are transverse vibrations.
- (ii) Waves produced on the surface of water.
- (iii) Light waves and infact, all electromagnetic waves are transverse in nature.
- (iv) Movement of the membrane of a tabla.

For the propagation of transverse waves, the medium must possess the same characteristics, viz., Elasticity of shape, inertia and minimum friction, uniform density and definite time interval for the motion to be handed on from one particle to the next particle as in the case of longitudinal waves.

A transverse wave travels in the form of crests and troughs.

Solids possess elasticity of shape, so transverse waves can travel through solids. Liquids do

not possess elasticity of shape but they possess another equally effective property *i.e.* they keep their level. So transverse waves can travel in liquid. But gases neither possess elasticity of shape nor do they keep their level, so transverse waves cannot travel in gases.

186. Define the terms compression, rarefaction, crest, trough, wavelength, frequency, time period.

Ans :

- 1. Compression** is a region of the medium in which particles come closer than normal. There is a decrease in volume and an increase of pressure/density of the medium.
- 2. Rarefaction** is a region of the medium when the particles get farther apart than the normal. There is a temporary increase in volume and a consequent decrease in density of the medium in this region.

Compressions and rarefactions are produced, when a longitudinal wave passes through a medium. These are represented in the figure(a). Longitudinal waves are therefore also called **pressure waves**.

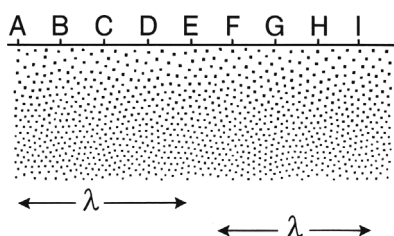


Figure: (a)

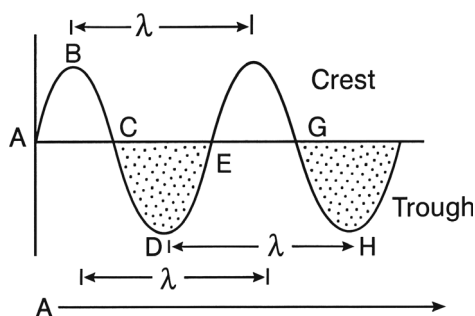


Figure: (b)

- 3. Crest** : It is a portion of medium which is raised above the normal position of rest of the particles of the medium, as a transverse wave passes through it

For example : The portion ABC, in the figure (b), represents a crest. B is the centre of the crest.

- 4. Trough** : It is a portion of the medium, which is depressed below the normal position of rest of the particles of the medium as a transverse wave passes through it.

For example, the portion CDE, in the figure (b) represent a trough D is the centre of the trough

- 5. Wavelength**

- (i) Wavelength of a wave is defined as the distance travelled by the wave during the time any one particle of the medium completes one vibration about its mean position.

It is represented by λ .

- (ii) Wavelength of a wave is also defined as the distance between any two nearest particles of the medium, vibrating in the same phase.

- 6. Frequency** : Frequency of vibration of a particle is defined as the number of vibrations completed by the particle in one second.

As one vibration is equivalent to one wavelength, therefore frequency is also equal to the number of complete wavelengths traversed by the wave in one second.

- 7. Time period** : It is the time taken by a particle of the medium to complete one vibration about its mean position.

It is denoted by T .

Time period, $T = \frac{1}{\nu}$

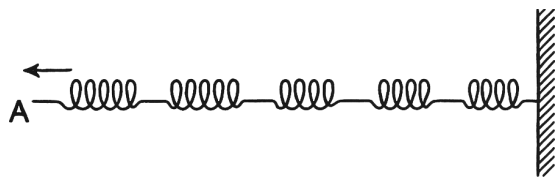
Where ν is the frequency

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- 187.** Explain the propagation of a wave through a medium using the analogy of a collection of springs connected to each other.

Ans :

Consider a collection of springs connected to one another as shown in figure. If the spring at one end is pulled suddenly, the disturbance travels to the other end. The first spring is disturbed from its equilibrium length. Since the second spring is connected to the first, it is also stretched or compressed, and so on. The disturbance moves from one end to the other, but each spring only executes small oscillations about its equilibrium length or position.



Case of a medium Now a small part of a medium through which a wave passes can be thought of as a spring. It is 'connected' to the neighbouring parts of the medium, *i.e.* to neighbouring 'springs'.

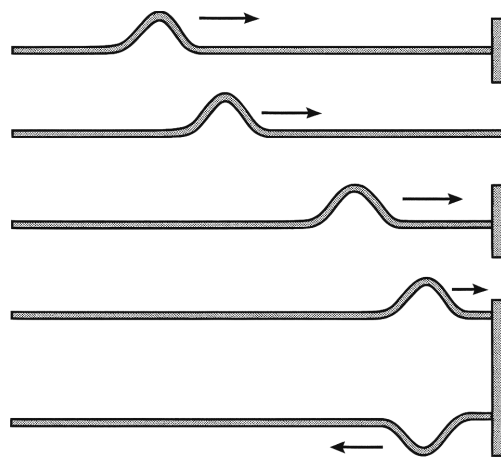
- (i) **Air :** Consider air through which a sound wave passes. The wave compresses or expands air as it passes through. If the density of a small region or volume of air is changed, say increased, by an amount of δP , there is an extra pressure δP , proportional to δP (Boyle's law). Pressure is force per unit area, so there is a restoring force proportional to the disturbance, just like in a spring. In this case, the quantity similar to extension or compression is the density change. Further the 'springs' are 'connected'. If a region is compressed relative to surroundings, the molecules in that region tend to move out. If it is comparatively rarefied the surrounding air will rush in. This is the 'connection'
- (ii) **Solid :** In this case, the spring model is even more appropriate, as in case of interatomic potentials and elastic behaviour. Each atom in a crystalline solid is in equilibrium, due to forces from other atoms cancelling out exactly. Displacing one atom keeping the others fixed leads to restoring forces exactly as in a spring. So, we can think of atom as end points, with springs between pairs of them. Since forces between atoms have a short range, only the nearest neighbour atoms need often be taken to be connected by springs

188. Show by an experiment that

- Speed of wave pulse increases as tension increases.
- Shape and speed of the wave pulse is constant.

Ans :

Take a coil spring and tie one of its ends to a rigid support. Build some tension in it by pulling the free end towards yourself. Give to the free end a sudden flipping motion by a jerk of the hand. A disturbance in the shape of a pulse can be seen travelling down the coil spring. The speed of the pulse can be measured by using a stop watch and a metre scale.



- By stretching the coil spring further, the tension in it can be increased. It is seen qualitatively that the speed of a wave increases with an increase in the tension in stretched strings or slinkly. The tension in a stretched string rigidly tied at one end can be varied by changing the pull on the other end. In figure we have shown graphically the phenomenon of a travelling wave pulse.
- The shape of the wave pulse remains unchanged as it travels down the coil spring except when it approaches the rigid boundary. It is reflected back with inverted shape by the rigid boundary.
- If the amplitude of the pulse is changed, the speed of pulse remains unchanged.

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189. Write the expression for the speed of propagation of transverse and longitudinal waves in different media and check these relations dimensionally.

Ans :

(a) **Speed of propagation of transverse wave**

- In a stretched string :** The speed of propagation of a transverse pulse in a string depends on the tension (a force) and the mass per unit length. If we denote tension by T and mass per unit length of the spring by m , the formula for speed of wave motion in strings can be expressed as

$$v = \sqrt{\frac{T}{m}} \quad (\text{transverse wave})$$

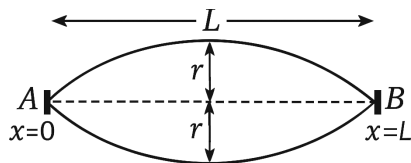
Dimensional method of checking the result

$$v = \sqrt{\frac{T}{m}}$$

$$[L^1 T^{-1}] = \left[\frac{M^1 L^1 T^{-2}}{M^1 L^{-1}} \right]^{\frac{1}{2}} = [L^2 T^{-2}]^{1/2}$$

$$[L^1 T^{-1}] = [L^1 T^{-1}]$$

The relation is dimensionally correct.



Let the displacement of incident and reflected waves be

$$y_1 = r \sin \frac{2\pi}{\lambda} (vt - x) \quad \dots(1)$$

$$y_2 = -r \sin \frac{2\pi}{\lambda} (vt - x) \quad \dots(2)$$

Negative sign means that a phase change of π takes place after reflection from a rigid support.

Resultant displacement due to superposition will be the linear sum of y_1 and y_2 .

$$\begin{aligned} y &= y_1 + y_2 \\ &= r \left[\sin \frac{2\pi}{\lambda} (vt - x) - \sin \frac{2\pi}{\lambda} (vt - x) \right] \\ &= 2r \cos \left(\frac{2\pi}{\lambda} \right) \sin \left(\frac{2\pi}{\lambda} \right) \left(-\frac{2x}{2} \right) \\ &= -2r \cos \frac{2\pi}{\lambda} vt \sin \frac{2\pi}{\lambda} x \quad \dots(3) \end{aligned}$$

The displacement given in equation (3) does not describe a moving wave because x and t are not in the combination $(x \pm vt)$, and it represents a stationary wave.

Boundary conditions

From figure, we find that

$$y = 0$$

at $x = 0$ and $x = L$

(i) At $x = 0$, from equation (3),

$$y = 0$$

(ii) At $x = L$, from equation (3),

$$y = -2r \cos \frac{2\pi}{\lambda} vt \sin \frac{2\pi}{\lambda} L$$

Now, $y = 0$ if $\sin \frac{2\pi}{\lambda} L = 0$

$$\frac{2\pi}{\lambda} L = 0, \pi, 2\pi, \dots$$

$$\frac{2\pi}{\lambda} L = n\pi (n = 0, 1, 2, \dots)$$

$$\lambda = \frac{2L}{n} \quad \dots(4)$$

First mode of vibration : For $n = 1$, let λ_1 is wavelength of standing wave set up

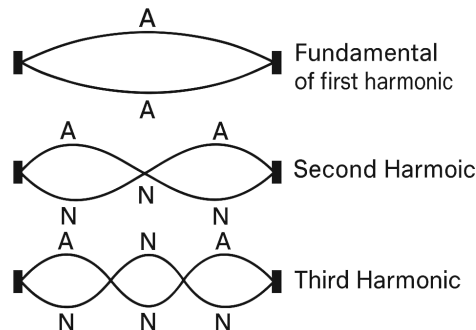
$$\lambda_1 = 2L \text{ or } L = \frac{\lambda_1}{2}$$

This is shown in figure (1)

If ν_1 = frequency of wave set up

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

This is called fundamental frequency of 1st harmonic.



Second mode of vibration : For $n = 2$, let the wavelength be λ_2

$$\lambda_2 = \frac{2L}{2} \text{ or } \lambda_2 = L$$

Then frequency of wave,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{\lambda}{L} = 2\nu_1$$

This is called first overtone or second harmonic.

Similarly, $\nu_3 = 2\nu_1$, called second overtone or third harmonic.

In general : $\nu_n = n\nu_1$ called $(n - 1)$ th overtone or n th harmonic.

Nodes (N)

Nodes are the points where displacement is least and strain is maximum.

In n th mode of vibration, there will be $(n + 1)$ nodes at positions.

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \dots, L$$

Antinodes (A)

Antinodes are the points where displacement is maximum and strain is minimum.

There will be n antinodes in n th mode of vibration at positions.

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \dots, \frac{(2n - 1)L}{2n}$$

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i.e. velocity of sound is independent of pressure provided the temperature of the medium remains constant.

Thus, when P changes, simultaneously ρ changes in a such a way that $\frac{P}{\rho}$ remains constant. So the

change in pressure has no effect on velocity of sound.

- (iv) **Effect of temperature :** Velocity of sound in air (or in a gas) is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \dots(1)$$

Since for one mole of a perfect gas

$$PV = RT$$

$$P \frac{M}{\rho} = RT$$

$$\frac{P}{\rho} = \frac{RT}{M} \quad \dots(2)$$

Putting equation (2) in (1), we get

$$v = \sqrt{\frac{\gamma RT}{M}}$$

If v_1 and v_2 are the velocities of sound at temperature T_1 and T_2 respectively, then

$$v_1 = \sqrt{\frac{\gamma RT_1}{M}}$$

$$v_2 = \sqrt{\frac{\gamma RT_2}{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$i.e., \quad v \propto \sqrt{T}$$

i.e., velocity of sound is directly proportional to the square root of absolute temperature of the medium.

Hence sound travels faster on a hot day than on a cold day.

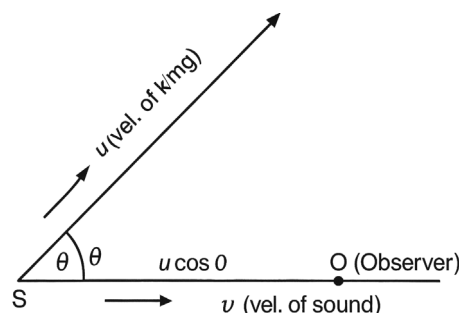
- (v) **Effect of wind velocity :** If v is the velocity of sound and u is the velocity of wind making an angle θ with the direction of propagation of sound as shown in the figure.

Resultant velocity of sound

$$V = v + u \cos \theta$$

(i) If $\theta = 0^\circ$, $V = v + u$

(ii) If $\theta = 180^\circ$, $V = v - u$



191. (a) What is the shape of the wave when two wave pulses travelling in the opposite direction cross each other ?
 (b) State superposition principle. Give examples by drawing suitable diagrams.
 (c) What is the limitation of the principle of superposition of waves ?

Ans :

- (a) Wave pulses continue to retain their individual shapes after they have crossed. However, at the moment when they are crossing each other the appearance of the wave form is different from the shape of either of the pulses (as shown in the following).
 (b) **Superposition principle :** When two or more pulses simultaneously move through a medium then,
 (i) each pulse moves as though others are not present and,
 (ii) the total displacement at any point at a given time is obtained by adding the corresponding displacements of the individual pulses.

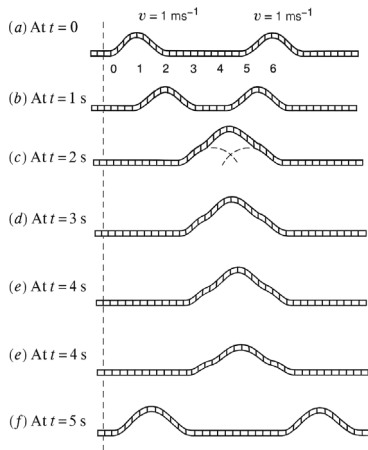
This principle explains the phenomenon of interference.

Illustration:

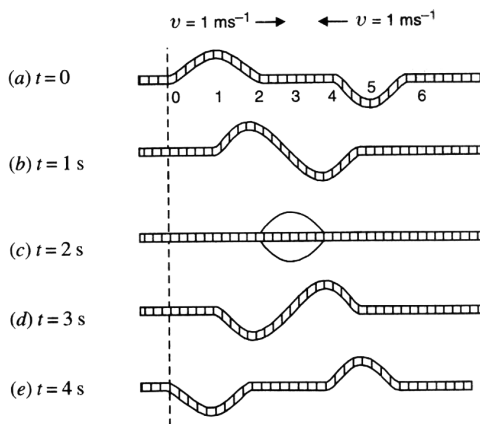
- (i) Superposition of similar pulses (Constructive interference). Consider the motion of two wave pulses of identical shapes moving towards each other with a speed of 1 m/s along a string. Figures (a) and Figure (f) have been drawn to show the waveform after every second. Figure (c) and Figure (d) show that when the wave functions overlap, the resultant waveform is the algebraic sum of the displacements due to each pulse.

The figures (c) and figure (d) are examples of constructive interference.

- (ii) When two wave pulses of equal and opposite shapes moving towards each other overlap, the interference is called destructive interference. Consider two wave pulses of opposite shape moving in the opposite direction at $t = 0$ in figure (a).



After $t = 2$ s the two waves overlap each other. Here resultant displacement becomes zero. This again shows that resultant waveform is the algebraic sum of individual wave.



Mathematical description of superposition of waves Consider two or more waves moving in the medium. The resultant waveform is the sum of wave functions of individual waves. That is if the wave functions of the moving waves are

$$\begin{aligned} y_1 &= f_1(x-vt) \\ y_2 &= f_2(x-vt) \\ &\dots\dots\dots \\ &\dots\dots\dots \\ y_n &= f_n(x-vt) \end{aligned} \quad \dots(1)$$

then the wave function describing the disturbance in the medium

$$y(x-vt) + f_2(x-vt) + \dots + f_n(x-vt) \dots(2)$$

where y is the resultant displacement.

- (c) **Limitation of the Principle of superposition :**
The principle of superposition holds as long as the size of the disturbance in the medium is not too large **Examples of Principle of superposition**
It is used in the study of :
(i) Interference
(ii) Reflection of waves
(iii) Standing waves or Stationary waves
(iv) Beats.

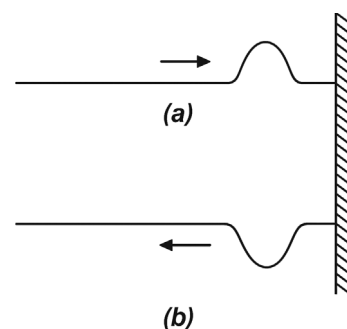
192. (a) Explain reflection of waves and find the general expression for the resultant displacement when direct and reflected waves overlap.
(b) Explain how does the reflection of wave from a rigid boundary changes the shape of the pulse ?

Ans :

- (a) When a wave pulse reaches a rigid boundary, it travels in the same medium with same speed in opposite direction with its shape inverted. This is called reflection of waves.

Consider a wave pulse moving from left to right as shown by solid lines in the left hand side on the figure (a). Its wave function in general is given by

$$y' = f(vt - x)$$



and it must satisfy the certain boundary conditions.

Let us choose the origin of the coordinate system such that the rigid boundary is at $x = 0$. The wave function given in $y' = f(vt - x)$ does not correctly satisfy the boundary condition at $x = 0$ at all times, since the displacement of the string at the rigid support is a function of time, i.e.

$$y'(x=0) = f(vt) \neq 0$$

which is not true. It shows that the description of the wave pulse by the wave function is

incomplete and breaks down near the rigid boundary. Clearly, we must add to equation y' ($x = 0$) $= f(vt)$ another term describing the displacement of the string, such that it is equal to $-f(vt)$ at $x = 0$. Only then will the end of the string be at rest at all times for all kinds of pulses. Now the only choices for travelling waves are $f(vt - x)$ or $-f(vt + x)$. The former is not admissible because it would lead to no displacement at all places in the string and at all times (including the time before the pulse hits the boundary).

We must therefore add a term

$$y'' = -f(vt - x)$$

to the displacement of the string so that the condition of rigid boundary at $x = 0$ is satisfied. With this, the total displacement is

$$y = f(vt - x) - f(vt + x)$$

- (b) The figures (a) to figure (e) in Q. 9 show how an incident wave pulse changes its shape as it approaches a rigid boundary on reaching the boundary it travels back in the opposite direction with an inverted shape. It may be noted that there are two types of reflection of sound.

- (i) **Reflection at a denser medium :** When reflection of sound waves takes place from denser medium, a wave of compression is reflected as a wave of compression and a wave of rarefaction is reflected as the wave of rarefaction *i.e.* the nature of wave does not change but the direction of motion of the particles in the reflected wave is opposite to that of the incident wave. It means, if the reflection of sound waves takes place from denser medium, it is without change of type, but with change in sign.
- (ii) **Reflection at a rarer medium :** When reflection of sound waves takes place from a rarer medium, a wave of compression is reflected as a wave of rarefaction and wave of rarefaction is reflected as a wave of compression *i.e.* the nature of wave changes but the direction of motion of the particles in the incident wave and reflected wave is in the same direction. It means if the reflection of sound waves takes place from rarer medium, it is with change of type but without change in sign.

193. Discuss various applications of reflection of waves.

Ans :

Various applications of reflection of waves are as follows :

1. **Echo :** When a word spoken by a person is repeated due to reflection of sound from buildings, trees, hills etc. we say echo is produced.
Echo is the phenomenon of repetition of sound of a source by reflection.
Since the effect of a sound lasts on human ear for 0.1 second, due to persistence of hearing, therefore to distinguish echo from the original sound, the reflected sound must reach the person after 0.1 second. It is only possible when reflecting surface is situated at a suitable distance from the person. If velocity of sound propagation is 340 m s^{-1} , then to hear an echo of simple sound, the distance (x) of reflecting surface from the person is given by

$$x = \frac{vt}{2} = \frac{340 \times 0.1}{2} = 17 \text{ m}$$

Therefore to hear an echo of simple sound, the reflecting surface should be situated at a distance of 17 m from the person otherwise no echo is possible.

Echo can be used to find the distance of a solid object, depth of an ocean or a lake and so on.

2. **Sound board :** In big halls or churches, concave reflecting boards are placed at the back of the speaker. The speaker stands near the focus of the reflecting board. The sound waves reflected back by the concave board are rendered parallel. Due to it, the voice of a speaker is made audible at a distance.
3. **Ear trumpet :** It is a sort of machine used by persons who are hard of hearing. The sound energy received by the wide end of the trumpet is reflected into a much smaller area increasing the intensity of sound in the ear.
4. **Speaking tube or Megaphone :** It is meant for concentrating sound energy in one direction by multiple reflections of sound from the sides of the tube.
5. This is the general principle of determining the distance of a distant object, depth ranging in lakes and oceans and air-crafts, navigations.

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- (ii) **In a solid :** The velocity of transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}},$$

where η is the modulus of rigidity and ρ is density of the material of the solid.

Dimensional method of checking the result.

$$v = \sqrt{\frac{\eta}{\rho}}$$

$$[L^1 T^{-1}] = \left[\frac{M^1 L^{-1} T^{-2}}{M^1 L^{-3}} \right]^{\frac{1}{2}}$$

$$[M^0 L^1 T^{-1}] = [M^0 L^1 T^{-1}]$$

The relation is dimensionally correct.

- (b) Speed of propagation of longitudinal wave

- (i) **In a solid :** The velocity of longitudinal pulse in a solid medium having bulk modulus K , modulus of rigidity η and density of material of medium ρ , is given by

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

Dimensional method of checking the result

$$v^2 = \frac{K}{\rho} + \frac{4\eta}{3\rho}$$

$$[L^1 T^{-1}]^2 = \frac{[M^1 L^{-1} T^{-2}]}{ML^{-3}} + \frac{[ML^{-1} T^{-2}]}{ML^{-3}}$$

$$[M^0 L^{-2} T^{-2}] = [M^0 L^2 T^{-2}] + [M^0 L^2 T^{-2}]$$

This relation is dimensionally correct.

- (ii) When solid is in the form of long rod, then

$$v = \sqrt{\frac{K}{\rho}} \quad (\text{longitudinal wave})$$

- (iii) **In liquids and gases :** The speed of propagation of a longitudinal pulse in a fluid depends only on the bulk modulus K and the density of the medium, ρ . The formula giving the speed of sound waves in fluids is

Dimensional method for checking the result

$$v = \sqrt{\frac{K}{\rho}}$$

$$[L^1 T^{-1}] = \left[\frac{M^1 L^{-1} T^{-2}}{M^1 L^{-3}} \right]^{\frac{1}{2}}$$

$$[L^1 T^{-1}] = [L^1 T^{-1}]$$

Hence, The relation is dimensionally correct.

- 190.** Discuss the effect of density, humidity, pressure, temperature and direction of wind, on the velocity of longitudinal waves in air.

Ans :

- (i) **Effect of density :** If ρ_1 and ρ_2 be the densities of two gases having same pressure P , then velocity of sound (or a longitudinal wave) in two gases will be given by

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \dots(1)$$

$$v_2 = \sqrt{\frac{\gamma P}{\rho_2}} \quad \dots(2)$$

Dividing equation (1) by equation (2), we get

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$$v \propto \frac{1}{\sqrt{\rho}}$$

i.e. pressure remaining the same, velocity of sound in a gas is inversely proportional to the square root of the density of the medium.

Hence, velocity of sound is smaller in oxygen than in hydrogen.

- (ii) **Effect of humidity or moisture :** Density of humid air is less than the density of dry air. Since velocity of longitudinal wave (*i.e.* sound) is inversely proportional to square of the density, so velocity of sound increases with the increase of humidity.

Hence, velocity of sound is more after rain than or a dry day.

- (iii) **Effect of pressure :** Since velocity of sound in air (or in a gas) is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \dots(1)$$

For 1 mole of a perfect gas at constant temperature,

$$PV = \text{constant}$$

$$P \frac{M}{\rho} = \text{constant}$$

[M is the molecular weight of the gas]

$$\frac{P}{\rho} = \frac{\text{constant}}{M} = \text{another constant}$$

$$\text{i.e.,} \quad \frac{P}{\rho} = \text{constant}$$

So equation (1) becomes,

$$v = \sqrt{\gamma \text{ constant}}$$

$$v = \text{constant}$$

[since, γ is constant for a given temperature]

where ϕ is the initial phase at $t = 0$ and $x = 0$.
The argument of the cosine in the equation (1) is called the phase angle or simple phase, ϕ .
Phase at any position x and time t

$$\phi_1 = \omega t - \kappa x + \phi \quad \dots(2)$$

Clearly it varies with x and t

When only t changes, say from t to $t + \Delta t$ and ϕ changes from ϕ to $\phi + \Delta\phi$ then

$$(\phi + \Delta\phi) - \phi = [\omega(t + \pi) - \kappa x + \phi_0] - [\omega t - \kappa x + \phi_0]$$

Phase difference,

$$\Delta\phi = \frac{2\pi}{T} \Delta t = 2\pi \nu \Delta T \quad \dots(3)$$

When only x changes in Δx , then a similar calculation gives phase difference,

$$\Delta\phi = -\frac{2\pi}{\lambda} \Delta x \quad \dots(4)$$

Significance of minus sign in equation (4),

When a wave travels in the $+x$ direction the forward points are behind in phase.

- (ii) Definition of Wavelength and Time Period in terms of Phase Difference.

From (4), if $\Delta x = \lambda$, neglecting minus sign.

$$\Delta\phi = \frac{2\pi}{\lambda} \lambda = 2\pi$$

Hence, λ is the distance between two points which have a phase difference of 2π .

Very often two particles having a phase difference of integral multiples of 2π are said to be in the same phase and those which have a phase difference of an odd multiple of π are said to be 'in opposite phase'.

Again, from equation (3), if $\Delta t = T$

$$\Delta\phi = \frac{2\pi}{T} T = 2\pi$$

Hence, 'T' is the time in which the phase at a given point changes by 2π or it is the time when the phase of a particle is the same.

- 196.** Explain that a periodic wave may have in general so many harmonic component waves. What are harmonics ?

Ans :

Let us consider each of the following harmonic waves :

$$y_1 = A \sin \frac{2\pi}{\lambda} (x - vt + \phi_1)$$

$$y_2 = A_2 \sin \frac{4\pi}{\lambda} (x - vt + \phi_2)$$

$$y_3 = A_3 \sin \frac{6\pi}{\lambda} (x - vt + \phi_3)$$

.....
.....

$$y_3 = A_n \sin \frac{2n\pi}{\lambda} (x - vt + \phi_3)$$

where

$$n = 1, 2, 3, 4, \dots$$

Here each y is a periodic function in x and also a periodic function in t . y_1 is periodic in x with period λ and periodic in t with period $\frac{\lambda}{v}$. y_2 is periodic in x with period $\frac{\lambda}{2}$ and periodic in t with period $\frac{\lambda}{2v}$. y_n is periodic in x with period λ/n and periodic in t with period λ/nv . The harmonic wave y_1 in equation $y_1 = A_1 \sin \frac{2\pi}{\lambda} (x - vt + \phi_1)$ is called fundamental or the first harmonic. The harmonic wave y_2 in equation $y_2 = A_2 \sin \frac{4\pi}{\lambda} (x - vt + \phi_2)$ is called the second harmonic. The harmonic wave y_n in equation $y_n = A_n \sin \frac{2n\pi}{\lambda} (x - vt + \phi_n)$ is called the n th harmonic. If the frequency of the fundamental is ν , the frequency of the second harmonic is 2ν and that of n th harmonic ν_n .

Let us assume that in a medium a harmonic wave and its higher harmonics are simultaneously propagating. According to the principle of superposition, the wave propagating in the medium will be the superposition of its constituent waves described by the wave motion

$$y = y_1 + y_2 + \dots + y_n$$

As each of the constituent harmonic waves repeat itself after the period of the fundamental, the resultant wave will be a periodic wave with period T .

Fourier analysis of periodic waves : It is a mathematical technique of breaking a periodic wave into a linear superposition of different harmonic components. It may be used in the study of :

- musical sound.
- stationary waves in string and in vibrating air columns.
- complex alternating potentials.

- 197.** Find expression for the various harmonics in a vibration string fixed at the two ends. Hence predict the position of nodes and anti-nodes.

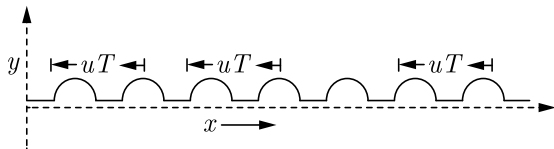
Ans :

Let us consider a stretched string of length L clamped at both ends. Let a harmonic wave be produced in the string. This harmonic wave will be reflected from two fixed ends A and B (as shown in the figure) and superimpose on each other and we get stationary waves

194. What is a harmonic wave ? Determine an expression for it.

Ans :

In a stretched string or in a coil spring let us imagine that identical wave pulses are being produced periodically by shaking the freehand. If a new wave pulse is produced after every T second then after some time the wave profile in the medium will appear as shown in the figure.



Let v be the speed of the wave motion in the medium. At any instant the distance between any two successive wave pulses will be vT . Also at any arbitrary point in the medium $v = 1/T$ pulses will pass every one second, v is the frequency of the particle wave at any point along the length of the string, the number of wave pulses crossing per second is $v = \frac{1}{T}$, where v is the frequency of the periodic wave. For a periodic wave the periodic length vT is called the wavelength (λ).

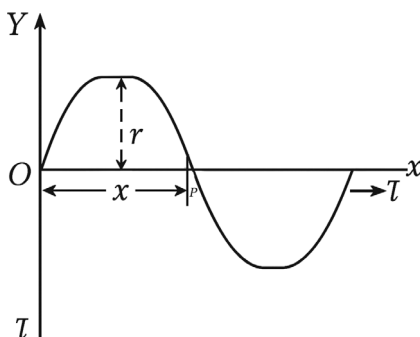
The general form of a wave function is

$$y(x, t) = f(x - vt)$$

where x and m are integers

Expression for a Plane Simple Harmonic Wave

Let a plane simple harmonic wave (longitudinal or transverse) start from a point O and travel along the positive X -direction along OX as shown in the figure. The displacement curve of the wave is shown by full line in the figure.



If we start counting time when the particle at O just crosses its mean position in the positive direction, then the equation of motion of the particle would be

$$y = r \sin \omega t \quad \dots(1)$$

where y is the displacement of the particle at an

instant t , ω is the angular frequency and r is the amplitude of motion

Consider a particle P at a distance x from the origin O . In wave motion, all successive particles of the medium to the right of O , receive the disturbance a little later than its preceding particle. Thus there is a definite phase lag between any two particles. If ϕ is the phase lag between the particles at O and P , then the equation of motion of the particle at P can be written as

$$y = r \sin(\omega t - \phi) \quad \dots(2)$$

As for a distance λ along a wave, the phase change is 2π , therefore, for the distance x along the wave, the phase lag would be

$$\phi = \frac{2\pi}{\lambda} x$$

Putting in (2), we get

$$y = r \sin \left(\omega t - \frac{2\pi x}{\lambda} \right) \quad \dots(3)$$

$$= r \sin \left(\frac{2\pi}{T} t - \frac{2\pi x}{\lambda} \right)$$

$$\left(\text{since, } \omega = \frac{2\pi}{T} \right)$$

$$= r \sin \left(\frac{2\pi}{\lambda} \lambda t - 2\pi x \right)$$

$$y = r \sin \kappa (vt - x) \quad \left(\text{since, } v = \frac{\lambda}{T} \right)$$

$$y = r \sin \kappa (vt - x)$$

$$\left[\kappa = \frac{2\pi}{\lambda} = \text{is phase constant} \right]$$

In general, if epoch is not zero, then

$$y = r \sin \kappa (vt - x + \phi)$$

The equation for a general harmonic wave of frequency ν , wavelength λ which is moving with speed v along the positive x -axis is

$$y(x, t) = A \sin[k(vt - x) + \phi]$$

λ is called the wavelength of the wave and $k(vt - x) + \phi_0$ is the phase at the point x , at time t .

195. Explain phase and phase difference in waves and hence define wavelength and time period.

Ans :

- (i) **Phase and Phase Difference :** Phase of a wave motion is the quantity which gives the complete information about the wave at any time and at any position.

A harmonic wave travelling towards the $+x$ -direction is given by

$$y = a \sin[\omega t - kx + \phi] \quad \dots(1)$$

198. What do you mean by an organ pipe ? Derive expression for stationary waves formed in a closed organ pipe and discuss normal modes of vibration of the pipe.

Ans :

An organ pipe is a wind instrument and consists of a long pipe one end of which is provided with or mouth piece to blow air and the other end is closed for closed organ pipe and open for open organ pipe. In an organ pipe (open or closed), the air column vibrates producing nodes and antinodes due to reflection from open end (for open organ pipe) and from closed end (for closed organ pipe).

Expression for standing waves in closed organ pipe

: Let L be length of closed organ pipe and the open end be considered at $x = 0$. The displacement at position x at time t in the incident wave be given by

$$y_1 = r \sin \frac{2\pi}{\lambda} (vt - x) \quad \dots(1)$$

On reflection from the closed end of the pipe, a phase change π takes place. So the displacement at the same position x and same time t in the reflected wave is given by

$$\begin{aligned} y_2 &= r \sin \left[\frac{2\pi}{\lambda} (vt + x) + \pi \right] \\ &= -r \sin \frac{2\pi}{\lambda} (vt + x) \quad \dots(2) \end{aligned}$$

Resultant displacement due to superposition is given by

$$\begin{aligned} y &= y_1 + y_2 \\ &= r \left[\sin \frac{2\pi}{\lambda} (vt - x) - \sin \frac{2\pi}{\lambda} (vt + x) \right] \\ y &= 2r \cos \frac{2\pi}{\lambda} vt \cdot \sin \frac{2\pi}{\lambda} x \quad \dots(3) \end{aligned}$$

This is the equation of a stationary wave.

From figure, we find that a node is always formed at the closed end.

i.e. $y = 0$ at $x = L$

From equation (3) at $x = L$, we have

$$y = -2r \cos \frac{2\pi}{\lambda} vt \cdot \sin \frac{2\pi}{\lambda} L$$

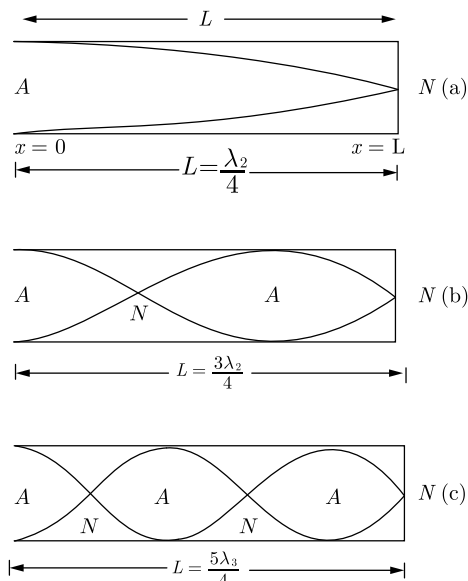
Now $y = 0$

$$\text{If } \sin \frac{2\pi}{\lambda} L = 0$$

$$\frac{2\pi}{\lambda} L = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\frac{2\pi}{\lambda} L = (2n + 1) \frac{\pi}{2}$$

$$\lambda = \frac{4L}{(2m + 1)} \quad \dots(4)$$



First normal mode of vibration : When $n = 0$ equation (4) gives first mode of vibration. If λ_1 is the wavelength, then

$$\lambda_1 = 4L$$

$$L = \frac{\lambda_1}{4}$$

The first mode is shown in figure (a). If ν_1 is the frequency of waves set up, then

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

It is called fundamental frequency or 1st harmonic

Second mode of vibration : When $n = 1$, equation (4) gives second mode of vibration. If λ_2 is wavelength, then

$$\lambda_2 = \frac{4L}{3}$$

$$L = \frac{3\lambda_2}{4}$$

This is shown in figure (b). If ν_2 is the frequency, then

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3} = 3 \frac{v}{4L} = 3\nu_1$$

It is called first overtone or third harmonic.

Third mode of vibration : For $n = 2$, equation (4) gives 3rd mode of vibration. If λ_3 is the wavelength of wave set up, then

$$\lambda_3 = \frac{4L}{5}$$

$$L = \frac{5\lambda_3}{4}$$

- 206.** One end of a long string of linear mass density $= 8.0 \times 10^{-3} \text{ kg m}^{-1}$ is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y -direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as a function of x and t that describes this wave on the string.

Ans :

Given,

$$\text{Amplitude, } A = 5 \text{ cm} = 0.05 \text{ m}$$

$$\begin{aligned} \nu &= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}} \\ &= \sqrt{\frac{90 \times 9.8}{8.0 \times 10^{-3}}} = 3.32 \times 10^2 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{So } \omega &= 2\pi\nu = 2 \times 3.142 \times 3.32 \times 10^2 \\ &= 1.61 \times 10^3 \text{ s}^{-1} \end{aligned}$$

$$\text{And } k = \frac{\omega}{\nu} = \frac{1.61 \times 10^3}{3.32 \times 10^2} = 4.84 \text{ m}^{-1}$$

Putting these values in

$$y = 0.05 \sin(\omega t - kx)$$

$$\text{we get } y = 0.05 \sin(1.61 \times 10^3 t - 4.84 x)$$

where x and y are in m.

- 207.** Earthquakes generates sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 kms^{-1} and that of P wave is 8.0 kms^{-1} . A seismograph records P and S waves from an earthquake. The first P wave arrives 4min before the first S wave. Assuming the waves travel in straight line, how far away does the earthquake occur ?

Ans :

Given,

Velocity of longitudinal sound wave,

$$v_s = 4 \text{ km s}^{-1}$$

Velocity of transverse sound wave, $v_p = 8 \text{ km s}^{-1}$

Let earthquake occurred at a distance $= x \text{ km}$

If longitudinal wave takes time t_s and transverse wave take t_p time to cover distance x , then :

$$t_s - t_p = 240$$

$$\frac{x}{v_s} - \frac{x}{v_p} = 240$$

$$x \left[\frac{v_p - v_s}{v_p v_s} \right] = 240$$

$$x = 240 \left[\frac{v_p v_s}{v_p - v_s} \right] = 240 \left[\frac{8 \times 4}{8 - 4} \right]$$

$$= 240 \times 8$$

$$x = 1920 \text{ km}$$

- 208.** At what temperature will the speed of sound in air become double of its value at 0°C ?

Ans :

$$\text{Since, } \frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{T}{273}}$$

$$2 = \sqrt{\frac{T}{273}}$$

$$T = 273 \times 4 =$$

$$1092 \text{ K} = 1092 - 273$$

$$= 819^\circ\text{C}$$

- 209.** What is the velocity of wave in a string of line or mass density m when, stretched to tension T ?

Ans :

Since velocity of sound v is given by

$$v \propto m^a$$

$$v \propto T^b$$

$$v \propto m^a T^b \quad \dots(1)$$

Writing dimensional formulae on both sides of equation (1), we get

$$[M^0 L^1 T^{-1}] [M^1 L^{-1}]^a [M^1 L^{-1} T^{-2}]^b$$

$$[M^0 L^1 T^{-1}] [M^{a+b} L^{-a-2b} T^{-a-2b}]$$

Comparing the powers of M and L on both sides, we get

$$a + b = 0 \quad \dots(1)$$

$$-a + b = 1 \quad \dots(2)$$

After adding equation (1) and (2), we get

$$2b = 1$$

$$b = 1/2$$

From equation (1), we get

$$a = -b = -1/2$$

Hence,

$$v \propto m^{-1/2} T^{1/2}$$

$$v \propto \sqrt{\frac{T}{m}}$$

$$\lambda_2 = \frac{2L}{2} =$$

$$L = \lambda_2$$

This is shown in figure(b).

If ν_2 is the frequency, then

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \left(\frac{v}{2L} \right) = 2 \nu_1$$

It is called first overtone or second harmonic.

Third mode of vibration

For $n = 3$, equation (4) gives third mode of vibration

If λ_3 is the wavelength of wave set up, then

$$\lambda_3 = \frac{2L}{3}$$

$$L = 3 \frac{\lambda_3}{2}$$

If ν_3 is the frequency of wave, then

$$\nu_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 \nu_1$$

This is called second overtone or third harmonic.

In general, for n th mode

$$\nu_n = n \nu_1$$

It is called $(n - 1)$ th overtone or n th harmonic.

Thus in an open organ pipe all harmonics are present.

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200. Give a comparison between Progressive and Stationary waves.

Ans :

The difference between progressive waves and stationary waves are as follows :

	Progressive Waves	Stationary Waves
1.	The disturbance produced in the medium travels forward, it being handed over from one particle to the next. Each particle executes the same type of vibration as the preceding one, though not at the same time.	There is no onward motion of the disturbance as no particle transfers its motion to the next. Each particle has its own vibration characteristics.

	Progressive Waves	Stationary Waves
2.	The amplitude of each particle is the same but the phase continuously varies along the medium i.e. a particle attains a particular stage of displacement at a time different from the neighbouring particles.	The amplitudes of different particles are different, ranging from zero at the nodes to the maximum at the antinodes. The medium gets split up into vibrating segments, all the particles in the same segment being in the same phase, though opposite to the particles in the neighbouring segment.
3.	No, particle remains permanently at rest. Twice during each vibration, however, a particle comes to momentary rest while at the extremes of its swing.	There are certain particles (called nodes) which remain permanently at rest.
4.	Different particles attain this position of momentary rest at different times.	All other particles attain the position of maximum displacement and momentary rest at the same time and they do so twice during each vibration.
5.	All the particles have the same maximum velocity which they attain while passing through their mean position one after the other.	All the particles attain their individual maximum velocities at the same time as they pass through their mean positions. This individual maximum velocity ranges from zero at the nodes to the maximum to the antinodes.

If ν_3 is the frequency of wave, then

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5\nu_1$$

This is called second overtone or fifth harmonic.

In general, for n th mode

$$\nu_n = (2n-1)\nu_1$$

It is called $(n-1)$ th overtone or $(2n-1)$ th harmonic.

Thus in a closed organ pipe only odd harmonics are present.

Nodes (N)

In n th mode of vibration, there will be n nodes at positions.

$$x = 0, \frac{2L}{2n+1}, \frac{4L}{2n+1}, \dots, \frac{2nL}{2n+1}$$

Antinodes (A)

In n th mode of vibration, there will be n antinode at positions

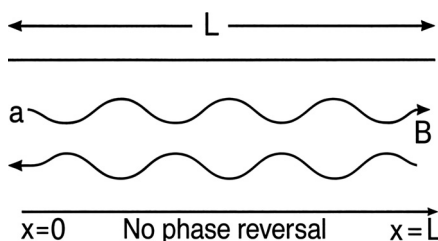
$$x = \frac{L}{2n+1}, \frac{3L}{2n+1}, \dots, L$$

- 199.** Derive expression for stationary waves formed in an open organ pipe and discuss normal modes of vibration of the pipe.

Ans :

Let L be length of open organ pipe. The displacement at position x at time t in the incident wave be given by

$$y_1 = r \sin \frac{2\pi}{\lambda} (vt - x) \quad \dots(1)$$



On reflection from the open end at $x=l$ of the pipe, there is no phase change, So the displacement at the position x and same time t in the reflected wave is given by

$$y = r \sin \frac{2\pi}{\lambda} (vt + x) \quad \dots(2)$$

The resultant displacement due to superposition is given by

$$y = y_1 + y_2 \\ = r \left[\sin \frac{2\pi}{\lambda} (vt - x) + \sin \frac{2\pi}{\lambda} (vt + x) \right]$$

$$y = 2r \sin \frac{2\pi}{\lambda} vt \cos \frac{2\pi}{\lambda} L \quad \dots(3)$$

This is the equation of a standing wave.

From figure, we find that antinodes are always formed on open ends of the pipe.

i.e., $y = \max.$ at $x = 0$ and at $x = L$

From equation (3), at $x = L$

$$y = 2r \sin \frac{2\pi}{\lambda} vt \cos \frac{2\pi}{\lambda} L$$

Now, $y = \max$

If $\cos \frac{2\pi}{\lambda} L = \pm 1$

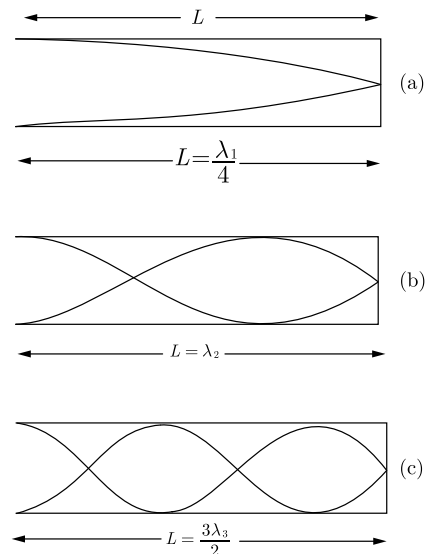
$$\frac{2\pi}{\lambda} L = n\pi \quad [n = 0, 1, 2, \dots]$$

$$\lambda = \frac{2L}{n} \quad \dots(4)$$

First normal mode of vibration

When $n = 1$ equation (4) gives first mode of vibration.

If λ_1 is the wavelength,



Then

$$\lambda_1 = 2L$$

$$L = \frac{\lambda_1}{2}$$

The first mode is shown in figure (a)

If ν_1 is the frequency of waves set up, then

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

It is called fundamental frequency or first harmonic.

Second mode of vibration : When $n = 2$, equation (4) gives second mode of vibration. If λ_2 is the wavelength, then

Where $n = 1, 2, 3, \dots$

$$\text{Hence, } 340 = \frac{(2n_1 - 1)v}{4l_1} = \frac{(2n_2 - 1)v}{4l_2}$$

$$\frac{(2n_1 - 1)v}{4l_1} = \frac{(2n_2 - 1)v}{4l_2}$$

$$\frac{(2n_1 - 1)}{(2n_2 - 1)} = \frac{l_1}{l_2}$$

For closed end pipe, $l_2 = 3l_1$

$$\frac{(2n_1 - 1)}{(2n_2 - 1)} = \frac{l_1}{3l_1} = \frac{1}{3}$$

which gives, $n_1 = 1$

$$n_2 = 2$$

Thus the fixed source of frequency 340 Hz is in resonance with the fundamental mode of the pipe for $l = 25.5$ cm, and with the third harmonic of the pipe for $l = 79.3$ cm.

$$\text{Now, } \nu = \frac{(2n - 1)v}{4l}$$

$$v = \frac{4l\nu}{(2n - 1)}$$

Here, for $n = 1$, $l = 25.5$ cm $= 25.5 \times 10^{-2}$ m

$$\text{Hence, } v = \frac{4 \times 25.5 \times 10^{-2} \times 340}{2 - 1} = 346.8 \text{ ms}^{-1}$$

204. A travelling harmonic wave on a string is described by, $y = 7.5 \sin(0.0050x + 12t + \pi/4)$

- (a) What are the displacement and velocity of oscillation of a point at $x = 1$ cm and $t = 1$ s? Is this velocity equal to the velocity of wave propagation?
- (b) Locate the points of the string which have the same transverse displacements and velocity as the $x = 1$ cm point at $t = 2$ s, 5 s, 11 s.

Ans :

$$(a) \quad y = 0.5 \sin(0.0050x + 12t + \pi/4) \quad \dots(1)$$

Here $x = 1$ cm and $t = 1$ s,

$$y = 7.5 \sin(0.0050 + 12 + 0.785)$$

$$y = 7.5 \sin(12.79) = 7.5 \times 0.2215$$

$$= 1.66 \text{ cm}$$

Differentiating (1) w.r.t. t

$$v = \frac{dy}{dt}$$

$$= 7.5 \cos(0.0050x + 12t + \frac{\pi}{4}) \times 12$$

$$= 9.0 \cos(0.0050x + 12t + \frac{\pi}{4})$$

For $x = 1$, $t = 1$ s

$$v = 90 \cos(0.0050 + 12 \times 1 + \frac{\pi}{4})$$

$$= 90 \cos(732.83^\circ)$$

$$= \cos 12.83^\circ$$

$$= 90 \times 0.9751 = 87.76 \text{ cm s}^{-1}$$

$$= 88 \text{ cm s}^{-1}$$

$$\text{Again, } y = 7.5 \sin 0.005 \left(x + \frac{12}{0.005} t + \frac{\pi}{0.005 \times 4} \right)$$

Hence, Velocity of propagation

$$= \frac{12}{0.005} = -24 \text{ ms}^{-1}$$

Then velocity is equal to velocity of wave propagation.

- (b) Comparing Equation (1) with the equation of a progressive wave

$$y = A \sin(\omega t + kx + \phi), \text{ we get}$$

$$k = 0.005 \text{ rad cm}^{-1}$$

$$\text{So } \lambda = \frac{2\pi}{k} = \frac{2 \times 3.142}{0.005} = 12.57 \text{ m} = 12.6 \text{ m}$$

All points are located at distances of $n\lambda$ ($n = \pm 1, \pm 2, \pm 3, \dots$) where $\lambda = 12.6$ m from the point $x = 1$ cm.

205. A narrow sound pulse (for example, a short pipe by a whistle) is sent across a medium

- (a) Does the pulse have a definite
- frequency,
 - wavelength
 - speed of propagation?
- (b) If the pulse rate is 1 after every 20 s, (i.e. the whistle is blown for a split second after every 20s), is the frequency of the note produced by the whistle equal to $\frac{1}{20} = 0.05 \text{ Hz}$?

Ans :

- (a) The pulse does not have a definite wavelength or frequency, but has a definite speed of propagation (in a non-dispersive medium).
- (b) The frequency of the note produced by the whistle is not equal to $\frac{1}{20} = 0.05 \text{ Hz}$

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4. Does every element of the medium through which this wave passes oscillate with the same amplitude and period? Do they oscillate in the same phase?

Ans :

1. $y(x, t)$ represents the displacement of the constituent of the medium at position x and time t from its equilibrium position.
2. If x is fixed at x_0 , the term $kx_0 + \phi$ is a constant. So, $y(x_0, t) = a \sin(-\omega t + \text{constant})$. This means the displacement at a fixed location varies sinusoidally with time; the element executes simple harmonic motion.
3. The period of oscillation of this element is

$$T = 2\pi/\omega.$$

4. Yes, every element oscillates with the same amplitude 'a' and the same period T . However, they do not oscillate in the same phase; the phase $(kx - \omega t + \phi)$ depends on the position x . (For a travelling wave).

219. A steel wire 0.72 m long has a mass of 5.0×10^{-3} kg. It is kept under a tension of 60 N. Transverse waves are generated on this wire.

1. What is linear mass density (μ) of a string?
2. On what properties of the string does the speed of transverse waves depend?
3. Calculate the linear mass density (μ) of this steel wire.
4. Calculate the speed of transverse waves on this wire. If the frequency of the wave generated is 200 Hz, what would be its wavelength?

Ans :

1. Linear mass density (μ) is the mass per unit length of the string (m/L).
2. The speed of transverse waves on a string depends on the tension (T) in the string and its linear mass density (μ).

3. $\mu = \text{mass/length}$

$$= (5.0 \times 10^{-3} \text{ kg}) / (0.72 \text{ m})$$

$$\approx 6.94 \times 10^{-3} \text{ kg/m.}$$

(Example 14.3 uses $6.9 \times 10^{-3} \text{ kg/m}$).

4. Speed

$$v = \sqrt{T/\mu}$$

$$= \sqrt{(60 \text{ N}) / (6.94 \times 10^{-3} \text{ kg/m})}$$

$$\approx \sqrt{8645.5} \approx 93.0 \text{ m/s.}$$

Wavelength

$$\lambda = v/\nu = (93.0 \text{ m/s}) / (200 \text{ Hz}) = 0.465 \text{ m.}$$

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CASE BASED QUESTIONS

210. A guitar string of length 0.75 m is fixed at both ends. When plucked, it vibrates. The speed of transverse waves on this string is determined by its tension and linear mass density. The string can vibrate in several modes, producing different musical notes.

1. What are standing waves, and how are they formed on a string fixed at both ends?
2. What are nodes and antinodes in a standing wave? What are the boundary conditions for a string fixed at both ends in terms of nodes?
3. What is the wavelength of the fundamental mode (first harmonic) of vibration for this guitar string?
4. If the speed of the transverse wave on the string is 300 m/s, what is the frequency of its fundamental mode? What would be the frequency of its second harmonic?

Ans :

1. Standing waves (or stationary waves) are wave patterns set up when a wave traveling in one direction gets reflected at boundaries and superimposes with the reflected wave traveling in the opposite direction.
2. Nodes are points in a standing wave where the amplitude is zero (no motion). Antinodes are points where the amplitude is the largest. For a string fixed at both ends, the ends of the string must be nodes.
3. For the fundamental mode ($n = 1$) on a string fixed at both ends, $L = n\lambda/2$, so

$$\lambda_1 = 2L = 2 \times 0.75\text{m} = 1.5\text{m}.$$

4. Fundamental frequency

$$\nu_1 = v/\lambda_1 = (300\text{m/s})/(1.5\text{m}) = 200\text{Hz}.$$

The frequency of the n th harmonic is

$$\nu_n = nv/(2L) = n\nu_1.$$

So, the second harmonic ($n = 2$) frequency

$$\nu_2 = 2\nu_1 = 2 \times 200\text{Hz} = 400\text{Hz}.$$

211. A pipe, 30.0 cm long, is open at both ends. It is made to resonate with an external sound source. The speed of sound in air is 330 m/s. We are interested in the harmonic modes of this pipe.

1. For a pipe open at both ends, what condition (node or antinode) applies at the ends for displacement of air molecules?
2. How are the possible wavelengths (λ) of standing waves in a pipe open at both ends related to its length L ?
3. Calculate the fundamental frequency (ν_1) for this pipe.

4. If one end of this pipe is now closed, what will be its new fundamental frequency? Will it still resonate with a source of frequency $2\nu_1$ (where ν_1 was the original fundamental of the open pipe)?

Ans :

1. For a pipe open at both ends, each end is an antinode for the displacement of air molecules.
2. For a pipe open at both ends, the possible wavelengths are given by $L = n\lambda/2$, so $\lambda = 2L/n$ for $n = 1, 2, 3, \dots$ (The text applies the string formula to open pipes).

3. Fundamental frequency

$$\begin{aligned}\nu_1 &= v/(2L) \\ &= (330\text{m/s})/(2 \times 0.30\text{m}) \\ &= (330/0.6)\text{Hz} = 550\text{Hz}.\end{aligned}$$

4. If one end is closed, the fundamental frequency is

$$\begin{aligned}\nu_1' &= v/(4L) \\ &= (330\text{m/s})/(4 \times 0.30\text{m}) \\ &= (330/1.2)\text{Hz} = 275\text{Hz}.\end{aligned}$$

The original

$$2\nu_1 = 2 \times 550\text{Hz} = 1100\text{Hz}.$$

For the closed pipe, possible frequencies are

$$(2n + 1)v/(4L) = (2n + 1) \times 275\text{Hz}.$$

If $2n + 1 = 4$, then $(2n + 1)$ is not odd, so 1100 Hz (which is $4 \times 275\text{Hz}$) will not be a resonant frequency for the pipe closed at one end as only odd harmonics are present. (Example 14.5 discusses this exact scenario where 1.1 kHz (1100 Hz) corresponds to $n = 2$ for open pipe, but for closed end, it is its fourth harmonic which is not possible).

212. A wave traveling along a string is described by the equation $y(x, t) = 0.005 \sin(80.0x - 3.0t)$, where numerical constants are in SI units (x, y in meters, t in seconds).

1. What is the general displacement relation for a sinusoidal wave traveling in the positive x -direction?
2. In the given equation, what physical quantities do the values 0.005, 80.0, and 3.0 represent?
3. Calculate the wavelength (λ) and the period (T) of this wave.
4. What is the speed of propagation (v) of this wave?

Ans :

1. The general displacement relation is

$$y(x, t) = a \sin(kx - \omega t + \phi).$$

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2. Comparing $y(x, t) = 0.005 \sin(80.0x - 3.0t)$ with $a \sin(kx - \omega t)$:
Amplitude $a = 0.005\text{m}$.
Angular wave number $k = 80.0\text{rad/m}$.
Angular frequency $\omega = 3.0\text{rad/s}$.

3. Wavelength

$$\lambda = 2\pi/k = 2\pi/(80.0\text{rad/m}) \\ \approx 0.0785\text{m or } 7.85 \text{ cm.}$$

Period

$$T = 2\pi/\omega = 2\pi/(3.0\text{rad/s}) \approx 2.09\text{s.}$$

4. Speed of propagation

$$v = \omega/k = (3.0\text{rad/s})/(80.0\text{rad/m}) \\ = 0.0375\text{m/s.}$$

Alternatively,

$$v = \lambda\nu = \lambda/T \\ = (0.0785\text{m})/(2.09\text{s}) \approx 0.0375\text{m/s.}$$

- 213.** Newton initially proposed that the speed of sound in a gas is given by $v = \sqrt{P/\rho}$, where P is the pressure and ρ is the density of the gas. This was based on the assumption that pressure variations during sound propagation are isothermal.

- What correction did Laplace make to Newton's formula for the speed of sound in a gas? What was the basis for this correction?
- For an ideal gas, the adiabatic bulk modulus B_{ad} is given by γP . What is γ ?
- Calculate the speed of sound in air at STP using Newton's formula, given $P = 1.01 \times 10^5\text{Pa}$ and $\rho_o = 1.29\text{kg/m}^3$.
- If $\gamma_{air} = 7/5$, calculate the speed of sound in air at STP using Laplace's corrected formula. How does this compare to the experimental value of approximately 331 m/s?

Ans :

- Laplace pointed out that pressure variations in sound waves are so fast that they are adiabatic, not isothermal. He corrected the formula to $v = \sqrt{\gamma P/\rho}$.
- γ is the ratio of the molar specific heat capacity at constant pressure (C_p) to the molar specific heat capacity at constant volume (C_v), i.e., $\gamma = C_p/C_v$.
- Using Newton's formula:

$$v = \sqrt{(1.01 \times 10^5\text{Pa})/(1.29\text{kg/m}^3)} \\ \approx \sqrt{78294.57} \approx 280\text{m/s.}$$

4. Using Laplace's formula:

$$v = \sqrt{(\gamma P)/\rho} \\ = \sqrt{((7/5) \times 1.01 \times 10^5\text{Pa})/(1.29\text{kg/m}^3)}$$

$$= \sqrt{1.4 \times 78294.57}$$

$$\approx \sqrt{109612.4} \approx 331.1\text{m/s.}$$

This value agrees well with the experimental value.

- 214.** A wave pulse travels along a stretched string. It encounters a boundary where the string is rigidly fixed to a wall. After interacting with the boundary, a reflected pulse is observed traveling back along the string.

- What happens to a traveling wave or pulse when it encounters a rigid boundary?
- What is the phase change suffered by a wave pulse upon reflection from a rigid boundary?
- If the incident pulse is described by $y_i(x, t) = a \sin(kx - \omega t)$, what is the mathematical form of the reflected pulse $y_r(x, t)$ from a rigid boundary?
- If the string was instead tied to a freely moving ring on a rod (an open boundary), what would be the phase change upon reflection, and how would the reflected pulse be described?

Ans :

- If the boundary is rigid, the pulse or wave gets reflected.
- A traveling wave or pulse suffers a phase change of π (or 180°) on reflection at a rigid boundary.
- The reflected wave at a rigid boundary is given by

$$y_r(x, t) = a \sin(kx + \omega t + \pi) \\ = -a \sin(kx + \omega t).$$

(Note: direction of travel for reflected wave implies $kx + \omega t$).

- On reflection at an open boundary, there is no phase change. The reflected pulse would be $y_r(x, t) = a \sin(kx + \omega t)$.

- 215.** Two sound waves of slightly different frequencies, ν_1 and ν_2 (with $\nu_1 > \nu_2$), and comparable amplitudes are heard simultaneously by a listener. The listener perceives a sound that waxes and wanes in intensity.

- What is the principle of superposition of waves?
- What is the phenomenon of "beats" in sound?
- If the two waves are $s_1 = a \cos(\omega_1 t)$ and $s_2 = a \cos(\omega_2 t)$, the resultant displacement $s = [2a \cos(\omega_b t)] \cos(\omega_a t)$. What are ω_a and ω_b in terms of ω_1 and ω_2 ?
- What is the beat frequency (ν_{beat}) heard by the listener in terms of ν_1 and ν_2 ?

Ans :

- The principle of superposition of waves states that when two or more waves traverse simultaneously in the same medium, the

displacement of any element of the medium is the algebraic sum of the displacements due to each wave.

- Beats is an interesting phenomenon arising from interference of waves, where two harmonic sound waves of close (but not equal) frequencies are heard, resulting in an audibly distinct waxing and waning of the intensity of the sound.
- $\omega_a = (\omega_1 + \omega_2)/2$ (average angular frequency) and $\omega_b = (\omega_1 - \omega_2)/2$.
- The intensity of the resultant wave waxes and wanes with a frequency which is $2\omega_b = \omega_1 - \omega_2$. Since $\omega = 2\pi\nu$, the beat frequency is $\nu_{beat} = \nu_1 - \nu_2$.

216. Two sitar strings A and B are played together. They are slightly out of tune and produce beats of frequency 5 Hz. The frequency of string A is known to be 427 Hz. The tension of string B is then slightly increased. After this adjustment, the beat frequency is found to decrease to 3 Hz.

- How is the speed of a transverse wave on a stretched string related to its tension T and linear mass density μ ?
- How does increasing the tension in a string affect its frequency of vibration?
- Before the tension in B was increased, what were the two possible frequencies for string B?
- What was the original frequency of string B? Explain your reasoning.

Ans :

- The speed of a transverse wave on a string is $v = \sqrt{T/\mu}$.
- The frequency of vibration of a string is proportional to the wave speed (v) (e.g., fundamental $\nu = v/2L$). Since $v \propto \sqrt{T}$, increasing the tension T increases the speed v , and thus increases its frequency of vibration.
- Since the beat frequency is 5 Hz and string A's frequency is 427 Hz, the possible frequencies for string B were $427\text{Hz} - 5\text{Hz} = 422\text{Hz}$ or $427\text{Hz} + 5\text{Hz} = 432\text{Hz}$.
- When the tension in string B was increased, its frequency (ν_B) increased.

If original $\nu_B = 432\text{Hz}$ (greater than ν_A), increasing ν_B further would increase the beat frequency ($\nu_B - \nu_A$).

If original $\nu_B = 422\text{Hz}$ (less than ν_A), increasing ν_B would decrease the beat frequency ($\nu_A - \nu_B$) as long as remains less than ν_A or does not exceed ν_A by more than the new beat frequency. Since the beat frequency decreased from 5 Hz to 3 Hz upon increasing tension (and thus frequency) of B, the original frequency of B

must have been lower than A. Thus, original $\nu_B = 422\text{Hz}$.

217. Steel is an elastic medium. When a disturbance is created in a steel rod, both transverse and longitudinal waves can propagate through it. Air, on the other hand, can primarily sustain only longitudinal waves.

- Differentiate between transverse and longitudinal waves based on the direction of oscillation of the constituents of the medium relative to the direction of wave propagation.
- What type of stress can fluids (like air) not sustain, which is necessary for the propagation of transverse waves?
- The speed of longitudinal waves in a steel bar is given by $v = \sqrt{Y/\rho}$ where Y is Young's modulus. What elastic property is primarily responsible for restoring force in this case?
- Why can steel support both types of waves while air primarily supports only longitudinal waves?

Ans :

- In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation. In longitudinal waves, they oscillate along the direction of wave propagation.
- Fluids offer very little resistance to shear stress; they cannot sustain shearing stress, which is required for transverse waves to propagate.
- Young's modulus (Y) is the relevant modulus of elasticity, which relates to longitudinal stress and strain. The restoring force is due to the material's resistance to elongation or compression.
- Steel, being a solid, can sustain both shearing stress (allowing for transverse waves) and compressive strain (allowing for longitudinal waves). Air, being a fluid, cannot sustain shearing stress and therefore primarily supports longitudinal waves (compressions and rarefactions).

218. Consider a harmonic wave described by $y(x, t) = a \sin(kx - \omega t + \phi)$. An observer focuses on a single element of the medium at a fixed location, say $x = x_0$.

Questions:

- What does the displacement function $y(x, t)$ represent?
- If x is fixed at x_0 , how does the displacement $y(x_0, t)$ change with time t ?
- What is the period of oscillation of this specific element of the medium?

	Progressive Waves	Stationary Waves
6.	All parts of the medium in case of longitudinal progressive waves undergo similar variation of density one after the other. There are no points where the density variation is zero.	The variation of density (in case of longitudinal stationary waves) is different at different points of the medium. At nodes the displacement is zero but the density variation is maximum whereas at the antinodes, the displacement is maximum though the density variation is zero.
7.	There is flow of energy across every plane along the direction of propagation of the waves.	There is no flow of energy across any plane.

NUMERICAL QUESTIONS

201. Derive the following relations by the method of dimensions

(i) $v = \sqrt{\frac{T}{\mu}}$

(ii) $v = \sqrt{\frac{E}{\rho}}$

T = Tension, μ = mass per unit length, E = coefficient of elasticity, ρ = density of medium.

Ans :

(i) Let $v \propto T^a \mu^b$

$$v = kT^a \mu^b \quad \dots(1)$$

k is dimensionless constant.

Writing dimension on both sides

$$L^1 T^1 = (M^1 L^1 T^{-2})^a (M^1 L^{-1})^b$$

Comparing dimensions on both sides

$$a + b = 0, \quad a - b = 1, \quad -2a - b = -1$$

Hence, $a = \frac{1}{2}, \quad b = -\frac{1}{2}$

From (1),

$$v = kT^{1/2} \mu^{-1/2}$$

$$v = k\sqrt{\frac{T}{\mu}}$$

Experimentally, $k = 1$

$$v = \sqrt{\frac{T}{\mu}}$$

(ii) Let, $v = kE^a \rho^b \quad \dots(2)$

$$L^1 T^1 = (M^1 L^1 T^{-2})^a (M^1 L^{-1})^b$$

$$[M^0 L^1 T^{-1}] = (M^{a+b} L^{a-b} T^{-2a})$$

Comparing the powers of M.L and T on both sides, we get

$$a + b = 0 \quad \dots(1)$$

$$-2a = -1$$

$$a = \frac{1}{2} \quad \dots(2)$$

$$a - b = 1$$

$$1/2 - b = 1$$

$$b = -\frac{1}{2}$$

From equation (1),

$$v = kE^{1/2} \rho^{-1/2}$$

$$v = \sqrt{\frac{E}{\rho}} \quad (\text{Experimentally } k = 1)$$

202. A steel wire 0.72 m long has a mass of 5.0×10^3 kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire ?

Ans :

Mass per unit length of the wire,

$$\begin{aligned} \mu &= \frac{5.0 \times 10^{-3} \text{ kg}}{0.72} \\ &= 6.9 \times 10^{-3} \text{ kg m}^{-1} \end{aligned}$$

Tension, $T = 60 \text{ N}$

The speed of transverse waves on the wire is given by

$$\begin{aligned} v &= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60}{6.9 \times 10^{-3} \text{ m s}^{-1}}} \\ &= 93 \text{ m s}^{-1} \end{aligned}$$

203. A metre long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.

Ans :

In case of closed end pipe, the frequency is given by,

$$\nu_n = \frac{(2n-1)v}{4l}$$

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